18.10.2012

FORMAL LANGUAGES AND AUTOMATA
Homework-2

1. Reduce the states of the incompletely specified Mealy machine below using:

   a. Complete cover

   b. Minimal closed cover

	00	01	10	11
A	A/0	-	F/1	-
B	E/0	-	C/0	D/1
C	-	B/1	E/0	-
D	-	C/0	-	B/1
E	C/0	C/1	-	D/0
F	A/0	E/0	A/1	-

Also, draw the state transition table of the reduced machine in Moore model.

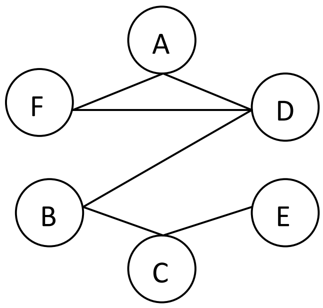
SOLUTION:

We start by building the dependency table:

	A	B	C	D	E
B	✘				
C	✘	(E-C)✔			
D	✔	✔	✘		
E	(A-C)✘	✘	(B-C)✔	✘	
F	✔	✘	✘	(E-C)✔	✘

Note: (E-C) and (B-C) pairs depend on each other. We can consider them as compatible.

Now we need to draw relation graph:



a. Complete Cover: We need to use the maximal compatibility class which is: {(A,D,F),(B,D),(B,C),(C,E)}

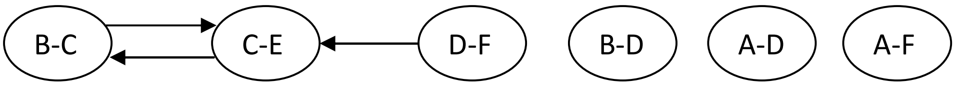
   New states are: S_1=(A,D,F) S_2=(B,D) S_3=(B,C) S_4=(C,E)

New state transition table is as follows:

	00	01	10	11
S_1	$S_1/0$	$S_4/0$	$S_1/1$	$S_2/1$
S_2	$S_4/0$	$S_3/0$	$S_3/0$	$S_2/1$
S_3	$S_4/0$	$S_2/1$	$S_4/0$	$S_2/1$
S_4	$S_4/0$	$S_3/1$	$S_4/0$	$S_1/0$

An example: when the transition from some compatible states is to only B we can choose either S_2 or S_3. But when it is to both B and D we have to choose S_2.

b. Minimum closed cover: We need to draw the dependency graph.



We need to cover all states and obey dependency rules.

For example, {(A,D,F),(C,E),(B,D)} is not a solution. It covers all the states but it disobeys a dependency (We included (E,C) but (E,C) depends on (B,C) so we have to include (B,C) as well.).

{(A,D,F), (B,C), (C,E)} is the minimum closed cover. Let's name our states:

$\hat{S}_1$=(A,D,F) $\hat{S}_2$=(B,C) $\hat{S}_3$=(C,E)

State transition table is as follows:

	00	01	10	11
$\hat{S}_1$	$\hat{S}_1/0$	$\hat{S}_3/0$	$\hat{S}_1/1$	$\hat{S}_2/1$
$\hat{S}_2$	$\hat{S}_3/0$	$\hat{S}_2/1$	$\hat{S}_3/0$	$\hat{S}_1/1$
$\hat{S}_3$	$\hat{S}_3/0$	$\hat{S}_2/1$	$\hat{S}_3/0$	$\hat{S}_1/0$

Moore model:

Let's consider the minimum closed cover and name the states as follows:

$\alpha = \hat{S}_1/0$
$\beta = \hat{S}_1/1$
$\gamma = \hat{S}_2/1$
$\delta = \hat{S}_3/0$

Moore state transition table:

	00	01	10	11	Output
α	α	δ	β	γ	0
β	α	δ	β	γ	1
γ	δ	γ	δ	β	1
δ	δ	γ	δ	α	0

2. Define the set of Fibonacci numbers F, using induction (recursion). What is the height of the element 13?

   SOLUTION:

   i. $F_1=1$ and $F_2=1$ $F_1, F_2 \in F$
   ii. $F_{n-1}, F_{n-2} \in F \Rightarrow (F_n = F_{n-1} + F_{n-2}) \in F$
   iii. No other number can be identified as a Fibonacci number, apart from the ones generated by the rule number (ii).

   $S_0=\{1,1\}$ $S_1=\{1,1,2\}$ $S_1=\{1,1,2\}$ $S_2=\{1,1,2,3\}$ $S_3=\{1,1,2,3,5\}$ $S_4=\{1,1,2,3,5,8\}$ $S_5=\{1,1,2,3,5,8,\mathbf{13}\}$ so its height is 5.

3. Let A be a language defined over Σ. What is the minimum possible value of X in the statement below:

$$(AA^+)^* \cup X = A^*$$

   SOLUTION:

   $AA^+ = \{AA, AAA, AAAA \dots\}$
   $(AA^+)^* = \{\Lambda, AA, AAA, AAAA \dots\}$
   $A^* = \{\Lambda, A, AA, AAA, AAAA \dots\}$
   $A^* \setminus (AA^+)^* = \{A\}$
   $X = A$