

## Hashing and Natural Parallism

Companion slides for  
The Art of Multiprocessor  
Programming  
by Maurice Herlihy & Nir Shavit

## Linked Lists

- We looked at a number of ways to make highly-concurrent list-based Sets:
  - Fine-grained locks
  - Optimistic synchronization
  - Lazy synchronization
  - Lock-free synchronization
- What's missing?

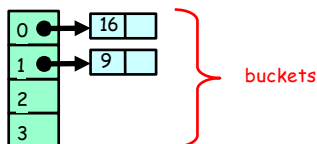
## Linear-Time Set Methods

- Problem is
  - add(), remove(), contains()
  - Take time linear in set size
- We want
  - Constant-time methods
  - (at least, on average)

## Hashing

- Hash function
  - $h$ : items  $\rightarrow$  integers
- Uniformly distributed
  - Different item most likely have different hash values
- Java hashCode() method

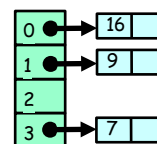
## Sequential Hash Map



2 Items

$$h(k) = k \bmod 4$$

## Add an Item



3 Items

$$h(k) = k \bmod 4$$

### Add Another: Collision

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4 Items

$h(k) = k \bmod 4$

7

### More Collisions

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5 Items

$h(k) = k \bmod 4$

8

### More Collisions

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5 Items

$h(k) = k \bmod 4$

Problem: buckets getting too long

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### Resizing

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5 Items

$h(k) = k \bmod 8$

Grow the array

10

### Resizing

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5 Items

$h(k) = k \bmod 8$

Adjust hash function

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### Resizing

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$h(4) = 0 \bmod 8$

$h(k) = k \bmod 8$

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## Resizing

$h(4) = 4 \bmod 8$

$h(k) = k \bmod 8$

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## Resizing

$h(15) = 7 \bmod 8$

$h(k) = k \bmod 8$

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## Resizing

$h(15) = 7 \bmod 8$

$h(k) = k \bmod 8$

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## Hash Sets

- Implement a Set object
  - Collection of items, no duplicates
  - `add()`, `remove()`, `contains()` methods
  - You know the drill ...

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## Simple Hash Set

```

public class SimpleHashSet {
    protected LockFreeList[] table;
    public SimpleHashSet(int capacity) {
        table = new LockFreeList[capacity];
        for (int i = 0; i < capacity; i++)
            table[i] = new LockFreeList();
    }
    ...
}

```

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## Add Method

```

public boolean add(Object key) {
    int hash = key.hashCode() % table.length;
    return table[hash].add(key);
}

```

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## No Brainer?



- We just saw a
  - Simple
  - Lock-free
  - Concurrent hash-based set implementation
- What's not to like?

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## No Brainer?



- We just saw a
  - Simple
  - Lock-free
  - Concurrent hash-based set implementation
- What's not to like?
- We don't know how to resize ...

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## Is Resizing Necessary?



- Constant-time method calls require
  - Constant-length buckets
  - Table size proportional to set size
  - As set grows, must be able to resize

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## Set Method Mix



- Typical load
  - 90% contains()
  - 9% add()
  - 1% remove()
- Growing is important
- Shrinking not so much

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## When to Resize?



- Many reasonable policies. Here's one.
- Pick a threshold on num of items in a bucket
- Global threshold
  - When  $\geq 1/4$  buckets exceed this value
- Bucket threshold
  - When any bucket exceeds this value

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## Coarse-Grained Locking



- Good parts
  - Simple
  - Hard to mess up
- Bad parts
  - Sequential bottleneck

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## Fine-grained Locking

Each lock associated with one bucket

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## Resize This

Make sure table reference didn't change between resize decision and lock acquisition

Acquire locks in ascending order

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## Resize This

Allocate new super-sized table

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## Resize This

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## Resize This

Striped Locks: each lock now associated with two buckets

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## Observations

- We grow the table, but not locks
  - Resizing lock array is tricky ...
- We use sequential lists
  - Not LockFreeList lists
  - If we're locking anyway, why pay?

