BLG311E – FORMAL LANGUAGES AND AUTOMATA

2013 SPRING

RECITEMENT 7

(Solutions for Midterm 2)

1) An automaton has given by the state transition matrix as below:

	5	10
Q_0	Q_5/0,-	Q_{10}/0,-
Q_5	Q_{10}/0,-	Q_0/0,S
Q_{10}	Q_0/0,S	Q_0/5,S

This matrix belongs to the chewing-gum automaton given as example during the courses. It only accepts 5 and 10 cents and gives 1 chewing-gum for 15 cents and returns the change.

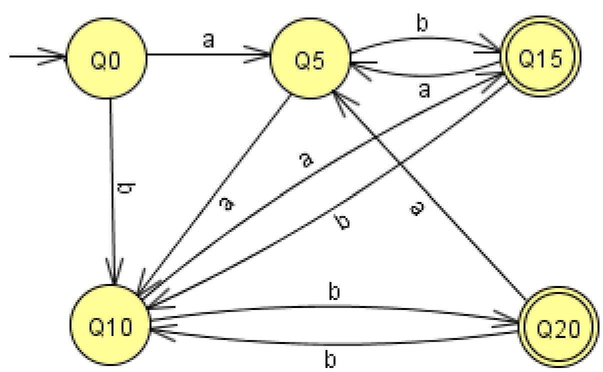
a) Let's model 5 cents as 'a' and 10 cents as 'b' as elements of the input alphabet. Draw the respective state transition chart by indicating accepted state (or states).

b) By applying the systematic methodology, obtain the regular expression of the automaton.

Solution:

a) First, we need to convert the given table into the Moore model.

	5 ('a')	**10 ('b')**	**Output**
$\mathbf{Q_0}$	Q_5	Q_{10}	0,-
$\mathbf{Q_5}$	Q_{10}	Q_{15}	0,-
$\mathbf{Q_{10}}$	Q_{15}	Q_{20}	0,-
$\mathbf{Q_{15}}$	Q_5	Q_{10}	0,S
$\mathbf{Q_{20}}$	Q_5	Q_{10}	5,S



b) <u>**Theorem:**</u> $\boldsymbol{x = xa \vee b \ \wedge \ \Lambda \notin A \ \Rightarrow x = ba^*}$

$L(M) = Q_{15} \vee Q_{20}$

$Q_0 = \Lambda$

$Q_5 = Q_0 a \vee Q_{15} a \vee Q_{20} a$

$Q_{10} = Q_0 b \vee Q_5 a \vee Q_{15} b \vee Q_{20} b$

$Q_{15} = Q_5 b \vee Q_{10} a$

$Q_{20} = Q_{10} b$

Place Q_0 in the expression of Q_5: $Q_5 = a \vee Q_{15} a \vee Q_{20} a$

Place Q_0 in the expression of Q_{10}: $Q_{10} = b \vee Q_5 a \vee Q_{15} b \vee Q_{20} b$

Place Q_5 in the expression of Q_{10}: $Q_{10} = b \vee (a \vee Q_{15} a \vee Q_{20} a) a \vee Q_{15} b \vee Q_{20} b$

Place Q_5 and Q_{10} in the expression of Q_{15}:

$$Q_{15} = (a \vee Q_{15}a \vee Q_{20}a)b \vee [b \vee (a \vee Q_{15}a \vee Q_{20}a)a \vee Q_{15}b \vee Q_{20}b]a$$
$$= (Q_{15} \vee Q_{20})(ab \vee aaa \vee ba) \vee ab \vee aaa \vee ba$$

Place Q_{10} in the expression of Q_{20}:

$$Q_{20} = [b \vee (a \vee Q_{15}a \vee Q_{20}a)a \vee Q_{15}b \vee Q_{20}b]b$$
$$= (Q_{15} \vee Q_{20})(aab \vee bb) \vee (aab \vee bb)$$

Place Q_{15} and Q_{20} in the expression of $L(M)$:

$$L(M) = Q_{15} \vee Q_{20} = (Q_{15} \vee Q_{20})(ab \vee aaa \vee ba \vee aab \vee bb) \vee (ab \vee aaa \vee ba \vee aab \vee bb)$$

