Linear and Nonlinear Regression

1) Suppose that a computer CPU (Central Processing Unit) system performance is related to the system temperature. A computer system has two distinc CPUs is serving in a local area. A system observer is monitoring the temperatures and clock frequencies of this system and records the data. System observer wants to model functions that describes all data.

Table 1 - CPU frequency (MHz) data on different temperatures (C)

Observation	Temp (C)	CPU – 1 Frequency (MHz)	CPU – 2 Frequency (MHz)
1	30.68	3102	3196
2	31.42	3245	3284
3	32.00	3596	3518
4	34.56	3612	3606
5	36.95	4484	4560
6	41.34	4608	4657
7	44.25	4937	4976
8	49.69	4912	4888
9	51.20	4846	4895
10	57.21	4573	4505

Considering the data on table 1;

a) Model linear regression functions CPUs using least square estimator method.

b) Plot the data of CPU frequencies at each temperature. Mark each data point on figure. (Display each CPU data on different figure. Get temperature values on x-axis and frequency values on y-axis.)

c) Plot linear regression functions that you modelled on your data points.

d) Calculate the coefficient of determination $\Re^2$ is also called ‘goodness of fit’ for your models and decide the which CPU frequency-temperature model is successfull.

e) Solve questiones (a), (b), (c), (d) for quadratic regression model.

f) Compare linear and quadratic regression models using coefficient of determination $\Re^2$ for each CPU.

g) Fill in the table below considering your model functions.

Temp (C)	CPU – 1 Frequency (MHz) Linear Regression Model	CPU – 1 Frequency (MHz) Quadratic Regression Model	CPU – 2 Frequency (MHz) Linear Regression Model	CPU – 2 Frequency (MHz) Quadratic Regression Model
25.34				
27.45				
29.56				
33.76				
35.43				
36.98				
43.29				
49.17				
55.89				
59.62				

Answer

1) $f(x) = ax + b$

a) $A = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}$ $\quad X = \begin{bmatrix} b \\ a \end{bmatrix}$ $\quad B = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix}$ $\quad AX = B$

For CPU-1 linear regression model;

$$\sum x_i = (30.68) + (31.42) + (32) + (34.56) + (36.95) + (41.34) + (44.25) + (49.69) + (51.20) + (57.21)$$

$$\sum x_i = 409.3$$

$$\sum x_i^2 = (30.68)^2 + (31.42)^2 + (32)^2 + (34.56)^2 + (36.95)^2 + (41.34)^2 + (44.25)^2 + (49.69)^2 + (51.20)^2 + (57.21)^2$$

$$\sum x_i^2 = 17542.7532$$

$$\sum y_i = (3102) + (3245) + (3596) + (3612) + (4484) + (4608) + (4937) + (4912) + (4846) + (4573)$$

$$\sum y_i = 41915$$

$$\sum x_i y_i = 1765484.56$$

$A = \begin{bmatrix} 10 & 409.3 \\ 409.3 & 17542.7532 \end{bmatrix}$ $\quad X = \begin{bmatrix} b \\ a \end{bmatrix}$ $\quad B = \begin{bmatrix} 41915 \\ 1765484.56 \end{bmatrix}$

$$\begin{bmatrix} 10 & 409.3 \\ 409.3 & 17542.7532 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 41915 \\ 1765484.56 \end{bmatrix} \qquad \begin{matrix} b = 1606.3286 \\ a = 63.1607 \end{matrix}$$

$$f(x) = ax + b$$
$$f(x) = (63.1607)x + 1606.3286$$

For CPU-2 linear regression model;

$$\sum y_i = (3196) + (3284) + (3518) + (3606) + (4560) + (4657) + (4976) + (4888) + (4895) + (4505)$$

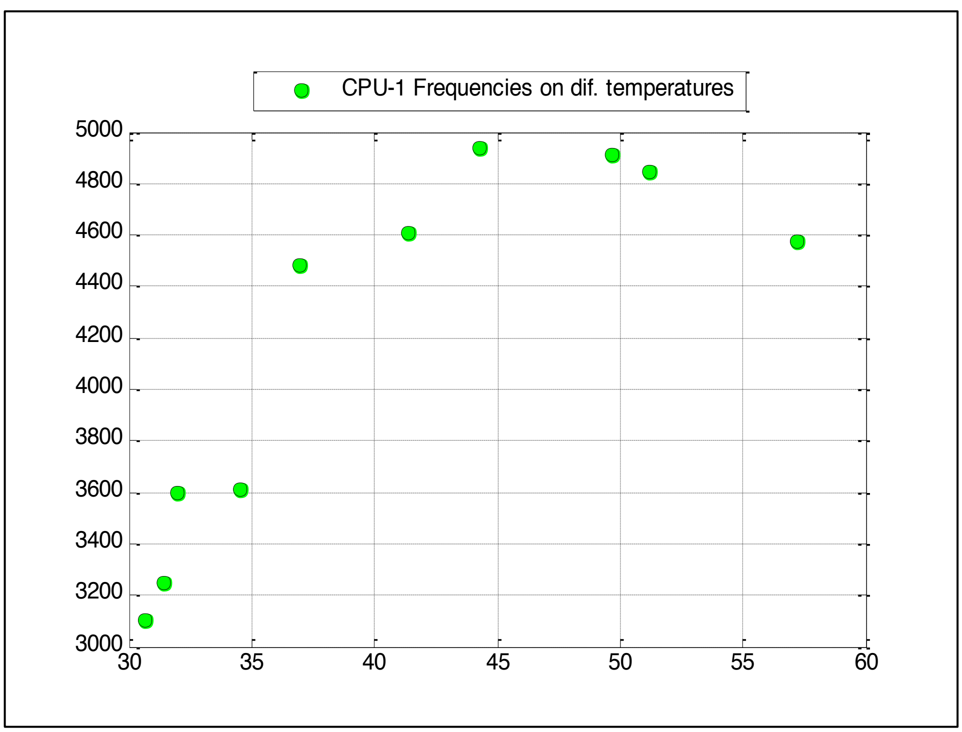
$$\sum y_i = 42085$$
$$\sum x_i y_i = 1770876.07$$

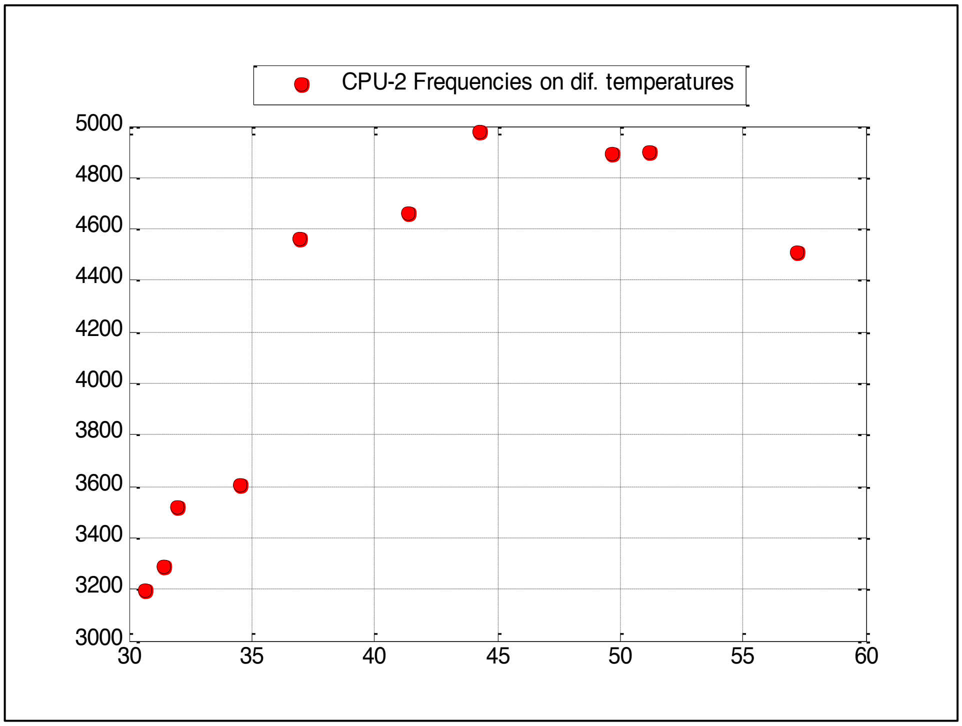
$$A = \begin{bmatrix} 10 & 409.3 \\ 409.3 & 17542.7532 \end{bmatrix} \quad X = \begin{bmatrix} b \\ a \end{bmatrix} \qquad B = \begin{bmatrix} 42085 \\ 1770876.07 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 409.3 \\ 409.3 & 17542.7532 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 42085 \\ 1770876.07 \end{bmatrix} \qquad \begin{matrix} b = 1704.4831 \\ a = 61.1780 \end{matrix}$$

