ISTANBUL TECHNICAL UNIVERSITY

COMPUTER ENGINERING DEPARTMENT

BLG 527E MACHINE LEARNING

CRN: 13817

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Homework #2

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Running Code

q1.r and q2.r R files can be directly run. They search "optdigits.tra" and "optdigits.tes.txt" files in the same directory. If R files and data files are different directory, “trainingFile” and “testFile” variables must be set accordingly. It took about 30 minutes for q1.r and about 5 minutes for q2.r to run.

Answers

Q1a-b)

Steps for multivariate analysis:

1- Eliminate feature column in training data set if whole column is 0 or fixed value to be sure within-class scatter matrix is invertible.

2- Calculate d-dimensional mean vectors per class. Create $mean_{ij}$ matrix.

$$m_i = \begin{bmatrix} \mu_{w_i(feature\ 1)} \\ \mu_{w_i(feature\ 2)} \\ \cdot \\ \mu_{w_i(feature\ 64)} \end{bmatrix}$$, with i = 0,1,2,3,...,9

3- Calculate common variance matrix

$$S = \sum_i P(C_i) S_i$$

where covariance matrix S_{ij}

$$S_{ij} = \frac{\sum_{t=1}^{N}(x_i^t - m_i)(x_j^t - m_j)}{N}$$

4- Get diagonal elements of common variance matrix as vector s
   a. Use vector s as it is if common covariance matrix and Σ is diagonal
   b. Use square root mean of s if common covariance matrix) and $\Sigma = s^2 I$ for some s>0

5- Discriminant function:

$$g_i(x) = -\frac{1}{2}\sum_{j=1}^{d}\left(\frac{x_j^t - m_{ij}}{s_j}\right)^2 + \log \mathrm{P}(C_i)$$

6- Run discriminant function for dataset and get prediction results.

7- Create k X k confusion matrix.

Q1c)

1- Training error of Q1a:

   8.579649 %

2- Test error of Q1a:

   10.68447 %

3- Training error of Q1b:

8.16113 %

4- Test error of Q1b:

10.62883 %

5- Test error for each class for Q1a:
1.117, 22.413, 13.812, 5.389, 6.077, 5.494, 3.333, 7.978, 19.883, 21.134

6- Test error for each class for Q1b:
1.129, 22.285, 10.285, 5.263, 6.250, 7.065, 3.846, 8.602, 16.867, 23.414

7- Confusion matrix of the Q1a:

truth

prediction	0	1	2	3	4	5	6	7	8	9
0	177	0	1	0	0	0	1	0	0	0
1	0	135	6	0	4	0	5	0	21	3
2	0	21	156	2	0	0	0	0	1	1
3	0	0	0	158	0	1	0	0	1	7
4	1	0	1	0	170	1	0	3	0	5
5	0	1	0	3	0	172	0	0	2	4
6	0	4	0	0	0	1	174	0	1	0
7	0	0	1	7	3	0	0	173	1	3
8	0	8	10	6	3	0	1	2	137	4
9	0	13	2	7	1	7	0	1	10	153

8- Confusion matrix of the Q1b:

truth

prediction	0	1	2	3	4	5	6	7	8	9
0	175	0	1	0	0	0	1	0	0	0
1	0	136	6	0	8	0	3	0	19	3
2	0	16	157	1	0	0	0	0	1	0
3	0	1	2	162	0	0	0	0	1	5
4	2	0	0	0	165	1	0	3	0	5

5	1	1	0	2	0	171	0	2	3	4
6	0	5	0	0	0	1	175	0	1	0
7	0	0	2	7	3	0	0	170	1	3
8	0	7	5	5	4	0	2	2	138	3
9	0	16	4	6	1	9	0	2	10	157

Training data sets gives better error than test sets as expected. Q1b (Σi = Σ (common covariance matrix) and Σ=s2I for some s>0) assumption gives best test error.

Classes (1 & 2) and (1 & 8) have confused each other most.

