Istanbul Technical University- Fall 2017
BLG527E Machine Learning
Homework 4

Purpose: Graphical Models, Hidden Markov Models.

Total worth: 6% of your grade.

Handed out: Thursday, Dec 8, 2017.

Due: Thursday, Dec 28, 2017 23.00. (through ninova!)

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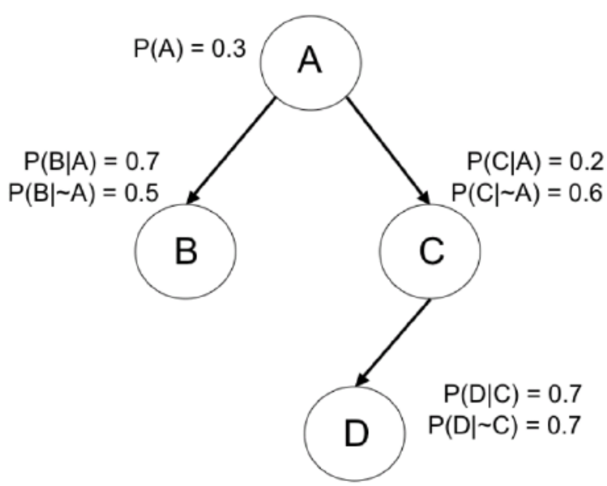
Policy: Collaboration in the form of discussions is acceptable, but you should write your own answer/code by yourself. Cheating is highly discouraged for it could mean a zero or negative grade from the homework. If a question is not clear, please let us know (via email, during office hour or in class).

Submission Instructions: Please submit through the class Ninova site.
Please write your answers to a report and upload it as a pdf file to Ninova. You do not need to write code for this homework.

QUESTIONS:

Q1) [3 points] For the Bayesian network shown below, compute the following:

a) **[1 points]** P(A,B,C,D)=?
b) **[1 points]** P(A|B) =?
c) **[1 points]** P(C|B) =?



Q1) [3 points] You are given the following HMM with N=2 hidden states: S1, S2, M=2 possible observations: a,b, and state transition probabilities (A) and observation probabilities (B) and initial state probabilities (P).

a) **[1.5 points]** Compute the probability that the observation sequence O = a,a,b was produced by this HMM.

b) **[1.5 points]** What is the most probable state sequence given O?

$$A = \begin{bmatrix} 0.8, & 0.2 \\ 0.2, & 0.8 \end{bmatrix} \quad B = \begin{array}{cc} a & b \\ \end{array} \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix} \quad P = [0.9, 0.1]$$

Hint: The forward and backward variables in an HMM are calculated as it was follows:

Forward variable:

$$a_t(i) \equiv P(O_1 ... O_t, q_t = S_i | \lambda)$$

Initialization:

$$a_t(i) = \pi_i b_i(O_1)$$

Recursion:

$$a_{t+1}(j) = \left[\sum_{i=1}^{N} a_t(i) a_{ij} \right] b_j(O_{t+1})$$

$$P(O|\lambda) = \sum_{i=1}^{N} a_T(i)$$

Backward variable:

$$\beta_t(i) \equiv P(O_{t+1} ... O_T | q_t = S_i, \lambda)$$

Initialization:

$$\beta_T(i) = 1$$

Recursion:

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} b_j(O_{t+1}) \beta_{t+1}(j)$$