BLG311E – FORMAL LANGUAGES AND AUTOMATA

2013 SPRING

RECITEMENT 4

(Solutions for Midterm 1)

1) A coffee vending machine that accepts coins of 5, 10 and 25 cents, gives coffee for 15 cents and returns the change. Coffee giving circuitry is made up of asynchronous circuits that is independent of the system clock and gives only one cup of coffee for every transition 0 to 1 at its input. There are also three change return systems for each type of coins and they give only one coin for each transition from 0 to 1 at their inputs, such as coffee giving circuitry. For this purpose,

a) (10p) Design ASM charts for Mealy and Moore approaches.

b) (10p) For Mealy and Moore approaches, design state control circuitry by performing state coding. DO NOT assign one flip-flop to each state.

c) (10p) For each approach, design the data unit.

SOLUTION:

For the Mealy model:

State table:

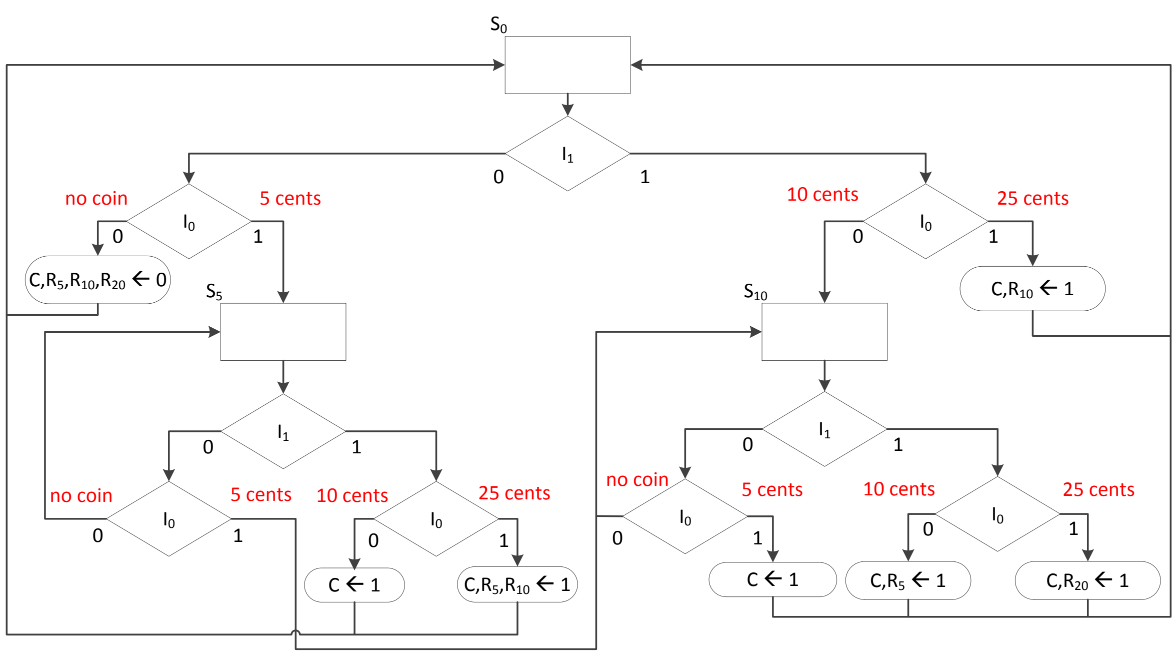
	5	**10**	**25**
S_0	S_5/0,-	S_{10}/0,-	S_0/10,C
S_5	S_{10}/0,-	S_0/0,C	S_0/15,C
S_{10}	S_0/0,C	S_0/5,C	S_0/20,C

Inputs:

I_1	**I_0**	
0	0	no coin
0	1	5 cents
1	0	10 cents
1	1	25 cents

This question can also be solved by using the inputs without encoding as I_5, I_{10} and I_{25} for each type of coins.

