1. i) $x(t) = e^{\cos(t/8)} = x(t+T) = e^{\cos((t+T)/8)}$

by taking ln of both sides

$$\cos(t/8) = \cos((t+T)/8)$$

$$\frac{T}{8} = 2\pi k \qquad T = 16\pi k$$

$$T_0 = 16\pi$$

ii) $x[n] = e^{j\frac{2\pi}{7}n} + e^{j\frac{1}{4}n}$

$$e^{j\frac{2\pi}{7}n} = e^{j\frac{2\pi}{7}(n+N)}$$

$$e^{j\frac{2\pi}{7}N} = e^{j2\pi r}$$

$$\frac{N}{7} = r$$

$$N = 7, \; r = 1.$$

$$e^{j\frac{1}{4}n} = e^{j\frac{1}{4}(n+N)}$$

$$e^{j2\pi r} = e^{j\frac{N}{4}}$$

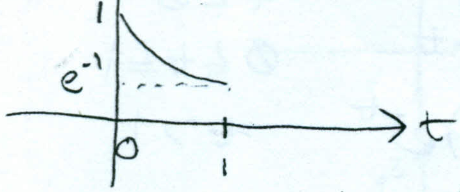
$$\frac{r}{N} = \frac{1}{8\pi}$$

no solution for $r \in \mathbb{Z}$ and $N \in \mathbb{Z}$.

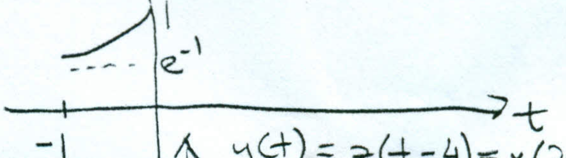
Since $e^{j\frac{1}{4}n}$ is not periodic $x[n]$ is not periodic.

2. $x(t) = e^{-t}(u(t) - u(t-1))$

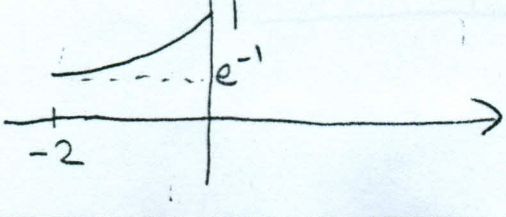
i)



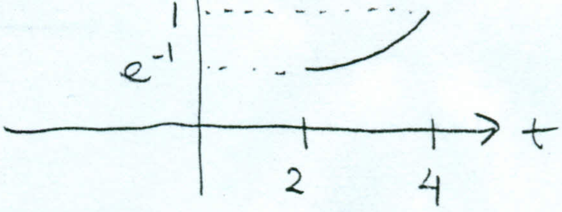
ii) $w(t) = x(-t)$



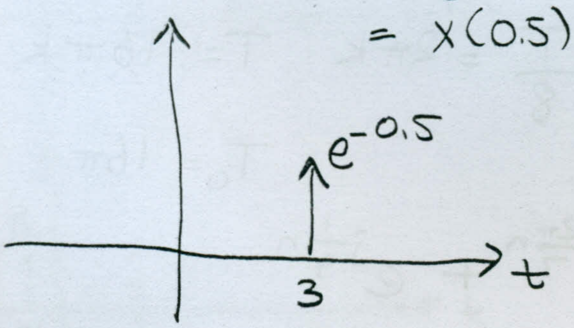
$z(t) = w\left(\frac{t}{2}\right) = x\left(-\frac{t}{2}\right)$



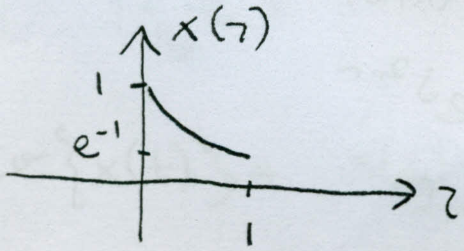
$y(t) = z(t-4) = x\left(2 - \frac{t}{2}\right)$

