KOM505E – Probability Theory and Stochastic Processes (CRNs 14105-6)

Homework 1: Due October 18th, 2016, TUESDAY, Noon (13:05pm) at the latest!

This is an individual homework. Although you can discuss the problems with your friends, you must write your own solutions, your own code and report independently.

Problems 3 and 4 from the Problems section will be graded. Those will constitute 50% of the total score.The rest of the questions will be checked for completeness (will constitute 50% of the total score).

Your textbook refers to: "Intuitive Probability and Random Processes using MATLAB", by Steven Kay.

For questions related to this homework, you can talk to or write to Prof. Gozde Unal at gozde.unal at itu.edu.tr

Submissions: Write a simple report that contains your name/number/email. Your written solutions for the problems should be clearly readable and neat. For Matlab problems, include your Matlab codes, and figures, if any, in your report. PRINT-OUT and submit your HARDCOPY report to the Box (KOM505E is indicated on the box) at Department Secretary Office of Computer Engineering. **No Late** reports are accepted. Furthermore, your emailed Homeworks are also **NOT** accepted.

Problems

1. Using Axioms of Probability, prove each of the statements below:

$$i)P(A^c) = 1 - P(A); \quad ii)P(\phi) = 0; \quad iii)P(B \cap A^c) = P(B) - P(A \cap B)$$

$$iv)P(A \cup B) = P(A) + P(B) - P(A \cap B); \quad v)\text{If } A \subseteq B \text{ then } P(A) \leq B;$$

$$vi)\text{If } \{B_i\} \text{ a partition of S then } P(A) = \sum_i P(A \cap B_i);$$

2. The sample space of an experiment is given as the real line:

$$S = \{v : -\infty < v < \infty\}$$

(a) Consider the events

$$A_1 = \{v : 0 \leq v < 1/2\} \;\; A_2 = \{v : 1/2 \leq v < 3/4\} \;\; A_i = \{v : 1-1/(2)^{i-1} \leq v < 1-1/(2)^i\}$$

Determine the events

$$\cup_{i=1}^{\infty} A_i \text{ and } \cap_{i=1}^{\infty} A_i$$

(b) Consider the events

$$B_1 = \{v : v \leq 1/2\} \;\; B_2 = \{v : v \leq 1/4\} \;\; B_i = \{v : v \leq 1/2^i\}\}$$

Determine the events

$$\cup_{i=1}^{\infty} B_i \text{ and } \cap_{i=1}^{\infty} B_i$$

3. Consider an experiment of drawing two balls at random from a bag containing four balls marked with integers 1 through 4.

(a) Find the sample space S_1 of the experiment if the first ball is replaced before the second is drawn. Then calculate the probability of getting the sum of the two numbers on the two drawn balls as "6".

(b) Find the sample space S_2 of the experiment if the first ball is not replaced. Then calculate the probability of getting the sum of the two numbers on the two drawn balls as "6".

4. Suppose that there are r = 20 students in our probability class.

   (a) What is the cardinality of the sample space for the birth dates of the students? That is find the possible number of birth dates for the students in the class.

   (b) What is the probability that at least two students in the class have the same birthday?

   (c) How large should the number of students in class, r, be for the probability you calculated in (b) to be greater than 0.5 ?

5. Prove the Axioms of Probability for the conditional probability: $P(A \mid B)$

6. Consider the experiment of throwing two fair dice.

   (a) Find the probability of the event that the two faces (on the dice) are the same.

   (b) Now, you are given that the sum on the two faces is not greater than 4. Find the probability of the same event in (a) with the information given. How did it change?

7. A company that produces bulbs has three plants where the bulbs are manufactured, where each plant produces 50, 30, and 20 percent of the total number of bulbs, respectively. Suppose that the probabilities that a bulb manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

   (a) If a bulb is selected at random from the output of the company, what is the probability that it is defective?

   (b) If a bulb selected at random turned out to be defective, what is the probability that it was manufactured by plant 2?

8. Suppose that you throw two fair dice. Let A be the event that the first die is odd, B be the event that the second die is odd, and C be the event that the sum is odd. Show that events A, B, and C are pairwise independent, but A, B, and C are not (mutually) independent.

Matlab assignments:

1. Install Matlab. See the following URL for guidelines to install MATLAB using university licence:

   https://bidb.itu.edu.tr/bilgi-bankasi/lisansli-ve-serbest-yazilimlar/lisansli-yazilimlar/matlab

2. The MATLAB function `rand()` generates a random number between 0 and 1 from a uniform distribution. Write a Matlab code that generates 10.000 random numbers in the interval (2,4). Plot the histogram of the numbers using 100 bins. You can use MATLAB command `hist()` for histogram generation.

3. The MATLAB function `randn()` generates a random number with a Gaussian distribution with zero mean and unit variance. Write a Matlab code that generates 100.000 random numbers with mean 10 and standard deviation 3. Plot the histogram with 100 bins.

4. Write a Matlab code that generates 100.000 random numbers (using `randn()`), multiplies each of these numbers by 20 and add 30. Plot a histogram with 100 bins.

5. You will write a Matlab script to simulate the quality control problem in Section 3.10. Assume a defective batch of 1000 chips contains 940 good chips and 60 bad chips. If we choose a sample of 100 chips, find the probability that there will be 95 or more good chips by using a computer simulation. To simplify the problem, assume sampling with replacement for the computer simulation and the theoretical probability. Compare your result to the theoretical probability you calculate. For the latter, again use Matlab to calculate the theoretical probability.

6. Solve Problem 4.28 and 4.29 from your textbook.

Problems from your textbook:

Solve the following problems from your textbook.

1. From Chapter 3, solve 3.7, 3.12, 3.45.
2. From Chapter 4, solve 4.3, 4.5, 4.13, 4.11, 4.15, 4.20, 4.30.
3. From Chapter 5, solve 5.2, 5.6, 5.9.