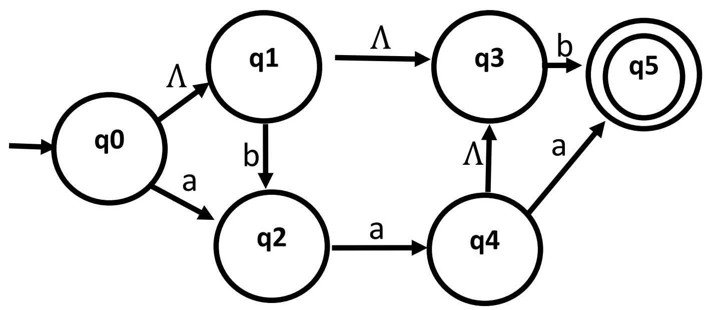
Using the stated theorem: $Q_{15} \vee Q_{20} = (ab \vee aaa \vee ba \vee aab \vee bb)(ab \vee aaa \vee ba \vee aab \vee bb)^*$

$$= (ab \vee aaa \vee ba \vee aab \vee bb)^+ \rightarrow L(M)$$

2) a) Transform from NFA to DFA the automaton given below.

b) Obtain the regular expression that is accepted.

c) Give the grammatical rules by using BNF notation.



Solution:

a) $R(q_0) = \{q_0, q_1, q_3\}$

$R(q_1) = \{q_1, q_3\}$

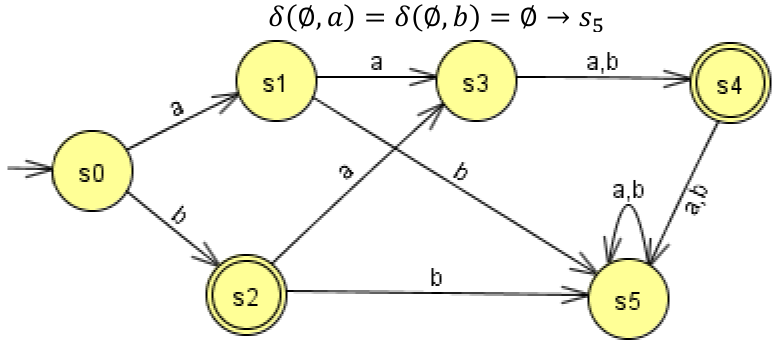
$R(q_2) = \{q_2\}$

$R(q_3) = \{q_3\}$

$R(q_4) = \{q_3, q_4\}$

$R(q_5) = \{q_5\}$

$s_0 = R(q_0) = \{q_0, q_1, q_3\}$
$\delta(s_0, a) = \delta(\{q_0, q_1, q_3\}, a) = \{R(q_2)\} = \{q_2\} \rightarrow s_1$
$\delta(s_0, b) = \delta(\{q_0, q_1, q_3\}, b) = \{R(q_2), R(q_5)\} = \{q_2, q_5\} \rightarrow \boldsymbol{s_2}$
$\delta(s_1, a) = \delta(q_2, a) = \{R(q_4)\} = \{q_3, q_4\} \rightarrow s_3$
$\delta(s_1, b) = \delta(q_2, b) = \emptyset$
$\delta(s_2, a) = \delta(\{q_2, q_5\}, a) = \{R(q_4)\} = \{q_3, q_4\} \rightarrow s_3$
$\delta(s_2, b) = \delta(\{q_2, q_5\}, b) = \emptyset$
$\delta(s_3, a) = \delta(\{q_3, q_4\}, a) = R(q_5) = \{q_5\} \rightarrow \boldsymbol{s_4}$
$\delta(s_3, b) = \delta(\{q_3, q_4\}, b) = R(q_5) = \{q_5\} \rightarrow s_4$
$\delta(s_4, a) = \delta(s_4, b) = \emptyset$
$\delta(\emptyset, a) = \delta(\emptyset, b) = \emptyset \rightarrow s_5$



b) The regular expression can be found heuristically as, $b \vee (a \vee b)a(a \vee b)$

c) The grammatical rules can be obtained from the DFA designed in **a**, as follows. Transitions to s_5 are omitted as it is a death state.

$$< s_0 >::= a < s_1 > |b < s_2 > | \, b$$
$$< s_1 >::= a < s_3 >$$
$$< s_2 >::= a < s_3 >$$
$$< s_3 >::= \; a| \, b$$

Eliminating one of $< s_1 >$ and $< s_2 >$ as they have the same production rules and assigning familiar labels to non-terminals with initial non-terminal as S.

$$< S >::= a < A > |b < A > | \, b$$
$$< A >::= a < B >$$
$$< B >::= \; a| \, b$$

3) If S is the initial non-terminal, which of the words *aaab*, *aabbaab*, and *abaaabb* can be derived using the following grammar. Draw the parse tree(s) of the derivable one(s).

$$S \rightarrow aT \mid baS$$
$$T \rightarrow bU \mid b$$
$$U \rightarrow S \mid aUb \mid a$$

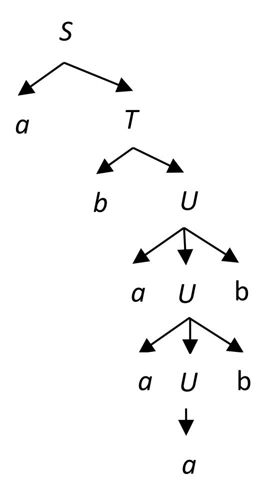
Provide an arithmetic rule between the number of *a*'s(*#a*) and *b*'s(*#b*) in the words derived using the grammar above.