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## Fine-Grained Hash Set



```
public class FGHashSet {

    protected RangeLock[] lock;
    protected List[] table;

    public FGHashSet(int capacity) {

        table = new List[capacity];
        lock = new RangeLock[capacity];

        for (int i = 0; i < capacity; i++) {
            lock[i] = new RangeLock();
            table[i] = new LinkedList();
        } ...
    }
}
```

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## The add() method



```
public boolean add(Object key) {

    int keyHash = key.hashCode() % lock.length;

    synchronized (lock[keyHash]) {

        int tabHash = key.hashCode() % table.length;

        return table[tabHash].add(key);

    }
}
```

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## Fine-Grained Locking



```
private void resize(int depth, List[] oldTab) {

    synchronized (lock[depth]) {

        if (oldTab == this.table){

            int next = depth + 1;

            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();

        }
    }
}
```

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## Fine-Grained Locking



```
private void resize(int depth,
                    List[] oldTab) {

    synchronized (lock[depth]) {
        if (oldTab == this.table){
            int next = depth + 1;
            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();
        }
    }
}
```

resize() calls  
resize(0, this.table)

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## Fine-Grained Locking



```
private void resize(int depth,
                    List[] oldTab) {

    synchronized (lock[depth]) {
        if (oldTab == this.table){
            int next = depth + 1;
            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();
        }
    }
}
```

Acquire next lock

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## Fine-Grained Locking



```
private void resize(int depth,
                    List[] oldTab) {

    synchronized (lock[depth]) {
        if (oldTab == this.table) {
            int next = depth + 1;
            if (next < lock.length)
                resize (next, oldTab);
            else
                sequentialResize();
        }
    }
}
```

Check that no one else has resized

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## Fine-Grained Locking



### Recursively acquire next lock

```
private synchronized (lock[depth]) {  
    if (oldTab == this.table){  
        int next = depth + 1;  
        if (next < lock.length)  
            resize (next, oldTab);  
        else  
            sequentialResize();  
    }  
}
```

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## Fine-Grained Locking



### Locks acquired, do the work

```
private synchronized (lock[depth]) {  
    if (oldTab == this.table){  
        int next = depth + 1;  
        if (next < lock.length)  
            resize (next, oldTab);  
        else  
            sequentialResize();  
    }  
}
```

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## Fine-Grained Locks



- We can resize the table
- But not the locks
- Debatable whether method calls are constant-time in presence of contention ...

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## Insight



- The contains() method
  - Does not modify any fields
  - Why should concurrent contains() calls conflict?

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## Read/Write Locks



```
public interface ReadwriteLock {  
    Lock readLock();  
    Lock writeLock();  
}
```

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## Lock Safety Properties



- No thread may acquire the write lock
  - while any thread holds the write lock
  - or the read lock.
- No thread may acquire the read lock
  - while any thread holds the write lock.
- Concurrent read locks OK

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## Read/Write Lock



- Satisfies safety properties
  - If readers > 0 then writer == false
  - If writer = true then readers == 0
- Liveness?
  - Lots of readers ...
  - Writers locked out?

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## FIFO R/W Lock



- As soon as a writer requests a lock
- No more readers accepted
- Current readers “drain” from lock
- Writer gets in

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## The Story So Far



- Resizing the hash table is the hard part
- Fine-grained locks
  - Striped locks cover a range (not resized)
- Read/Write locks
  - FIFO property tricky

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## Optimistic Synchronization



- If the contains() method
  - Scans without locking
- If it finds the key
  - OK to return true
  - Actually requires a proof ....
- What if it doesn't find the key?

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## Optimistic Synchronization



- If it doesn't find the key
  - May be victim of resizing
- Must try again
  - Getting a read lock this time
- Makes sense if
  - Keys are present
  - Resizes are rare

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## Stop The World Resizing



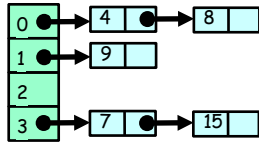
- The resizing we have seen up till now stops all concurrent operations
- Can we design a resize operation that will be incremental
- Need to avoid locking the table
- A lock-free table with incremental resizing

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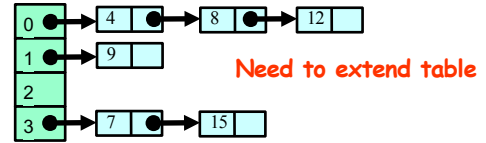
## Lock-Free Resizing Problem



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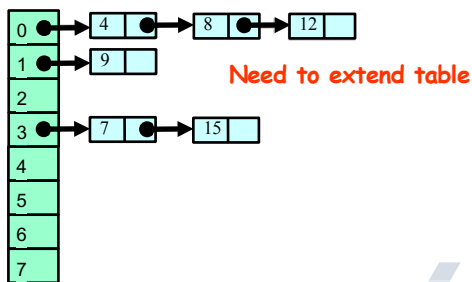
## Lock-Free Resizing Problem



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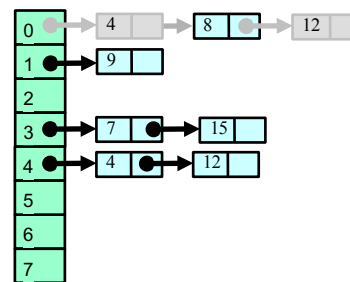
## Lock-Free Resizing Problem