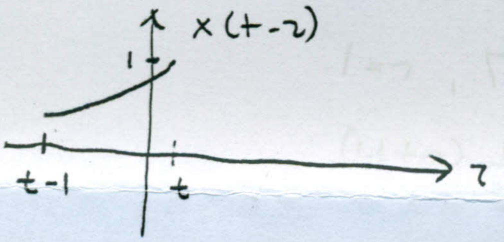


$$y(t)\left[\delta(t)+\delta(t-3)\right] = y(\emptyset)\delta(t) + y(3)\delta(t-3)$$
$$= y(3)\delta(t-3)$$
$$= x(0.5)\delta(t-3) = e^{-0.5}\delta(t-3)$$



iii)





$x(\tau)x(t-\tau) = \emptyset$ for $t < \emptyset$

$x(\tau)x(t-\tau) = e^{-\tau}e^{-(t-\tau)}$ for $0 \le \tau \le t,\ 0 \le t \le 1$

$x(\tau)x(t-\tau) = e^{-\tau}e^{-(t-\tau)}$ for $t-1 \le \tau \le 1,\ t > 1$

$$x(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)\,d\tau$$

$$= \begin{cases} \emptyset & t < \emptyset \\ \int_0^t e^{-t}\,d\tau & \emptyset \le t \le 1 \quad e^{-\tau} \cdot e^{-(t-\tau)} \\ \int_{t-1}^{1} e^{-t}\,d\tau & t > 1 \end{cases}$$

$$= \begin{cases} \emptyset & t < \emptyset \\ t\,e^{-t} & \emptyset \le t \le 1 \\ (2-t)e^{-t} & t > 1 \end{cases}$$

3 i)

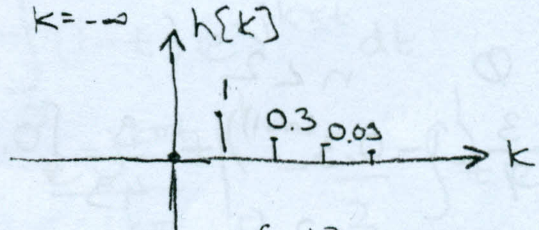
$$y[n] = h[n] * x[n]$$
$$= h[n] * (0.5\delta[n+1] + \delta[n] + 0.5\delta[n-1])$$
$$= 0.5h[n+1] + h[n] + 0.5h[n-1]$$
$$= 0.5(0.3^{(n+1)}u[n] + 0.3^{(n-1)}u[n-2])$$
$$+ 0.3^{n}u[n-1]$$

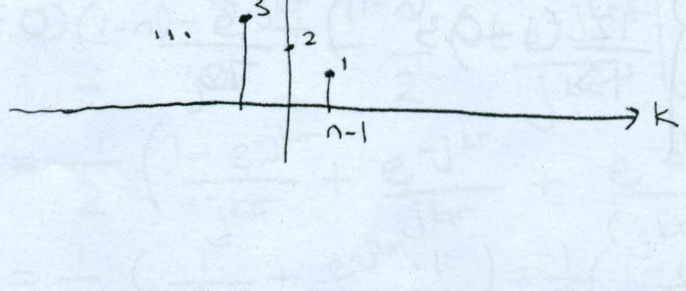
ii)

$$y[n] = h[n] * x[n]$$
$$= \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$
$$= \sum_{k=-\infty}^{\infty} h[k](0.5)^{n-k}$$
$$= (0.5)^n \sum_{k=-\infty}^{\infty} (0.3)^k u[k-1](0.5)^{-k}$$
$$= (0.5)^n \sum_{k=1}^{\infty} \left(\frac{0.3}{0.5}\right)^k \qquad r = k-1$$
$$= (0.5)^n \sum_{r=0}^{\infty} (0.6)^{r+1}$$
$$= (0.5)^n \frac{0.6}{1-0.6} = (1.5)(0.5)^n$$