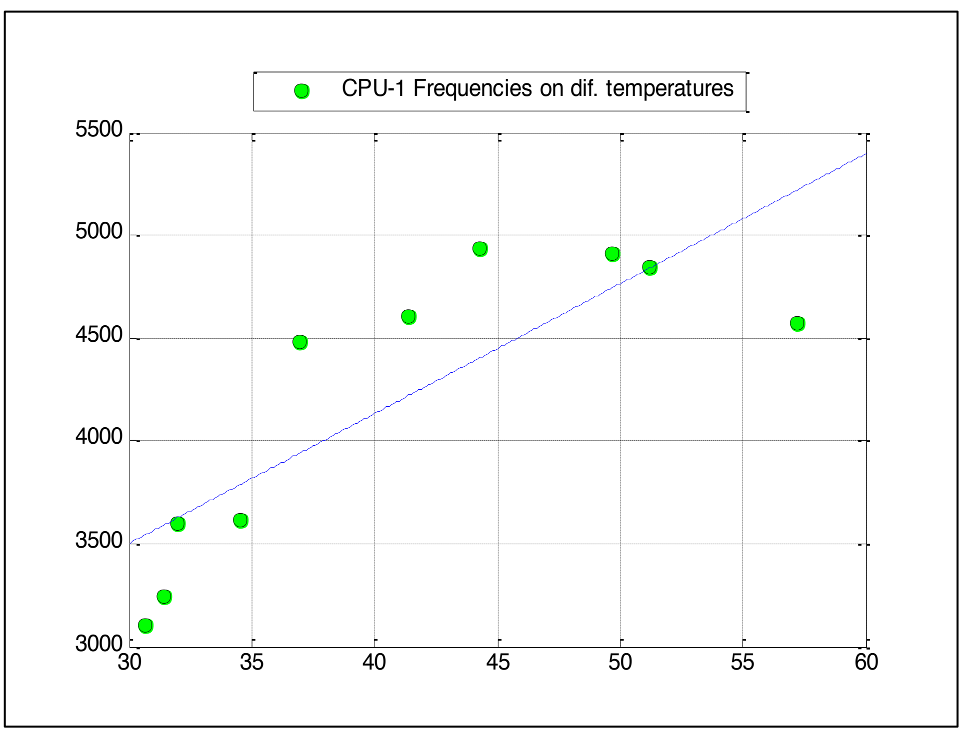
$$f(x) = ax + b$$
$$f(x) = (61.1780)x + 1704.4831$$

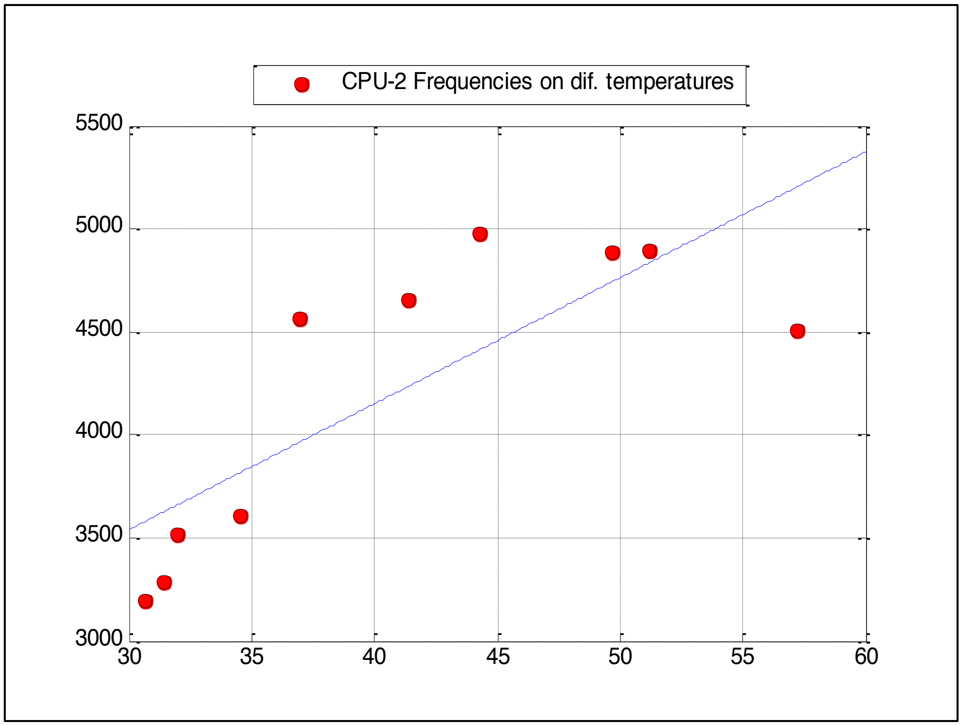
b)




CPU-2 Frequencies on dif. temperatures
5000
4800
4600
4400
4200
4000
3800
3600
3400
3200
3000
30
35
40
45
50
55
60

c)


CPU-1 Frequencies on dif. temperatures
5500
5000
4500
4000
3500
3000
30
35
40
45
50
55
60



d) Sum of squared errors $SS_{err} = \sum_i (y_i - f_i)^2$

Sum of squares total $SS_{tot} = \sum_i (y_i - \bar{y})^2$

$$\Re^2 = 1 - \frac{SS_{err}}{SS_{tot}} = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2}$$

For CPU-1 linear model;
$\bar{y} = 4191.5$

x_i	y_i	f_i	$y_i - f_i$	$(y_i - f_i)^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
30.68	3102	3544.0988	-442.0988	195451.4161	-1089.5	1187010.25
31.42	3245	3590.8377	-345.8377	119603.7797	-946.5	895862.25
32.00	3596	3627.4710	-31.47100	990.4238410	-595.5	354620.25
34.56	3612	3789.1623	-177.1623	31386.51313	-579.5	335820.25
36.95	4484	3940.1164	543.8835	295809.2996	292.5	85556.25

