**BLG311E – FORMAL LANGUAGES AND AUTOMATA**

**2013 SPRING**

**RECITEMENT 3**

**1.** Let A and B be languages defined over $\Sigma$ and $\Lambda \notin B$,

a) Propose a solution to the equation $A \cup XB = X$.

b) Show that your solution is correct.

c) Let A={a, b} and B={aa, ab, ba, bb}. Write the solution set and give 5 example words from it.

**Solution:**

a) Solution is $X=AB^*$

b) When we replace X in the equation:

$$A \cup (AB^*)B = A \cup AB^+ = A(\{\Lambda\} \cup B^+) = AB^*$$

Solution is correct.

c) $AB^* = \{a, b\}\{aa, ab, ba, bb\}^*$

Examples: {a}, {aaa}, {baa}, {bbbaa}, {aabbabb}

**2.** Let A and B be languages defined over $\Sigma$

Show that equation $A^*B^* \cap B^*A^* = A^* \cup B^*$ holds.

**Solution:**

$A^*B^* = ( \{\Lambda\} \cup A^+) (\{\Lambda\} \cup B^+) = (\{\Lambda\} \cup A^+ \cup B^+ \cup A^+B^+)$

Same for $B^*A^* \rightarrow B^*A^* = (\{\Lambda\} \cup A^+ \cup B^+ \cup B^+A^+)$

Intersection of 2 sets:

$$A^*B^* \cap B^*A^* = (\{\Lambda\} \cup A^+ \cup B^+)$$

$A^* \cup B^* = (\{\Lambda\} \cup A^+) \cup (\{\Lambda\} \cup B^+) = (\{\Lambda\} \cup A^+ \cup B^+)$

It holds.

**3.** Show that following expressions hold. If they do not hold give a counterexample.

**a)** $A^+A^+ = A^+$

Does not hold.

Let A = {1} :

$A^+ = \{1, 11, 111, 1111, ...... 1^n, ...\}$

$A^+A^+ = \{11, 111, 1111, ...... 1^n, ...\}$

**b)** $(A^*B^*)^* = (B^*A^*)^*$

By using Theorem 13 on page 28 of the slides:

$(A^*B^*)^* = (A \cup B)^* \rightarrow (B^*A^*)^* = (B \cup A)^* \rightarrow (B \cup A)^* = (A \cup B)^*$

Holds

**c)** $(AB)^* = (BA)^*$

Does not hold

Let A ={0} and B ={1}

$(AB)^* = \{\Lambda, 01, 0101, 010101, ..... (01)^n,....\}$

$(BA)^* = \{\Lambda, 10, 1010, 101010, ..... (10)^n,....\}$

**4.** Matrix below is a relation defined on the set {a, b, c}. Draw the relation graph of the relation itself, its powers, reflexive, symmetric, transitive closures as well as reflexive closure of its symmetric closure.

| | **a** | **b** | **c** |
|---|---|---|---|
| **a** | 0 | 1 | 0 |
| **b** | 1 | 0 | 1 |
| **c** | 0 | 0 | 0 |

**Solution:**

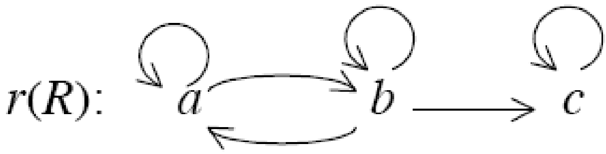
**a)** $R = \{(a, b), (b, a), (b, c)\}$

Relation graph R:


**b)** Powers of the relation will be found in (e).

**c)** Reflexive closure:

$r(R) = R \cup R^0 = R \cup E$, $E = R^0$ (E is the unit relation)



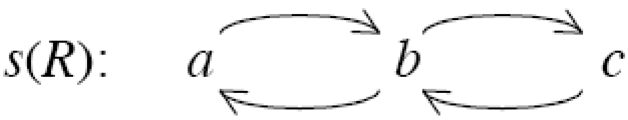

$R$ = {(a,b), (b,a), (b,c)}

$E$ = {(a,a), (b,b), (c,c)}

$r(R)$ = {(a,b), (b,a), (b,c), (a,a), (b,b), (c,c)}

**d)** Symmetric closure:

$s(R) = R \cup R^{-1}$




$R$ = {(a,b), (b,a), (b,c)}

$R^{-1}$ = {(b,a) | (a,b) ∈ $R$}

$R^{-1}$ = {(a,b), (b,a), (c,b)}

$R \cup R^{-1} = s(R)$ = {(a,b), (b,a), (b,c), (c,b)}

**e)** Transitive closure:

$$t(R) = \bigcup_{i=1}^{\infty} R^i$$

We need to find the powers of the relation for the transitive closure.

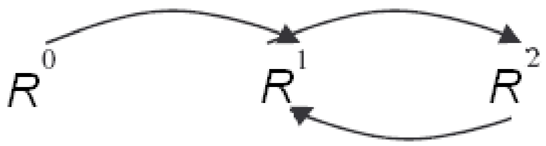

$R$ = {(a,b), (b,a), (b,c)}

$R^2 = RR$ = {(a,b), (b,a), (b,c)}{(a,b), (b,a), (b,c)} = {(a,a), (b,b), (a,c)}


$R^3 = R^2R$ = {(a,a), (b,b), (a,c)}{(a,b), (b,a), (b,c)} = {(a,b), (b,a), (b,c)}


$R^1=R^3$
$RR= R^3R \rightarrow R^2=R^4$
$R^{2n+1}=R^1$ and $R^{2n}=R^2$ (n>0)

Powers of the relation graph:




Transitive closure → $t(R) = R \cup R^2$:

$t(R)$ = {(a,b), (b,a), (b,c)} ∪ {(a,a), (b,b), (a,c)}
= {(a,b), (b,a), (b,c), (a,a), (b,b), (a,c)}


**f)** Reflexive closure of the symmetric closure

$rs(R)$ = ? Let $P = s(R)$
We know that: $s(R)$ = {(a,b), (b,a), (b,c), (c,b)}
We need to find $r(P)$.
$r(P)$ = {(a,b), (b,a), (b,c), (c, b), (a,a), (b,b), (c,c)}