Analysis of Algorithms II

PROBLEM SOLVING 6

PROBLEM 1

- A supermarket is trying to find a diverse set of at least k customers so that no two customers selected in the subset have ever bought the same item. This problem is called the Diverse Subset (DIV -SUBSET) Problem and an instance of it is given below.

- In this problem instance, if a customer i has bought item j, the table entry at (i,j) is 1, otherwise it is 0

PROBLEM 1

	Detergent (d)	Milk (m)	Cherry (c)	Bread (b)
Ahmet (A)	0	1	0	1
Bahri (B)	1	1	0	0
Cemal (C)	0	0	0	1

(a)Show that the DIV -SUBSET problem is NP-Complete, by computing a polynomial-time reduction which can convert any given instance of the DIV-SUBSET problem to an instance of the INDEPENDENT-SET problem, or anyone of the other NP Complete problems (3-SAT, VERTEX-COVER, SET-COVER, CIRCUIT-SAT) we have seen in the class

PROBLEM 1

	Detergent (d)	Milk (m)	Cherry (c)	Bread (b)
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Cemal (C)	0	0	0	1

- Let S(u,j), if customer u bought item j
- Construct a graph G(V,E) as follows
- V={set of customers}
- E(u,v): customer u and v have bought same item
- There exist j that S(u,j)=1 and S(v,j)=1
- If there exists a DIV-SUBSET of size k
- Then there is an independent set of size k

PROBLEM 1

(b) Write down the complexity of your polynomial-time reduction algorithm in terms of m (number of products) n (number of customers) and k (the subset size), explaining why: O($\mathbf{mn^2}$)

PseudoCode	**Complexity**
To construct E	
For each item j	**(m)**
V_j**: set of nodes to be connected to item i**	
V_j**=0**	
For each customer u	**(n)**
If u bought j	
$V_j = V_j$ **U {u}**	
Connect all the nodes in V_j	**(n^2)**

PROBLEM 1

(c) Apply the polynomial-time reduction that you have found to the example DIV-SUBSET problem instance below

	Detergent (d)	Milk (m)	Cherry (c)	Bread (b)
Ahmet (A)	0	1	0	1
Bahri (B)	1	1	0	0
Cemal (C)	0	0	0	1

PROBLEM 1

	Detergent (d)	Milk (m)	Cherry (c)	Bread (b)
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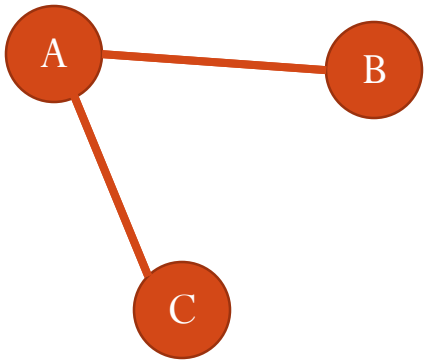
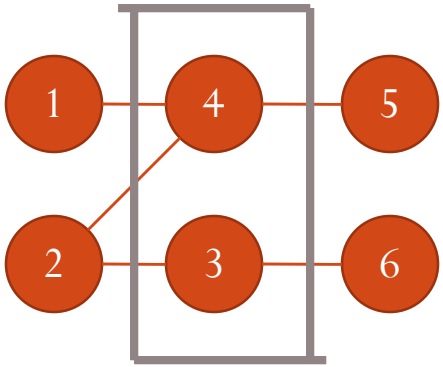


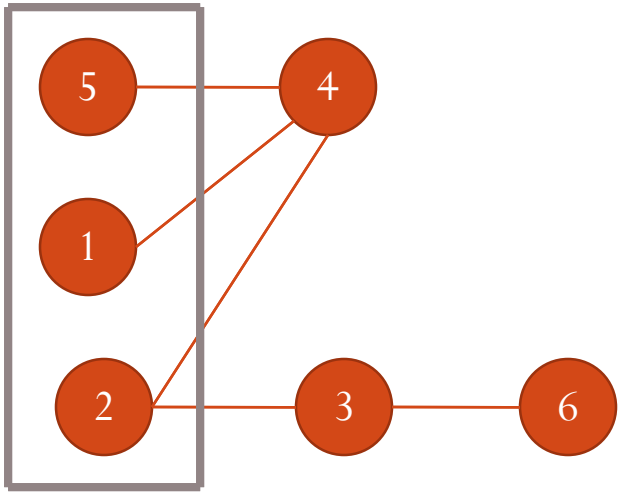
- If there is an independent set of size k, the there is a set of k customers who haven't ever bought the same item