ASM diagram:



State control circuit:

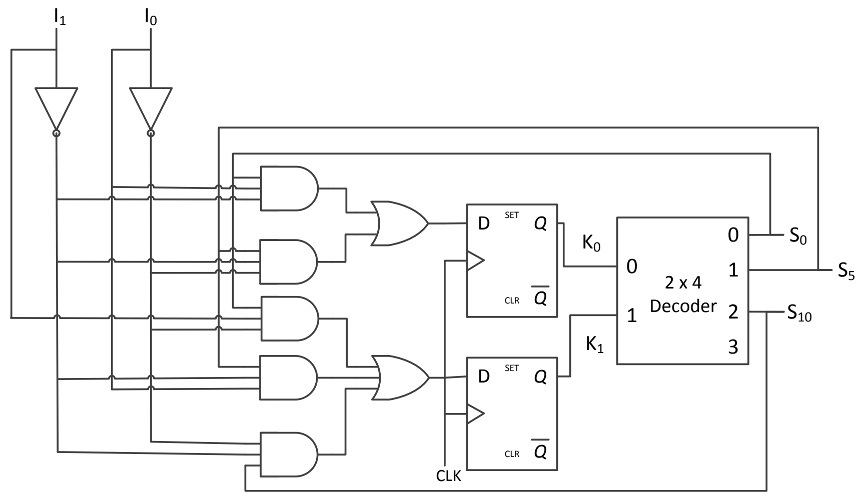
$S_0 = (I_1 \odot I_0)S_0 + I_1S_5 + (I_1+I_0)S_{10}$

$S_5 = I_1'I_0S_0 + I_1'I_0'S_5$

$S_{10} = I_1I_0'S_0 + I_1'I_0S_5 + I_1'I_0'S_{10}$

For state coding:

- $S_0 \rightarrow K_1'K_0'$
- $S_5 \rightarrow K_1'K_0$
- $S_{10} \rightarrow K_1K_0'$
- $K_1 = I_1I_0'S_0 + I_1'I_0S_5 + I_1'I_0'S_{10}$
- $K_0 = I_1'I_0S_0 + I_1'I_0'S_5$



Data unit:

$C = I_1I_0S_0 + I_1S_5 + (I_1+I_0)S_{10}$

$R_5 = I_1I_0S_5 + I_1I_0'S_{10}$

$R_{10} = I_1I_0(S_0 + S_5)$

$R_{20} = I_1I_0S_{10}$

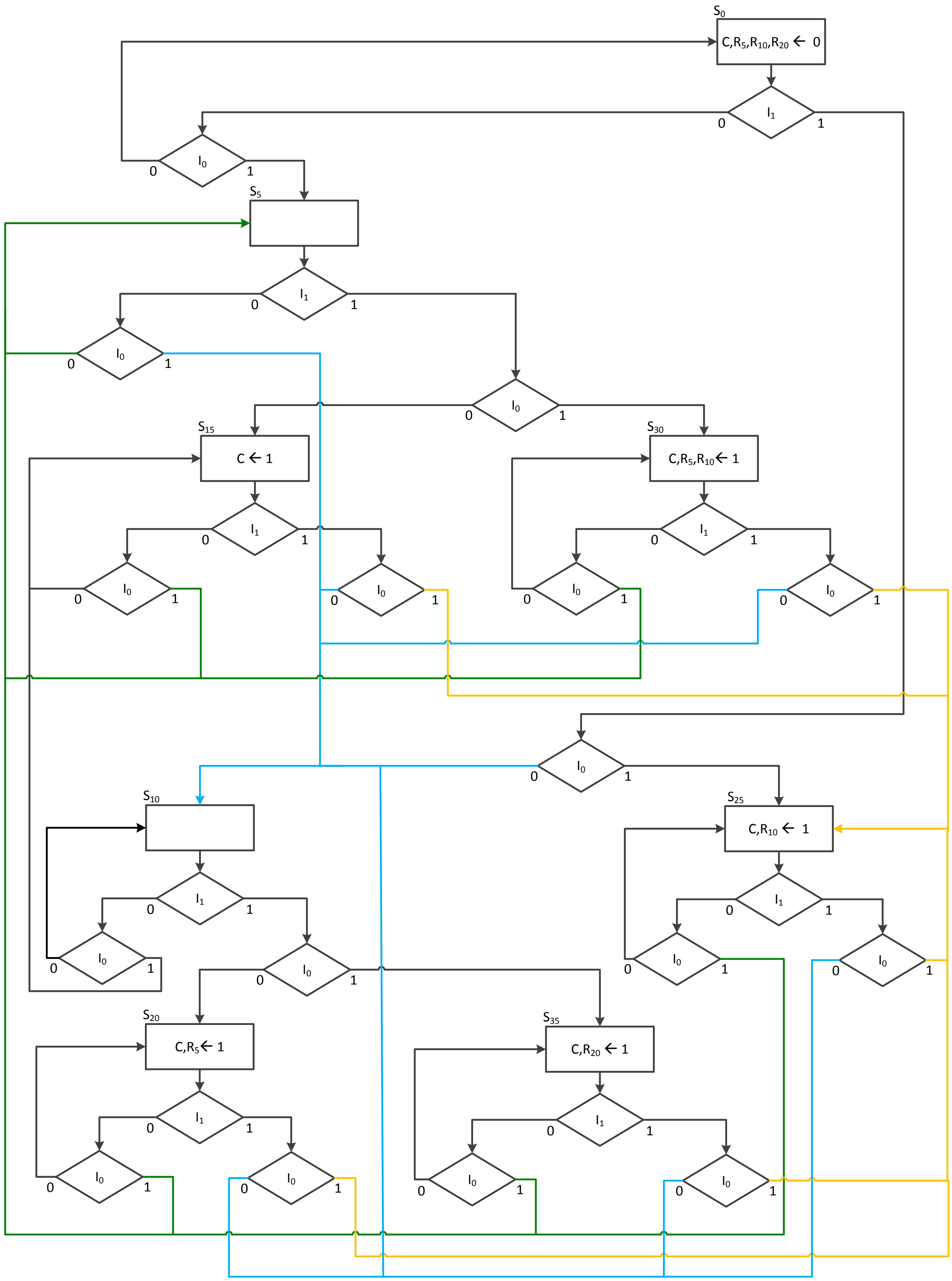
For the Moore model:

State table:

	5	**10**	**25**	**Output**
$\mathbf{S_0}$	S_5	S_{10}	S_{25}	0,-
$\mathbf{S_5}$	S_{10}	S_{15}	S_{30}	0,-
$\mathbf{S_{10}}$	S_{15}	S_{20}	S_{35}	0,-
$\mathbf{S_{15}}$	S_5	S_{10}	S_{25}	0,C
$\mathbf{S_{20}}$	S_5	S_{10}	S_{25}	5,C
$\mathbf{S_{25}}$	S_5	S_{10}	S_{25}	10,C
$\mathbf{S_{30}}$	S_5	S_{10}	S_{25}	15,C
$\mathbf{S_{35}}$	S_5	S_{10}	S_{25}	20,C

Inputs I_1 and I_0 are the same as the Mealy model.

ASM diagram:



S0
C,R5,R10,R20 ← 0
I1
0
1
I0
0
1
S5
I1
0
1
I0
0
1
I0
0
1
S15
C ← 1
S30
C,R5,R10 ← 1
I1
0
1
I0
0
1
S10
S25
C,R10 ← 1
S20
C,R5 ← 1
S35
C,R20 ← 1

State control circuit:

$S_0 = I_1'I_0'S_0$

$S_5 = I_1'I_0(S_0 + S_{15} + S_{20} + S_{25} + S_{30} + S_{35}) + I_1'I_0'S_5$

$S_{10} = I_1I_0'(S_0 + S_{15} + S_{20} + S_{25} + S_{30} + S_{35}) + I_1'I_0S_5 + I_1'I_0'S_{10}$

$S_{15} = I_1I_0'S_5 + I_1'I_0S_{10} + I_1'I_0'S_{15}$

$S_{20} = I_1I_0'S_{10} + I_1'I_0'S_{20}$

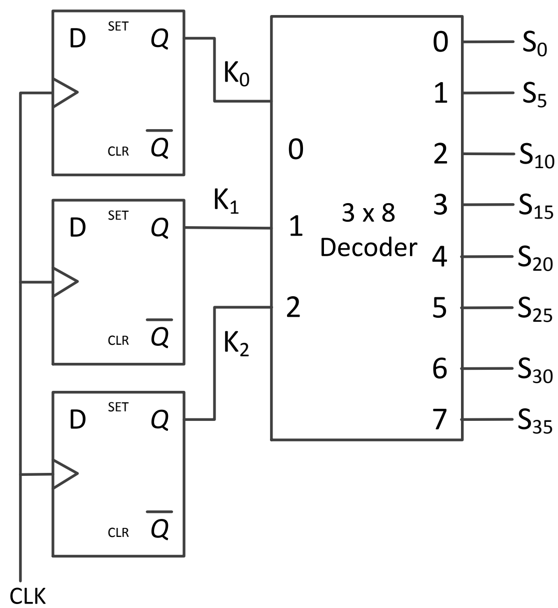
$S_{25} = I_1I_0(S_0 + S_{15} + S_{20} + S_{25} + S_{30} + S_{35}) + I_1'I_0'S_{25}$

