

BLG 335E – Analysis of Algorithms I

Fall 2013, Recitation 5

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Question 1

- Insert the following sequence of numbers into a 2-3-4 tree
 - {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}



- **2-3-4 tree:** Perfect balance by allowing 1, 2, or 3 keys per node:
 - 2-node: one key, two children.
 - 3-node: two keys, three children.
 - 4-node: three keys, four children.
- Every path from root to leaf has the same length.



Solution 1 (Cont.)

- {**53**, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}

53



Solution 1 (Cont.)

- {53, **27**, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}

27	53
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Solution 1 (Cont.)

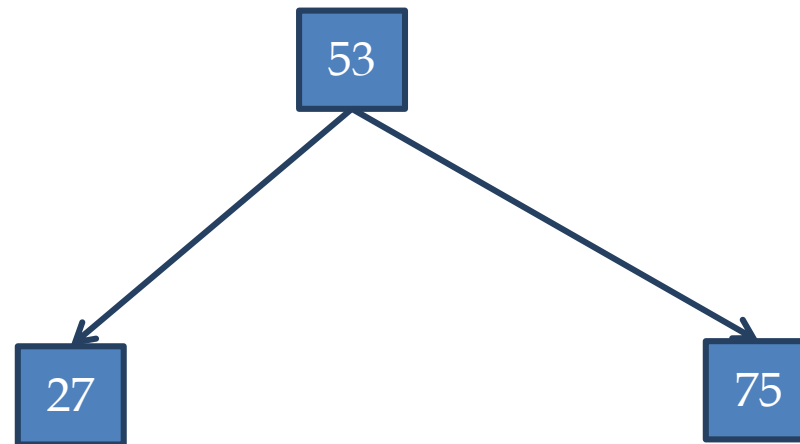
- {53, 27, **75**, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}

27	53	75
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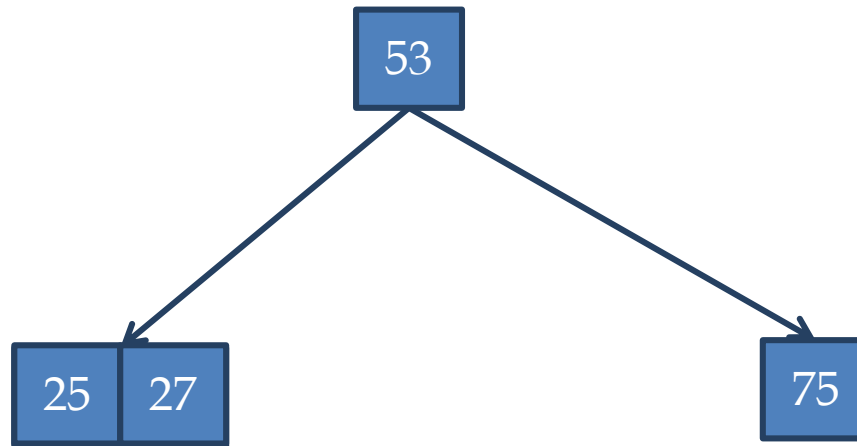
Solution 1 (Cont.)

- {53, 27, 75, **25**, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



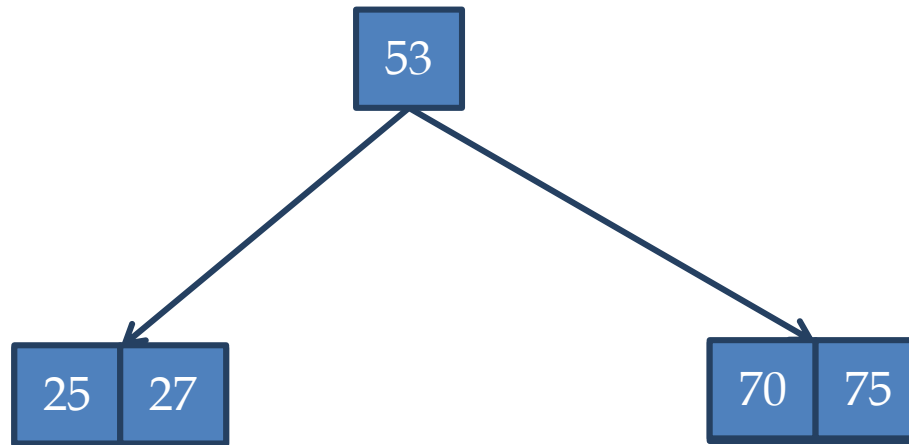
Solution 1 (Cont.)

- {53, 27, 75, **25**, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



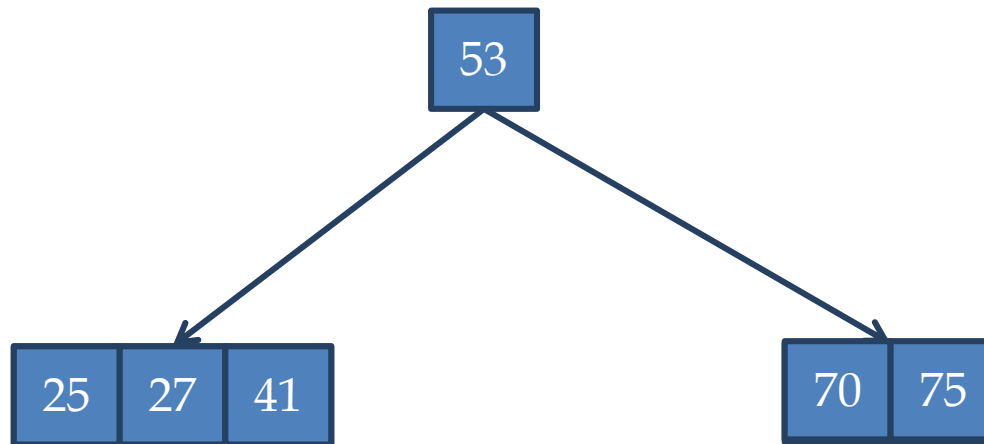
Solution 1 (Cont.)

- {53, 27, 75, 25, **70**, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}



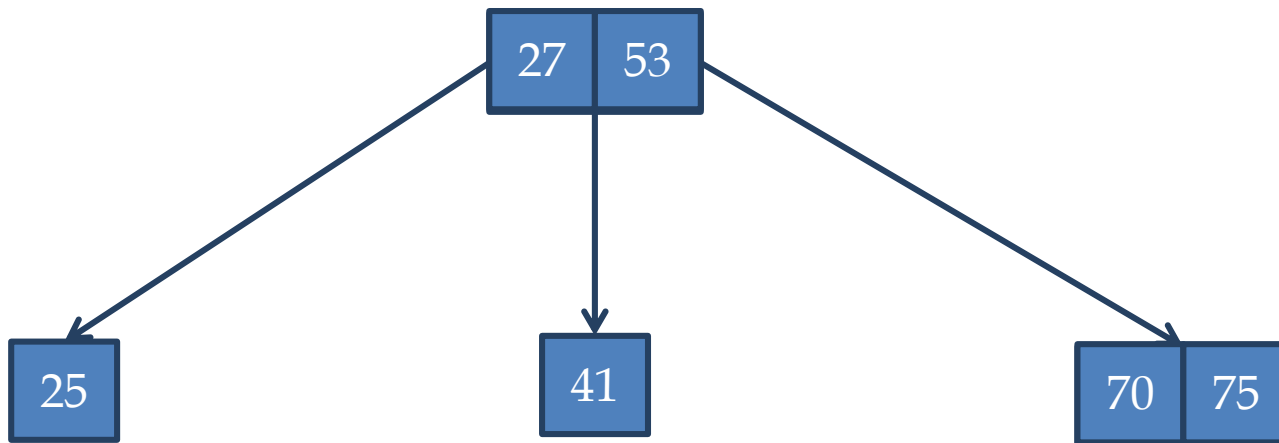
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, **41**, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}



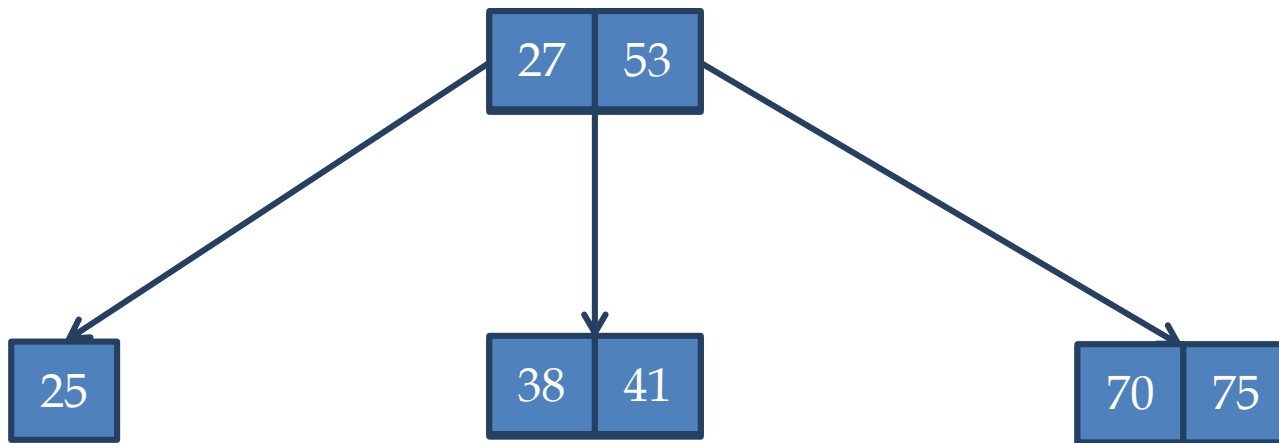
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, **38**, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



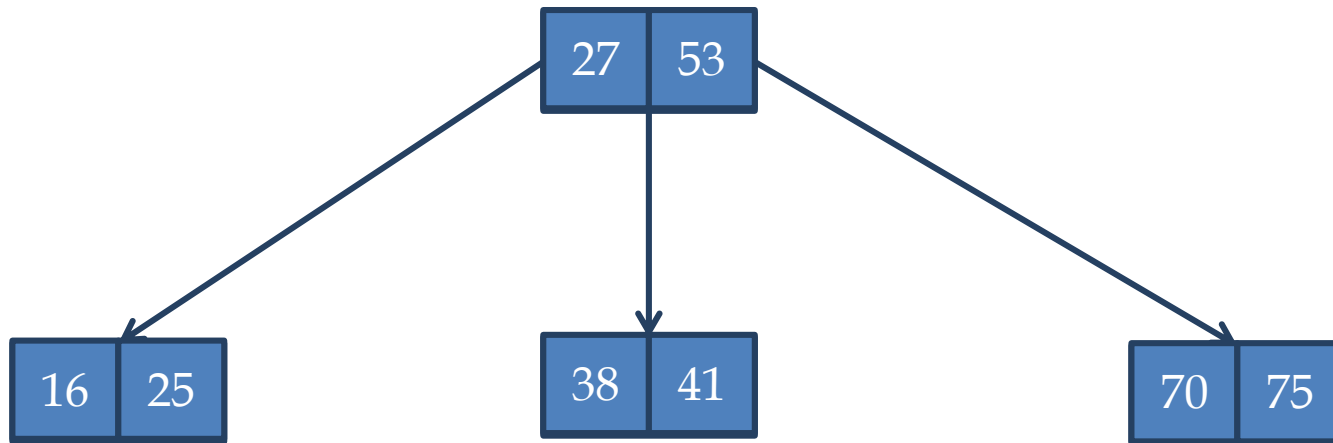
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, **38**, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



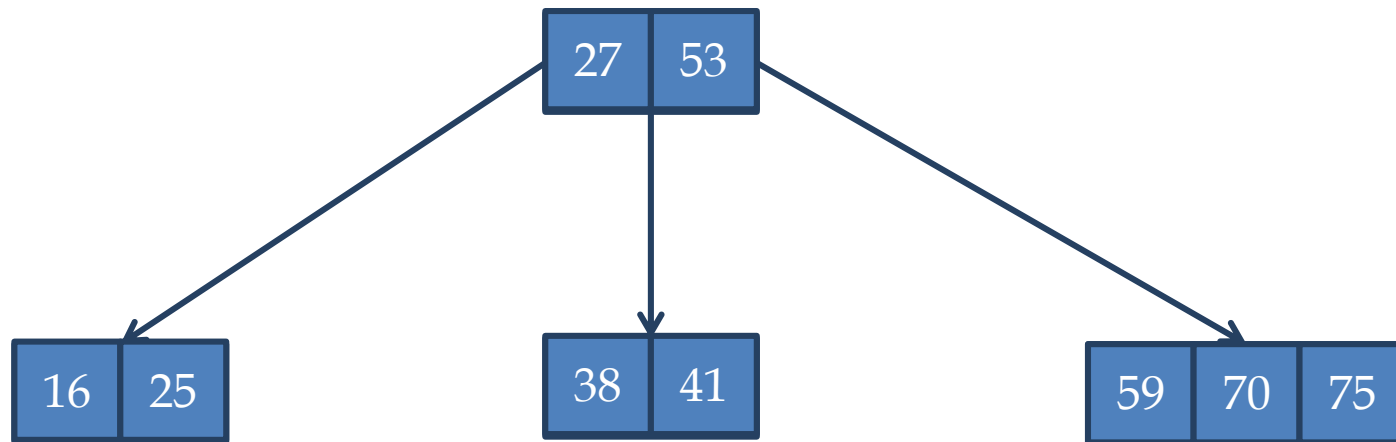
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, **16**, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}