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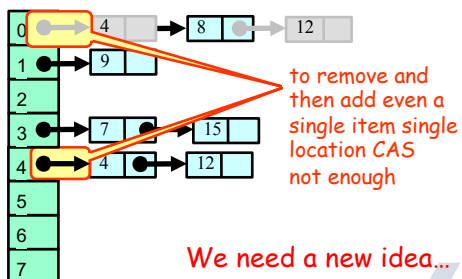
## Lock-Free Resizing Problem



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## Lock-Free Resizing Problem



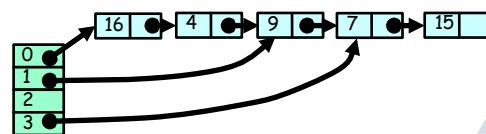
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## Don't move the items



- Move the buckets instead
- Keep all items in a single lock-free list
- Buckets become "shortcut pointers" into the list



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### Recursive Split Ordering

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### Recursive Split Ordering

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### Recursive Split Ordering

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### Recursive Split Ordering

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List entries sorted in order that allows recursive splitting. How?

### Recursive Split Ordering

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### Recursive Split Ordering

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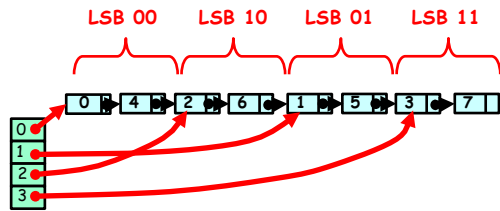
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LSB 0

LSB 1

LSB = Least significant Bit

## Recursive Split Ordering



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## Split-Order



- If the table size is  $2^i$ ,
  - Bucket  $b$  contains keys  $k$ 
    - $k = b \pmod{2^i}$
  - bucket index consists of key's  $i$  LSBs

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## When Table Splits

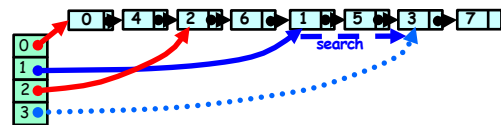


- Some keys stay
  - $b = k \pmod{2^{i+1}}$
- Some move
  - $b+2^i = k \pmod{2^{i+1}}$
- Determined by  $(i+1)^{\text{st}}$  bit
  - Counting backwards
- Key must be accessible from both
  - Keys that will move must come later

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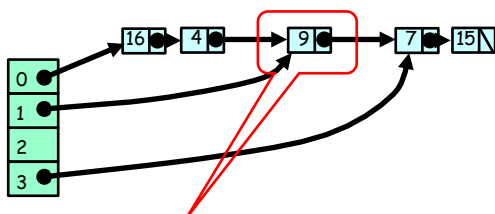
## Parent Always Provides a Short Cut



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## Sentinel Nodes

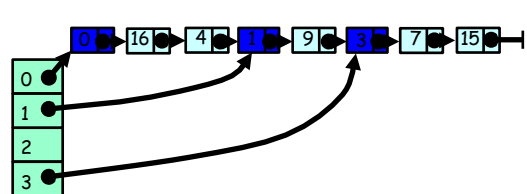


Problem: how to remove a node pointed by 2 sources using CAS

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## Sentinel Nodes



Solution: use a Sentinel node for each bucket

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## Sentinel vs Regular Keys



- Want sentinel key for  $i$  ordered
  - before all keys that hash to bucket  $i$
  - after all keys that hash to bucket  $(i-1)$

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## Splitting a Bucket

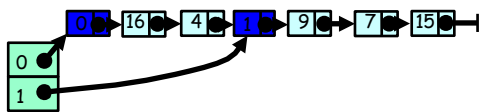


- We can now split a bucket
- In a lock-free manner
- Using two CAS() calls ...

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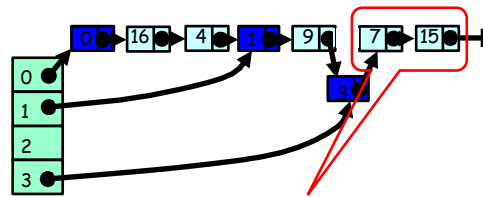
## Initialization of Buckets



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## Initialization of Buckets

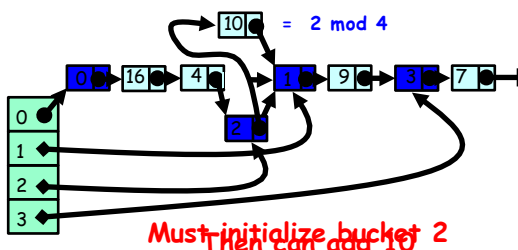


Need distribution to place 3rd bucket key!!  
Now 5 points to sentinel - bucket has been split

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## Adding 10



Must initialize bucket 2  
Then can add 10

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## Main List



- Lock-Free List from earlier lecture
- With some minor variations

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## Split-Ordered Set: Fields



```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(2);
        setSize = new AtomicInteger(0);
    }
}
```

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## Fields



```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(2);
        setSize = new AtomicInteger(0);
    }
}
```

For simplicity treat table as big array ...