Solution:

aaab and *aabbaab* cannot be derived using the given grammar.

abaaabb can be derived as

$$S \rightarrow aT \rightarrow abU \rightarrow abaUb$$
$$\rightarrow abaaUbb \rightarrow abaaabb.$$



Starting from the initial non-terminal S and eliminating parts where $\#a = \#b$:

$$S \rightarrow aT \vee baS \rightarrow (ba)^{*}aT \rightarrow aT$$
$$S \rightarrow a(bU \vee b) \rightarrow abU \vee ab \rightarrow U \vee ab$$
$$S \rightarrow a^{n}(S \vee a)b^{n} \vee ab \rightarrow S \vee a \vee ab$$
$$S \rightarrow a \vee ab$$

So $\#a = (\#b + 1) \vee \#b$

4) Design a PDA that can recognize the words produced by the following grammar:

$$S \rightarrow aSb \mid aTb$$
$$T \rightarrow bTa \mid \Lambda$$

Give executions of PDA for the following words: *aaabbaabbb*, *aaabbb*, *aba*.

Solution:

First, check the format of the strings accepted by this PDA:

$$S \to aSb \lor aTb \to a^n aTbb^n, n \geq 0 \to a^n T b^n, n > 0$$
$$T \to bTa \lor \Lambda \to b^m a^m, m \geq 0$$
$$S \to a^n T b^n, n > 0 \to a^n b^m a^m b^n, m \geq 0 \text{ and } n > 0$$

Design of the PDA:

$$M = (S, \Sigma, \Gamma, \Delta, s_0, F)$$

$$S = \{q_0, q_1, q_2, q_3, q_4\},\ \Sigma = \{a, b\},\ \Gamma = \{a, b\},\ s_0 = q_0,\ F = q_4$$

$$\Delta = \{ \underbrace{[(q_0, a, \Lambda), (q_1, a)]}_{a},$$

$$m = 0 \to \underbrace{[(q_1, a, \Lambda), (q_1, a)]}_{a^{n-1}}, \underbrace{[(q_1, b, a), (q_4, \Lambda)]}_{b}, \underbrace{[(q_4, b, a), (q_4, \Lambda)]}_{b^{n-1}},$$

$$m > 0 \to \underbrace{[(q_1, b, \Lambda), (q_2, b)], [(q_2, b, \Lambda), (q_2, b)]}_{b^m}, \underbrace{[(q_2, a, b), (q_3, \Lambda)], [(q_3, a, b), (q_3, \Lambda)]}_{a^m}, \underbrace{[(q_3, b, a), (q_4, \Lambda)]}_{b}$$

$$\}$$

Executions for the given words:

State	Tape	Stack	Transition Rule
q_0	*aaabbaabbb*	Λ	$[(q_0, a, \Lambda), (q_1, a)]$
q_1	*aabbaabbb*	*a*	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	*abbaabbb*	*aa*	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	*bbaabbb*	*aaa*	$[(q_1, b, \Lambda), (q_2, b)]$
q_2	*baabbb*	*baaa*	$[(q_2, b, \Lambda), (q_2, b)]$
q_2	*aabbb*	*bbaaa*	$[(q_2, a, b), (q_3, \Lambda)]$
q_3	*abbb*	*baaa*	$[(q_3, a, b), (q_3, \Lambda)]$
q_3	*bbb*	*aaa*	$[(q_3, b, a), (q_4, \Lambda)]$
q_4	*bb*	*aa*	$[(q_4, b, a), (q_4, \Lambda)]$
q_4	*b*	*a*	$[(q_4, b, a), (q_4, \Lambda)]$
q_4	Λ	Λ	

State	Tape	Stack	Transition Rule
q_0	*aaabbb*	Λ	$[(q_0, a, \Lambda), (q_1, a)]$
q_1	*aabbb*	*a*	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	*abbb*	*aa*	$[(q_1, a, \Lambda), (q_1, a)]$
q_1	*bbb*	*aaa*	$[(q_1, b, a), (q_4, \Lambda)]$
q_4	*bb*	*aa*	$[(q_4, b, a), (q_4, \Lambda)]$
q_4	*b*	*a*	$[(q_4, b, a), (q_4, \Lambda)]$
q_4	Λ	Λ	

aba is not accepted by this PDA.