41.34	4608	4217.3919	390.6080	152574.6580	416.5	173472.25
44.25	4937	4401.1895	535.8104	287092.8115	745.5	555770.25
49.69	4912	4744.7837	167.2162	27961.26322	720.5	519120.25
51.20	4846	4840.1564	5.84356	34.14719347	654.5	428370.25
57.21	4573	5219.7522	-646.7522	418288.4689	381.5	145542.25

$$\Re^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \overline{y})^2} = 1 - \frac{1529192.7816}{4681144.5} = 0.6733$$

For CPU-2 linear model;
$\overline{y} = 4208.5$

x_i	y_i	f_i	$y_i - f_i$	$(y_i - f_i)^2$	$y_i - \overline{y}$	$(y_i - \overline{y})^2$
30.68	3196	3581.4241	-385.4241	148551.7676	-1012.5	1025156.25
31.42	3284	3626.6958	-342.6958	117440.4524	-924.5	854700.25
32.00	3518	3662.1791	-144.1791	20787.6128	-690.5	476790.25
34.56	3606	3818.7947	-212.7947	45281.6183	-602.5	363006.25
36.95	4560	3965.0102	594.9898	354012.8621	351.5	123552.25
41.34	4657	4233.5816	423.4183	179283.1245	448.5	201152.25
44.25	4976	4411.6096	564.3904	318536.5236	767.5	589056.25
49.69	4888	4744.4179	143.5820	20615.8136	679.5	461720.25
51.20	4895	4836.7967	58.20330	3387.6241	686.5	471282.25
57.21	4505	5204.4764	-699.4764	489267.3460	296.5	87912.25

$$\Re^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \overline{y})^2} = 1 - \frac{1697164.7455}{4654328.5} = 0.6354$$

e) $$A = \begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \qquad X = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \qquad B = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \\ \sum (x_i^2 y_i) \end{bmatrix}$$

$$AX = B$$

$$f(x) = a_0 + a_1 x + a_2 x^2$$

For CPU-1 quadratic regression model;

$$\sum x_i = (30.68) + (31.42) + (32) + (34.56) + (36.95) + (41.34) + (44.25) + (49.69) + (51.20) + (57.21)$$

$$\sum x_i = 409.3$$

$$\sum x_i^2 = (30.68)^2 + (31.42)^2 + (32)^2 + (34.56)^2 + (36.95)^2 + (41.34)^2 + (44.25)^2 + (49.69)^2 + (51.20)^2 + (57.21)^2$$

$$\sum x_i^2 = 17542.7532$$

$$\sum x_i^3 = (30.68)^3 + (31.42)^3 + (32)^3 + (34.56)^3 + (36.95)^3 + (41.34)^3 + (44.25)^3 + (49.69)^3 + (51.20)^3 + (57.21)^3$$

$$\sum x_i^4 = (30.68)^4 + (31.42)^4 + (32)^4 + (34.56)^4 + (36.95)^4 + (41.34)^4 + (44.25)^4 + (49.69)^4 + (51.20)^4 + (57.21)^4$$

$$\sum x_i^4 = 36635256.9725$$

$$\sum y_i = (3102) + (3245) + (3596) + (3612) + (4484) + (4608) + (4937) + (4912) + (4846) + (4573)$$

$$\sum y_i = 41915$$

$$\sum x_i y_i = 1765484.56$$
$$\sum x_i^2 y_i = 77582844.1358$$

$$A = \begin{bmatrix} 10 & 409.3 & 17542.7532 \\ 409.3 & 17542.7532 & 785839.1172 \\ 17542.7532 & 785839.1172 & 36635256.9725 \end{bmatrix} \qquad X = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$B = \begin{bmatrix} 41915 \\ 1765484.56 \\ 77582844.1358 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 409.3 & 17542.7532 \\ 409.3 & 17542.7532 & 785839.1172 \\ 17542.7532 & 785839.1172 & 36635256.9725 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 41915 \\ 1765484.56 \\ 77582844.1358 \end{bmatrix}$$

$a_0 = -8547.3023$
$a_1 = 558.6792$
$a_2 = -5.7732$

$$f(x) = a_0 + a_1 x + a_2 x^2$$
$$f(x) = -(8547.3023) + (558.6792)x - (5.7732)x^2$$

For CPU-2 linear regression model;

$$\sum y_i = (3196) + (3284) + (3518) + (3606) + (4560) + (4657) + (4976) + (4888) + (4895) + (4505)$$

$$\sum y_i = 42085$$
$$\sum x_i y_i = 1770876.07$$
$$\sum x_i^2 y_i = 77733283.4261$$

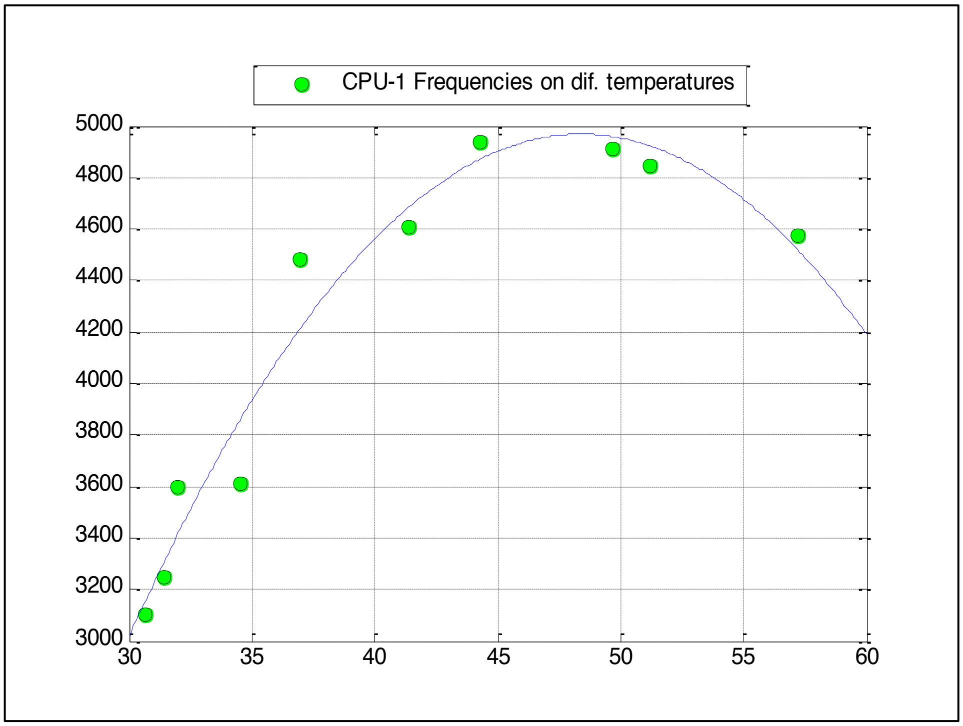
$$A = \begin{bmatrix} 10 & 409.3 & 17542.7532 \\ 409.3 & 17542.7532 & 785839.1172 \\ 17542.7532 & 785839.1172 & 36635256.9725 \end{bmatrix} \qquad X = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