Q2a)

1- Eliminate feature column in training data set if whole column is 0 or fixed value to be sure within-class scatter matrix is invertible.
2- Calculate d-dimensional mean vectors per class. Create $mean_{ij}$ matrix.

$$m_i = \begin{bmatrix} \mu_{w_i(feature\ 1)} \\ \mu_{w_i(feature\ 2)} \\ \cdot \\ \mu_{w_i(feature\ 64)} \end{bmatrix}$$, with i = 0,1,2,3,...,9

3- Calculate scatter matrices
   a. Calculate within-class scatter matrix S_W

$$S_W = \sum_{i=1}^{c} S_i$$

$$S_i = \sum_{x \in D_i}^{n} (x - m_i)(x - m_i)^T$$

   b. Calculate between-class scatter matrix S_B

$$S_B = \sum_{i=1}^{c} N_i (x - m_i)(x - m_i)^T$$

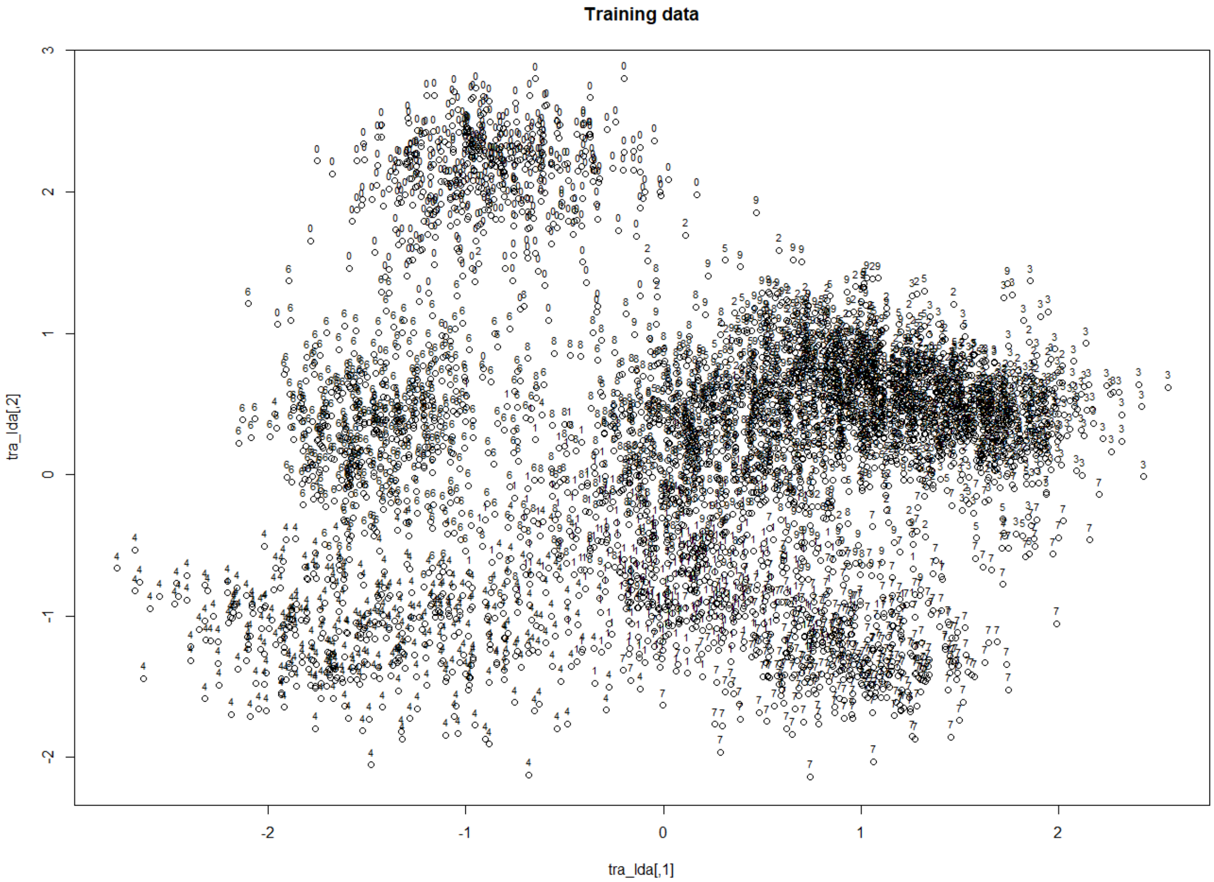
4- Solving the generalized eigenvalue problem for the matrix

