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BLG517E Formula sheet

$\sum_{i=1}^{n} i = \frac{n.(n+1)}{2}$ | $\sum_{i=1}^{n} i^2 = \frac{n.(n+1).(2n+1)}{6}$ | $\sum_{k=0}^{n} a.r^k = a\frac{1-r^{n+1}}{1-r}$ | $\sum_{k=0}^{\infty} a.r^k = \frac{a}{1-r}$ $|r|<1$

$e = \sum_{n=0}^{\infty} \frac{1}{n!}$ | $e^x = \sum_{n=0}^{\infty} x^n . \frac{1}{n!}$ | $e^{\frac{x}{k}} = \sum_{n=0}^{\infty} \left(\frac{x}{k}\right)^n \frac{1}{n!}$ | $e^{-\alpha}\frac{\alpha^n}{n!} \to$ Poisson

$A/B/m/n \to$ #servers + queue

A → arrival dist. func; B → service dist. func; m → #servers

$n>m$, buf $= n-m$

$n=m$, no buf.

Little's Rule

$N = \lambda T$

- N → avg. # of customers in the sys.
- λ → avg arrival rate
- T → avg. delay (time spent in sys)

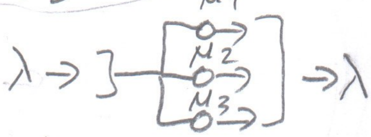
$\rho = \frac{E[N_s]}{m}$ → Exp. # of people at server

$E[N] = \sum_{n=1}^{m} n . P_n$ ← Expected num. of customer

$\lambda_1, \lambda_2, \lambda_2 \to \lambda = \sum \lambda_i$

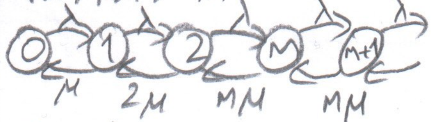
$\lambda \xrightarrow{P_1} \lambda P_1$, $\xrightarrow{P_2} \lambda P_2$, $\xrightarrow{P_3} \lambda P_3$

$\sum P_i = 1$



$\lambda < \mu = \sum \mu i$

M/M/m → # of servers



$\rho = \frac{\lambda}{m.\mu}$

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n . P_0, & n \le m \\ \frac{1}{m!} . \frac{1}{m^{(n-m)}} . \left(\frac{\lambda}{\mu}\right)^n . P_0, & n > m \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{m-1} \frac{(m.\rho)^n}{n!} + \frac{(m.\rho)^m}{m!(1-\rho)}\right]^{-1}$$

$$P_Q = \frac{P_0 . (m.\rho)^m}{m! . (1-\rho)} \quad \text{(Erlang C Formula)}$$

$$N_Q = P_Q . \frac{\rho}{1-\rho} = \lambda . T_Q$$

$$T = \frac{1}{\mu} + T_Q$$

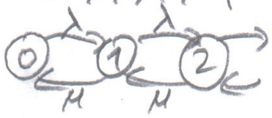
$$N = \lambda . T$$

M/M/∞

$P_n = P_0 . \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n$ | $P_0 = e^{-\frac{\lambda}{\mu}}$

$N = \frac{\lambda}{\mu}$ | $T = \frac{1}{\mu}$

M/M/1 $\quad \lambda < \mu$



$\sum P_i = 1$

$P_1(\lambda + \mu) = P_0.\lambda + P_2.\mu$

$\rho = \frac{\lambda}{\mu}$

$P_n = \left(\frac{\lambda}{\mu}\right)^n . \left(1 - \frac{\lambda}{\mu}\right)$

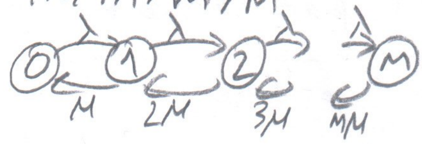
$N = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1-\rho}$

$T = \frac{N}{\lambda} = \frac{1}{\mu - \lambda}$

$\underbrace{T - \frac{1}{\mu}}_{\text{service time}} = \underbrace{T_q}_{\text{queue time}}$

$N_q = \lambda . T_q$

M/M/m/m



$P_n = P_0 . \left(\frac{\lambda}{\mu}\right)^n . \frac{1}{n!}$ (Erlang B formula)

$P_m = P_0 . \left(\frac{\lambda}{\mu}\right)^m . \frac{1}{m!}$ Prob. of incoming pac to be lost

