IMPORTANT: Besides your calculator and the sheets you use for calculations you are only allowed to have an A4 sized “copy sheet” during this exam. Notes, problems and alike are not permitted. Please submit your “copy sheet” along with your solutions. You may get your “copy sheet” back after your solutions have been graded. ***Do not forget to write down units and convert units carefully! Cell phones are not allowed and should be placed on the front desk before the exam.***

ELE222E INTRODUCTION TO ELECTRONICS (11245)
Final Exam ✎ 20 January 2012 ⌛ 12.00-14.00
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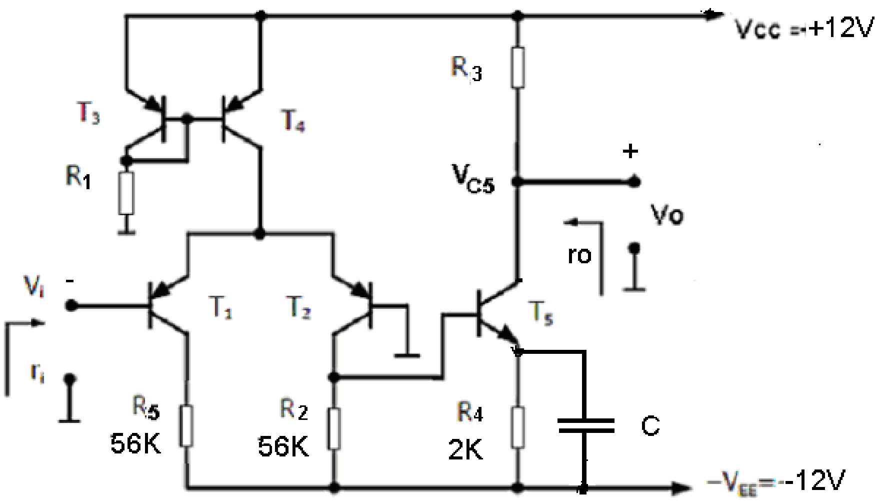
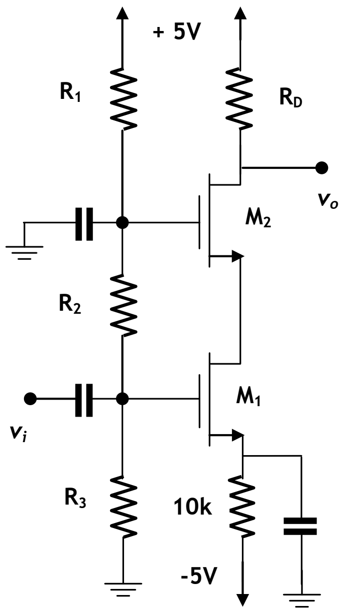
1. For the circuit shown below $h_{FE} = h_{fe} = \beta_F = 200$, $|V_{BE}| = 0{,}6V$, $V_T = 25mV$, and $V_{CEsat} \approx 0V$. **ASSUME** ideal capacitor and $V_A = 150$ V for T_3 & T_4, and $V_A = \infty$ for T_1, T_2 and T_5.



   a. Required are $r_o = 12k$ and $V_{C5} = 0$ V for $V_i = 0$. Calculate R_1 and R_3. (15 points) **ASSUME** $V_A=\infty$ for T_3 & T_4.
   b. Find r_i, $A_v = \dfrac{v_o}{v_i}$ and CMRR. (15 points)
   c. For what range of a sinusoidal input voltage will the output signal be clipped? (5 points)
   d. Draw comparatively the input and output voltages for a sinusoidal input voltage with 5 mV amplitude. (5 points)

2. Analyze the MOS circuit shown on the right:



   a. Design the biasing and find the unknown resistor values for both MOS operate in saturation (for example, assume $V_{DS} = V_{GS}$ for both MOS) and for the following parameters

$$R_1 + R_2 + R_3 = 100k\,,\; V_{tn1} = V_{tn2} = V_{tn} = 1{,}2V\,,\; V_{A1/2} \to \infty\,,$$