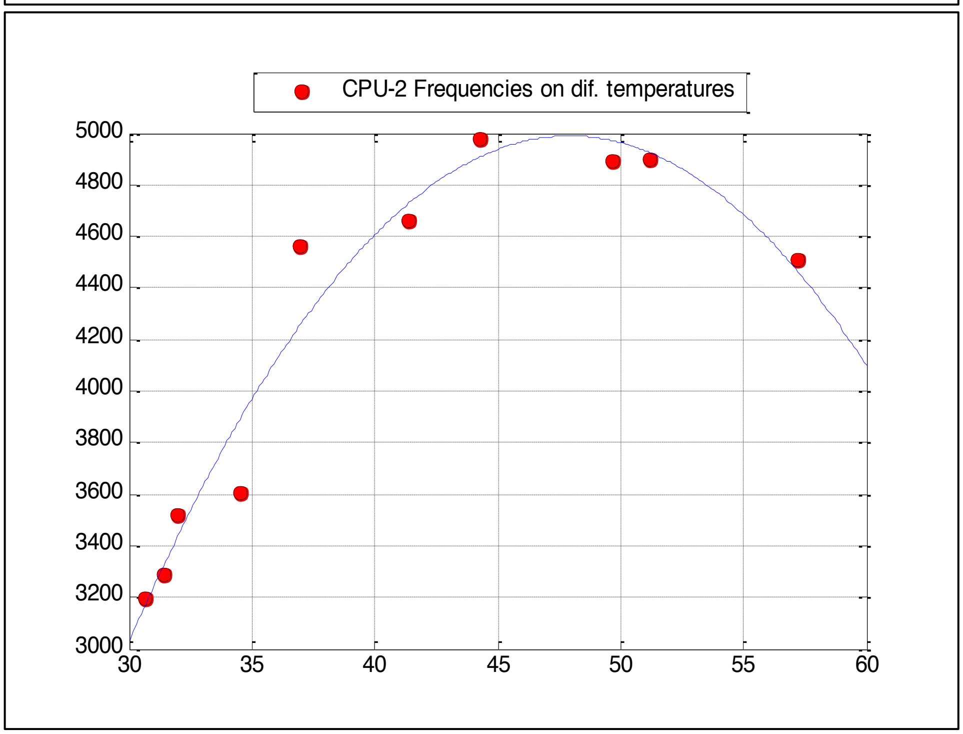
$$B = \begin{bmatrix} 42085 \\ 1770876.07 \\ 77733283.4261 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 409.3 & 17542.7532 \\ 409.3 & 17542.7532 & 785839.1172 \\ 17542.7532 & 785839.1172 & 36635256.9725 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 42085 \\ 1770876.07 \\ 77733283.4261 \end{bmatrix}$$

$a_0 = -9035.4689$
$a_1 = 585.3102$
$a_2 = -6.1066$

$$f(x) = a_0 + a_1 x + a_2 x^2$$
$$f(x) = -(9035.4689) + (585.3102)x - (6.1066)x^2$$





Sum of squared errors $SS_{err} = \sum_i (y_i - f_i)^2$

Sum of squares total $SS_{tot} = \sum_i (y_i - \bar{y})^2$

$$\Re^2 = 1 - \frac{SS_{err}}{SS_{tot}} = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2}$$

For CPU-1 quadratic model;

$$f(x) = -(8547.3023) + (558.6792)x - (5.7732)x^2$$
$$\bar{y} = 4191.5$$

x_i	y_i	f_i	$y_i - f_i$	$(y_i - f_i)^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
30.68	3102	3158.8794	-56.8794	3235.2739	-1089.5	1187010.25
31.42	3245	3307.0004	-62.0004	3844.0549	-946.5	895862.25
32.00	3596	3418.6753	177.3246	31444.0492	-595.5	354620.25
34.56	3612	3865.1777	-253.1777	64098.9581	-579.5	335820.25
36.95	4484	4213.7297	270.2702	73046.0096	292.5	85556.25
41.34	4608	4682.1224	-74.1224	5494.1346	416.5	173472.25
44.25	4937	4869.9658	67.0341	4493.5739	745.5	555770.25
49.69	4912	4958.8815	-46.8815	2197.8791	720.5	519120.25
51.20	4846	4922.9753	-76.9753	5925.2017	654.5	428370.25
57.21	4573	4519.1429	53.8570	2900.5844	381.5	145542.25

$$\Re^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2} = 1 - \frac{196679.7197}{4681144.5} = 0.9580$$

For CPU-2 quadratic model;

$$f(x) = -(9035.4689) + (585.3102)x - (6.1066)x^2$$
$$\bar{y} = 4208.5$$

x_i	y_i	f_i	$y_i - f_i$	$(y_i - f_i)^2$	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
30.68	3196	3173.9350	22.0649	486.8613	-1012.5	1025156.25
31.42	3284	3326.4419	-42.4419	1801.3162	-924.5	854700.25
32.00	3518	3441.2991	76.7008	5883.0280	-690.5	476790.25

34.56	3606	3899.1676	-293.1676	85947.2734	-602.5	363006.25
36.95	4560	4254.3867	305.6132	93399.4625	351.5	123552.25
41.34	4657	4725.1022	-68.1022	4637.9146	448.5	201152.25
44.25	4976	4907.4029	68.5970	4705.5501	767.5	589056.25
49.69	4888	4970.8126	-82.8126	6857.9422	679.5	461720.25
51.20	4895	4924.3278	-29.3278	860.1219	686.5	471282.25
57.21	4505	4463.3229	41.6770	1736.9775	296.5	87912.25

$$\Re^2 = 1 - \frac{\sum_i (y_i - f_i)^2}{\sum_i (y_i - \bar{y})^2} = 1 - \frac{206316.4483}{4654328.5} = 0.9557$$

f) Linear regression and quadratic regression models are better for CPU-1 because of greater value of $\Re^2$ for CPU-1 in each regression model.

g) Cpu-1 regression models

$f_1(x) = (63.1607)x + 1606.3286$

$f_2(x) = -(8547.3023) + (558.6792)x - (5.7732)x^2$

Cpu-2 Regression models

$f_1(x) = (61.1780)x + 1704.4831$

$f_2(x) = -(9035.4689) + (585.3102)x - (6.1066)x^2$

Temp	CPU1 f_1	CPU1 f_2	CPU2 f_1	CPU2 f_2
25.34	3206.8207	1902.5668	3254.7336	1875.1484
27.45	3340.0898	2438.3211	3383.8192	2429.9577
29.56	3473.3588	2922.6696	3512.9047	2930.3926
33.76	3738.6338	3733.7743	3769.8523	3764.6818
35.43	3844.1122	3999.6909	3872.0196	4036.5487
36.98	3942.0112	4217.6857	3966.8455	4258.4022
43.29	4340.5553	4818.8043	4352.8787	4858.6940
49.17	4711.9402	4965.1524	4712.6053	4980.3745
55.89	5136.3801	4643.5789	5123.7215	4602.3800

59.62	5371.9695	4240.0558	5351.9154	4154.5443

2) Dataset that given below, is constructed using some observations over an event. A researcher wants to model a function using non-linear regression (exponential model).