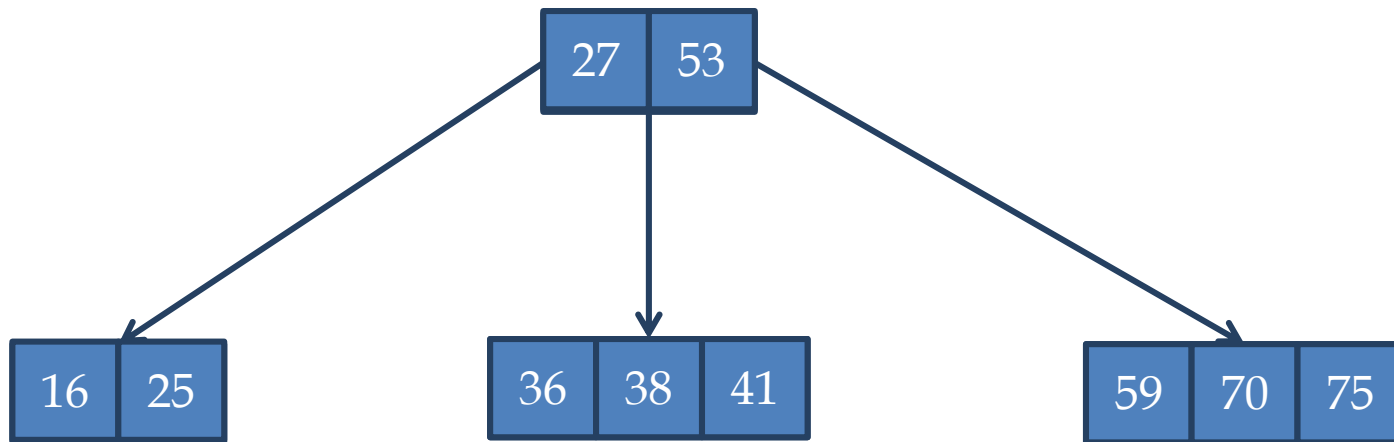
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, **59**, 36, 73, 65, 60, 46, 55, 33, 68, 79, 48}



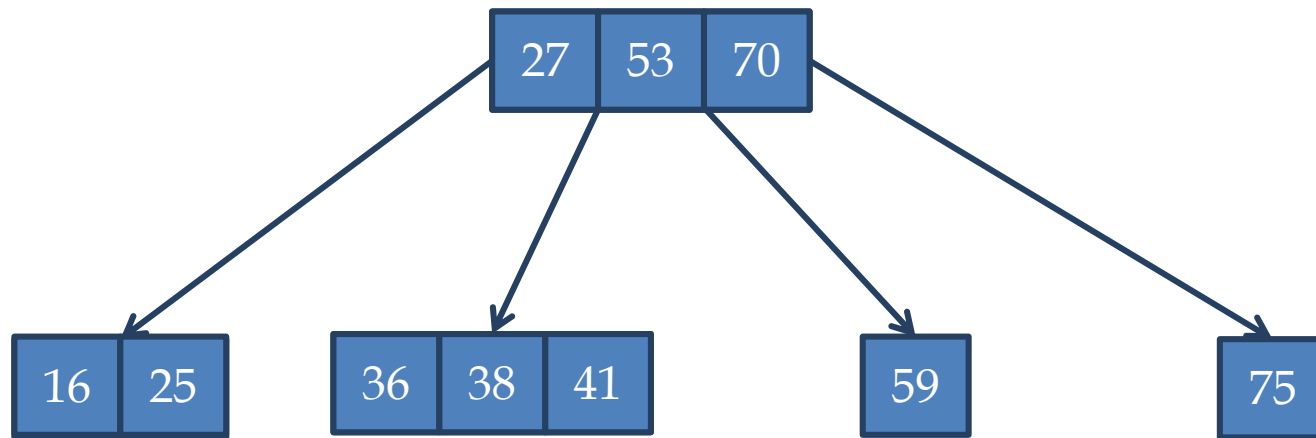
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, **36**, 73, 65, 60, 46, 55, 33, 68, 79, 48}



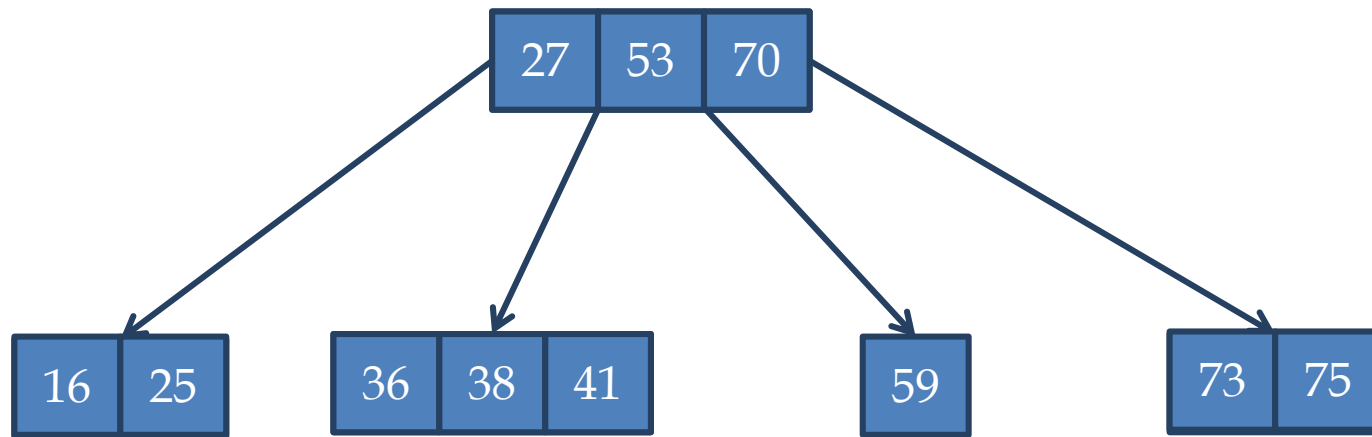
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, **73**, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



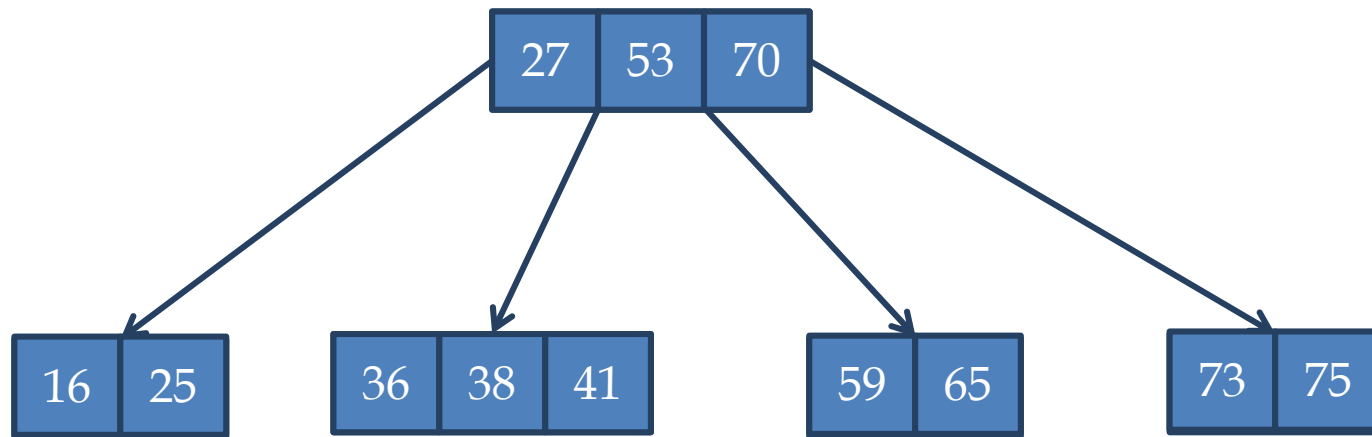
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, **73**, 65, 60, 46, 55, 33, 68, 79, 48} → causes a split



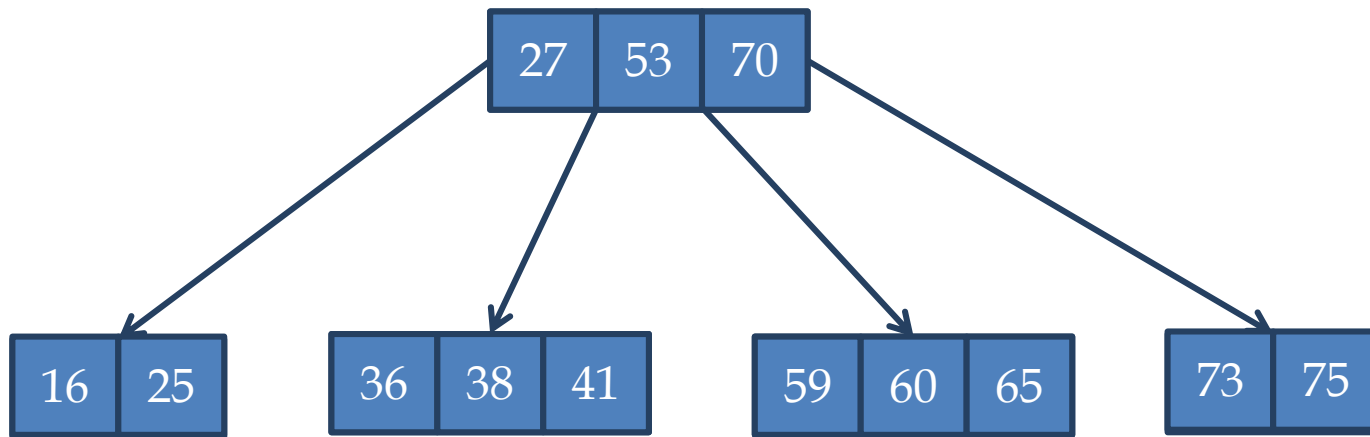
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, **65**, 60, 46, 55, 33, 68, 79, 48}



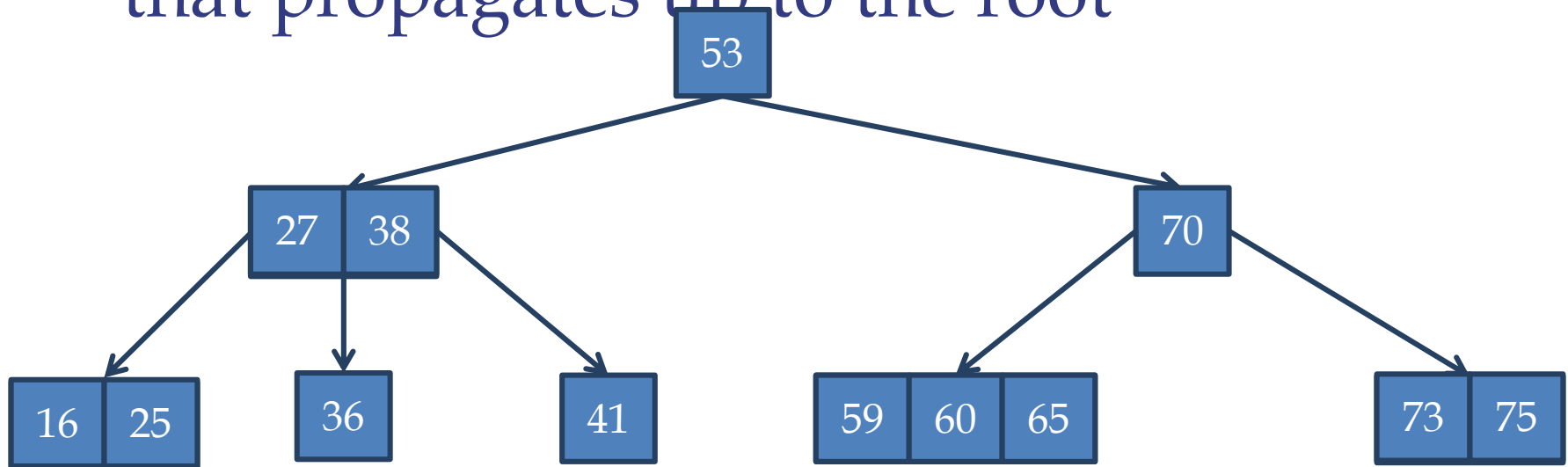
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, **60**, 46, 55, 33, 68, 79, 48}



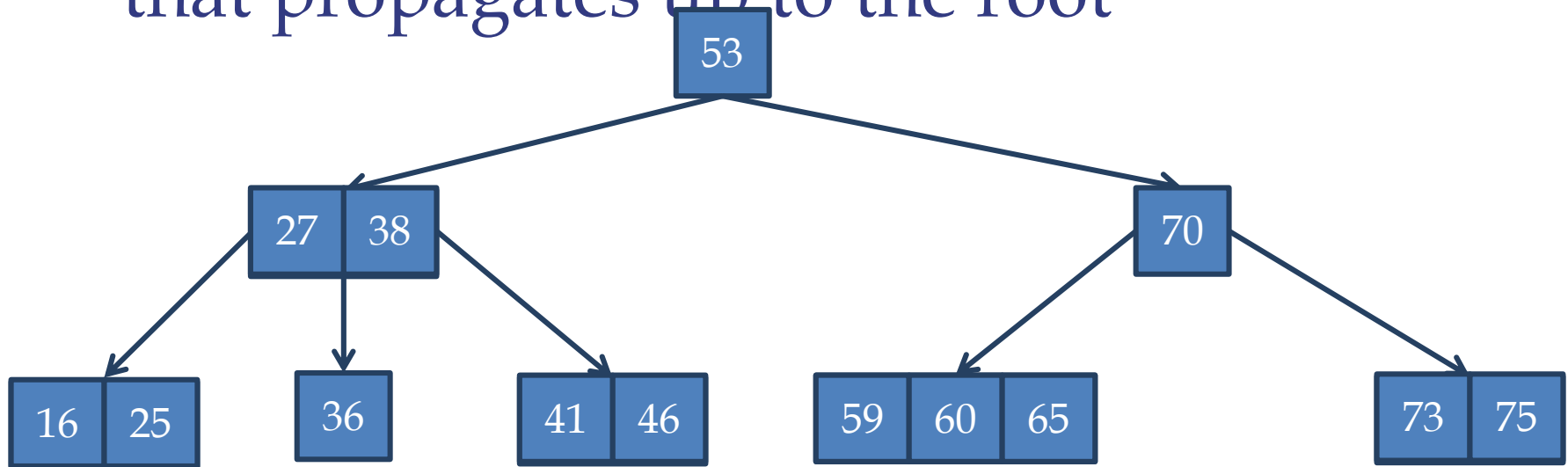
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, **46**, 55, 33, 68, 79, 48} → causes a split that propagates up to the root