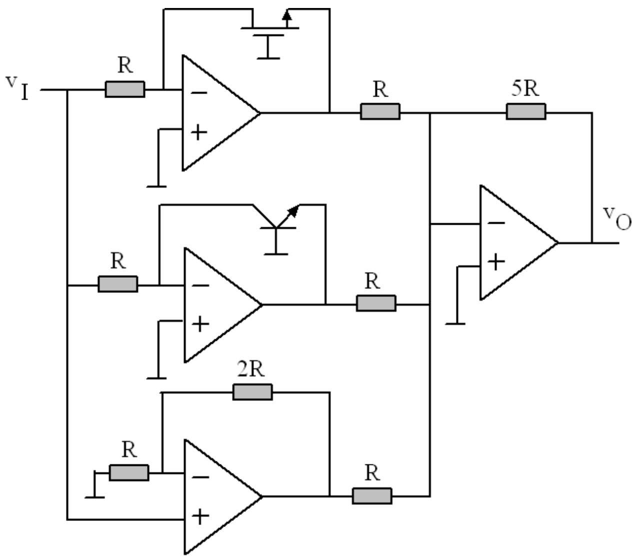
$$K_{n1} = K_{n2} = K_n = \beta = \frac{1}{2}\mu_n C_{ox}\frac{W}{L} = 800\mu A/V^2\text{, and } I_{DQ} = 0{,}4mA.$$

   ASSUME all capacitors are ideal. (20 points)

   b. Determine the small signal voltage gain

$$A_v = \frac{v_o}{v_i}\text{ . (10 points)}$$

   HINT: You need to draw the small signal circuit.



3. Find the formula that relates the output v_o of the OPAMP circuit to the input v_i for $v_i > 0$. Parameters are: NMOS: $\beta = 400\ \mu A/V^2$, $V_{th} = 1V$; BJT: $I_s = 10nA$, $V_T = 26$ mV. (20 points)

   a. Why does this circuit not work for $v_i < 0$? (5 points)

   b. Why does the MOS work in saturation? (5 points)

GOOD LUCK!

SOLUTIONS:

BJT PROBLEM:

$V_{A5}=\infty \Rightarrow r_{o5}=\infty \Rightarrow r_o = R_3 = \underline{\underline{12k}}$ 2 POINTS

$$I_{C5}=\frac{V_{CC}-V_{C5}}{R_3}=\frac{12V-0V}{12k}=\underline{\underline{1mA}}$$

$$-\left[V_{C2}-\left(-V_{EE}\right)\right]+V_{BE5}+I_{E5}R_4=-R_2\left(I_{C2}-I_{B5}\right)+V_{BE5}+I_{E5}R_4=0\Rightarrow$$

$$I_{C2}=\frac{V_{C2}-\left(-V_{EE}\right)}{R_2}+I_{B5}=\frac{I_{E5}R_4+V_{BE5}}{R_2}+\frac{I_{C5}}{\beta_F}=\frac{I_{C5}\left(\frac{\beta_F}{\beta_F+1}\right)R_4+V_{BE5}}{R_2}+\frac{I_{C5}}{\beta_F}=\underline{\underline{51{,}6\mu A}}$$ 2+2+2 POINTS

$I_{C4}=I_{E1}+I_{E2}=2I_{C2}\dfrac{\beta_F+1}{\beta_F}=\underline{\underline{0{,}103mA}}$, 1 POINT since T_3 and T_4 are identical,

$I_{C4}=I_{R1}=0{,}103mA \Rightarrow I_{C4}=\dfrac{V_{CC}-V_{EB3}}{R_1}\Rightarrow R_1=\dfrac{V_{CC}-V_{EB3}}{I_{C4}}=\dfrac{12V-0{,}6V}{0{,}103mA}=\underline{\underline{110k}}$ 2 POINTS

Also since $V_{A4} \neq 0$ the current mirror has a finite output resistance $r_{o4}=\dfrac{V_{A5}}{I_{C4}}=\dfrac{150V}{0{,}103mA}=\underline{\underline{1M446}}$ which appears between the emitters of the two BJT in the differential stage. 2 POINTS