Table 2 – Table of dataset.

x	0	1.12	1.96	2.38	2.80	3.46	4.25	6.74	8
y=F(x)	4.45	9.37	14.78	13.26	23.98	17.64	25.88	69.51	66.15

Considering the data on table 2;

a) Using non-linear regression model (exponential model) fit a function to dataset.
b) Using Matlab polyfit function perform the operation mentioned in section (a).
c) Plot the regression model using polyval fuction for the range [0,100].
d) Fill in the table below considering your model function.

x	0.06	3.87	1.54	10.56	23.67	17.49	4.25	9.66	19.23
y=F(x)									

Answers

2)

x	0	1.12	1.96	2.38	2.80	3.46	4.25	6.74	8
y=F(x)	4.45	9.37	14.78	13.26	23.98	17.64	25.88	69.51	66.15

a) For exponential regression model general formula notation;

$$y = ae^{bx}$$

$$\ln(y) = \ln(ae^{bx})$$
$$\ln(y) = \ln(a) + \ln(e^{bx})$$
$$\ln(y) = \ln(a) + bx$$

$$z = \ln(y)\ ,\ a_0 = \ln(a)$$
$$z = a_0 + bx \text{ (Linear model)}$$

$$f(x) = ax + b$$

$$A=\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \qquad X=\begin{bmatrix} b \\ a \end{bmatrix} \qquad B=\begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix} \qquad AX=B$$

For CPU-1 linear regression model;

$$\sum x_i = (0)+(1.12)+(1.96)+(2.38)+(2.80)+(3.46)+(4.25)+(6.74)+(8)$$
$$\sum x_i = 30.71$$

$$\sum x_i^2 = (0)^2+(1.12)^2+(1.96)^2+(2.38)^2+(2.80)^2+(3.46)^2+(4.25)^2+(6.74)^2+(8)^2$$
$$\sum x_i^2 = 158.0621$$

$$\sum y_i = \ln(4.45)+\ln(9.37)+\ln(14.78)+\ln(13.26)+\ln(23.98)+\ln(17.64)+\ln(25.88)+\ln(69.51)+\ln(66.15)$$
$$\sum y_i = 26.7427$$

$$\sum x_i y_i = 108.7137$$

$$A=\begin{bmatrix} 9 & 30.71 \\ 30.71 & 158.0621 \end{bmatrix} \qquad X=\begin{bmatrix} b \\ a \end{bmatrix} \qquad B=\begin{bmatrix} 26.7427 \\ 108.7137 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 30.71 \\ 30.71 & 158.0621 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 26.7427 \\ 108.7137 \end{bmatrix} \qquad \begin{aligned} b &= 1.8530 \\ a &= 0.3278 \end{aligned}$$

$$\begin{aligned} b &= 1.8530 \\ a &= 0.3278 \end{aligned} \qquad f(x) = (0.3278)x + 1.8530$$

For $z = a_0 + bx$ linear model;

$$\begin{aligned} a_0 &= 1.8530 \\ b &= 0.3278 \end{aligned} \qquad a_0 = \ln(a) \qquad a = e^{a_0} \qquad a = e^{1.8530} = 6.3789$$

So;

$$y = ae^{bx}$$
$$y = (6.3789) \cdot e^{0.3278x}$$

b) polyfit(x, log(y),1)

$$z = a_0 + bx$$

$$\begin{aligned} a_0 &= 1.8530 \\ b &= 0.3278 \end{aligned} \qquad a_0 = \ln(a) \qquad a = e^{a_0} \qquad a = e^{1.8530} = 6.3789$$

$$y = (6.3789) \cdot e^{0.3278x}$$

c) Figure x= [0, 100]

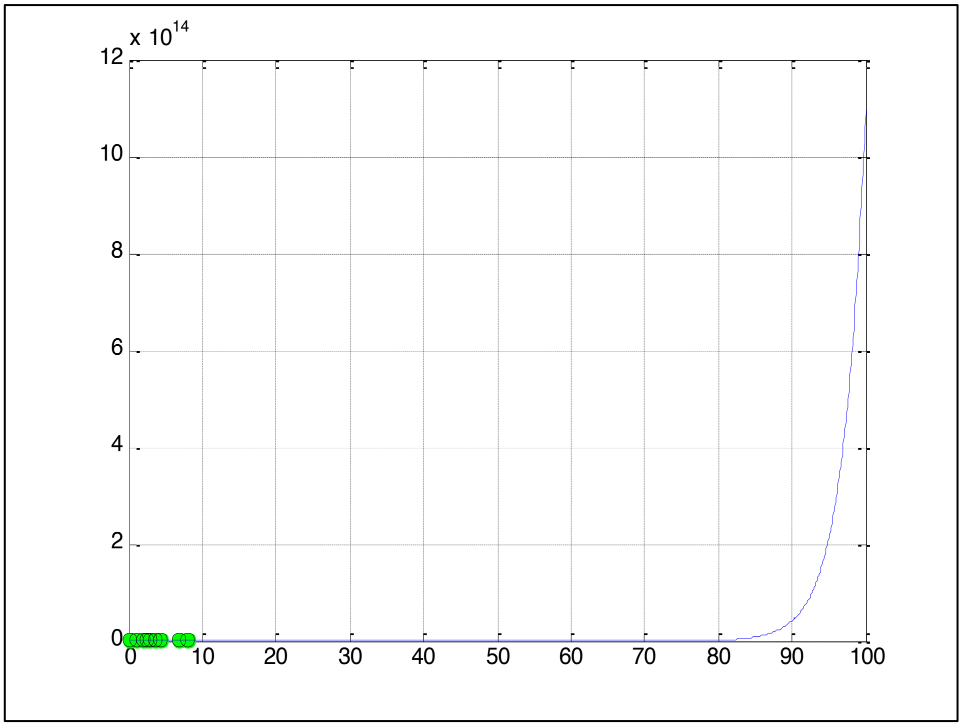
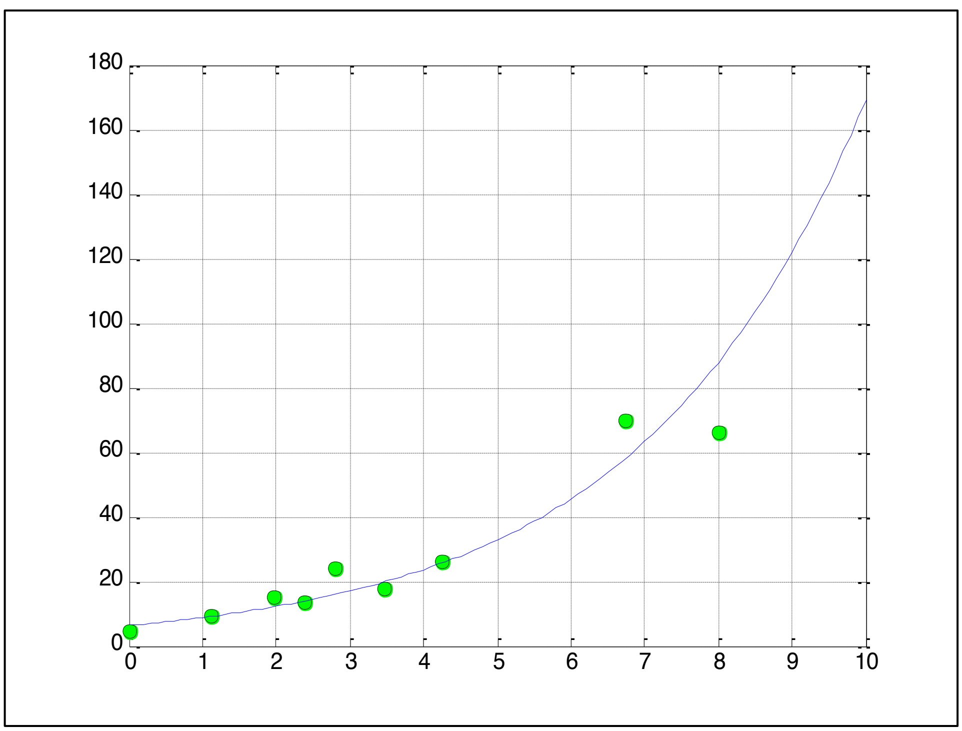


