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DIGITAL CIRCUTIS 1st MIDTERM EXAM (Question 1)

QUESTION 1 (30 Points):

a. A is a 4-bit and B is an 8-bit, **signed**, binary integer, which are given as follows:
$A=(8)_{16}$ and $B=(FA)_{16}$.

Perform necessary operations on **binary numbers** to compare the **absolute values** ($|A|$, $|B|$) of two numbers. Decide which number has the greater absolute value by interpreting the obtained binary result. Explain the operations and interpretation of the result.

b. Assume that the numbers A and B are **unsigned**, and answer the same question.

Solution:

Note: Base 10 values are not necessary. Digital circuits can only operate on binary numbers. All operations must be performed (as written in the question) on **binary numbers**.

a.

A= 1000 Sign extension to obtain 8-bit A= 1111 1000
B= 1111 1010 (Because Hex F=1111, Hex A=1010)
Both numbers are negative (sign =1)
To obtain absolute values we apply 2's complement operations.
2's complement of A= 1111 1000 : 0000 0111 + 1 = 0000 1000 = $|A|$
2's complement of B= 1111 1010: 0000 0101 + 1 = 0000 0110 = $|B|$

To compare the absolute values we perform $|A| - |B| = |A| +$ 2's complement of $|B|$

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                      |A| :    0000 1000
2's complement of |B| :  + 1111 1010
                             1 000  0010
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Interpretation:
Absolute values are unsigned numbers. Therefore we investigate the carry (barrow).
Carry=1, that means no barrow. Consequently $|A| > |B|$

b.

A= 1000 Extension to obtain 8-bit A= 000 1000 (A is unsigned)
B= 1111 1010
As the numbers are unsigned A=$|A|$, and B= $|B|$
To compare the numbers we perform A- B = A+ 2's complement of B

