1. a. Y is necessarily discrete, the realizations $y_1=1$, $y_2=3$ can be enumerated.

b. $p_1 = Pr\{Y=y_1\} = Pr\{Y=1\} = Pr\{X \le 4\} + Pr\{X > 8\}$

$$= F_X(4) + 1 - F_X(8)$$

$p_2 = Pr\{Y=y_2\} = Pr\{Y=3\} = Pr\{4 < X \le 8\}$

$$= F_X(8) - F_X(4)$$

c) $$F_Y(y) = \sum_{i=1}^{2} p_i \, u(y-y_i)$$

$$= [F_X(4) + 1 + F_X(8)]\, u(y-1) + [F_X(8) - F_X(4)]\, u(y-3)$$

$$F_X(4) = F_U\left(\frac{4-6}{2}\right) = F_U(-1) = 1 - F_U(1) = 0.1587$$

$$F_X(8) = F_U\left(\frac{8-6}{2}\right) = F_U(1) = 0.8413$$

$$F_Y(y) = 0.3174\, u(y-1) + 0.6826\, u(y-3)$$

2. a) $$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = \int_0^1 a(x-1)^2\,dx = a\frac{(x-1)^3}{3}\Big|_0^1$$

$$= a\frac{1}{3}$$

$$\boxed{a=3}$$

b) $Pr\{X=0.5\} = \emptyset$ since X is a cont. r.v. with density given.

c). $Pr\{X>0.5\} = Pr\{X \ge 0.5\}$ since X is cont.

$$= \int_{0.5}^{1} 3(x-1)^2\,dx = 3\frac{(x-1)^3}{3}\Big|_{0.5}^{1}$$

$$= -(-0.5)^3 = \frac{1}{8}$$

d) $$Pr\{X>0.5 \mid X<0.75\} = \frac{Pr\{0.5 < X \le 0.75\}}{Pr\{X<0.75\}} = \frac{F_X(0.75) - F_X(0.5)}{F_X(0.75)}$$

$$F_X(0.5) = 1 - Pr\{X > 0.5\} = \frac{7}{8}$$

$$F_X(0.75) = \int_0^{0.75} 3(x-1)^2 dx = 3\frac{(x-1)^3}{3}\Big|_0^{0.75} = 1 - \frac{1}{64}$$

$$= \frac{63}{64}$$

$$Pr\{X > 0.5 \mid X < 0.75\} = \frac{\frac{63}{64} - \frac{7}{8}}{\frac{63}{64}}$$

$$= \frac{\frac{7}{64}}{\frac{63}{64}} = \frac{1}{9}$$

$$Pr\{X > 0.5 \mid X < 0.25\} = \frac{P(\emptyset)}{Pr\{X < 0.25\}} = \emptyset$$

e) $$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^1 x\, 3(x-1)^2 dx$$

$$= 3\left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right)\Big|_0^1$$

$$= 3\left(\frac{1}{4} - \frac{2}{3} + \frac{1}{2}\right)$$

$$= \frac{3}{12}$$

f) $$\sigma_X^2 = E[X^2] - E[X]^2$$

$$E[X^2] = \int_0^1 x^2 3(x-1)^2 dx = 3\left(\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3}\right)\Big|_0^1$$

$$= 3\left(\frac{1}{5} - \frac{2}{4} + \frac{1}{3}\right)$$

$$= \frac{3}{30} = \frac{1}{10}$$

$$\sigma_X^2 = \frac{1}{10} - \left(\frac{1}{4}\right)^2 = \frac{6}{160}$$

3\. a) $\Pr\left\{\begin{array}{c}\text{arrives late after}\\ \text{Wednesday}\end{array}\right\} = 1 - \Pr\left\{\begin{array}{c}\text{arrives late}\\ \text{on or before Wednesday}\end{array}\right\}$

$$= 1 - \Pr\left\{\begin{array}{c}\text{arrives late}\\ \text{Monday}\end{array}\right\} - \Pr\left\{\begin{array}{c}\text{arrives}\\ \text{late}\\ \text{Tuesday}\end{array}\right\} - \Pr\left\{\begin{array}{c}\text{arrives}\\ \text{late}\\ \text{Wednesday}\end{array}\right\}$$

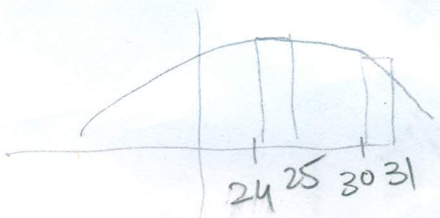
$$= 1 - \frac{1}{5} - \frac{4}{5}\cdot\frac{1}{5} - \left(\frac{4}{5}\right)^2\frac{1}{5}$$

$$= 1 - \frac{1}{5}\left(1 + \frac{4}{5} + \frac{16}{25}\right)$$

$$= \frac{64}{125}$$

b) $Np = 52\,\frac{64}{125} = 26.62$, $\sqrt{Np(1-p)} = 3.6$ } so we can use De-Moivre Laplace

$$\Pr\left\{\begin{array}{c}\text{24-30 weeks}\\ \text{first time late after}\\ \text{wednesday}\end{array}\right\} = F_U\underbrace{\left(\frac{31-26.62}{3.6}\right)}_{1.2166} - F_U\underbrace{\left(\frac{24-26.62}{3.6}\right)}_{-0.727}$$



$$= 0.891 - (1 - 0.766)$$

$$= 0.657$$

4\. if so $P(A)P(\bar{A}) = P(A \cap \bar{A}) = \emptyset$

either $P(A) = \emptyset$ or $P(A) = 1$.

Yes, but either $P(A) = 0$ or $P(\bar{A}) = \emptyset$.