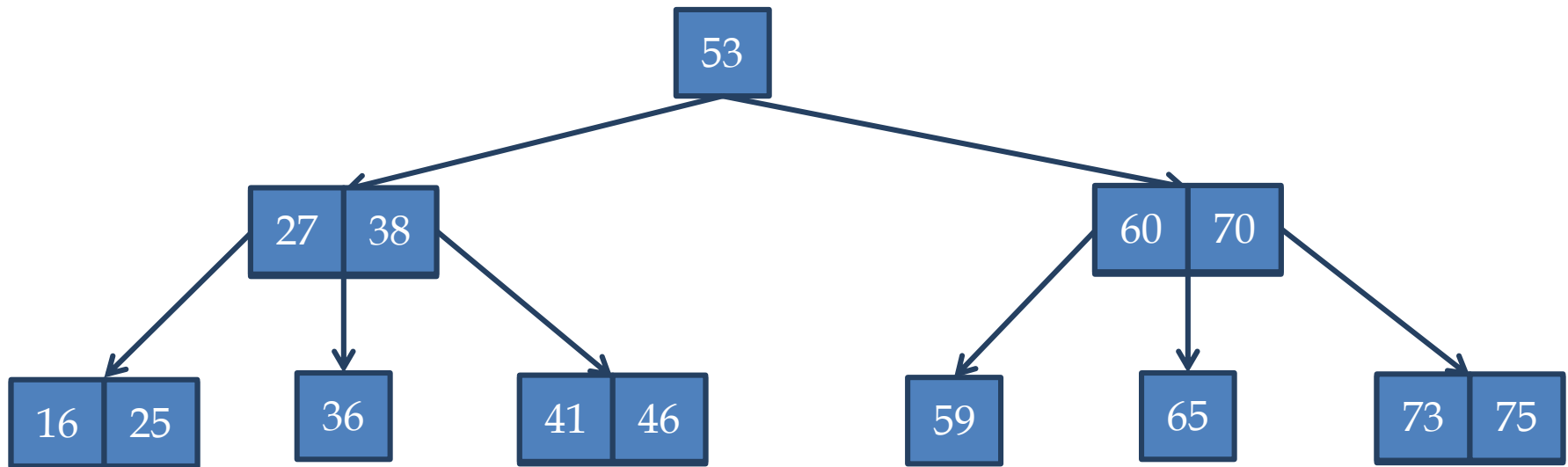
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, **46**, 55, 33, 68, 79, 48} → causes a split that propagates up to the root



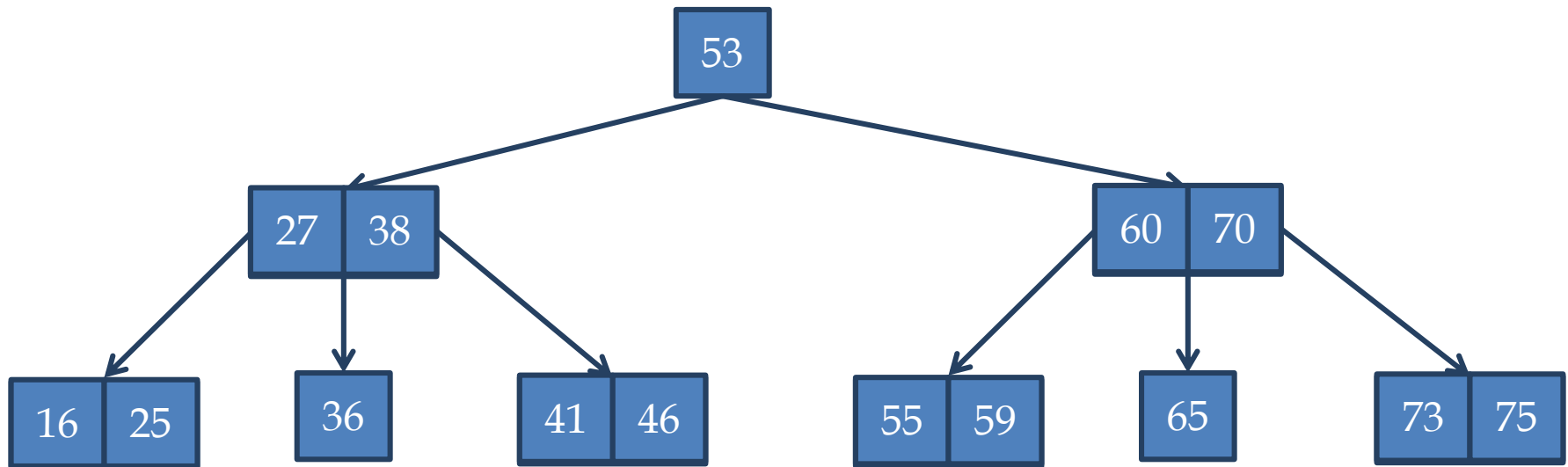
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, **55**, 33, 68, 79, 48} → causes a split



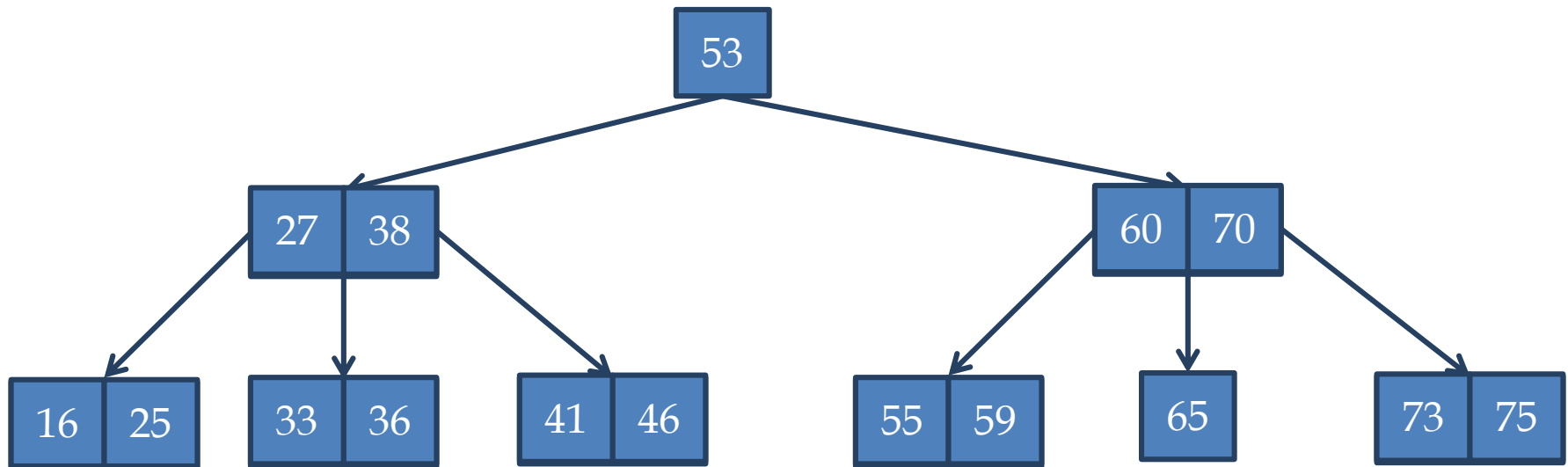
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, **55**, 33, 68, 79, 48} → causes a split



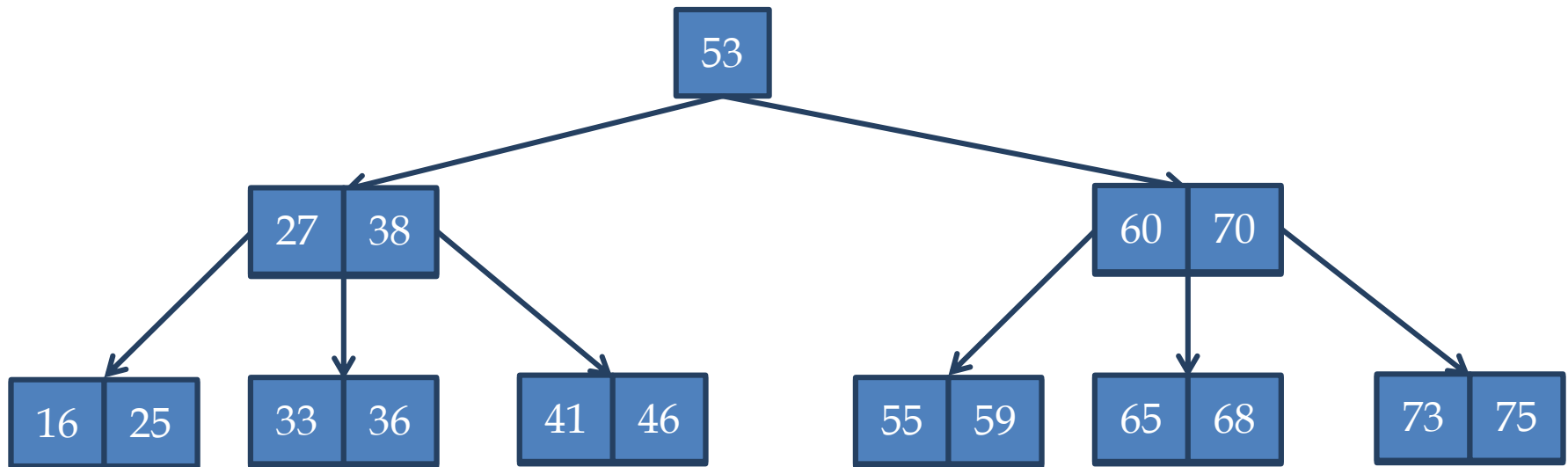
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, **33**, 68, 79, 48}



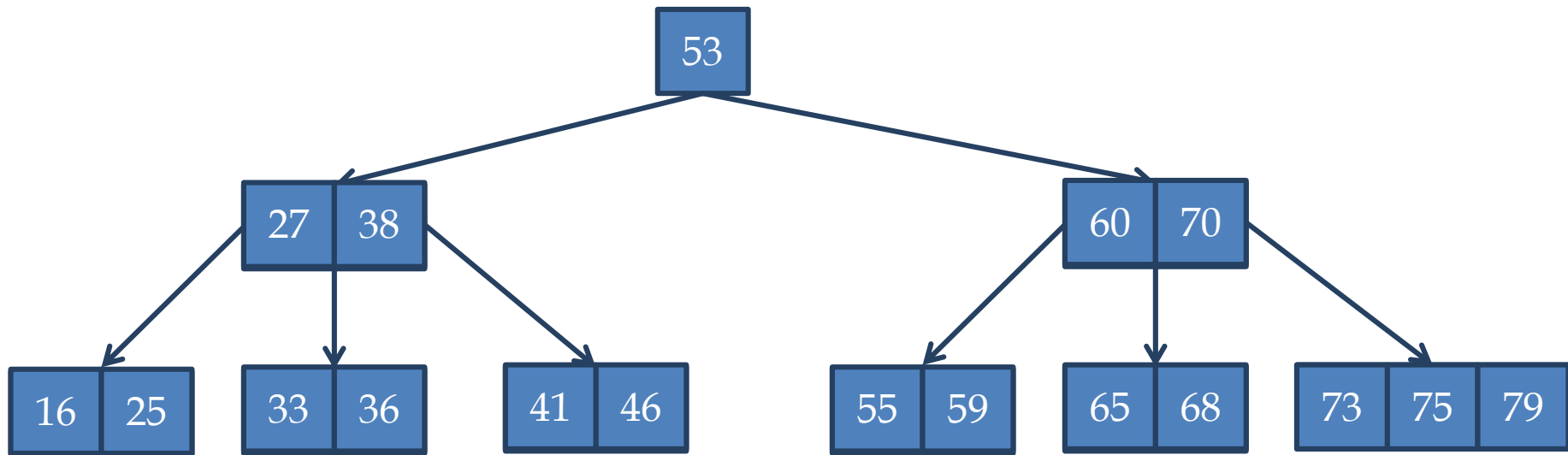
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, **68**, 79, 48}



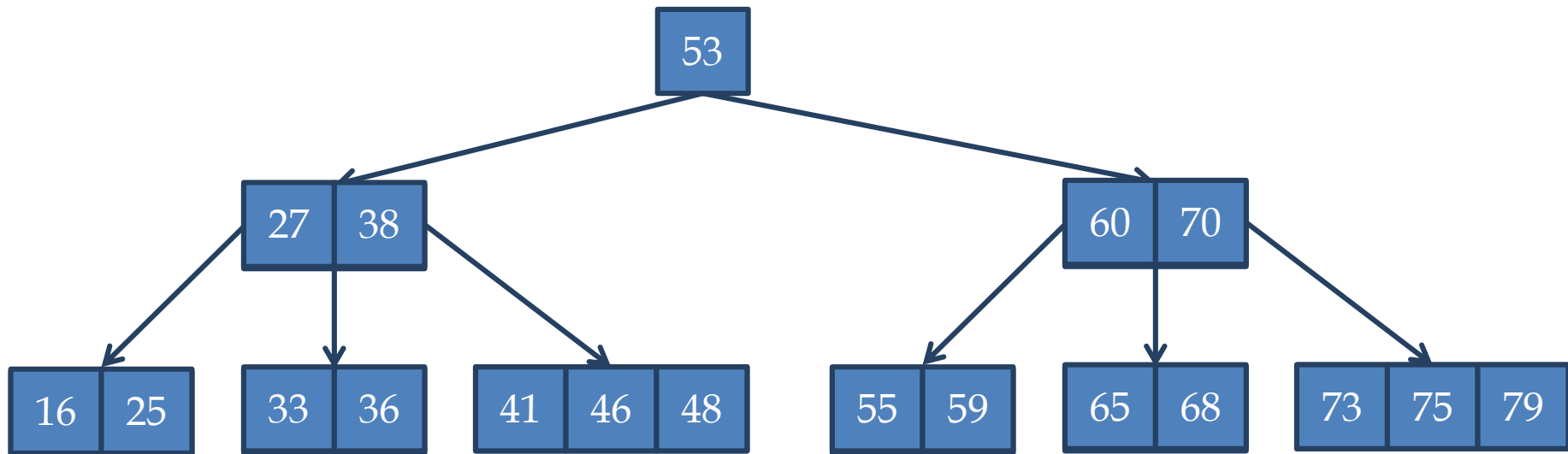
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, **79**, 48}