Figure x = [0, 10]



d)

x	0.06	3.87	1.54	10.56	23.67	17.49	4.25	9.66	19.23
y=F(x)	**6.5056**	**22.6822**	**10.5677**	**203.2757**	**14943.3500**	**1970.8450**	**25.6911**	**151.3422**	**3486.2769**

$$F(0.06) = (6.3789) \cdot e^{0.3278*(0.06)} = 6.5056$$

$$F(3.87) = (6.3789) \cdot e^{0.3278*(3.87)} = 22.6822$$

$$F(1.54) = (6.3789) \cdot e^{0.3278*(1.54)} = 10.5677$$

$$F(10.56) = (6.3789) \cdot e^{0.3278*(10.56)} = 203.2757$$

$$F(23.67) = (6.3789) \cdot e^{0.3278*(23.67)} = 14943.3500$$

$$F(17.49) = (6.3789) \cdot e^{0.3278*(17.49)} = 1970.8450$$

$$F(4.25) = (6.3789) \cdot e^{0.3278*(4.25)} = 25.6911$$

$$F(9.66) = (6.3789) \cdot e^{0.3278*(9.66)} = 151.3422$$

$$F(19.23) = (6.3789) \cdot e^{0.3278*(19.23)} = 3486.2769$$

3) Over a computer sub-network, three end-users connected to a wireless media via smart-phones. These users download data using the internet connection of the wireless router. In a time interval, internet data download speed statistics (KB/sn) of these three users recorded and modelled with functions for each user. Modelled functions for data download speed statistics of these users are given in the following lines.

$$F_{USER1}(t) = -0.0189t^2 + 1.3406t + 80.7743$$
$$F_{USER2}(t) = -0.0043t^2 + 0.4259t + 105.2688$$
$$F_{USER3}(t) = 0.0181t^2 - 0.8689t + 105.9157$$

Data download speed functions are avaible for a time interval is 100 seconds. Considering the given functions answer the following questions;

a) Using single Trapezoidal rule calculate the downloaded data size (kilobytes) in [10,70] time range for each smart-phone user.
b) Calculate the true errors and absolute relative true errors considering the values you find in section (a).
c) Using multiple Trapezoidal rule calculate data sizes (kilobytes) in [10,70] time range for each smart-phone user by getting Trapezoidal segment counts as 2,4,6,10,20.
d) Calculate the true errors and absolute relative true errors considering the values you find in section (c).
e) Write your segments, approximate values, true values, true errors, absolute true errors as tabulated form. (Segments : 1,2,4,6,10. Note that you should use previous section results, do not calculate necessary elements again.)
f) Plot the data download speeds of users for time range [0, 100] and compare your calculations with graphics.
g) Using Simpson's method for integration solve section (a), (b), (c), (d), (e).

Answers

3)

a) $\int_a^b f(x)dx \approx Area\, of\, trapezoid$ $\qquad$ $\int_a^b f(x)dx \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \quad [10,70]$$
$$\int_{10}^{70} F_1(t)dt \approx (70-10)\left[\frac{82.0063+92.2903}{2}\right] = 5228.8980$$

$$F_2(t) = -0.0043t^2 + 0.4259t + 105.2688 \quad [10,70]$$

$$\int_{10}^{70} F_1(t)dt \approx (70-10)\left[\frac{114.0118+109.0978}{2}\right] = 6693.2880$$

$$F_3(t) = 0.0181t^2 - 0.8689t + 105.9157 \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt \approx (70-10)\left[\frac{133.7827+99.0367}{2}\right] = 6984.5820$$

True integration results;

$$F(t) = -0.0063t^3 + 0.6703t^2 + 80.7743t \quad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70) - F(10) = 6777.7710 - 868.4730 = 5909.2980$$

$$F(t) = -0.00143t^3 + 0.21295t^2 + 105.2688t \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70) - F(10) = 7921.7810 - 1072.5530 = 6849.2280$$

$$F(t) = 0.00603t^3 - 0.43445t^2 + 105.9157t \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70) - F(10) = 7.3535840 - 1.0217420 = 6331.8420$$

b)

	True Error	**Relative Abs. True Error**
User - 1	$5909.2980 - 5228.8980 = 680.4$	$(680.4 / 5909.2980) x100 = \%11.5140$
User - 2	$6849.2280 - 6693.2880 = 155.94$	$(155.94 / 6849.2280) x100 = \%2.2767$
User - 3	$6331.8420 - 6984.5820 = -652.74$	$(652.74 / 6331.8420) x100 = \%10.3088$

c) $$\int_a^b f(x)dx \approx \frac{(b-a)}{2n}\left[f(a) + 2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\} + f(b)\right]$$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \qquad [10,70]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(2)}\left[f(10) + 2\{f(40)\} + f(70)\right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(4)}\left[f(10)+2\left\{f(25)+f(40)+f(55)\right\}+f(70)\right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(6)}\left[f(10)+2\left\{f(20)+f(30)+f(40)+f(50)+f(60)\right\}+f(70)\right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(10)}\left[f(10)+2\left\{\begin{matrix} f(16)+f(22)+f(28)+f(34)+f(40)+ \\ f(46)+f(52)+f(58)+f(64) \end{matrix}\right\}+f(70)\right]$$

$$\int_a^b f(x)dx \approx \frac{(70-10)}{2(20)}\left[f(10)+2\left\{\begin{matrix} f(13)+f(16)+f(19)+f(22)+f(25)+f(28)+f(31)+ \\ f(34)+f(37)+f(40)+f(43)+f(46)+f(49)+f(52)+ \\ f(55)+f(58)+f(61)+f(64)+f(67) \end{matrix}\right\}+f(70)\right]$$

	Approximate values		
	User-1	User-2	User-3
2-segments	**5739.1980**	**6809.3880**	**6495.8820**
4-segments	**5866.7730**	**6838.4130**	**6373.7069**
6-segments	**5890.3980**	**6843.7880**	**6351.0820**
10-segments	**5902.4940**	**6846.5400**	**6339.4980**
20-segments	**5907.5970**	**6847.7009**	**6334.6109**

True values		
User-1	User-2	User-3
5909.2980	6849.2280	6331.8420

d) For user-1;

