**BLM 336 TUTORIAL FOR CHAPTER 7 – SIGNALS AND SYSTEMS**

**Problem 1.**
The frequency which under the sampling theorem must be exceeded by the sampling frequency is called the Nyquist rate. Determine the Nyquist rate corresponding to each of the following signals.

**a)** $x(t) = 1 + \cos(2000\Pi t)$

**b)** $x(t) = \frac{\sin(4000\Pi t)}{\Pi t}$

**Solution.**

**a)** $X(j\omega) = 0$ for $|\omega| > 2000\Pi$
Therefore, the Nyquist rate for using this signal is

$\omega_N = 2.(2000\Pi) = 4000\Pi$

**b)** $X(j\omega)$ is a rectangular pulse for which $X(j\omega) = 0$ for $|\omega| > 4000\Pi$
Therefore, the Nyquist rate for using this signal is

$\omega_N = 2.(4000\Pi) = 8000\Pi$

**Problem 2.**
Impulse train sampling of x[n] is used to obtain

$$g[n] = \sum_{k=-\infty}^{+\infty} x[n].\delta[n - kN]$$

If $X(e^{j\omega}) = 0$ for $\frac{3\Pi}{7} \le |\omega| \le \Pi$

Determine the largest value fort he sampling interval N which ensures that no aliasing takes place while sampling x[u].

**Solution.**
We are interested in the lowest rate which x[u] may be sampled without the possibility of aliasing.

To find the lowest rate at which x[n] may be sampled while avoiding the possibility of aliasing we must find a N such that

$\frac{2\Pi}{N} \geq 2\left(\frac{9\Pi}{7}\right)$

$N \leq \frac{7}{3}$

Therefore, N can be at most 2.

**Problem 3.**
Let x(t) be a signal with Nyquist rate $W_0$.
Also, let

$$y(t) = x(t)p(t-1),$$

Where

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT), \text{ and } T < \frac{2\Pi}{\omega_0}$$

Specify the constraints on the magnitude and phase of the frequency response of $\Omega$ fitler that gives x(t) as its output when y(t) is the input.

**Solution.**

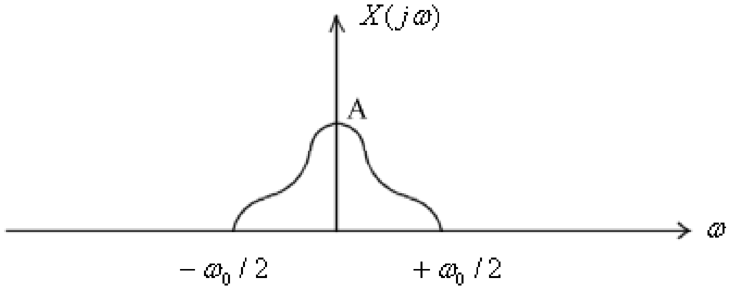
$$p(t) \xleftrightarrow{FT} \frac{2\Pi}{T} \sum_{n=-\infty}^{+\infty} \delta(\omega - 2k\Pi / T)$$

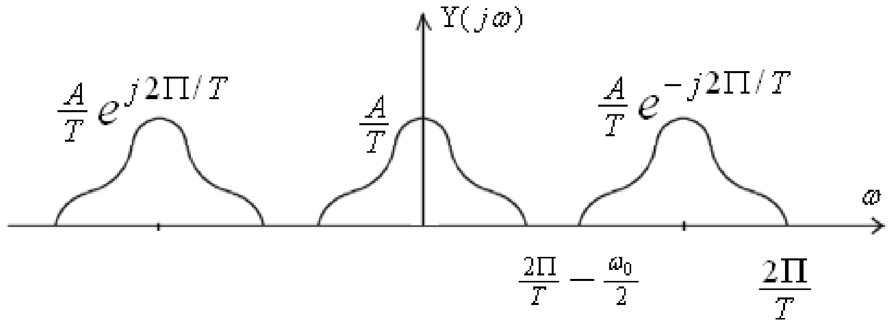
$$p(t-1) \longleftrightarrow \frac{2\Pi}{T}.e^{-j\omega} \sum_{k=-\infty}^{+\infty} \delta(\omega - k.\tfrac{2\Pi}{T}) = \frac{2\Pi}{T} \sum_{k=-\infty}^{+\infty} \delta(\omega - k.\tfrac{2\Pi}{T}).e^{-jk\frac{2\Pi}{T}}$$

y(t) = x(t).p(t-1)

$$Y(j\omega) = \frac{1}{2\Pi}\left[X(j\omega) * FT\{p(t-1)\}\right]$$
$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X\left[j(w - k.\tfrac{2\Pi}{T})\right] e^{-jk\frac{2\Pi}{T}}$$

Y( $j\omega$) consists of replicas of X( $j\omega$) shifted by $K\frac{2\Pi}{T}$ and added to each other.




**Problem 4.**
The signal y(t) is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$y(t) = x_1(t)*x_2(t)$

where

$$X_1(j\omega) = 0 \text{ for } |\omega| > 1000\Pi$$
$$X_{21}(j\omega) = 0 \text{ for } |\omega| > 2000\Pi$$

Impulse-train sampling is performed on y(t) to obtain

$$y_p(t) = \sum_{n=-\infty}^{+\infty} y(nT)\delta(t - nT)$$

Specify the range of values for the sampling period T which ensures that y(t) is recoverable from $y_p(t)$.

**Solution.**
Using the properties of the Fourier transform, we obtain

$$Y(j\omega) = X_1(j\omega).X_2(j\omega)$$

Therefore, Y( $j\omega$ )=0 for $|\omega| > 1000\Pi$ . This implies that the Nyquist rate for y(t) is $2x1000\Pi = 2000\Pi$ .

Therefore, the sampling period T can at most be $\frac{2\Pi}{(2000\Pi)} = 10^{-3}$ sec .

Therefore, we have to use T<$10^{-3}$ sec in order to be able to recover y(t) from $y_p(t)$.