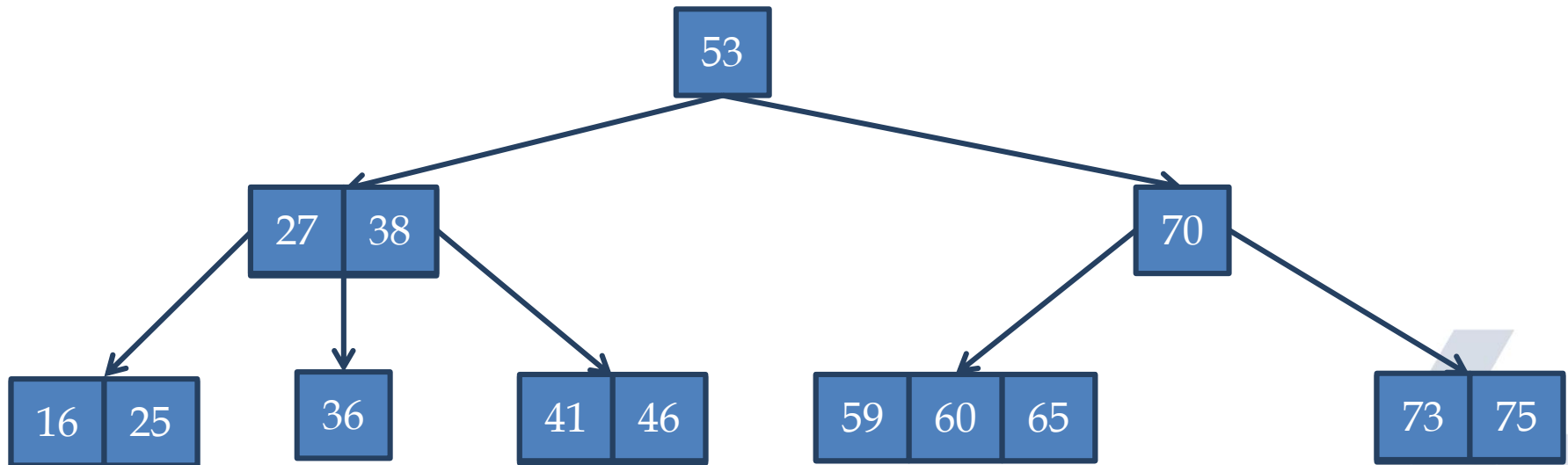
Solution 1 (Cont.)

- {53, 27, 75, 25, 70, 41, 38, 16, 59, 36, 73, 65, 60, 46, 55, 33, 68, 79, **48**}



Question 2

- 2-3-4 trees are balanced and can be searched in $O(\log n)$, but they have different node structures.
- To get 2-3-4 tree advantages in a binary tree format, we can represent it as a red-black tree.
- Convert the following 2-3-4 tree to a red-black tree

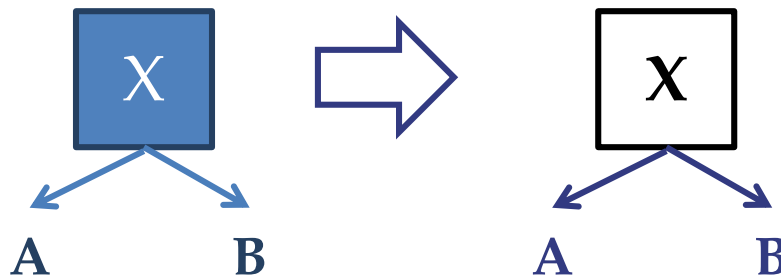


- **Properties of a red-black tree:**
 - the root is always black
 - **black condition:** every path from the root to a leaf node has the same number of black nodes
 - **red condition:** every red node has a black parent

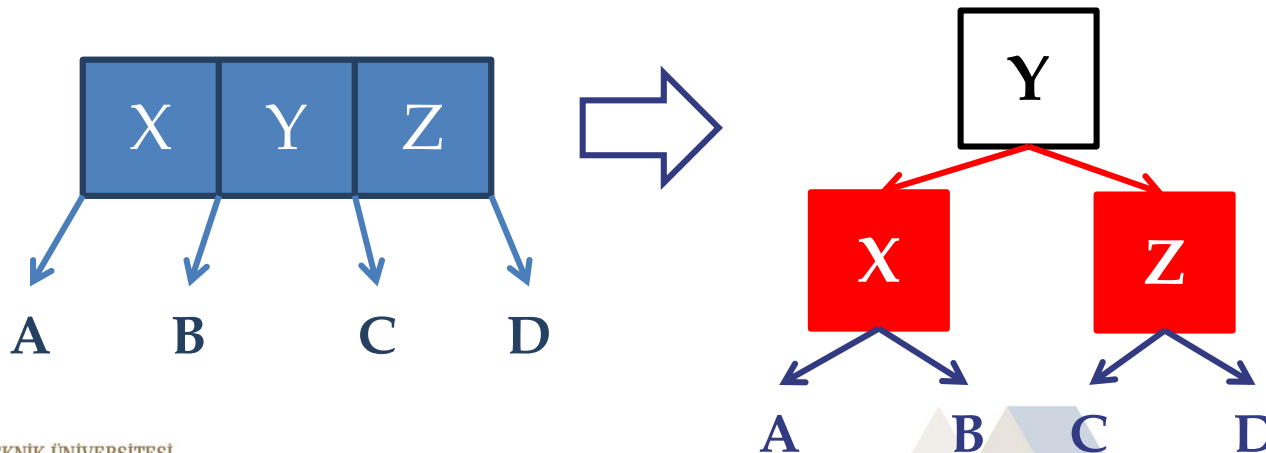


Solution 2 (Cont.)

- **2-nodes:** can be represented with a **black** node

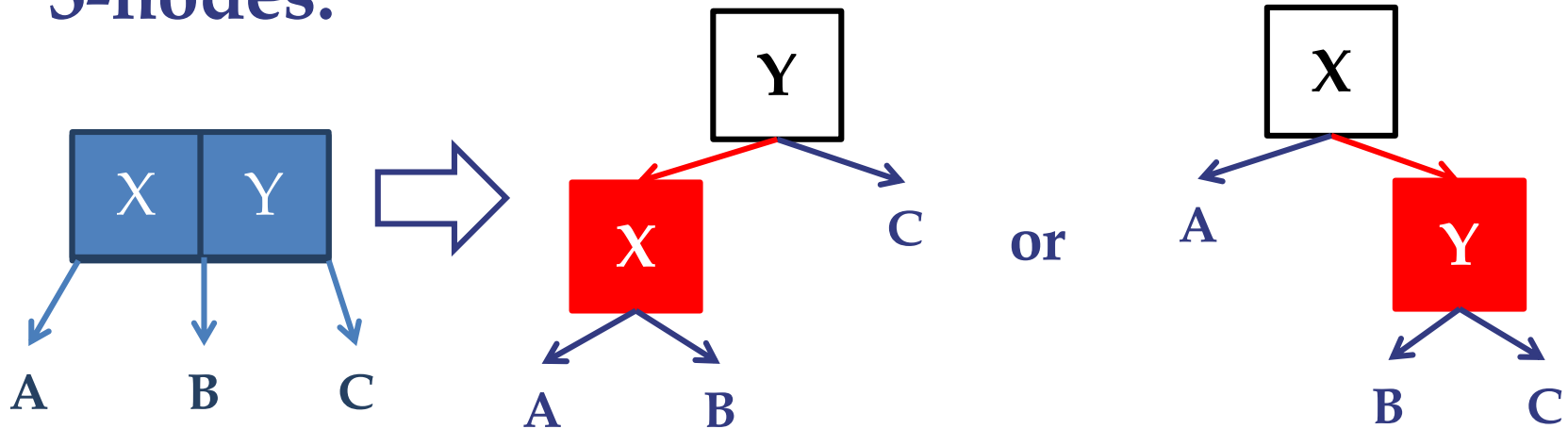


- **4-nodes:** center value becomes the parent (**black**) and the others become children (**red**)



Solution 2 (Cont.)

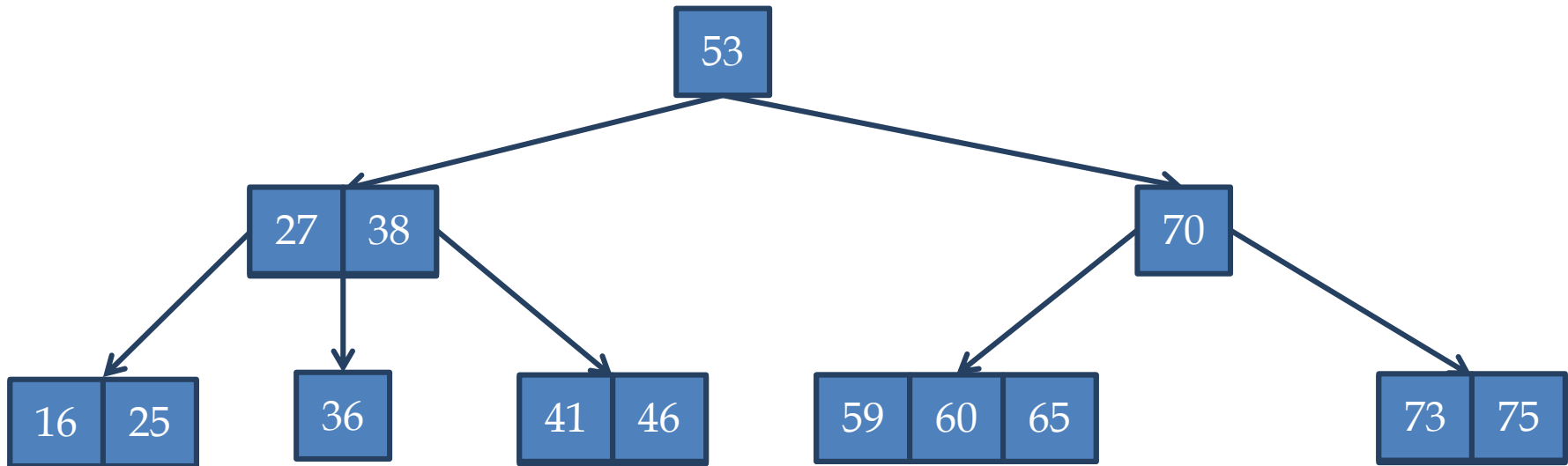
- 3-nodes:



- Note:

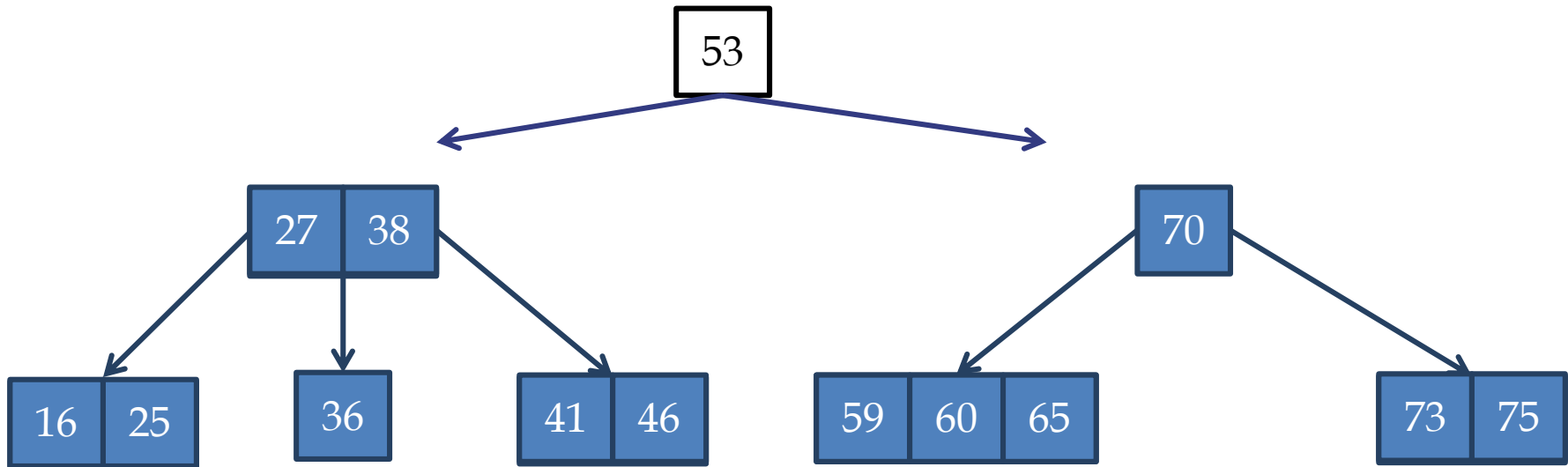
1. Red-black trees are not unique
2. However, the corresponding 2-3-4 tree is unique

Solution 2 (Cont.)