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                     A :    0000 1000
2's complement of B:  + 0000 0110
                            0000 1110
```

Interpretation:
Absolute values are unsigned numbers. Therefore we investigate the carry (barrow).
Carry=0, that means barrow. Consequently A< B and $|A| < |B|$

QUESTION 2 (35 Points):
a. Using axioms and theorems of Boolean algebra show that
(a⨁b⨁c) + (a⨀b⨀c) = a⨁b⨁c
Note:
i. The output of the x⨁y function is 1 if the operands are different (x≠y).
ii. The output of the x⨀y is 1 if the operands are same (equal) (x=y).
b. A=(a_1a_0) and B=(b_1b_0) are two 2-bit, binary integers.
Find the logical expression F, that outputs 1 if A = B by using bitwise comparison.
Implement the expression using **only** 2-input NAND gates.

Solution:
a) Note:

$a\oplus b \ = \ a\bar{b} \ + \bar{a}b$ output is 1 if the operands are different (x≠y).

$a \odot b = ab + \bar{a}\bar{b}$ output is 1 if the operands are same (equal) (x=y).

Also:

$\overline{(a\oplus b)} \ = \ a \odot b$

$\overline{(a \odot b)} \ = \ a\oplus b$

$(a\oplus b\oplus c) + (a \odot b \odot c) \ = \ a\oplus b\oplus c \quad ?$

$$= \left(a\overline{(b\oplus c)} + \bar{a}(b\oplus c) \right) + \left(a\underbrace{(b \odot c)}_{\overline{(b\oplus c)}} + \bar{a}\underbrace{\overline{(b \odot c)}}_{(b\oplus c)} \right)$$

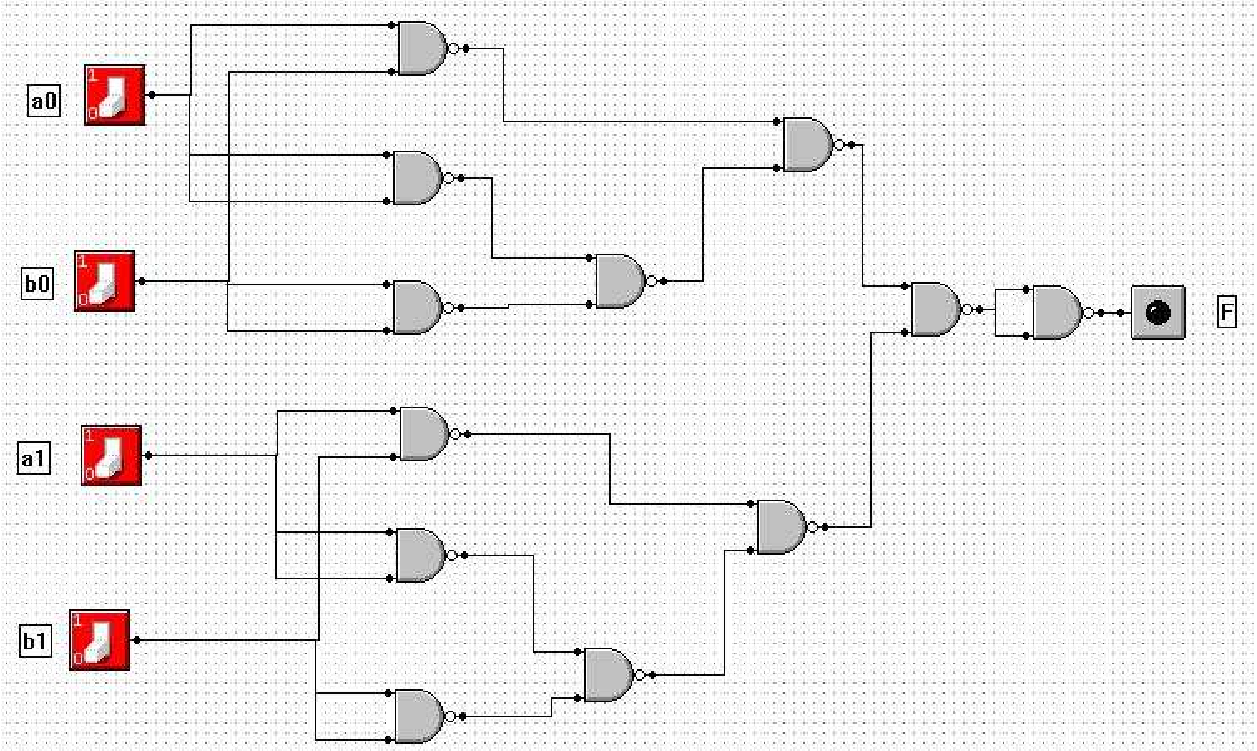
$$= \ a\overline{(b\oplus c)} + \bar{a}(b\oplus c) + \ a\overline{(b\oplus c)} + \bar{a}(b\oplus c)$$

$$= \underbrace{a\overline{(b\oplus c)} + \ a\overline{(b\oplus c)}}_{a\overline{(b\oplus c)}} + \underbrace{\bar{a}(b\oplus c) + \bar{a}(b\oplus c)}_{\bar{a}(b\oplus c)}$$

$$= \ a\overline{(b\oplus c)} + \bar{a}(b\oplus c) \ = \ a\oplus b\oplus c \quad \surd$$

Note: There are also other possible solutions.

b) $F \ = \ (a_0 \odot b_0)(a_1 \odot b_1) = \left(a_0\, b_0 + \overline{a_0\, b_0}\,\right)\left(a_1\, b_1 + \overline{a_1\, b_1}\,\right)$



QUESTION 3 (35 Points):

