



## ADVANCED DATA STRUCTURES

### HOMEWORK 1

1) Give asymptotic upper and lower bounds  $T(n)$  in each of the following recurrences. You may assume that  $T(1)$  is constant.

a)  $T(n) = T(n-1) + 1/n$

b)  $T(n) = 2T(n/2) + n \lg n$

c)  $T(n) = 5T(n/5) + n/\lg n$

2) Use and draw a recursion tree to give an asymptotically tight solution to the recurrence  $T(n) = T(n-a) + T(a) + cn$  where  $a \geq 1$  and  $c > 0$  are constants.

3) Write an algorithm(pseudocode or C++ code fragment) which takes a linked list of integers and sums up the odd numbers in the list and sums up the even numbers in the list and returns the product of two sums. You should use recursion in your answer. What is the asymptotic running time of your algorithm?

4) Consider the following algorithm. Given a list of numbers  $A$  and a target number  $t$ . The algorithm returns 1 if  $t$  is in the list, and 0 otherwise.

```
unsortedSearch(A, t, p, q)
if q - p < 1
    if A[p] = t return 1 else return 0
if unsortedSearch(A, t, p,  $\lfloor \frac{p+q}{2} \rfloor$ ) = 1 return 1
if unsortedSearch(A, t,  $\lfloor \frac{p+q}{2} \rfloor + 1, q$ ) = 1 return 1
return 0
```

Analyze the algorithm with respect to worst-case asymptotic complexity and give the worst case running time in terms of  $\theta$  notation with necessary explanation.

5)  $X[1..n]$  is an array of  $n$  distinct numbers which have been permuted randomly. If  $i < j$  and  $X[i] > X[j]$ , then  $(i, j)$  pair is called an inversion of  $X$ . Use indicator random variables to compute the expected number of inversions of  $X$ .

**Submission:** Due to **21.10.2011 17:00**, submissions should be done to the "Advanced Data Structure" box in the Computer Engineering Department secretarial office. Manual delivery will not be accepted.