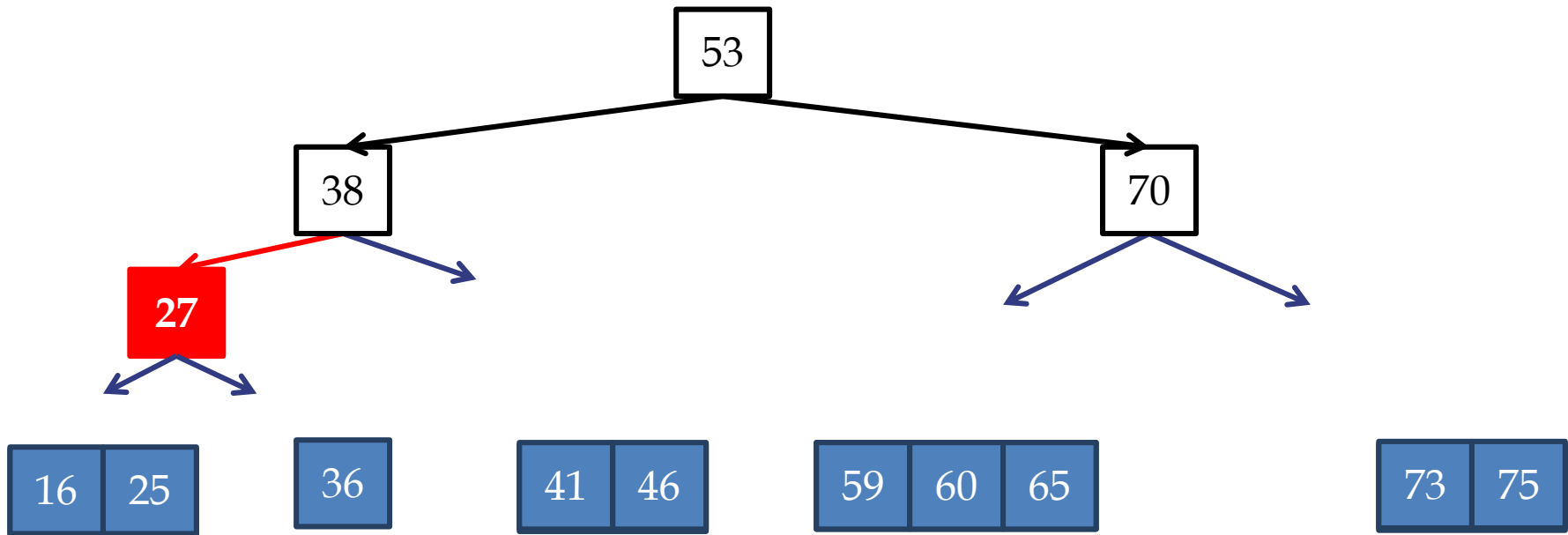


- Top-down conversion algorithm (start at the root):
 1. Apply red-black tree representation to each node
 2. Repeat for next level...

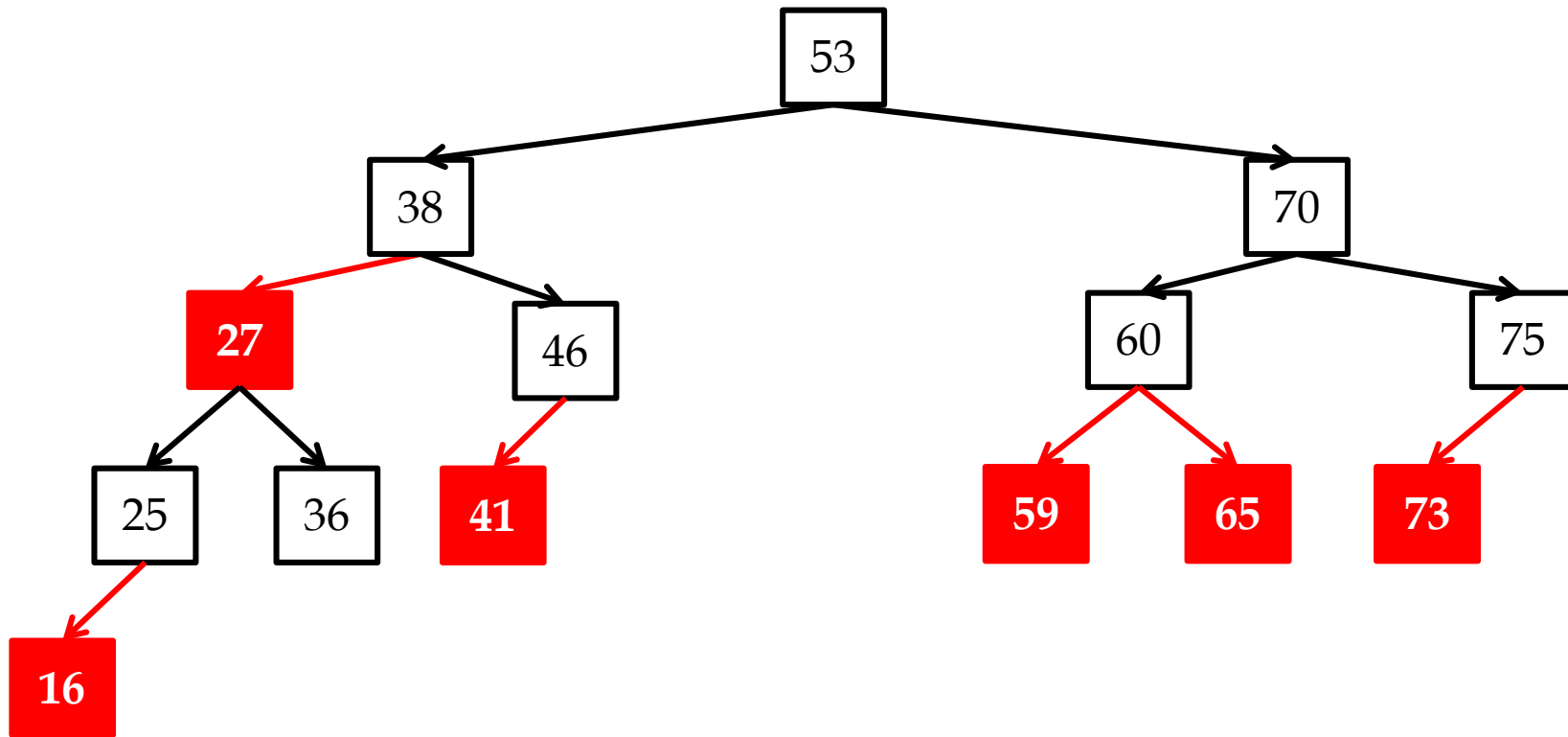
Solution 2 (Cont.)



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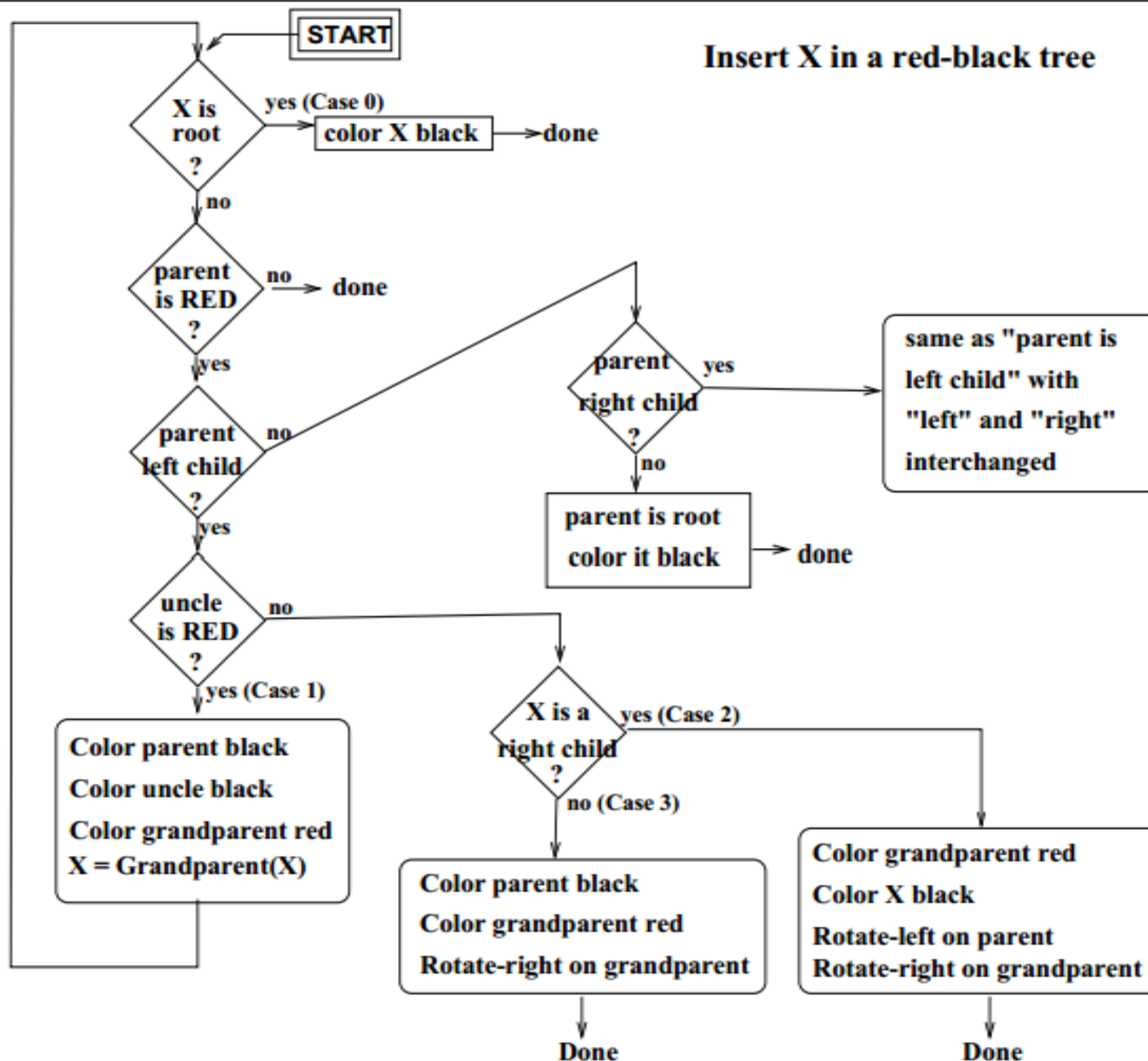


Question 3

- Insert the following sequence of numbers into a red-black tree
 - {2, 1, 4, 5, 9, 3, 6, 7}



Insert X in a red-black tree



Solution 3

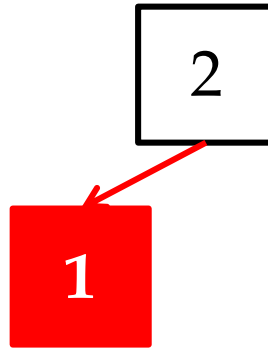
□ {2, 1, 4, 5, 9, 3, 6, 7}

2



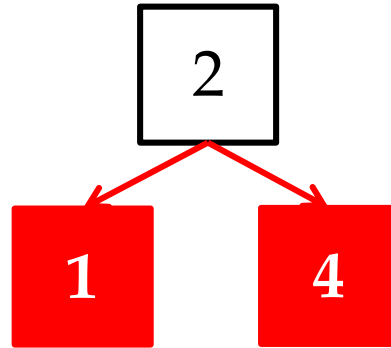
Solution 3 (Cont.)

□ {2, **1**, 4, 5, 9, 3, 6, 7}



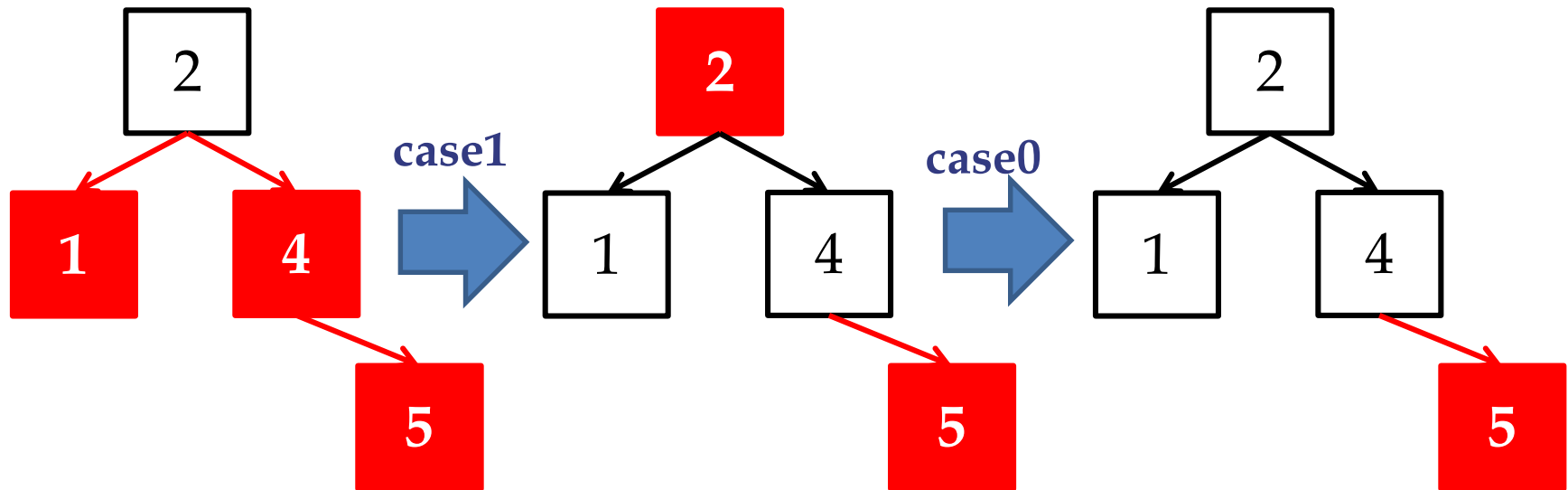
Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, 3, 6, 7}



