analysis of algorithms

The 1^{st} Practice Session

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Algorithms

Algorithm: A step by step procedure for soloving a problem or accomplishing some end especially by a computer [1]

Besides correctness efficiency is the main criteria to decide whether an algorithm is good or bad [2]

- Time efficiency indicates how fast an algorithm in question runs
- Space efficiency deals with the extra memory the algorithm requires

An algorithm can be expressed in (*i*) natural language, (*ii*) pseudocode or (*iii*) programming language[3].

Analysis of algorithms

Analysis: the seperation of an intellectual or substantial whole into its constituent parts of individual study [5]

Different algorithm types with several examples

- Graph algorithms - bipartite matching, djikstra, ...
- Greedy algorithms - interval scheduling, ...
- Divide & conquer - mergesort, ...
- Dynamic algorithms - weighted interval scheduling, knapsack algorithm, ...

The first chapter consists introduction to analysis of algorithms with stable matching problem [1]

- A problem is stated, formulazed and finally solved with algorithm of Gale and Shapley

[1] consider to take a look at [4]

2004 National Computer Olympics - Tubitak[6]

There are ten restaurants in Ankara $(a, b, c, d, e, f, g, h, i, j)$ Ahmet will visit all of them between monday and friday for lunch and dinner. however, according to meals served in these restaurants Ahmet determined some rules

- d must be visited after one day of visiting j
- a, b, c, d, e must not be visited for dinner
- a and f must be visited on the same day but not monday
- e and i must be visited on the same day
- d and g must not be visited on the same day

Which One Is a Stable Matching?

m	tu	w	th	f
b	e	c	d	a
h	i	j	g	f

m	tu	w	th	f
b	i	a	c	d
g	e	f	j	h

m	tu	w	th	f
c	d	a	e	b
j	h	f	i	g

m	tu	w	th	f
e	c	d	a	b
i	j	h	g	f

m	tu	w	th	f
c	d	e	a	b
j	h	g	f	i

Which One Can Not Be True?

- e is visited on monday
- d is visited on tuesday
- f is visited on wednesday
- g is visited on thursday
- j is visited on friday

Which One Has To Be True?

- b and j are visited on the same day
- c and g are visited on the same day
- d and h are visited on the same day
- f is visited one day after e's visit
- g is visited one day after a's visit

If b is visited one day after d's visit, which one has to be true?

- b is visited on wednesday
- b is visited on thursday
- e is visited one day before f's visit
- f is visited one day before j's visit
- j and c are visited on the same day

problem # 1

True or False? In every instance of stable matching problem, there is a stable matching containing a pair (m, w) such that m is ranked first on the preference list of w and w is ranked first on preference list of m.

problem # 2

True or False? Consider an instance of the stable matching problem in which there exists a man m and a woman w such that m is ranked first on the preference list of w and w is ranked first on the preference list of m. Then in every stable matching S for this instance, the pair (m, w) belongs to S.

problem # 3

Consider two television networks a and d. There are n prime-time programming slots in the schedule and each television has n shows. each show has fixed rating. A network wins a time slot if its program has larger rating than other's. Both televisions reveal a schedule and they're allowed to change it. For every set of tv shows and rating, is there always a stable pair of schedules?

problem # 4

There are m hospitals and n medical students graduating in a given year. Each student is interested in joining one of the hospitals. Each hospital has a ranking of the students in order of preference and each student has a ranking of the hospitals in order to preference. We assume that there are more students graduating than slots available in m hospitals. Assignment of students to hospitals is instable if,

- s is assigned to h, s' is not assigned and h prefers s' to s
- s is assigned to h, s' is assigned to h', h prefers s' to s and, s' prefers h to h'

Show that there is always a stable matching between students and hospitals. Give an efficient algorithm to obtain a stable matching.

problem # 5

Now we allow ties in the ranking. For instance a woman w can say that, m_1 ranked in the first place, second place is tie between m_2 and m_3 and m_4 is in last place. Now there are two different instability types

- A strong instability in a perfect matching, a man m and a woman w prefers each other to their partners. Does there always exist a perfect marching with no strong instablity?
- A weak instability in a perfect matching, a man m with partner w' prefers the women w with partner m'. For woman w, m an m' are indifferent. Does there always a perfect matching with no weak instability?

problem # 6

Peripatetic shipping lines inc., is a shipping company that owns n ships and provides service to n ports. There is a stable monthly schedule (you can assume the month here m days and $m > n$) with following strict requirement:

- No two ships can be in the same port on the same day

The company wants to perform maintenance on all the ships this month, via following scheme. They want to truncate each ship's schedule: for each ship s_i, there will be some day when it arrives in its scheduled port and simply remains there for the rest of the month. Show that such a set of truncations can always be found and give an efficiient algorithm to find them.

- miriam-webster dictionary, 'algorithm' matter
- anany levitin, introduction to the design and analysis of algorithms, p42
- steven s. skiena, the algorithm design manual, p9-10
- george polya, how to solve it
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- 2000-2010 national computer olympics questions and solutions, altin nokta yayinlari