$S_{30} = I_1I_0S_5 + I_1'I_0'S_{30}$

$S_{35} = I_1I_0S_{10} + I_1'I_0'S_{35}$

For state coding:

- $S_0 \rightarrow K_2'K_1'K_0'$
- $S_5 \rightarrow K_2'K_1'K_0$
- $S_{10} \rightarrow K_2'K_1K_0'$
- $S_{15} \rightarrow K_2'K_1K_0$
- $S_{20} \rightarrow K_2K_1'K_0'$
- $S_{25} \rightarrow K_2K_1'K_0$
- $S_{30} \rightarrow K_2K_1K_0'$
- $S_{35} \rightarrow K_2K_1K_0$



$K_2 = I_1I_0(S_0 + S_5 + S_{15}) + I_1S_{10} + (I_1 \odot I_0)(S_{20} + S_{25} + S_{30} + S_{35})$

$K_1 = I_1I_0'(S_0 + S_{20} + S_{25}) + I_0(S_5 + S_{10}) + I_1I_0'S_5 + I_1'I_0'S_{10} + I_0'(S_{15} + S_{30} + S_{35})$

$K_0 = I_0(S_0+S_{10}+S_{15}+S_{20}+S_{25}+S_{30}+S_{35})+I_0'S_5 + (I_1'I_0')(S_{15}+S_{25}+S_{35})$

Data unit:

$C = S_{15} + S_{20} + S_{25} + S_{30} + S_{35}$

$R_5 = S_{20} + S_{30}$

$R_{10} = S_{25} + S_{30}$

$R_{20} = S_{35}$

2) (20p) Simplify the given state transition table

State\inputs	a	b	Output
Q_0	Q_1	Q_2	0
Q_1	Q_1	Q_3	0
Q_2	Q_1	Q_2	0
Q_3	Q_1	Q_4	0
Q_4	Q_1	Q_2	1

SOLUTION:

	Q_0	**Q_1**	**Q_2**	**Q_3**
Q_1	(Q_2, Q_3) X			
Q_2	**OK**	(Q_2, Q_3) X		
Q_3	(Q_2, Q_4) X	(Q_3, Q_4) X	(Q_2, Q_4) X	
Q_4	X	X	X	X

State\inputs	a	b	Output
$S_0=\{Q_0,Q_2\}$	S_1	S_0	0
$S_1=\{Q_1\}$	S_1	S_2	0
$S_2=\{Q_3\}$	S_1	S_3	0
$S_3=\{Q_4\}$	S_1	S_0	1

3) (20p) Let A be a language defined over Σ. What is the minimum possible value of X in the statement below

$$(\mathrm{AA}^{+})^{*} \cup \mathrm{X} = \mathrm{A}^{*}$$

SOLUTION:

$AA^{+} = \{AA, AAA, AAAA \ldots\}$

$(AA^{+})^{*} = \{\Lambda, AA, AAA, AAAA \ldots\}$

$A^{*} = \{\Lambda, A, AA, AAA, AAAA \ldots\}$

$A^{*} \setminus (AA^{+})^{*} = \{A\}$

$X = A$

4) (30p) Let α be defined over set A. The reflexive closure α' of this relation should hold the following properties

(a) α' is reflexive

(b) $\alpha \subseteq \alpha'$

(c) If $\alpha \subseteq \alpha''$ and α'' is reflexive then $\alpha' \subseteq \alpha''$.

Suppose R is a relation on A

- Prove that R is reflexive if and only if (⇔) R is its own reflexive closure, using the properties defined above.
- Do similar theorems hold for symmetry and transitivity? Justify your answers with proofs and counterexamples.

SOLUTION:

Let S be the reflexive closure of R

- (→) Suppose R is reflexive. By clause (b) $R \subseteq S$ and by clause (c) (with $\alpha'' = R$) $S \subseteq R$. Therefore $R = S$
- (←) Suppose $R = S$. By clause (a) in the definition, R is reflexive.

Same properties exists for symmetry and transitivity, their proofs are similar.