KOM505E – Probability Theory and Stochastic Processes

Homework 2

This is an individual homework. Although you can discuss the project with your friends, you must write your code and report independently.

One question from Matlab assignments and one question from the textbook assignments will be graded. These two questions will be selected **randomly** (will constitute 80% of the total score). The rest of the questions will be checked for only completeness (will constitute 20% of the total score).

If you have any questions about this assignments, please send an email to `kamasak@itu.edu.tr`

Matlab assignments:

1. Write a Matlab function that generates random numbers from Poisson distribution using the uniform random number generator of Matlab (ie. `rand`). Generate 100.000 Poisson distributed random numbers with $\lambda = 3$ using your code, and draw their histogram.

   Include your Matlab codes, and figures in your report.

2. Write a Matlab function that generates uniformly distributed random numbers using the Gaussian distributed random number generator of Matlab (ie. `randn`). Generate 100.000 uniformly distributed random numbers between 0 and 1 using your code, and draw their histogram.

   Include your Matlab codes, and figures in your report.

3. Assume that x is a discrete random variable with range [-10, 10] and its probability mass function is given as:

$$pmf(x) = \begin{cases} 0.025 & \text{if } x \text{ is negative} \\ 0 & \text{if } x \text{ is zero} \\ 0.075 & \text{if } x \text{ is positive} \end{cases}$$

   Write a Matlab that computes the pmf of a new random variable y that is computed as $y = x^2 + 2x + 7$. Draw the cdf and pmf of y.

   Include your Matlab codes, and figures in your report.

4. Numeric computation of continuous integrals.
   Matlab has functions that can take indefinite and definite integrals over symbolic variables (see `int` function of Matlab). However in some cases, it is required to compute a continuous integral numerically. Although Matlab has functions for numeric integration (ie. `integral` function), you need to write a function that computes the following integral numerically using Riemann sum:

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

   this integral can be discretized as:

$$erf(x) \approx \frac{2}{\sqrt{\pi}} \sum_{k=0}^{x/\Delta t} e^{-(k\Delta t)^2} \Delta t$$

   where Δt is a small number that determines the precision of this computation. If Δt is chosen very large the result will not be precise. On the other hand, if Δt is chosen very small, the computation will take longer time.

   Write a Matlab script that computes $erf(1.35)$ numerically using different Δt values of 0.1, 0.01, 0.001, 0.0001, 0.00001. In addition estimate the required execution time using `cputime` function of Matlab. Draw a graph of estimated execution time versus Δt using `semilogx` function. This function should use linear scale for execution time and logarithmic scale for Δt.

In addition compare your numeric result with the result of `erf` function of Matlab. Plot the difference (your numeric result and Matlab function result) versus Δt using `semilogx` function. This function should use linear scale for difference and logarithmic scale for Δt.

Include your Matlab codes, and figures in your report.

5. Assume a continuous random variable has the following probability density function:

$$f(x) = Kt^2 e^{-Nt}$$

where $x \in [0, \infty]$ and K is a constant real number. Write a Matlab script that computes the approximate value of K for different values of N in range [0.1, 1] with increments of 0.1. Plot the value of K versus N.

This script should also compute approximate value of mean (μ_x) and variance (σ_x^2) of x.

Plot the value of μ_x versus N.

Plot the value of σ_x^2 versus N.

Include your Matlab codes, and figures in your report.

Textbook assignments:

Solve the following questions from "Intuitive Probability and Random Processes using MATLAB".

1. From Chapter 3, solve 3.9, 3.12, 3.32
2. From Chapter 4, solve 4.3, 4.13, 4.20
3. From Chapter 5, solve 5.9, 5.20, 5.27
4. From Chapter 6, solve 6.12, 6.21, 6.28
5. From Chapter 10, solve 10.14, 10.30, 10.36
6. From Chapter 11, solve 11.17, 11.36, 11.45

Following questions will not be graded but we suggest you to solve these questions:

1. From Chapter 3, solve 3.23, 3.45
2. From Chapter 4, solve 4.2, 4.4, 4.5, 4.7, 4.19, 4.22, 4.24
3. From Chapter 5, solve 5.5, 5.31
4. From Chapter 6, solve 6.16, 6.20, 6.27
5. From Chapter 10, solve 10.5, 10.6, 10.19, 10.20, 10.54, 10.55, 10.56
6. From Chapter 11, solve 11.19, 11.41, 11.46