BLG 335E – Analysis of Algorithms I
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Outline

- Elementary Data Structures
- Medians and Order Statistics
- Hash Tables

Extra Exercises from Rec. 3

- Implement a **queue** by a singly linked list **L**. The operations **Enqueue** and **Dequeue** should still take **O(1)** time.
- Using a tail pointer beside head pointer !

Extra Exercises from Rec. 3

- Write an **O(n)-time** recursive procedure that, given an n-node **binary tree**, prints out the key of each node in the tree.

Tree Traversal (Recursively)

- Inorder
- Preorder
- Postorder

Tree Traversal (Recursively)

- Inorder

```
void Inorder(node *nptr){
  if(nptr){
    Inorder(nptr->left);
    cout << nptr->number << endl;
    Inorder(nptr->right);
  }
}
```

Tree Traversal (Recursively)

- Preorder

```
void Preorder(node *nptr){
   if(nptr){
      cout << nptr->number << endl;
      Preorder(nptr->left);
      Preorder(nptr->right);
   }
}
```

Tree Traversal (Recursively)

- Postorder

```
void Postorder(node *nptr){
   if(nptr){
      Postorder(nptr->left);
      Postorder(nptr->right);
      cout << nptr->number << endl;
   }
}
```

Question 2

- Show that the second smallest of n elements can be found with

  $n + \lceil \lg n \rceil - 2$

comparisons in the worst case.

(*Hint:* Also find the smallest element.)

Question 2

- What about the minimum element?
- Conduct a tournament by using the pairs all the time.
- Consider a tree structure. Leaves are numbers, each inner node corresponds to comparions.
- By doing so, the minimum element can be found with n – 1 comparisons.

Question 2

- What about the second minimum?
- In the search for the smallest number, the second smallest number must have come out smallest in every comparison made with it until it was eventually compared with the smallest.
- So the second smallest is one of them !
- What to do now?

Question 2

- Second tournament is applied to this subset again.
- At most $\lceil \lg n \rceil$ elements (Depth of the tree).
- What about the number of comparisons?
- $\lceil \lg n \rceil$ - 1

Question 2

- What about total number of comparisons?
- n – 1 + $\lceil \lg n \rceil$ – 1
- n + $\lceil \lg n \rceil$ – 2

Question 4

- Consider inserting the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 into a hash table of length $m = 11$ using open addressing with the primary hash function $h'(k) = k \bmod m$.
- Illustrate the result of inserting these keys using **linear probing**, using **quadratic probing** with $c_1 = 1$ and $c_2 = 3$, and using **double hashing** with $h_2(k) = 1 + (k \bmod (m - 1))$.

Answer 4: Using Linear Probing

- **Linear probing:** $h(k, i) = (h'(k) + i) mod\, m$
- $h'(k) = k\, mod\, m$
- $m = 11,\ i = \{0, 1, 2, \ldots, m-1\}$
- Set of keys: $\{10, 22, 31, 4, 15, 28, 17, 88, 59\}$

$h(10{,}0) = 10$	$h(28{,}0) = 6$	$h(59{,}0) = 4$
$h(22{,}0) = 0$	$h(17{,}0) = 6$	$\boldsymbol{h(59, 1) = 5}$
$h(31{,}0) = 9$	$\boldsymbol{h(17, 1) = 7}$	$\boldsymbol{h(59, 2) = 6}$
$h(4{,}0) = 4$	$h(88{,}0) = 0$	$\boldsymbol{h(59, 3) = 7}$
$h(15{,}0) = 4$	$\boldsymbol{h(88, 1) = 1}$	$\boldsymbol{h(59, 4) = 8}$
$\boldsymbol{h(15, 1) = 5}$		

- The resulting hash table:
  - $H = \{22, 88, nil, nil, 4, 15, 28, 17, 59, 31, 10\}$

Answer 4: Using Quadratic Probing

- **Quadratic probing:** $h(k,i) = (h'(k) + c_1 i + c_2 i^2) mod\ m$
- $h'(k) = k\ mod\ m, c_1 = 1, c_2 = 3$
- $m = 11,\ i = \{0, 1, 2, \dots, m-1\}$
- Set of keys: $\{10, 22, 31, 4, 15, 28, 17, 88, 59\}$

$h(10,0) = 10$	$h(17,0) = 6$	$\boldsymbol{h(88,3) = 8}$
$h(22,0) = 0$	$\boldsymbol{h(17,1) = 10}$	$\boldsymbol{h(88,4) = 8}$
$h(31,0) = 9$	$\boldsymbol{h(17,2) = 9}$	$\boldsymbol{h(88,5) = 3}$
$h(4,0) = 4$	$\boldsymbol{h(17,3) = 3}$	$\boldsymbol{h(88,6) = 4}$
$h(15,0) = 4$	$h(88,0) = 0$	$\boldsymbol{h(88,7) = 0}$
$\boldsymbol{h(15,1) = 8}$	$\boldsymbol{h(88,1) = 4}$	$\boldsymbol{h(88,8) = 2}$
$h(28,0) = 6$	$\boldsymbol{h(88,2) = 3}$	$h(59,0) = 4$
		$\boldsymbol{h(59,1) = 8}$
		$\boldsymbol{h(59,2) = 7}$

- The resulting hash table:
  - $H = \{22, nil, 88, 17, 4, nil, 28, 59, 15, 31, 10\}$

Answer 4: Using Double Hashing

- **Double Hashing:** $h(k,i) = \left(h_1(k) + ih_2(k)\right) mod\ m$
- $h_1(k) = k\ mod\ m$ and $h_2(k) = 1 + k\ mod\ (m-1)$
- $m = 11,\ i = \{0, 1, 2, \ldots, m-1\}$
- Set of keys: $\{10, 22, 31, 4, 15, 28, 17, 88, 59\}$

$h(10,0) = 10$	$\boldsymbol{h(15,2) = 5}$	$\boldsymbol{h(88,2) = 7}$
$h(22,0) = 0$	$h(28,0) = 6$	$h(59,0) = 4$
$h(31,0) = 9$	$h(17,0) = 6$	$\boldsymbol{h(59,1) = 3}$
$h(4,0) = 4$	$\boldsymbol{h(17,1) = 3}$	$\boldsymbol{h(59,2) = 2}$
$h(15,0) = 4$	$h(88,0) = 0$	
$\boldsymbol{h(15,1) = 10}$	$\boldsymbol{h(88,1) = 9}$	

- The resulting hash table:
  - $H = \{22, nil, 59, 17, 4, 15, 28, 88, nil, 31, 10\}$