Consider the logical function given below:
$f(a,b,c,d)=U_1(0,1,3,6,9)+U_\Phi(4,7,8,13,15)$

a. Draw the Karnaugh map of the function f(a,b,c,d) and find all prime implicants.
b. Assuming 2 units cost for each variable and 1 unit cost for each complement, find the expression of f(a,b,c,d) with the lowest cost.
c. Implement the expression (found in b.) with **only** 2-input NAND gates.

a)

ab \ cd	00	01	11	10
00	1	1	1	
01	Φ		Φ	1
11		Φ	Φ	
10	Φ	1		

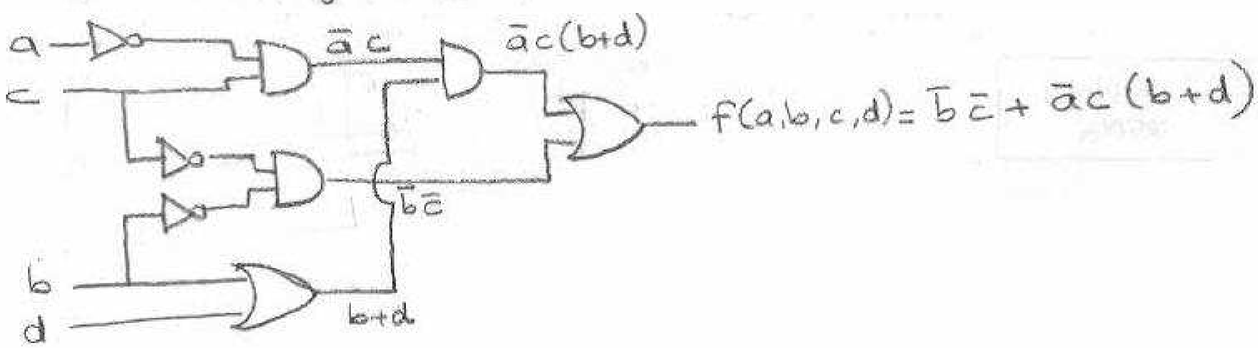
	Prime implicants	Covered points	Cost
A	$\bar{b}\bar{c}$	0, 1, 9	6
B	$\bar{a}bc$	6	7
C	$\bar{a}cd$	3	7
D	$\bar{a}b\bar{d}$	6	8
E	$a\bar{c}d$	9	7
F	$\bar{a}\bar{c}\bar{d}$	0	9
G	$\bar{a}\bar{b}d$	1, 3	8

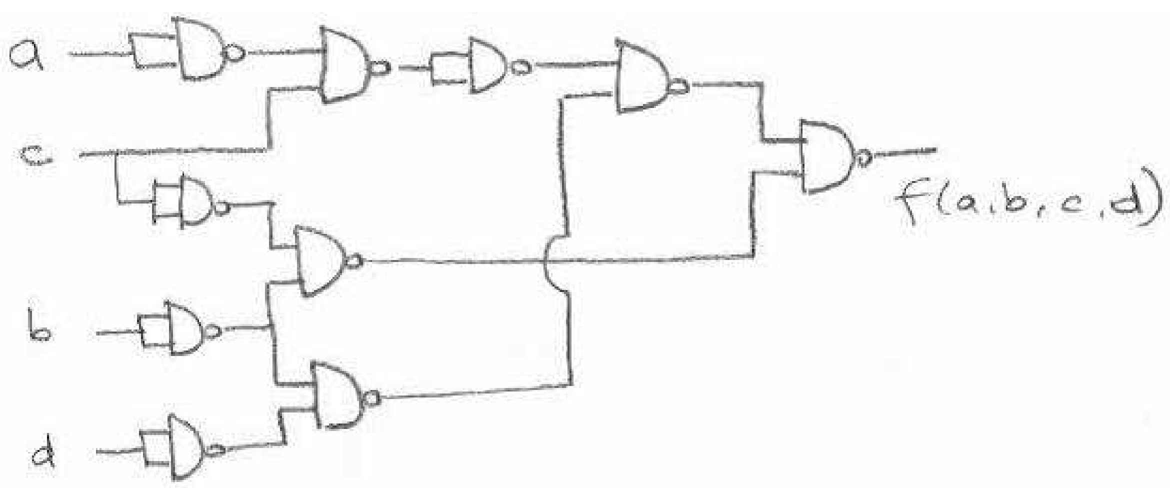
b) Prime implicant Chart

	0	1	3	6	9	Cost
A	X	X			X	6
B				X		7
C			X			7
D				X		8
E					X	7
F	X					9
G		X	X			8

1) A covers F and has lower cost. F is deleted
2) A covers E and has lower cost. E is deleted
3) B covers D and has lower cost. D is deleted
4) '3' is covered by C and G. As C has lower cost, it is selected.

$$f(a,b,c,d) = \underbrace{\bar{b}\bar{c}}_{6}^{A} + \underbrace{\bar{a}bc}_{7}^{B} + \underbrace{\bar{a}cd}_{7}^{C}$$ ← total cost = 20 units





Or

$$f(a,b,c,d) = \left[\left(\bar{b}\bar{c} + \bar{a}c(b+d)\right)'\right]'$$

$$= \left[(\bar{b}\bar{c})' \cdot \left(\bar{a}c(b+d)\right)'\right]'$$

$$= \left[(\bar{b} \downarrow \bar{c}) \cdot \left[(\bar{a}c)' + (b+d)'\right]\right]'$$

$$= \left[(\bar{b} \downarrow \bar{c}) \cdot \left\{\left[(\bar{a} \downarrow c) + (\bar{b}\,\bar{d})\right]'\right\}'\right]'$$

$$= \left[(\bar{b} \downarrow \bar{c}) \cdot \left\{(\bar{a} \downarrow c)' \cdot (\bar{b} \cdot \bar{d})'\right\}'\right]'$$

$$= \left[(\bar{b} \downarrow \bar{c}) \cdot \left\{(\bar{a} \downarrow c)' \downarrow (\bar{b} \downarrow \bar{d})\right\}\right]'$$

$$= (\bar{b} \downarrow \bar{c}) \downarrow \left\{(\bar{a} \downarrow c)' \downarrow (\bar{b} \downarrow \bar{d})\right\}$$