MAT 271E Probability and Statistics

Homework 1

Assigned: February 11, 2012

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No late homework will be accepted!

Do not copy from solutions from your classmates. All work must be your own!

Read: "Probability and Stochastic Processes", Yates and Goodman, Ch. 1 and Ch. 2

1) An integrated circuit factory has three machines X, Y, and Z. Test one integrated circuit produced by each machine. Either a circuit is acceptable (*a*) or it fails (*f*). An observation is a sequence of three test results corresponding to the circuits from machines X, Y, and Z, respectively. For example, *aaf* is the observation that the circuits from X and Y pass the test and the circuit from Z fails the test.

- **a)** What are the elements of the sample space of this experiment?
- **b)** What are the elements of the sets:
  Z_F = {circuit from Z fails}
  X_A = {circuit from X is acceptable} .
- **c)** Are Z_F and X_A mutually exclusive?
- **d)** Are Z_F and X_A collectively exhaustive?
- **e)** What are the elements of the sets
  C = {more than one circuit acceptable}
  D = {at least two circuits fail}
- **f)** Are C and D mutually exclusive?
- **g)** Are C and D collectively exhaustive?

2) Computer programs are classified by the length of the source code and by the execution time. Programs with more than 150 lines in the source code are big (*B*). Programs with $\leq$ 150 lines are little (*L*). Fast programs (*F*) run in less than 0.1 seconds. Slow programs (*W*) require at least 0.1 seconds. Monitor a program executed by a computer. Observe the length of the source code and the run time. The probability model for this experiment contains the following information: P[LF] = 0.5, P[BF] = 0.2, and P[BW] = 0.2. What is the sample space of the experiment? Calculate the following probabilities:

- **a)** P[W]
- **b)** P[B]
- **c)** $P[W \cup B]$

3) Cellular telephones perform *handoffs* as they move from cell to cell. During a call, a telephone either performs zero handoffs (H_0), one handoff (H_1), or more than one handoff (H_2). In addition, each call is either long (L), if it lasts more than 3 minutes, or brief (B). The following table describes the probabilities of the possible types of calls.

	H_0	H_1	H_2
L	0.1	0.1	0.2
B	0.4	0.1	0.1

What is the probability $P[H_0]$ that a phone makes no handoffs? What is the probability a call is brief? What is the probability a call is long or there are at least two handoffs?

4) You have a six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even-numbered.

- **a)** What is $P[R_3|G_1]$, the conditional probability that 3 is rolled given that the roll is greater than 1?
- **b)** What is the conditional probability that 6 is rolled given that the roll is greater than 3?
- **c)** What is $P[G_3|E]$, the conditional probability that the roll is greater than 3 given that the roll is even?
- **d)** Given that the roll is greater than 3, what is the conditional probability that the roll is even?

5) Assume that M, N, Q, and R are events with probabilities $P[M \cup N] = 5/8$, $P[M] = 3/8$, $P[QR] = 1/3$, and $P[Q] = 1/2$. M and N are disjoint, and Q and R are independent.

- **a)** What is $P[M \cap N]$?
- **b)** What is $P[N]$?
- **c)** What is $P[M \cap N^c]$?
- **d)** What is $P[M \cup N^c]$?
- **e)** Are M and N independent?
- **f)** What is $P[R]$?
- **g)** What is $P[Q \cup R]$?
- **h)** What is $P[Q|R]$?
- **i)** What is $P[Q \cap R^c]$?
- **j)** What is $P[Q \cup R^c]$?
- **k)** What is $P[Q^c \cap R^c]$?
- **l)** Are Q and R^c independent?

6) Suppose that for the general population, 1 in 5000 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 99% of the time.

a) What is P[-|H], the conditional probability that a person tests negative given that the person does have the HIV virus?

b) What is P[H|+], the conditional probability that a randomly chosen person has the HIV virus given that the person tests positive?