PROBLEM 2

- Given an undirected graph G(V,E) with V={1,2,3,4,5,6} and E={(1;4),(2;3),(2;4),(3;6),(4;5)}. Consider two different subset of vertices: S1={3,4} And S2={1,2,5}

(a) Draw G, show S1. Seperately draw G and show S2





PROBLEM 2

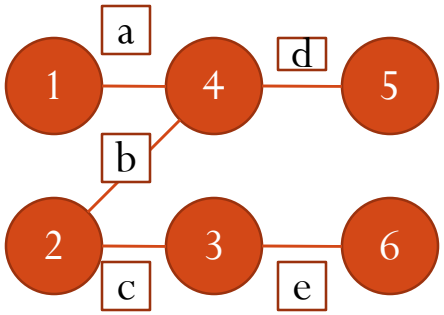
(b) Is S1 or S2 a VERTEX-COVER for the graph G? Why?

S1 is a VERTEX-COVER because the nodes {3,4} cover all edges

S2 is not a VERTEX-COVER since the edge (3,6) is not covered

PROBLEM 2

- For the subset which is a VERTEX-COVER, show the corresponding SET-COVER instance.



$U = \{a,b,c,d,e\}$

$S1 = \{a\}$

$S2 = \{b,c\}$

$S3 = \{c,e\}$

$S4 = \{a,b,d\}$

$S5 = \{d\}$

$S6 = \{e\}$

$S3 \cup S4 = \{a,b,c,d,e\} = U$

PROBLEM 3

- Suppose you are helping to organize a summer camp. The camp suppose to have at least one counsellor who is skilled at each of the n sports that are covered by the camp. (baseball, voleyball and so on)
- They have received job applications from m potential counsellors. For each of the n sports, there is some subset of m applicants qualified in that sport.
- The question is for a given; $k<m$, is it possible to hire at most k of the counsellors and have at least one counsellor qualified in each of n sports?
- We call this Efficient Recruiting Problem
- Show the Efficient Recruiting is NP Complete.

PROBLEM 3

- A set of k counsellor can cover all the sports??
- We would solve an instance of VERTEX-COVER

G(V,E): graph

S_e: sport for each edge e

C_v: counsellor for each vertex v

C_v is qualified in a sport S_e if only if e has an endpoint equal to v

PROBLEM 3

- There are k counsellors that together are qualified in all sports, the corresponding vertices in G have the propoerty that each edge has an end in at least one of them; so they define a VERTEX-COVER of size k.

- If there is a vertex cover of size k, this set of counsellors has the property that each sport is contained in the list of qualifications of at least one of them.

PROBLEM 3

- The instance of Efficient Recruiting has size polynomial in the size of G.

- We could determine the answer to the Efficient Recruiting instance in polynomial time, we could also solve the instance of Vertex Cover in poloynomial time.

PROBLEM 4-Quiz 3

- You and your friends Ali and Ayşe rank the 4 songs from the most liked to the least liked:
- 1: "Hey Jude", Beattles
- 2: "We Are the Champions", Queen
- 3: "Iron Man", Black Sabbath
- 4: "Uzun İnce Bir Yoldayım", Aşık Veysel

	Most Liked			Least Liked
You	3	4	2	1
Ali	2	3	1	4
Ayşe	3	1	4	2

- Ali and Ayşe separately invite you for a concert, you do not know who is singing. Which one would you go with?

PROBLEM 4- Quiz 3

- Find the number of inversions between you-Ali, you-Ayşe. Whichever is minimum, that could be your concert partner

	MostLiked 1	 2	 3	Least Liked 4
Ali	3	1	4	2
Ayşe	1	4	2	3

Find the number of inversions between you-Ali

- Inv(3, 1, 4, 2)
- Inv(3, 1) = 1+ Inv(4,2)=1 + Inv((1,3),(2,4))= 1 (inversion: 3,2): TOTAL: 3

Problem 4- Quiz 3

	MostLiked 1	2	3	Least Liked 4
Ali	3	1	4	2
Ayşe	1	4	2	3

Between you-Ayse

- Inv(1,4,2,3)
- Inv(1,4)=0 + Inv(2,3)=0 + Inv((1,4),(2,3)) = 2 (Inversion: (4,2): TOTAL: 2

You should go to the concert with Ayşe.

QUESTIONS??