BLG 336E – Analysis of Algorithms II

Practice Session 5

Atakan Aral

Istanbul Technical University – Department of Computer Engineering

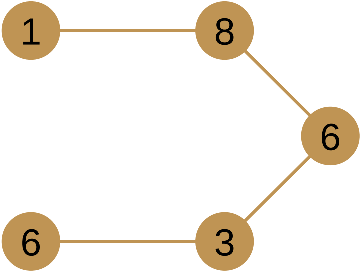
29.04.2014

Outline

- Find an independent set in a path G whose total weight is as large as possible.

Independent Set

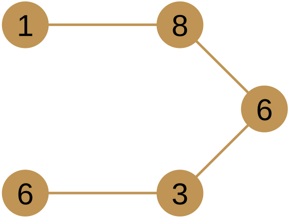
Subset of nodes such that no two of them are adjacent (connected by an edge).



- **Attempt 1:** Odds vs. evens

```
Let S_1 be the set of all v_i where i is an odd number
Let S_2 be the set of all v_i where i is an even number
(Note that S_1 and S_2 are both independent sets)
Determine which of S_1 or S_2 has greater total weight,
  and return this one
```

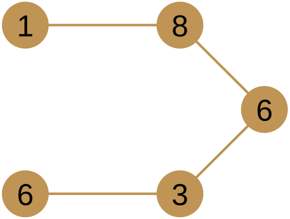
- $1 + 6 + 6$ vs. $8 + 3$
- Yields 13 which is sub-optimal.



- **Attempt 2:** Heaviest-first

```
Start with S equal to the empty set
While some node remains in G
  Pick a node v_i of maximum weight
  Add v_i to S
  Delete v_i and its neighbors from G
Endwhile
Return S
```

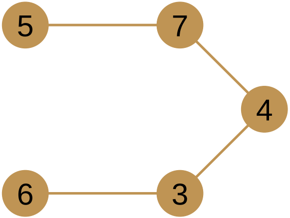
- Take 8, remove first 3 nodes then take 6.
- Yields 14 which is optimal.



- **Attempt 2:** Heaviest-first

```
Start with S equal to the empty set
While some node remains in G
  Pick a node v_i of maximum weight
  Add v_i to S
  Delete v_i and its neighbors from G
Endwhile
Return S
```

- What about now?
- Yields 13 which is sub-optimal.



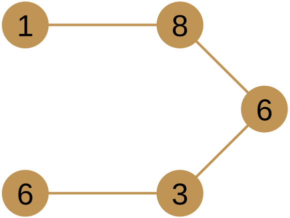
- **Attempt 3:** Dynamic Programming

- S_i: Independent set on $v_1, v_2, ...v_i$
- X_i: Weight of S_i

- $X_0 = 0$
- $X_1 = w_1$
- $i > 1$:
  1. v_i does not belong to $S_i \rightarrow X_i = X_{i-1}$
  2. v_i belongs to $S_i \rightarrow X_i = w_i + X_{i-2}$

- **So:** $X_i = max(X_{i-1}, w_i + X_{i-2})$

- **Attempt 3:** Dynamic Programming

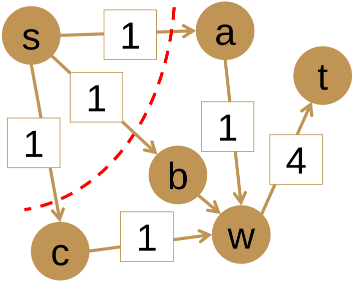


- $X_0 = 0 \; S = ()$
- $X_1 = 1 \; S = (v_1)$
- $X_2 = max(X_1, w_2 + X_0) = max(1, 8 + 0) = 8 \; S = (v_2)$
- $X_3 = max(X_2, w_3 + X_1) = max(8, 6 + 1) = 8 \; S = (v_2)$
- $X_4 = max(X_3, w_4 + X_2) = max(8, 3 + 8) = 11 \; S = (v_2, v_4)$
- $X_5 = max(X_4, w_5 + X_3) = max(11, 6 + 8) = 14 \; S = (v_2, v_5)$

- Let G be an arbitrary flow network, with a source s, a sink t, and a positive integer capacity c_e on every edge e;
- let (A, B) be a minimum $s - t$ cut with respect to these capacities c_e.
- Now suppose we add 1 to every capacity; then (A, B) is still a minimum $s - t$ cut with respect to these new capacities $c_e + 1$.

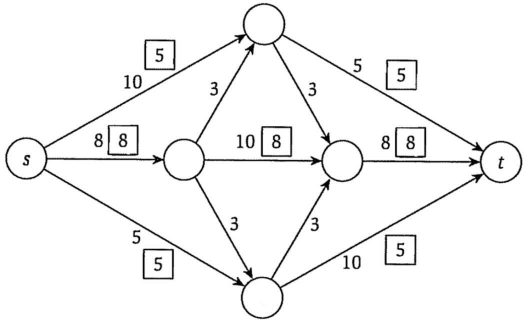
Solution

- False:



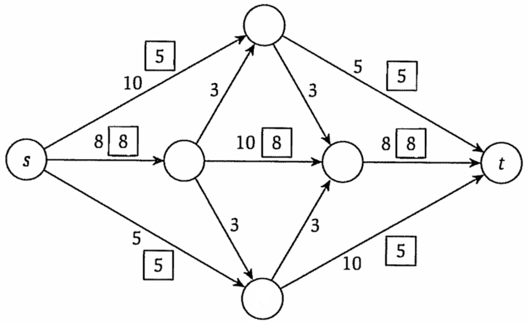
- Consider the flow network below.
- An $s - t$ flow is computed.

- What is the value of the flow?



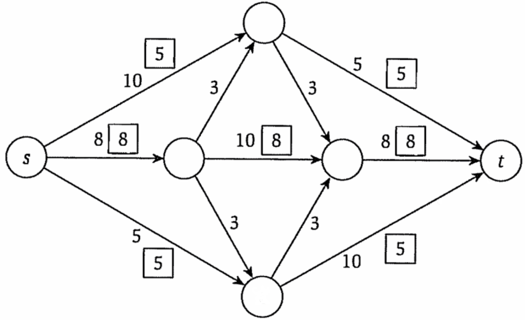
- Consider the flow network below.
- An $s - t$ flow is computed.

- Is this a maximum $s - t$ flow?

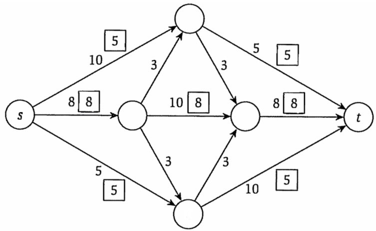


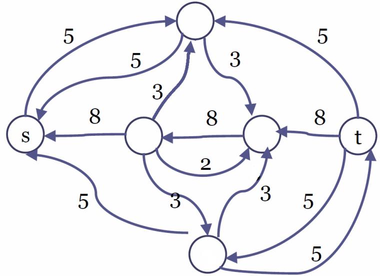
- Consider the flow network below.
- An $s - t$ flow is computed.

- Find the maximum $s - t$ flow.



- Value is 18.
- We need to draw the residual graph to see if it is the maximum flow.





- It is not the maximum flow.
- An augmenting path with a value of 3 can be found.
- Since no more augmenting paths are available the maximum flow is 18 + 3 = 21.