$r_{e1}=r_{e2}=r_{e1/2}=\dfrac{V_T}{I_{C1/2}}=\underline{\underline{484{,}4\Omega}};r_{e5}=\dfrac{V_T}{I_{C5}}=\underline{\underline{25\Omega}}$ 2 POINTS

$r_i=2\beta_F r_{e1}=\underline{\underline{193k8}}$ 2 POINTS

$A_v=\dfrac{v_o}{v_i}=\dfrac{v_o}{v_{c2}}\dfrac{v_{c2}}{v_i}=\dfrac{-R_3}{r_{e5}}\cdot\dfrac{R_2\parallel r_{i5}}{2r_{e1/2}}=-480\cdot 4{,}74\cong\underline{\underline{-2274}}$ 3+3+1 POINTS

with $r_{i5}=\beta_F\left(r_{e5}+R_{e5}\right)=\beta_F r_{e5}=\underline{\underline{5k}}$ 2 POINTS

$$CMRR=20\log\left[\frac{2R_{e1/2}+r_{e1/2}}{r_{e1/2}}\right]=20\log\left[\frac{2r_{o4}+r_{e1/2}}{r_{e1/2}}\right]=20\log\left[\frac{2\cdot 1M446+484{,}4\Omega}{484{,}4\Omega}\right]\cong\underline{\underline{76dB}}$$ 4 POINTS

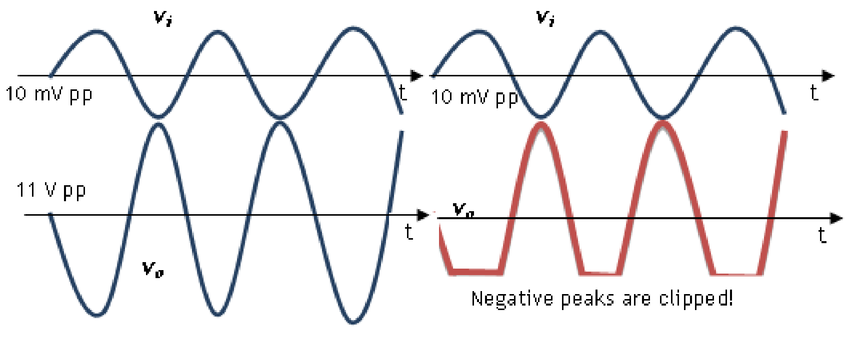
When T_5 is in saturation $V_{C5}=V_{CEsat}+V_{E5}=0+\left(-V_{EE}+I_{E5}R_4\right)=0+\left(-12+1mA\cdot 2k\right)=-10V$. When T_5 is in cut-off $I_{C5}=0V\Rightarrow V_{C5}=V_{CC}=12V$. That means the output swings between -10 and +12 V. 5 POINTS

When we apply an input voltage with 5 mV amplitude, we need to calculate the swing of the output:

$v_i=5mV\Rightarrow v_o=-11V$

$v_i=-5mV\Rightarrow v_o=+11V$

BUT we need to take the clipping into account. Before being clipped the signal will look like the two blue curves, yet after clipping, the output signal will be clipped by 1 V on the negative side as seen on the brown curve… 5 POINTS



MOS PROBLEM:

Since $I_{DQ1} = I_{DQ2} = I_{DQ} = K_n (V_{GS} - V_{tn})^2 \Rightarrow 0{,}4mA = 0{,}8(V_{GS} - 1{,}2V)^2 [mA]$, from

$V_{GS1} = V_{GS2} = V_{GS} = \pm\sqrt{\frac{0{,}4}{0{,}8}} + 1{,}2 = \begin{cases} -0{,}71 + 1{,}2V \\ +0{,}71 + 1{,}2V \end{cases}$, using the assumption suggested above, the correct solution is $V_{DS} = V_{GS} = \underline{\underline{1{,}91V}}$ 2 POINTS