2-segment $5909.2980 - 5739.1980 = 170,01$

$(170,0100 / 5909.2980)*100 = 2.8769$

4-segment $5909.2980 - 5866.7730 = 42.525$

$(42.5250 / 5909.2980)*100 = 0.7196$

6-segment $5909.2980 - 5890.3980 = 18,9$

$(18,9000 / 5909.2980)*100 = 0.3198$

10-segment $5909.2980 - 5902.4940 = 6.804$

$(6.8040 / 5909.2980)*100 = 0.1151$

20-segment $5909.2980 - 5907.5970 = 1.701$

$(1.7010 / 5909.2980)*100 = 0.0287$

2-segment $6849.2280 - 6809.3880 = 39.84$

$(39.8400 / 6849.2280)*100 = 0.5816$

4-segment $6849.2280 - 6838.4130 = 10.815$

$(10.8150 / 6849.2280)*100 = 0.1579$

6-segment $6849.2280 - 6843.7880 = 5.44$

$(5.4400 / 6849.2280)*100 = 0.0794$

10-segment $6849.2280 - 6846.5400 = 2.688$

$(2.6880 / 6849.2280)*100 = 0.0392$

20-segment $6849.2280 - 6847.7009 = 1.5271$

$(1.5271 / 6849.2280)*100 = 0.0222$

2-segment $6331.8420 - 6495.8820 = -164.04$

$(164.0400 / 6331.8420)*100 = 2.5907$

4-segment $6331.8420 - 6373.7069 = -41.8649$

$(41.8649 / 6331.8420)*100 = 0.6611$

6-segment $6331.8420 - 6351.0820 = -19.24$

$(19.2400 / 6331.8420)*100 = 0.3038$

10-segment $6331.8420 - 6339.4980 = -7.656$

$(7.6560 / 6331.8420)*100 = 0.1209$

20-segment $6331.8420 - 6334.6109 = -2.7689$

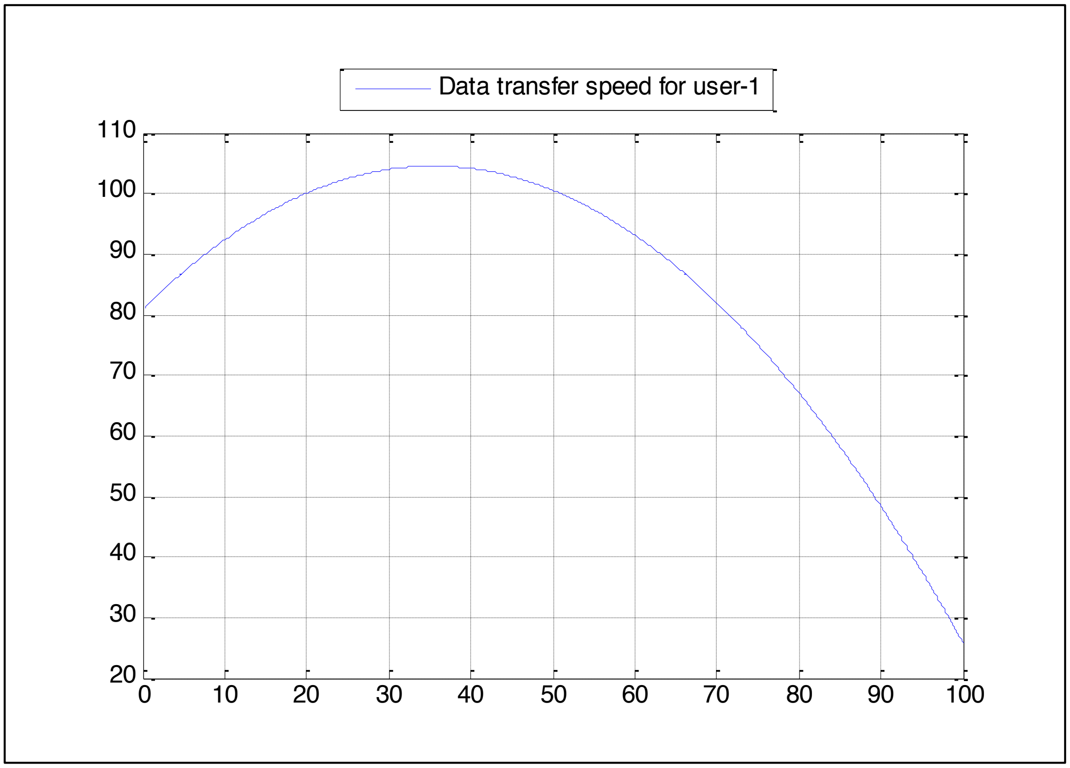
$(2.7689 / 6331.8420)*100 = 0.0437$

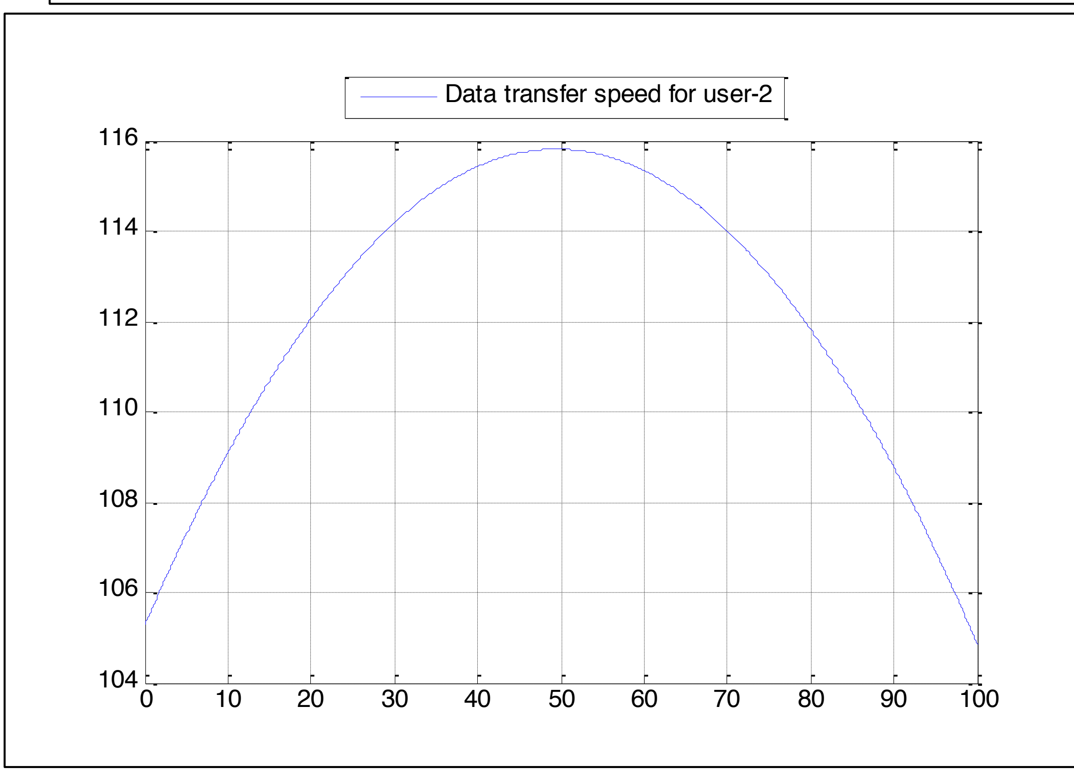
e)

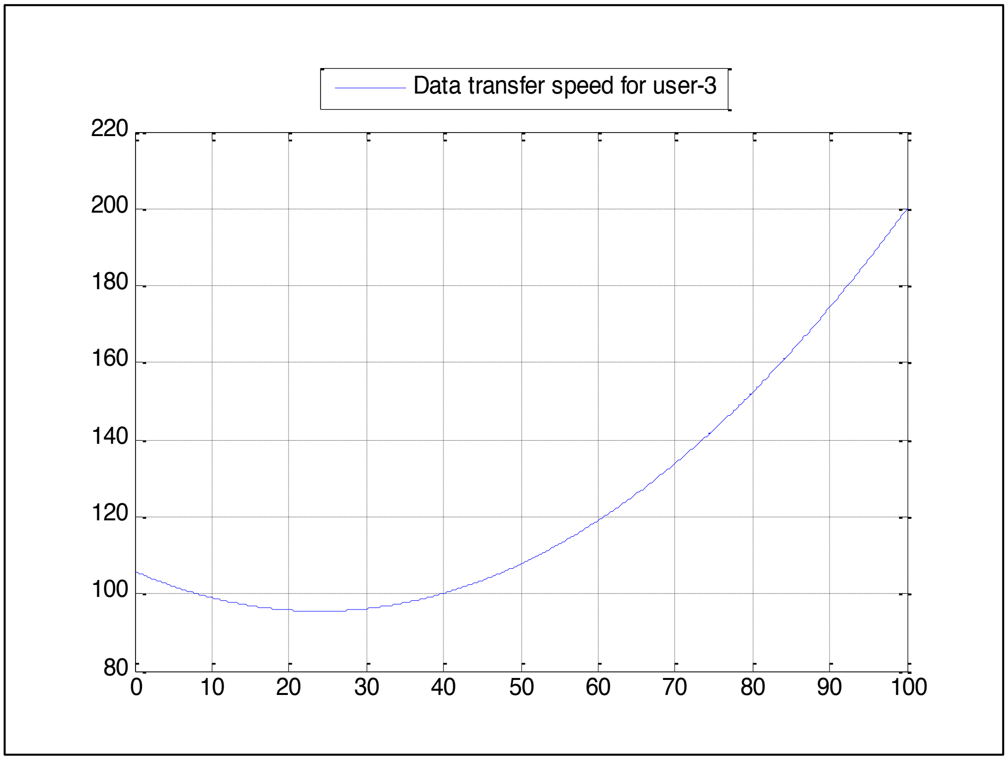
	Approximate values		
Segments	User-1	User-2	User-3
2	**5739.1980**	**6809.3880**	**6495.8820**
4	**5866.7730**	**6838.4130**	**6373.7069**