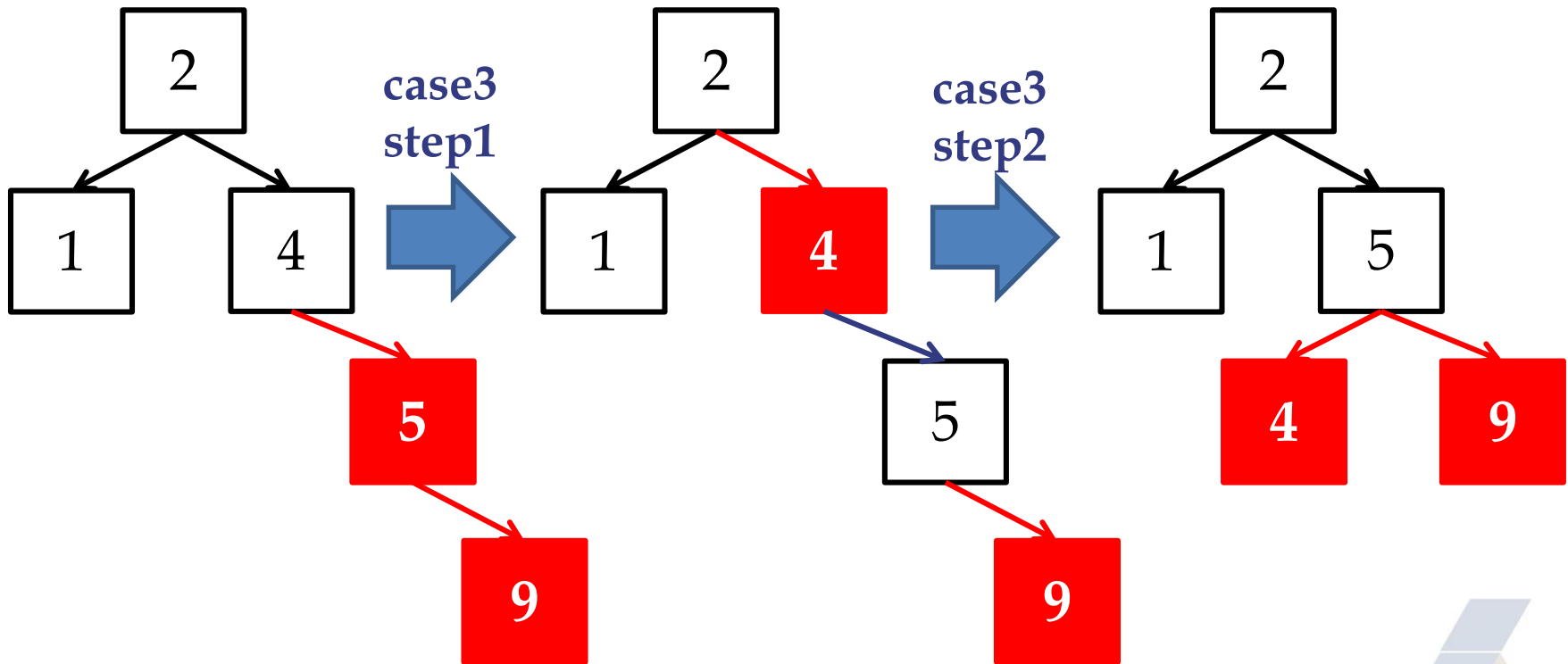
Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, 3, 6, 7} → recoloring



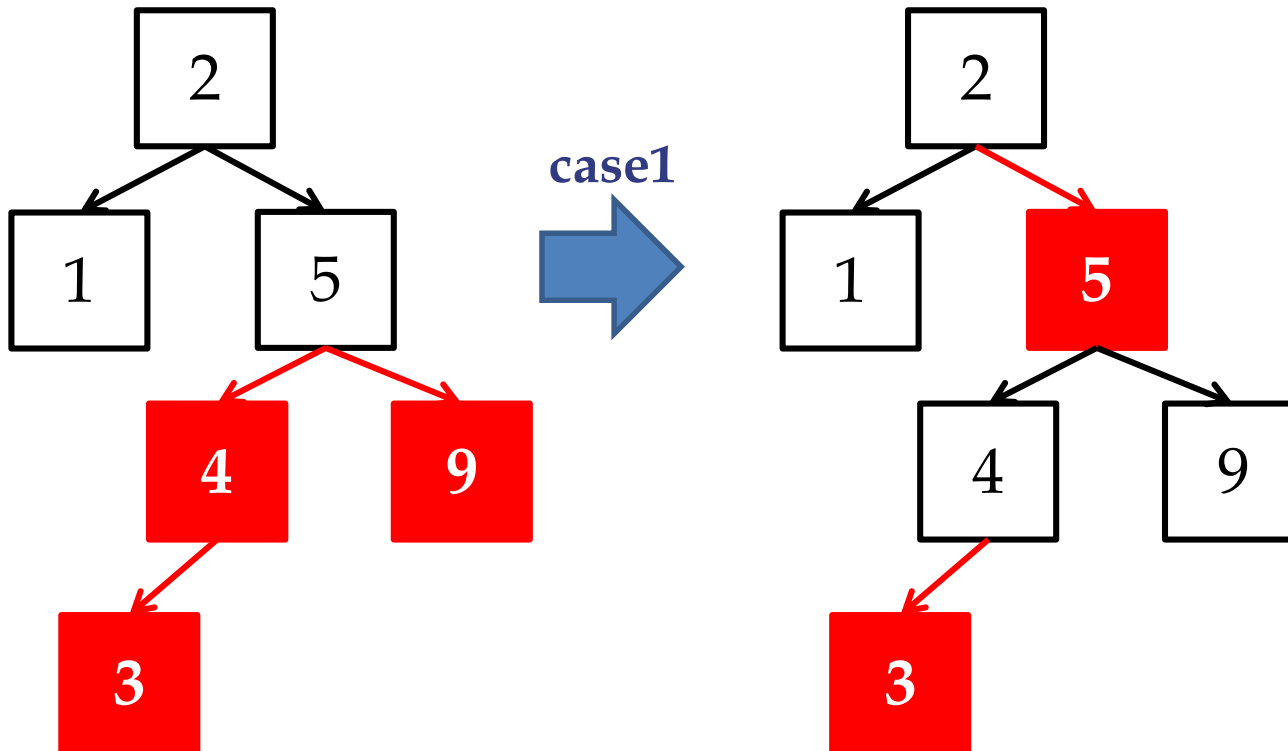
Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, 3, 6, 7} → recoloring and rotation



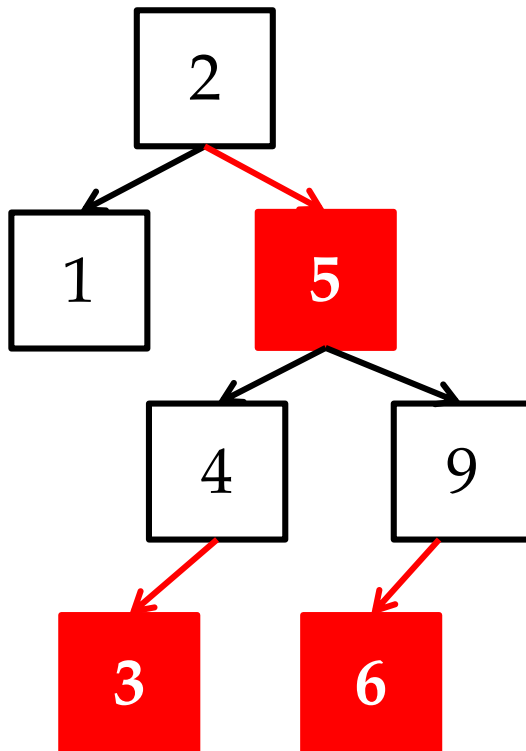
Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, **3**, 6, 7} → recoloring



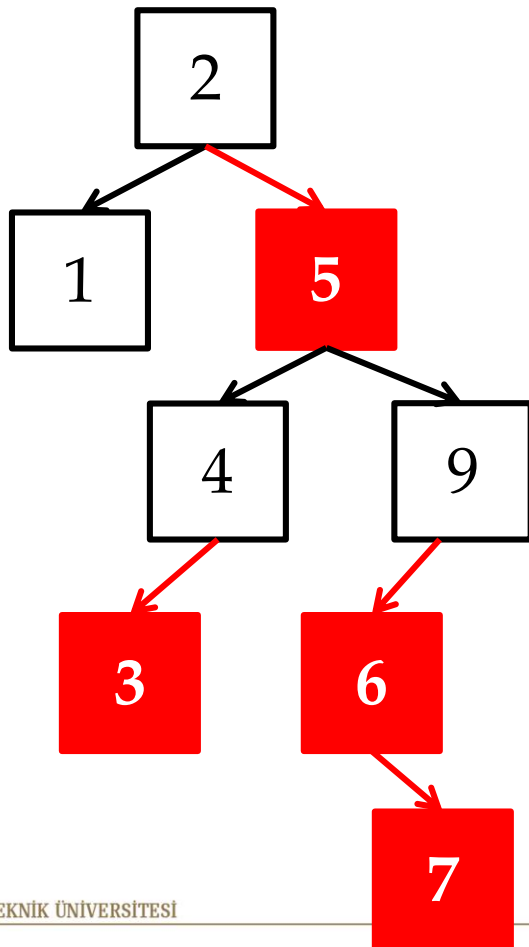
Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, 3, **6**, 7}

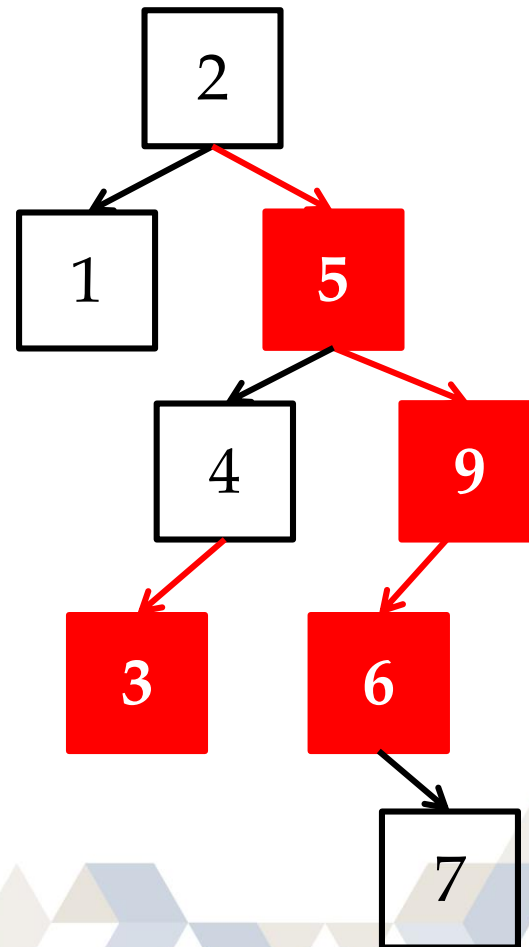


Solution 3 (Cont.)