$V_2 = R_2 I = V_{GS} = \underline{\underline{1{,}91V}}$ 2 POINTS

On the other hand $V_3 = R_3 I = -5V + V_{GS} + 10k \cdot I_{DQ} = -5V + 1{,}91V + 10k \cdot 0{,}4mA = \underline{\underline{0{,}91V}}$ 2 POINTS

Since $V_1 = R_1 I = I_{DQ} R_D = 0{,}4mA \cdot R_D$, $V_1 = 5V - V_2 - V_3 = 5 - 1{,}91 - 0{,}91 = \underline{\underline{2{,}18V}}$ 2 POINTS

Since $V_1 = R_1 I = I_{DQ} R_D = 0{,}4mA \cdot R_D = 2{,}18V$, $R_D = \frac{V_1}{I_{DQ}} = \frac{2{,}18V}{0{,}4mA} = \underline{\underline{5k45}}$ 2 POINTS

$V_1 + V_2 + V_3 = I[R_1 + R_2 + R_3] = 5V \Rightarrow I = \frac{5V}{100k} = \underline{\underline{50\mu A}} \Rightarrow R_3 = \frac{V_3}{I} = \frac{0{,}91V}{50\mu A} = \underline{\underline{18k2}}$

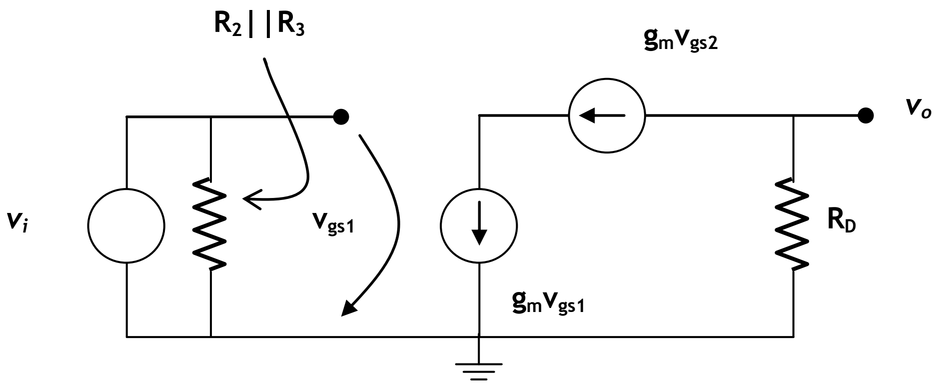
$V_2 = R_2 I = V_{GS} = 1{,}91V \Rightarrow R_2 = \frac{V_2}{I} = \frac{1{,}91V}{0{,}5\mu A} = \underline{\underline{38k2}}$ 8 POINTS

$V_1 = 2{,}18V \Rightarrow R_1 = \frac{V_1}{I} = \frac{2{,}18V}{0{,}5\mu A} = \underline{\underline{43k6}}$

Double check the values: $R_1 + R_2 + R_3 = 18k2 + 38k2 + 43k6 = 100k$ ✓

M_1 and M_2 are identical, $g_{m1} = g_{m2} = g_m = 2K_n (V_{GS} - V_{tn}) = 2 \cdot 0{,}8m \cdot (1{,}91 - 1{,}2) = \underline{\underline{1{,}13mA/V}}$. 2 POINTS

Now we have to draw the small signal circuit:

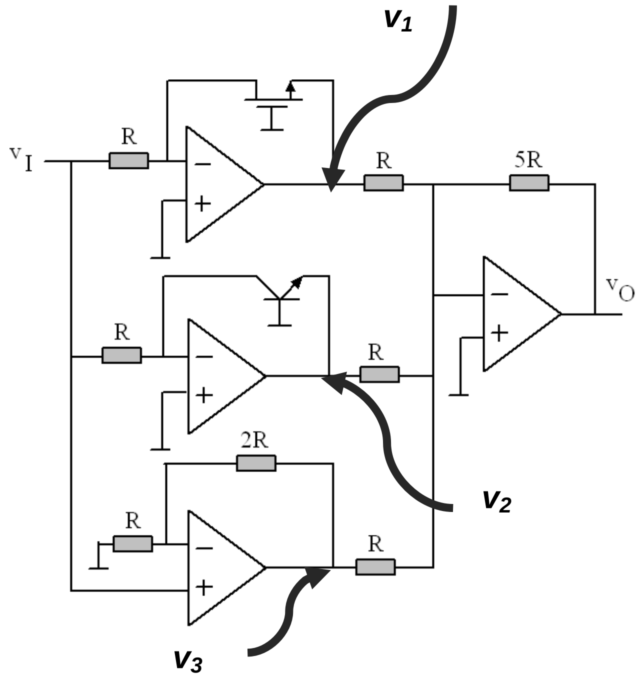


8 POINTS

Realize that $g_m v_{gs2} = g_m v_{gs1} \Rightarrow v_o = -g_m v_{gs1} R_D$

Also realize $v_i = v_{gs1} \Rightarrow A_v = \frac{v_o}{v_i} = -g_m R_D = -1{,}13m \cdot 5k45 = \underline{\underline{-6{,}15}}$ 2 POINTS

OPAMP PROBLEM:



Let's start with the last OPAMP (#4). Obviously this is an inverting amplifier as we have seen in class, thus:

$$v_o = -\frac{5R}{R}[v_1 + v_2 + v_3] = -5[v_1 + v_2 + v_3]$$

To find **v_1**, **v_2** and **v_3**, it will be easier to start with the lowermost OPAMP (#3). This is another an non-inverting amplifier:

$$v_3 = \left(\frac{2R}{R} + 1\right)v_i = 3v_i$$ 5 POINTS

For the middle OPAMP (#2): 5 POINTS

$$i_c = \frac{v_i}{R} = I_s \exp\left[\frac{V_{BE}}{V_T}\right] = 10^{-8} e^{V_{BE}/V_T} = 10^{-8} \exp\left[\frac{-V_2}{0{,}026}\right]$$

$$\Leftrightarrow \ln\left[\frac{v_i}{10^{-8} R}\right] = \frac{-V_2}{0{,}026}$$

$$\Rightarrow V_2 = -0{,}026 \ln\left[\frac{v_i}{10^{-8} R}\right] = 0{,}026 \ln\left[\frac{10^{-8} R}{v_i}\right]$$

Finally the uppermost OPAMP (#1):

$$I_D = \frac{V_i}{R} = K_n (V_{GS} - V_{tn})^2 = \beta(-V_1 - 1)^2 \Leftrightarrow \frac{V_i}{400\mu R} = (-V_1 - 1V)^2$$

The meaningful solution for the NMOS will be $V_1 = -V_{GS} = -\sqrt{\frac{V_i}{400\mu R}} - 1V$, thus $V_{GS} > 1V$ 5 POINTS

$$v_o = -5[v_1 + v_2 + v_3] = -5\left\{-\sqrt{\frac{v_i}{400\mu R}} - 1V - V_T \ln\left[\frac{v_i}{10^{-8} R}\right] + 3v_i\right\} = 5\sqrt{\frac{v_i}{400\mu R}} + 5V_T \ln\left[\frac{v_i}{10^{-8} R}\right] - 15v_i + 5V$$

5 POINTS

a) For $v_i < 0$ neither the BJT nor the MOS will operate, because they both will be in cut-off. Let's see their situation in detail: 5 POINTS

- **BJT**: For $v_i < 0$ the CB junction of the npn transistor will be forward biased and the BE junction will be reverse biased, because the output of OPAMP #1, that now operates as a comparator, will be forced to +V_{CC}. There is NO way for the BJT to operate. It will be in cut-off.
- **MOS**: The output of OPAMP #2, that now operates as a comparator, will be forced to +V_{CC}. Therefore, $V_{GS} < 0$ and the MOS is in cut-off.

b) Since the drain of the MOS is connected to the non-inverting input of OPAMP #1, and the gate of the MOS is also connected to ground, $V_D = V_G \Leftrightarrow V_{DS} = V_{GS}$, i.e., the MOS is in saturation region. 5 POINTS