25.04.2013

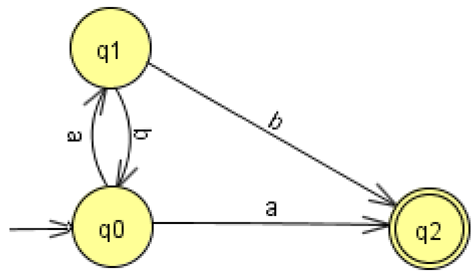
BLG311E – FORMAL LANGUAGES AND AUTOMATA

2013 SPRING

RECITEMENT 6

1) For the given automata,

a) Heuristically, find its regular expression
b) Find determinist equivalent
c) Systematically produce the regular expression for the language defined by the DFA you have designed. Show that it is equivalent to the expression you define in a).



2)

$$L(M) = \{a^n b^m, \qquad 0 \leq n \leq m \leq 2n\}$$

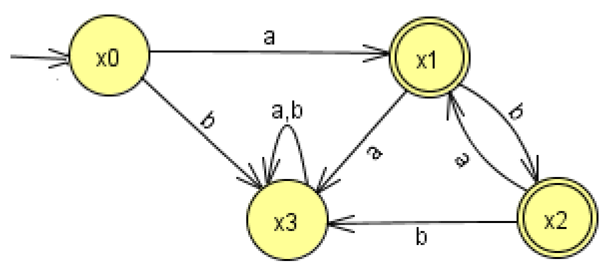
a) Construct DFA accepting this language
b) Choose an example string and show that it is accepted by the PDA you designed.

Solutions:

1)

a) L(M) = (ab)*a v (ab)*ab = (ab)*(a v ab)

b) $q_0 = x_0$
$\delta(x_0,a) = \delta(q_0,a) = \{ q_1, q_2\} = x_1$
$\delta(x_0,b) = \delta(q_0,b) = \emptyset$
$\delta(x_1,a) = \delta(\{ q_1, q_2\},a) = \emptyset$
$\delta(x_1,b) = \delta(\{ q_1, q_2\},b) = \{ q_0, q_2\} = x_2$
$\delta(x_2,a) = \delta(\{ q_0, q_2\},a) = \{ q_1, q_2\} = x_1$
$\delta(x_2,b) = \delta(\{ q_0, q_2\},b) = \emptyset$
$\delta(\emptyset,a) = \delta(\emptyset,b) = \emptyset = x_3$



c)

<u>**Teorem:**</u> $x = xa \, v \, b \, \wedge \, \Lambda \notin A \Rightarrow x = ba^*$

$x_1 \, v \, x_2 = ?$

Place x_2 in the expression of x_1:
$x_1 = x_0a \, v \, x_2a = x_0a \, v \, x_1ba$

$x_0 = \Lambda$

$x_1 = x_0a \vee x_2a$
$x_2 = x_1b$
$x_3 \rightarrow$ kuyu

Place x_0 and according to the Theroem defined above:
$x_1 = x_0a \vee x_1ba = a \vee x_1ba = a(ba)^*$

Place x_1 in the expression of x_2:
$x_2 = x_1b = a(ba)^*b$

$L(M)= x_1 \vee x_2 = a(ba)^* \vee a(ba)^*b = a(ba)^* (\Lambda \vee b)$

Language defined in a was: (ab)*(a v ab)

$L(M) = (ab)^*(a \vee ab) = (ab)^*a (\Lambda \vee b)$

$(ab)^* a \stackrel{?}{=} a(ba)^* \rightarrow$ can be proved by induction

Induction: $(ab)^n a \stackrel{?}{=} a(ba)^n$

n=0 : $a = a$ ✓

n=k : $(ab)^k a = a(ba)^k$ Assume True

n=k+1 : $(ab)^{k+1}a \stackrel{?}{=} a(ba)^{k+1}$

$(ab)^k aba \stackrel{?}{=} a(ba)^k ba$

It was assumed that: $(ab)^k a = a(ba)^k$. Then,

$[(ab)^k a]ba \stackrel{?}{=} [a(ba)^k]ba \rightarrow ba = ba$ ✓

2)

a) Production rules:

S → aSB | Λ
B → bb | b

PDA definition

$M = (S, \sum, \Gamma, \Delta, s, F)$
$S = \{ q0, q1, q2, q3 \}$, $\sum = \{a, b\}$, $\Gamma = \{a\}$, $F = \{q2, q3\}$
Δ = { [(q0, Λ, Λ), (q3, Λ)],
[(q0, a, Λ), (q1, a)],
[(q0, a, Λ), (q0, a)],
[(q0, b, a), (q2, Λ)],
[(q1, Λ, Λ), (q0, a)],
[(q2, b, a), (q2, Λ)]
}

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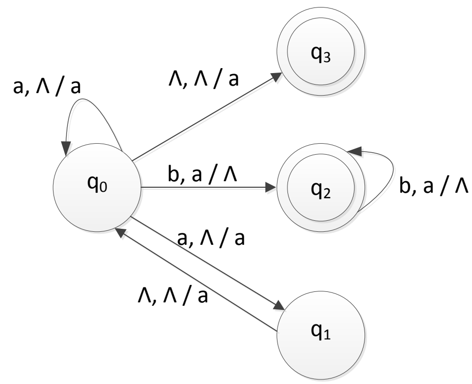

b) Example String: aabbbb

State	Tape	Stack	Transition Rule
q0	aabbbb	Λ	(q0, a, Λ), (q1, a)
q1	abbbb	a	(q1, Λ, Λ), (q0, a)
q0	abbbb	aa	(q0, a, Λ), (q1, a)
q1	bbbb	aaa	(q1, Λ, Λ), (q0, a)
q0	bbbb	aaaa	(q0, b, a), (q2, Λ)
q2	bbb	aaa	(q2, b, a), (q2, Λ)
q2	bb	aa	(q2, b, a), (q2, Λ)
q2	b	a	(q2, b, a), (q2, Λ)
q2	Λ	Λ	(q2, b, a), (q2, Λ)