□ {2, 1, 4, 5, 9, 3, 6, **7**} → recoloring

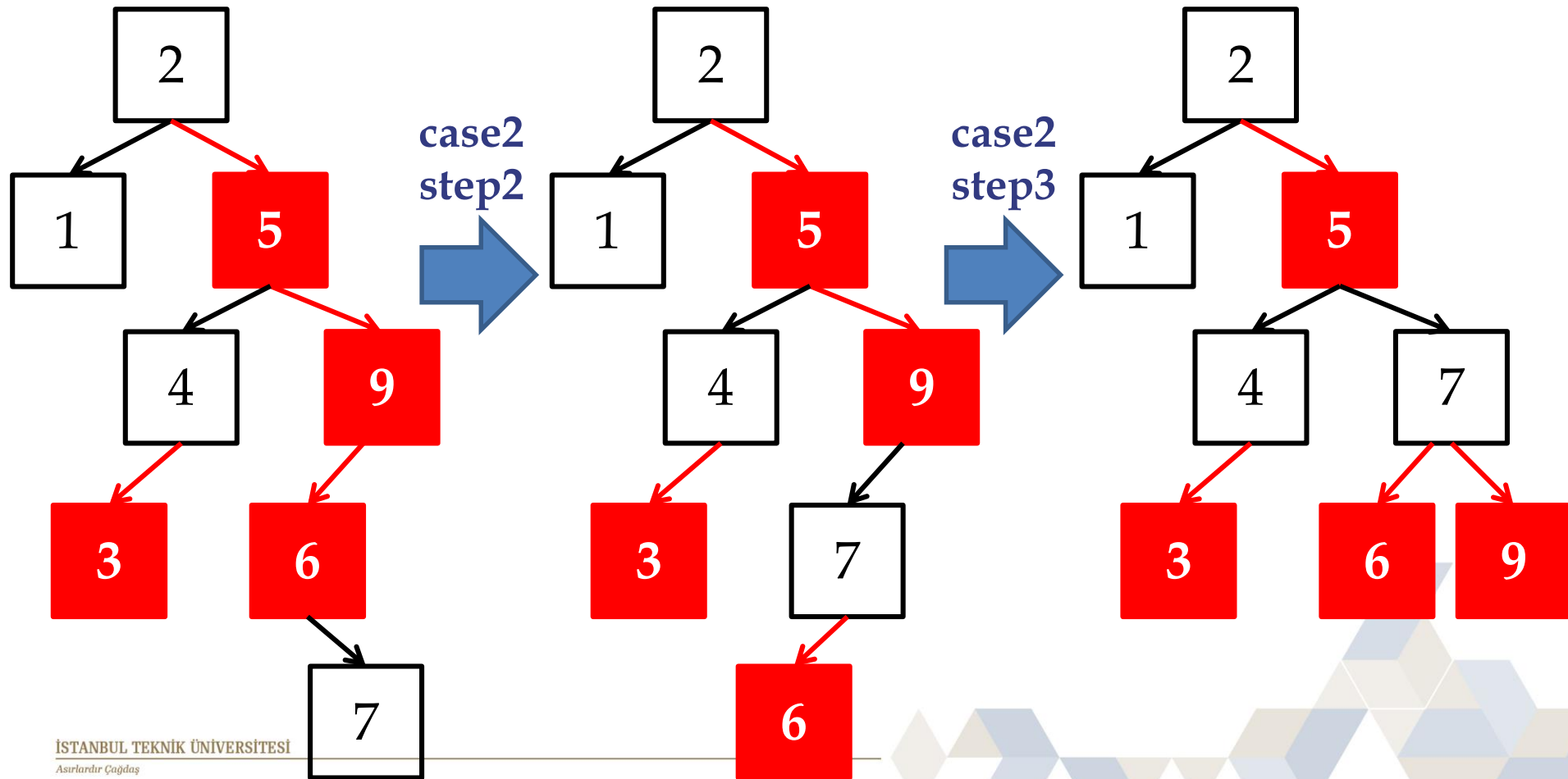


case2
step1



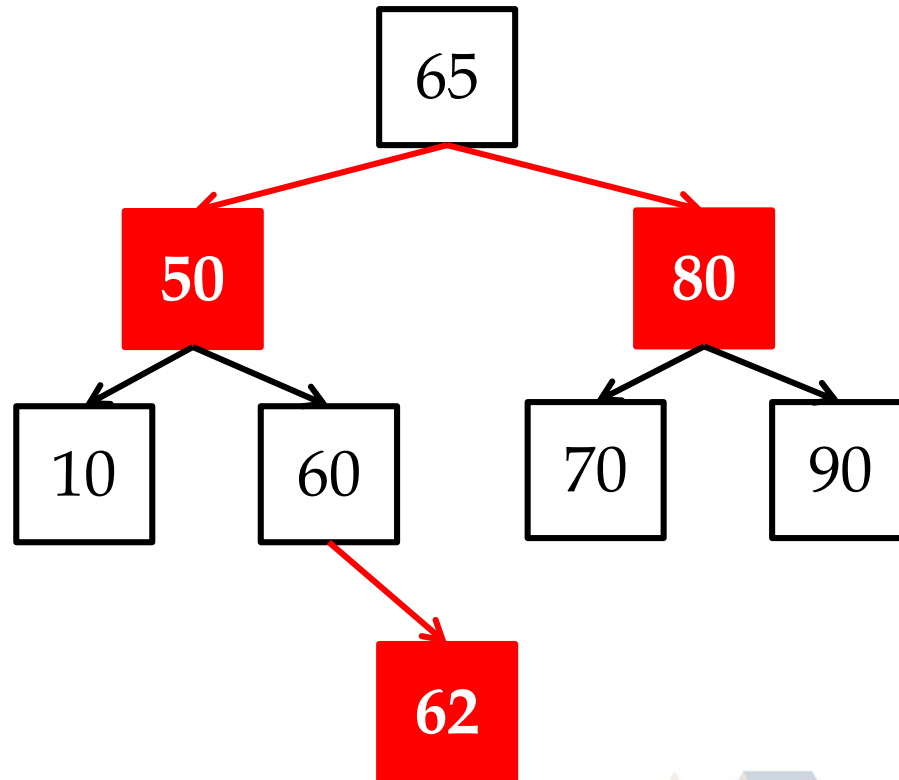
Solution 3 (Cont.)

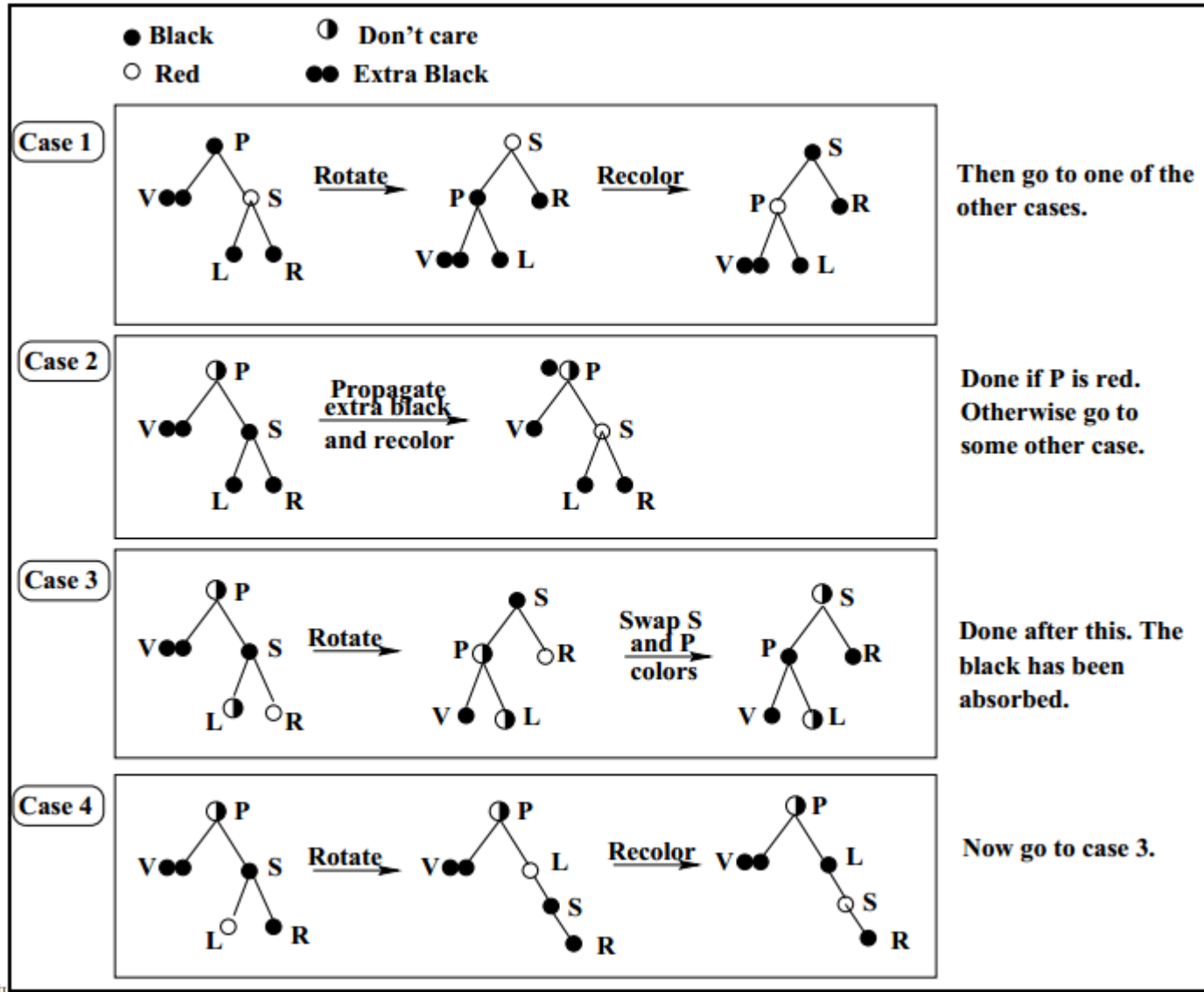
□ {2, 1, 4, 5, 9, 3, 6, 7} → 2 rotations



Question 4

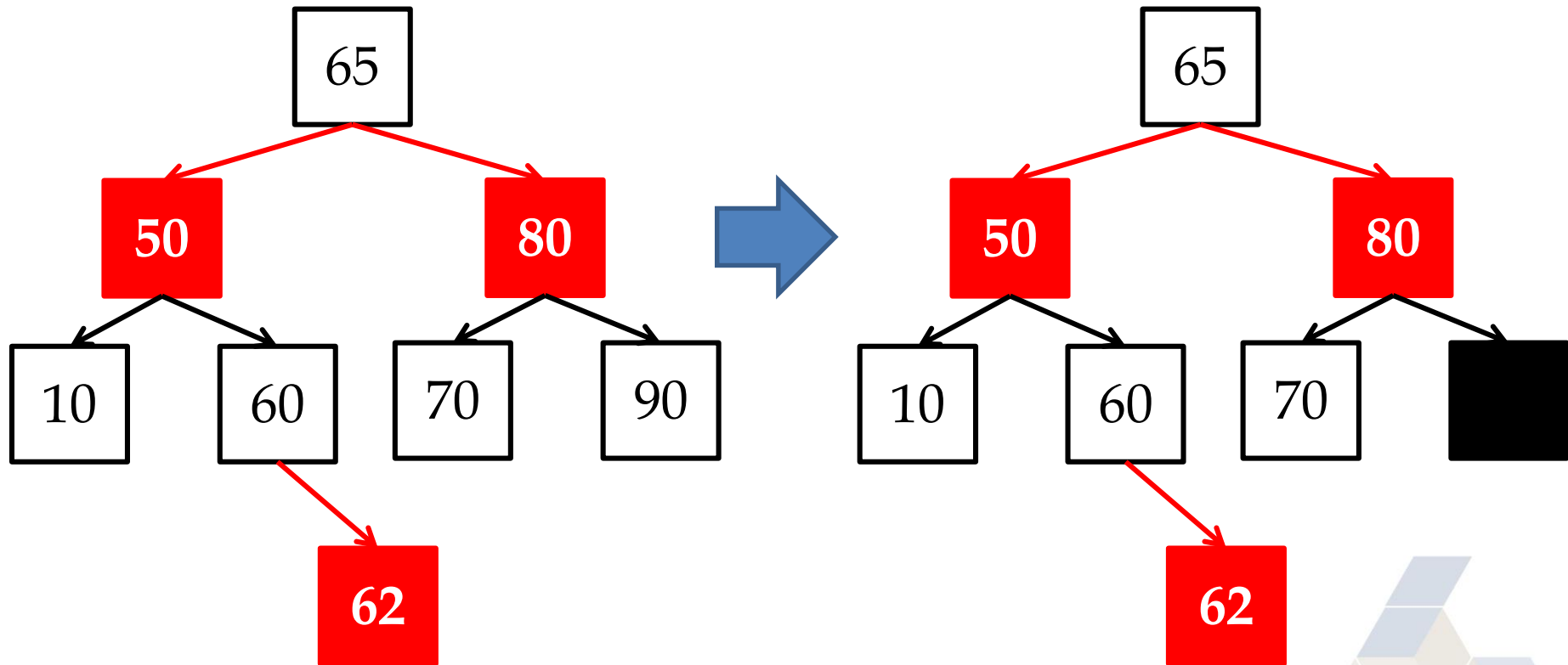
- Delete 90, 80 and 70 from the following red-black tree in the given order





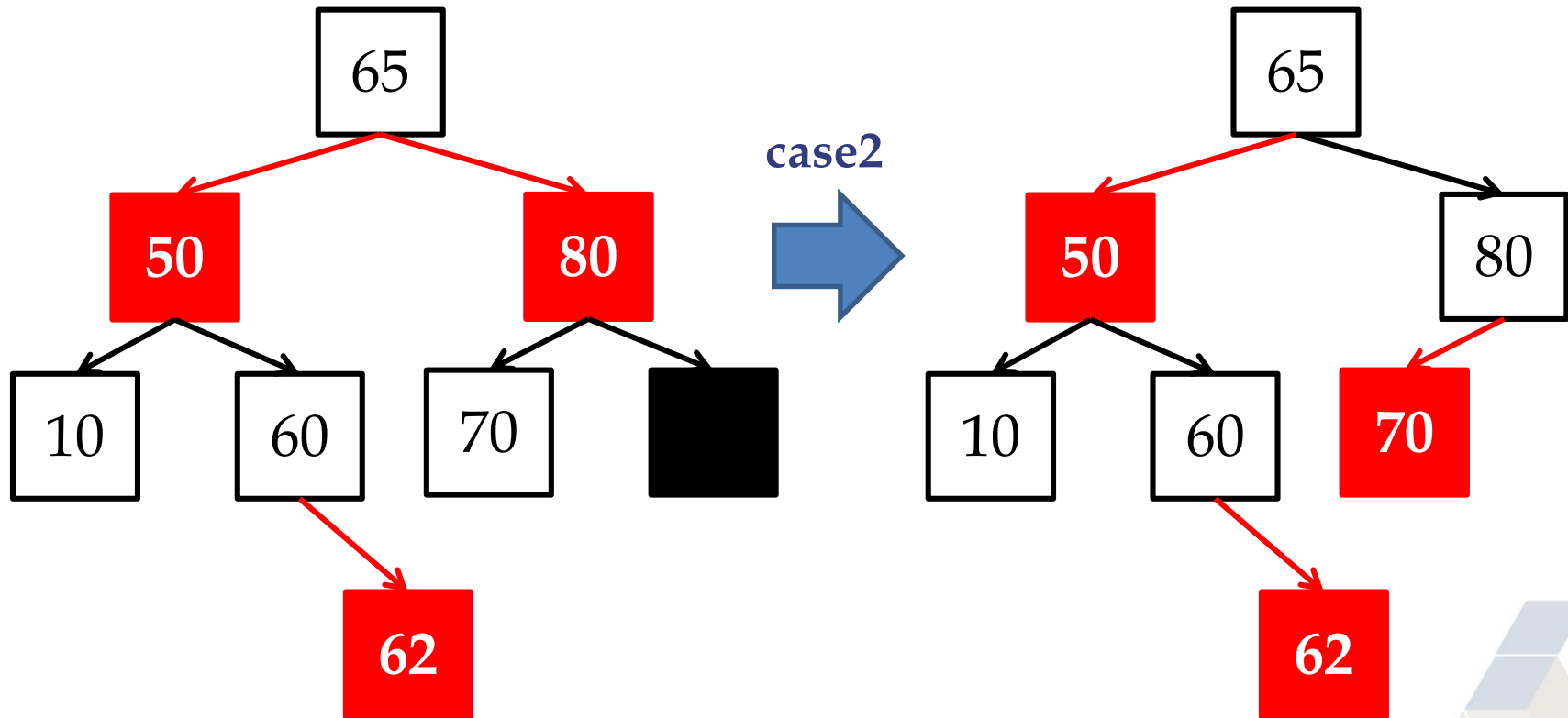
Solution 4 (Cont.)

