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2012-2013 Spring Term
MAT 202E
NUMERICAL METHODS
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MIDTERM EXAM-II SOLUTIONS

Q1) (35 pts)

We need to take natural logarithm both sides of equation.

$\ln(-r) = \ln k + n * \ln C$
$z = \ln(-r)$
$w = \ln C$
$a_0 = \ln k$
$a_1 = n$
We get $z = a_0 + a_1 w$

$$a_1 = \frac{n\sum_{i=1}^{n} w_i z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} z_i}{n\sum_{i=1}^{n} w_i^2 - (\sum_{i=1}^{n} w_i)^2}$$

$$a_0 = \frac{\sum_{i=1}^{n} z_i}{n} - a_1 \frac{\sum_{i=1}^{n} w_i}{n}$$

$n = 7$

$$\sum_{i=1}^{n} w_i = -4.3643 \quad \sum_{i=1}^{n} z_i = -12.391 \quad \sum_{i=1}^{n} w_i z_i = 16.758 \quad \sum_{i=1}^{n} {w_i}^2 = 30.998$$

$a_1 = 0.31941$
$a_0 = -1.5711$

$k = e^{-1.5711}$
$n = a_1 = 0.31941$
$-r = 0.20782 * C^{0.31941}$

For C=3, -r=0.29517

2) (30 pts)

Temperature vs. depth for a lake

Temperature $T\ (°C)$	Depth $z\ (m)$
19.1	0
19.1	–1
19	–2
18.8	–3
18.7	–4
18.3	–5
18.2	–6
17.6	–7
11.7	–8
9.9	–9
9.1	–10

a) (5 pts)

Sudden change where (T_n-T_{n+1}>5 ^{o}C) is between {(17.6 ^{o}C, 11.7 ^{o}C), (-7 m, -8 m)}

b) (25 pts)

$$z_{wanted} = \frac{z_n^{featured} + z_{n+1}^{featured}}{2} = (-7 + (-8))/2 = -7.5 \text{ m}$$

$$T(z) = T(z_0) + \frac{T(z_1) + T(z_0)}{z_1 - z_0}(z - z_0)$$

$$T(z) = 58.9 + 5.9z$$

$$T(z_{wanted}) = 14.65\ ^0C$$

Q3) (35 pts) Refers to course slides. (week-10 pages 60-65)