MAT 271E Probability and Statistics

Homework 2

Assigned: February 19, 2011

Due: February 22, 2011 (in class, before class starts)

No late homework will be accepted!

Do not copy from solutions from your classmates. All work must be your own!

Show all your work!

Read: "Probability and Stochastic Processes", Yates and Goodman, Ch. 2

1) A particular operation has six components. Each component has a failure probability q, independent of any other component. The operation is successful if and only if all three hold:

- Component 1 or component 2 works.
- Components 3 and 4 both work or component 5 works.
- Component 6 works.

a) Sketch a block diagram for this operation.
b) What is the probability $P[W]$ that the operation is successful?

2) Assume that people try to access your website five times and then give up. You want to make sure that you are able to serve at least 98% of the people who want to access the site. Let p be probability that a person accesses your site. What should be the smallest value for p?

3) Assume that we have two classes that both undergraduate and graduate students are allowed to take. Class A contains 30 undergraduate students and 5 graduate students, while Class B contains 40 undergraduate and 8 graduate students. We toss a biased coin that has a 0.7 probability of turning up heads. If the result of the toss is heads, we pick a student from Class A; if it is tails, we pick a student from Class B. Find the probability that a graduate student is chosen.

4) Suppose you flip a coin twice. On any flip, the coin comes up heads with probability 1/3. Use H_i and T_i to denote the result of flip i.

a) What is the probability, $P[T_1 \mid T_2]$, that the first flip is tails given that the second flip is tails?
b) What is the probability that the first flip is tails and the second flip is heads?

5) Assume you are allowed to use only six numbers to create your PIN for the student registration system: 1, 2, 3, 4, 5, and 6. How many five-digit PINs are possible? How many six-digit PINs are possible if each number appears only once in each PIN?

6) Consider 6 tosses of a coin. A possible sequence of outcomes is HTHTHT. Asume that the probability of getting heads is 0.6 and each toss is independent.

a) What is the probability of the sequence THTTHT?
b) What is the probability that a sequence contains exactly three tails?