- Delete **90**, 80 and 70



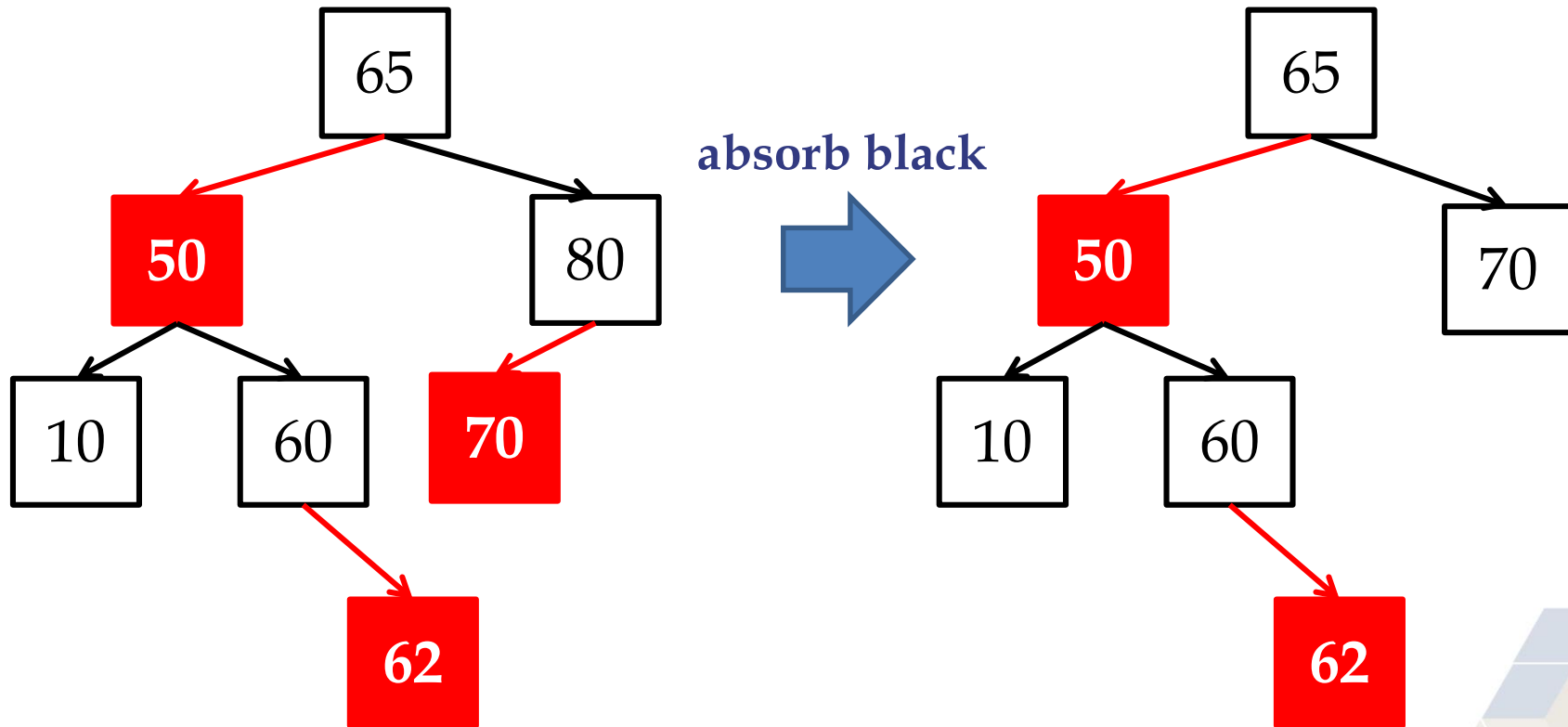
Solution 4 (Cont.)

- Delete **90**, 80 and 70



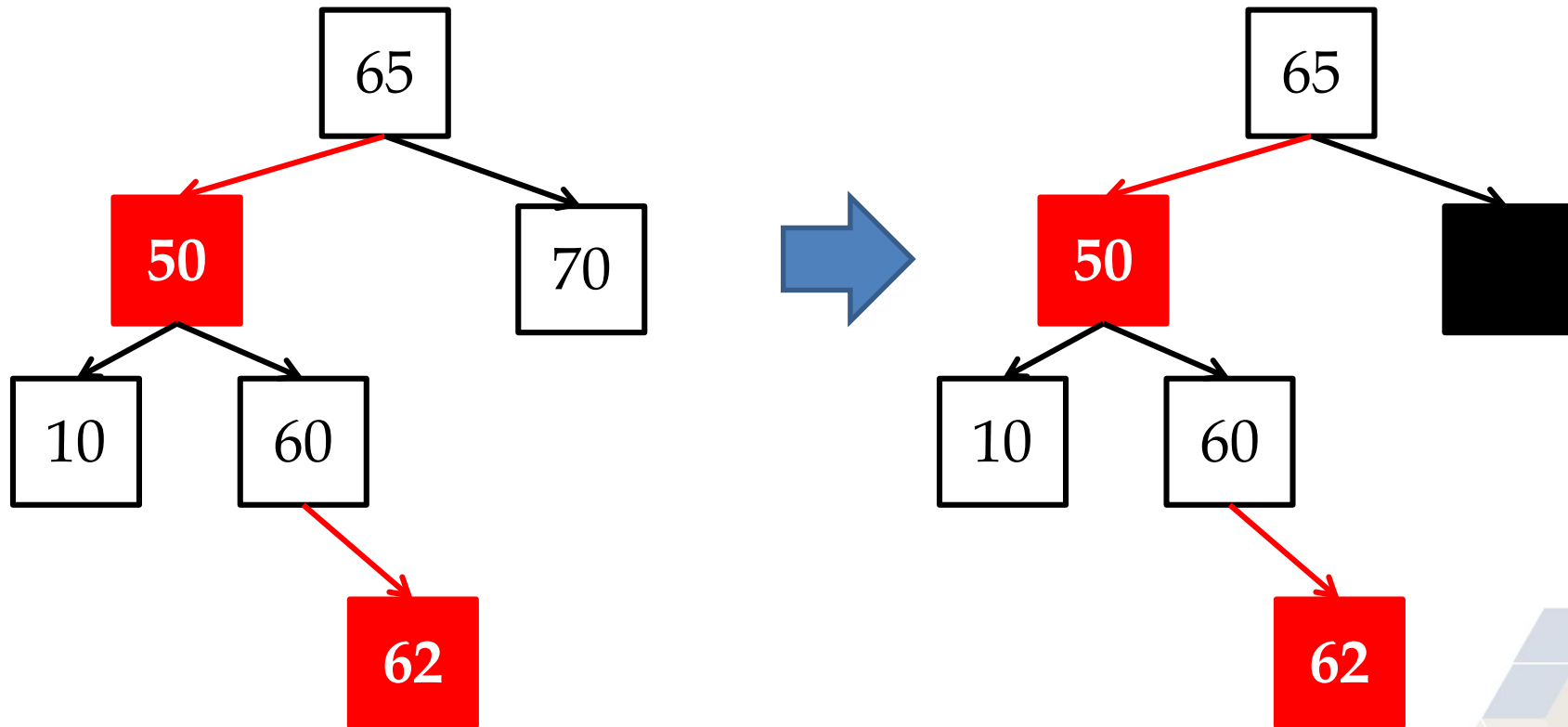
Solution 4 (Cont.)

- Delete 90, **80** and 70



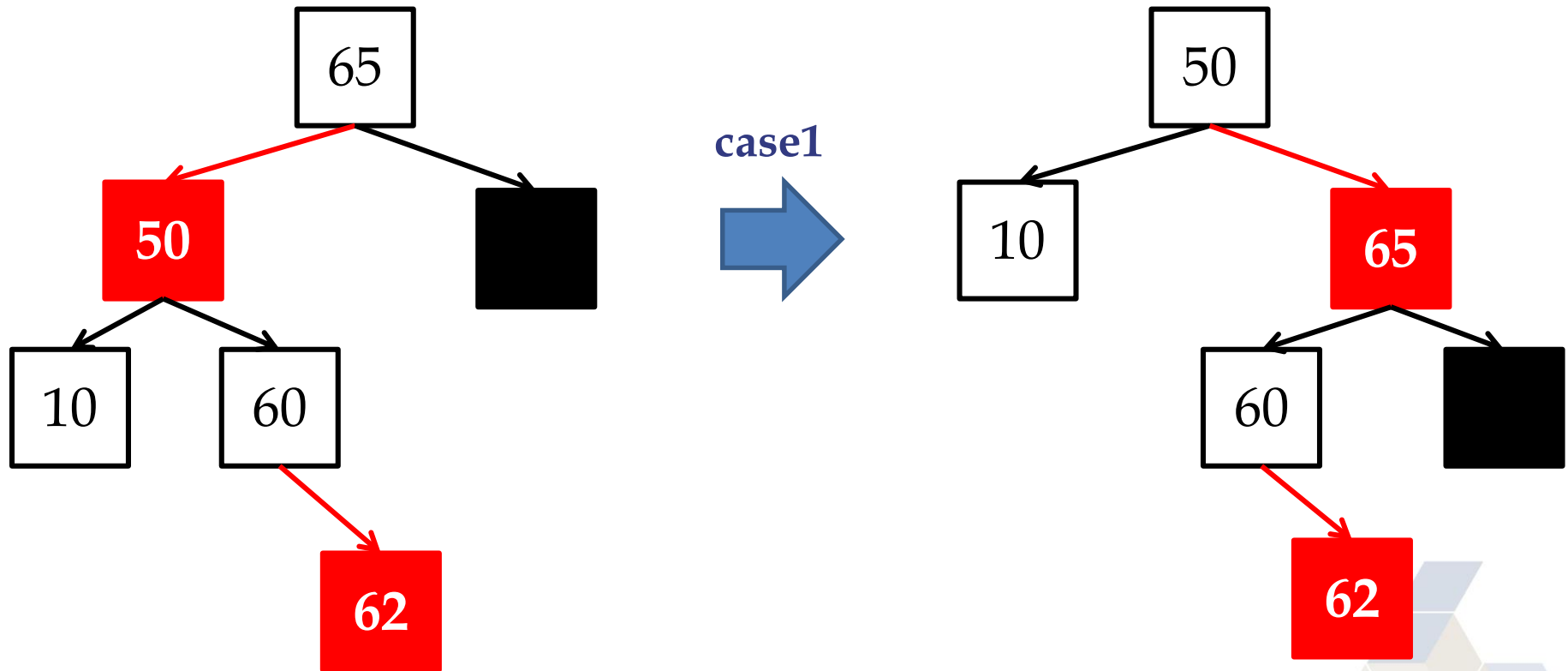
Solution 4 (Cont.)

- Delete 90, 80 and **70**



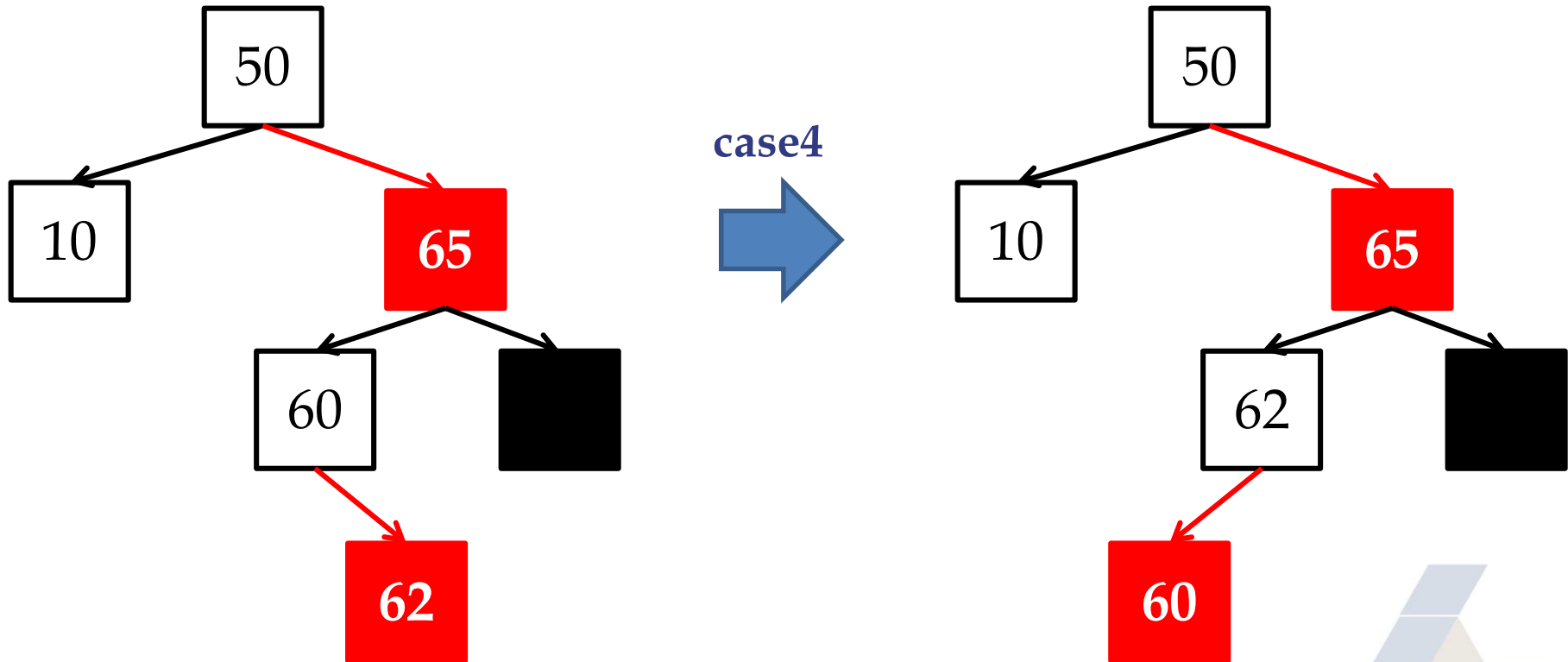
Solution 4 (Cont.)

- Delete 90, 80 and **70**



Solution 4 (Cont.)

- Delete 90, 80 and **70**



Solution 4 (Cont.)

- Delete 90, 80 and **70**