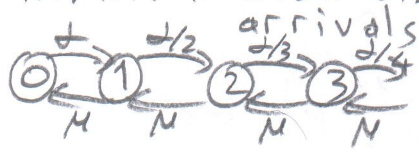
P_m → last state

$P_0 = \left[\sum_{n=0}^{m} \left(\frac{\lambda}{\mu}\right)^n . \frac{1}{n!}\right]^{-1}$

$N = \sum_{n=1}^{m} n.P_n = \lambda' T$

$T' = ?$ $\lambda' = \lambda(1 - P_m)$

M/M/1 with discouraged arrivals



$P_n = P_0 \left(\frac{\alpha}{\mu}\right)^n . \frac{1}{n!}$ | $P_0 = e^{-\frac{\alpha}{\mu}}$

TDM		SM	divided
$N_{sys} = N.k$	>	$N_{sys} = N$	$N_{tot} = n_1.N_1 + n_2.N_2$
T	>	T	$T_{avg} = T_1.\frac{\# pac T_1}{\# pac} +$

Kleinrock Ind. Appr.

link(i,j) $\lambda_{ij} = \sum X_{streams}$

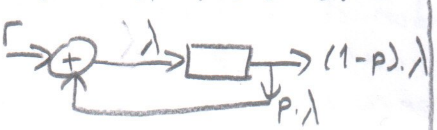
$$N = \sum_{(i,j)} \Big(\underbrace{\frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}}}_{N_{ij}} + \underbrace{\lambda_{ij} . d_{ij}}_{\text{prop. del.}} \Big)$$

$$T = \frac{1}{\gamma} . \sum_{ij} \left(\frac{\lambda_{ij}}{\mu_{ij} - \lambda_{ij}} + \lambda_{ij} . d_{ij} \right)$$

$\frac{1}{\mu_{ij}}$ = avg. transmis. time

$$T_{path} = \sum_{i,j \in path} \left(\frac{1}{\mu_{ij} - \lambda_{ij}} + d_{ij} \right)$$

Jackson Network



$\lambda = r + \lambda . p$ $\quad \mu >> \lambda$

$\lambda = \frac{r}{1-p}$

Directed network

$$A_{ij} = \begin{cases} 1, & \text{if there is edge from } j \text{ to } i \\ 0, & \text{otherwise} \end{cases}$$

Bi-partite Networks

$$B_{ij} = \begin{cases} 1, & \text{if vertex } j \text{ belongs to group } i \\ 0, & \end{cases}$$

$P = B^T . B$ → diagonal element

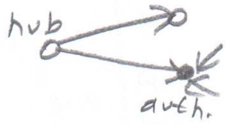
$P' = B . B^T$

Pagerank

$$x_i = \alpha . \sum_j A_{ij} . \frac{x_j}{k_j^{out}} + \beta$$

α ↓ const.

β ↓ free part const.



Closeness centrality

$$l_i = \frac{1}{n} \sum_j d_{ij} \qquad l_i = \frac{1}{n-1} \sum_{j \neq i} d_{ij}$$

↑ mean geodesic dis.

$$\to C_i = \frac{1}{l_i} = \frac{n-1}{\sum d_{ij}}$$

Clustering coefficient

$$C = \frac{\#\text{of closed paths of length } 2}{\#\text{of paths of length } 2} = \frac{(\#\text{of triangles}) \times 6}{\#\text{of paths of length } 2}$$

R_i of vertex

$$R_i = \frac{1}{4}(0+1+2+1)$$

↓ Redundancy

local clustering

$$C_i = \frac{R_i}{k_i - 1}$$

↑ degre of vertex i

clusterin coef. for netw:

$$\frac{1}{n} . \sum_{i=1}^{n} C_i$$

Random graphs

$n \to$ # of vertices

$m \to$ # of edges

$\langle k \rangle = (n-1) . p = c$

avg. deg

$\langle m \rangle = \binom{n}{2} . p$

$P(k) = \binom{n-1}{k} . p^k (1-p)^{n-1-k}$

Fraction of vertices in giant:

$S = 1 - e^{-cS}$

Confidence Interval

Mean:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} x_i$$

$$s^2 = \frac{1}{n-1} \sum_{1}^{n} (x_i - \bar{x})^2$$

↓ varience

$$\bar{x} \mp \frac{\sigma}{\sqrt{n}} . V_{\alpha/2}$$

Diameter:

$$\frac{\ln n}{\ln c}$$

Clustering coef:

$$C = \frac{c}{n-1}$$

π_s - prop. of random chosen vertex belongs to a compn. size s.

$$\pi_s = \frac{s . n_s}{n}$$

$n_s \to$ # of compn. of size s