iii)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$





$$h[k]x[n-k] = \emptyset \qquad n < 2$$

$$h[k]x[n-k] = (n-k)(0.3)^k \qquad 2 \le n,\ 1 \le k \le n-1$$

$$y[n] = \begin{cases} \emptyset & n < 2 \\ \displaystyle\sum_{k=1}^{n-1} (n-k)(0.3)^k & n \geq 2. \end{cases}$$

$$y[n] = \begin{cases} \emptyset & n < 2 \\ n \displaystyle\sum_{r=\emptyset}^{n-2} (0.3)^{r+1} - \sum_{r=0}^{n-2} (r+1)(0.3)^{r+1} & n \geq 2 \end{cases}$$

$$= \begin{cases} \emptyset \\ (n-1)(0.3)\dfrac{1-0.3^{(n-1)}}{(1-0.3)} - (0.3)^2 \dfrac{d}{da}\Big(\underbrace{\sum_{r=0}^{n-2} a^r}_{\frac{1-a^{n-1}}{1-a}}\Big)\Big|_{a=0.3} \end{cases}$$

$$= \begin{cases} \emptyset \\ \dfrac{3}{7}\left(1-0.3^{(n-1)}\right) - (0.09)\dfrac{\left[(n-1)a^{n-2}(1-a) + (1-a^{n-1})\right]}{(1-a)^2} \end{cases}$$

$$= \begin{cases} \emptyset & n < 2 \\ \dfrac{3}{7}\left(1-0.3^{(n-1)}\right) + \dfrac{9}{49}\left[0.7(n-1)(0.3)^{n-2} - (1-0.3^{(n-1)})\right] & n \geq 2 \end{cases}$$

$$= \begin{cases} \emptyset & n < 2 \\ \dfrac{12}{49}\left(1-0.3^{(n-1)}\right) + \dfrac{9}{70}(n-1)(0.3)^{n-2} & n \geq 2. \end{cases}$$

4

$$y(t) = \int_{-\infty}^{\infty} \underbrace{e^{-\beta\tau} u(\tau)}_{h(\tau)} \underbrace{e^{-\alpha(t-\tau)} u(t-\tau)}_{x(t-\tau)} d\tau$$

$$= \int_{0}^{t} e^{-\beta\tau} \cdot e^{+\alpha\tau} \cdot e^{-\alpha t} d\tau$$

a) $\alpha = \beta$ then

$$y(t) = e^{-\alpha t} \int_0^t 1 \, d\tau$$

$$= t e^{-\alpha t} = t e^{-\beta t} u(t) \text{ as } \alpha = \beta$$

b) $\alpha \neq \beta$ then

$$= e^{-\alpha t} \int_0^t e^{(\alpha-\beta)\tau} d\tau$$

$$= \frac{e^{-\alpha t}}{\alpha - \beta} \left\{ e^{t(\alpha-\beta)} - 1 \right\} = \frac{e^{-\beta t} - e^{-\alpha t}}{\alpha - \beta} u(t)$$

c) Causal as $h(t) = 0 \quad \forall t < 0$

d) $$\int_0^{\infty} |e^{-\beta t}| \, dt = \frac{-1}{\beta} e^{-\beta t} \Big|_0^{\infty} = 1/\beta < \infty$$

BIBO stable if $\beta > 0$

4. i). While $x[n] = \emptyset$ for $n < \emptyset$

$y[-1] \neq \emptyset$

Therefore the system is not causal.

ii) Even though both $x[n]$ & $y[n]$ are bounded we have to generalize this result to all inputs Therefore we need to determine $h[n]$

$y[n] = h[n]$ when $x[n] = \delta[n]$

We know that $y[n] = u[n] + u[n-5] + 0.5u[n-6]$ when $x[n] = u[n-1]$ due to time invariance

Due to linearity $y[n] = u[n+1] + u[n-4] + 0.5\,u[n-5]$
$- u[n] - u[n-5] - 0.5u[n-6]$

when $x[n] = u[n] - u[n-1] = \delta[n]$

$$\delta[n] \longrightarrow \boxed{\quad} \longrightarrow h[n] = \delta[n+1] + \delta[n-4] + 0.5\delta[n-5]$$

Since $\sum_{n=-\infty}^{\infty} |h[n]| = 2.5 < \infty$

The system is BIBO stable.

