Inclusion-Exclusion Question

In the summer school of mathematical logic that lasted for 12 weeks, Sharon was participating with 7 of her friends. During the summer school Sharon had her lunch with each of her friends 35 times. She had lunch with each couple 16 times, 8 times with each group of three, 4 times with each group of four, twice with each group of five and once with six of her friends. She never had lunch with 7 of her friends altogether. Assuming that she had lunch every day, use inclusion-exclusion to find out if Sharon ever ate alone.

Answer

Let's think that we have a different set for each of his friends. We are trying to find the number of elements in a union of seven-sets.
"*had lunch with each of her friends 35 times*" : $|F_1| = |F_2| = ... = |F_7| = 35$
So $|F_1 \cup F_2 \cup ... \cup F_7| = 7 \times 35 = 245$. However we count F_n twice whenever she had lunch with couples.
"*had lunch with each couple 16 times*" : $|F_i \cap F_j| = 16$ and we have $\binom{7}{2} = 21$ different couples which gives us $16 \times 21 = 336$ lunches. But again we have excluded groups of three this time which we shouldn't.
Let's continue like this and perform inclusion-exclusion:
$|F_i \cap F_j \cap F_k| = 8$ and we have $\binom{7}{3} = 35$ different groups and $8 \times 35 = 280$ lunches
$|F_i \cap F_j \cap F_k \cap F_l| = 4$ and we have $\binom{7}{4} = 35$ different groups and $4 \times 35 = 140$ lunches
$|F_i \cap F_j \cap F_k \cap F_l \cap F_m| = 2$ and we have $\binom{7}{5} = 21$ different groups and $2 \times 21 = 42$ lunches
$|F_i \cap F_j \cap F_k \cap F_l \cap F_m \cap F_n| = 1$ and we have $\binom{7}{6} = 7$ different groups and $1 \times 7 = 7$ lunches
When we put all of those together we can find the total number of lunches she had with her friends as $245 - 336 + 280 - 140 + 42 - 7 = 567 - 483 = 84$. In 12 weeks she had a total number of 84 days to lunch. She never had lunch alone!