6	**5890.3980**	**6843.7880**	**6351.0820**
10	**5902.4940**	**6846.5400**	**6339.4980**
20	**5907.5970**	**6847.7009**	**6334.6109**
	True Errors		
2	**170.01**	**39.84**	**0.5816**
4	**42.525**	**10.815**	**0.1579**
6	**18.9**	**5.44**	**0.0794**
10	**6.804**	**2.688**	**0.0392**
20	**1.701**	**1.5271**	**0.0222**
	Abs. Rel. True Errors		
2	**2.8769**	**-164.04**	**2.5907**
4	**0.7196**	**-41.8649**	**0.6611**
6	**0.3198**	**-19.24**	**0.3038**
10	**0.1151**	**-7.656**	**0.1209**
20	**0.0287**	**-2.7689**	**0.0437**

f)


Data transfer speed for user-1
110
100
90
80
70
60
50
40
30
20
0
10
20
30
40
50
60
70
80
90
100


Data transfer speed for user-2
116
114
112
110
108
106
104
0
10
20
30
40
50
60
70
80
90
100



g) $\int_a^b f_2(x)dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right] \approx \frac{(b-a)}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$

$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \quad [10,70]$

$\int_{10}^{70} F_1(t)\,dt \approx \frac{70-10}{6}\left[82.0063+4f\left(40\right)+92.2903\right] = 10\left[82.0063+416.6332+92.2903\right] = 5909.2980$

$F_2(t) = -0.0043t^2 + 0.4259t + 105.2688 \quad [10,70]$

$\int_{10}^{70} F_1(t)dt \approx \left(70-10\right)\left[\frac{+}{2}\right] = 6693.2880$

$$\int_{10}^{70} F_1(t)\,dt \approx \frac{70-10}{6}\left[114.0118+4f\left(40\right)+109.0978\right]=10\left[114.0118+461.6992+109.0978\right]=$$

6848.0880

$$F_3(t) = 0.0181t^2 - 0.8689t + 105.9157 \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt \approx \frac{70-10}{6}\left[133.7827+4f\left(40\right)+99.0367\right]=10\left[133.7827+400.4788+99.0367\right]=$$

6332.9820

True integration results;

$$F(t) = -0.0063t^3 + 0.6703t^2 + 80.7743t \quad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70)-F(10) = 6777.7710 - 868.4730 = 5909.2980$$

$$F(t) = -0.00143t^3 + 0.21295t^2 + 105.2688t \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70)-F(10) = 7921.7810 - 1072.5530 = 6849.2280$$

$$F(t) = 0.00603t^3 - 0.43445t^2 + 105.9157t \qquad [10,70]$$

$$\int_{10}^{70} F_1(t)dt = F(70)-F(10) = 7.3535840 - 1.0217420 = 6331.8420$$

	True Error	**Relative Abs. True Error**
User - 1	5909.2980-5909.2980=0	(0 / 5909.2980)x100 = %0.00
User - 2	6849.2280-6848.0880=1.14	$(1.14/6849.2280)x100 = \%0.0166$
User - 3	6331.8420-6332.9820=−1.14	$(1.14/6849.2280)x100 = \%0.0166$

$$\int_a^b f(x)dx \approx \frac{(b-a)}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i) + f(x_n) \right]$$

$$F_1(t) = -0.0189t^2 + 1.3406t + 80.7743 \qquad [10,70]$$

	Approximate values		
	User-1	User-2	User-3
2-segments	**5909.2980**	**6848.0880**	**6332.9820**
4-segments	**5909.2980**	**6848.0880**	**6332.9820**
6-segments	**5909.2980**	**6848.0880**	**6332.9820**
10-segments	**5909.2980**	**6848.0880**	**6332.9820**
20-segments	**5909.2980**	**6848.0880**	**6332.9820**

True values		
User-1	User-2	User-3
5909.2980	**6849.2280**	**6331.8420**

For user-1;

2-segment $5909.2980 - 5909.2980 = 0$ $(0/5909.2980)*100 = \%0$

4-segment $5909.2980 - 5909.2980 = 0$ $(0/5909.2980)*100 = \%0$

6-segment $5909.2980 - 5909.2980 = 0$ $(0/5909.2980)*100 = \%0$

10-segment $5909.2980 - 5909.2980 = 0$ $(0/5909.2980)*100 = \%0$

20-segment $5909.2980 - 5909.2980 = 0$ $(0/5909.2980)*100 = \%0$

For user-2;

2-segment $6849.2280 - 6849.2280 = 0$ $(0/6849.2280)*100 = \%0$

4-segment $6849.2280 - 6849.2280 = 0$ $(0/6849.2280)*100 = \%0$

6-segment	$6849.2280 - 6849.2280 = 0$	$(0/6849.2280)*100 = \%0$
10-segment	$6849.2280 - 6849.2280 = 0$	$(0/6849.2280)*100 = \%0$
20-segment	$6849.2280 - 6849.2280 = 0$	$(0/6849.2280)*100 = \%0$

For user-3;

2-segment	$6331.8420 - 6331.8420 = 0$	$(0/6331.8420)*100 = \%0$
4-segment	$6331.8420 - 6331.8420 = 0$	$(0/6331.8420)*100 = \%0$
6-segment	$6331.8420 - 6331.8420 = 0$	$(0/6331.8420)*100 = \%0$
10-segment	$6331.8420 - 6331.8420 = 0$	$(0/6331.8420)*100 = \%0$
20-segment	$6331.8420 - 6331.8420 = 0$	$(0/6331.8420)*100 = \%0$

	Approximate values		
Segments	User-1	User-2	User-3
2	**5909.2980**	**6849.2280**	**6331.8420**
4	**5909.2980**	**6849.2280**	**6331.8420**
6	**5909.2980**	**6849.2280**	**6331.8420**
10	**5909.2980**	**6849.2280**	**6331.8420**
20	**5909.2980**	**6849.2280**	**6331.8420**
	True Errors		
2	**0.00**	**0.00**	**0.00**
4	**0.00**	**0.00**	**0.00**
6	**0.00**	**0.00**	**0.00**
10	**0.00**	**0.00**	**0.00**
20	**0.00**	**0.00**	**0.00**
	Abs. Rel. True Errors		
2	**0.00**	**0.00**	**0.00**
4	**0.00**	**0.00**	**0.00**
6	**0.00**	**0.00**	**0.00**

10	**0.00**	**0.00**	**0.00**
20	**0.00**	**0.00**	**0.00**