$$S_W^{-1} S_B$$

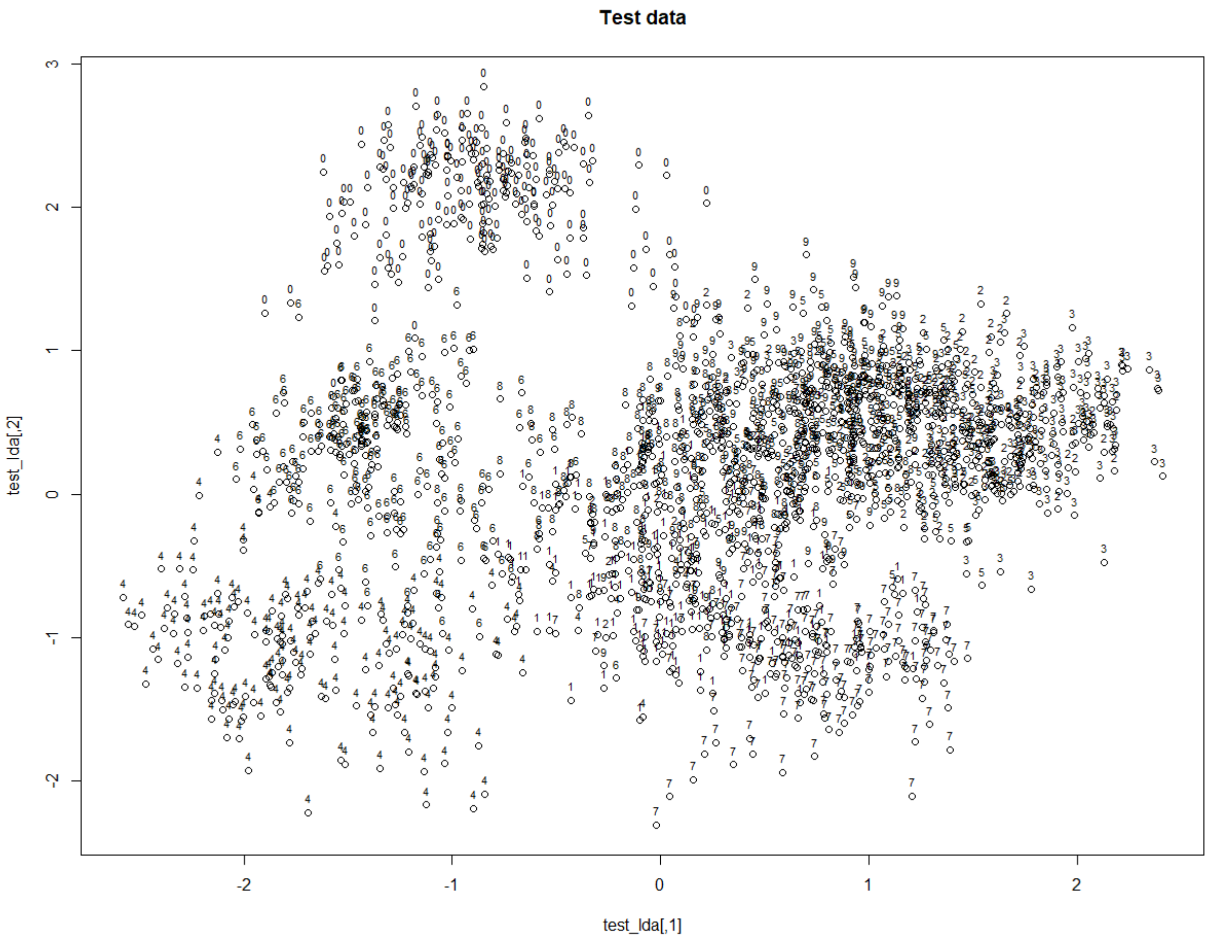
5- Choosing 2 eigenvectors with the largest eigenvalues
6- Transforming the samples onto the new subspace for training and test data set by calculated eigenvectors from training data set.

$$Y = X \times W$$

7- Visualize training data set and test data set LDA results.
   a. Training data result:



b. Test data result:



Q3)

LDA gives better for dimensionality reduction over PCA since, PCA doesn't use class information to reduce dimension.

We are interested in the matrix W that maximizes:

$$J(W) = \frac{|W^T S_B W|}{|W^T S_W W|}$$

The largest eigenvectors $S_W^{-1} S_B$ is the solution, and PCA within-class scatter matrix S_W gives bigger value over LDA which leads to worse dimension reduction.