5. i)

$$a_k = \frac{1}{2}\int_0^1 (1-t)\,e^{-jk\pi t}\,dt$$

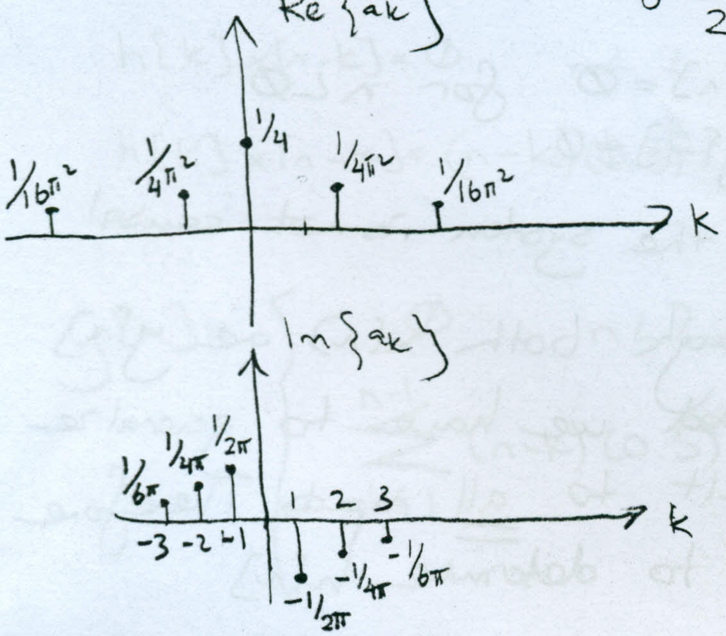
$$= \frac{1}{2}\,\frac{e^{-jk\pi t}}{-jk\pi}\Big|_0^1 - \frac{1}{2}\int_0^1 t\,e^{-jk\pi t}\,dt \qquad \begin{array}{l} u = t \quad du = dt \\ dv = e^{-jk\pi t}dt \\ v = -\dfrac{e^{-jk\pi t}}{jk\pi} \end{array}$$

$$= \frac{1}{2}\,\frac{1 - e^{-jk\pi}}{jk\pi} - \frac{1}{2}\left(-t\,\frac{e^{-jk\pi t}}{jk\pi}\Big|_0^1 + \int_0^1 \frac{e^{-jk\pi t}}{jk\pi}dt\right)$$

$$= \frac{1}{2}\left(\frac{1 - e^{-jk\pi}}{jk\pi} + \frac{e^{-jk\pi}}{jk\pi} + \frac{e^{-jk\pi t}}{(jk\pi)^2}\Big|_0^1\right)$$

$$= \frac{1}{2}\left(\frac{1}{jk\pi} + \frac{e^{-jk\pi} - 1}{(jk\pi)^2}\right) = \frac{1}{2}\left(\frac{1-(-1)^k}{k^2\pi^2} - j\,\frac{1}{k\pi}\right)$$

$$a_0 = \frac{1}{2}\int_0^1 (1-t)\,dt = \frac{1}{4}$$



ii) $$\mathcal{E}v\{x(t)\} = \frac{x(t)+x(-t)}{2} \longleftrightarrow \frac{a_k + a_k^*}{2} = Re\{a_k$$

$$= \frac{1}{2}\left(\frac{1-(-1)^k}{k^2\pi^2}\right.$$