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## Fields



```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(2);
        setSize = new AtomicInteger(0);
    }
}
```

In practice, want something that grows dynamically

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## Fields



```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(2);
        setSize = new AtomicInteger(0);
    }
}
```

How much of table array are we actually using?

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## Fields



```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(2);
        setSize = new AtomicInteger(0);
    }
}
```

Track set size so we know when to resize

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## Fields



Initially use 1 bucket and size is zero

```
public class SSet {
    protected LockFreeList[] table;
    protected AtomicInteger tableSize;
    protected AtomicInteger setSize;

    public SSet(int capacity) {
        table = new LockFreeList[capacity];
        table[0] = new LockFreeList();
        tableSize = new AtomicInteger(1);
        setSize = new AtomicInteger(0);
    }
}
```

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## Add() Method



```
public boolean add(Object object) {
    int hash = object.hashCode();
    int bucket = hash % tableSize.get();
    int key = makeRegularKey(hash);

    LockFreeList list = getBucketList(bucket);

    if (!list.add(object, key))
        return false;

    resizeCheck();

    return true;
}
```

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## Resize



- Divide set size by total number of buckets
- If quotient exceeds threshold
  - Double tableSize field
  - Up to fixed limit

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## Initialize Buckets

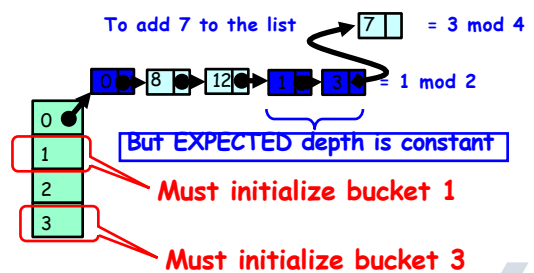


- Buckets originally null
- If you find one, initialize it
- Go to bucket's parent
  - Earlier nearby bucket
  - Recursively initialize if necessary
- Constant expected work

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## Recall: Recursive Initialization



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## Initialize Bucket



```
void initializeBucket(int bucket) {
    int parent = getParent(bucket);

    if (table[parent] == null)
        initializeBucket(parent);

    int key = makeSentinelKey(bucket);

    LockFreeList list = new LockFreeList(
        table[parent], key);
}
```

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## Initialize Bucket



```
void initializeBucket(int bucket) {
    int parent = getParent(bucket);
    if (table[parent] == null)
        initializeBucket(parent);
    int key = makeSentinelKey(bucket);
    LockFreeList list =
        new LockFreeList(table[parent],
            key);
}
```

Find parent, recursively  
initialize if needed

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## Initialize Bucket



```
void initializeBucket(int bucket) {
    int parent = getParent(bucket);
    if (table[parent] == null)
        initializeBucket(parent);
    int key = makeSentinelKey(bucket);
    LockFreeList list =
        new LockFreeList(table[parent],
                        key);
}
```

Prepare key for new sentinel

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## Initialize Bucket



Insert sentinel if not present, and  
get back reference to rest of list

```
void initializeBucket(int bucket) {
    int parent = getParent(bucket);
    if (table[parent] == null)
        initializeBucket(parent);
    int key = makeSentinelKey(bucket);
    LockFreeList list =
        new LockFreeList(table[parent],
                        key);
}
```

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## Correctness



- Linearizable concurrent set implementation
- Theorem:  $O(1)$  expected time
  - No more than  $O(1)$  items expected between two dummy nodes on average
  - Lazy initialization causes at most  $O(1)$  expected recursion depth in `initializeBucket()`

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## Empirical Evaluation

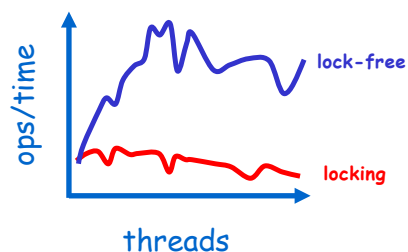


- On a 30-processor Sun Enterprise 3000
- Lock-Free vs. fine-grained (Lea) optimistic
- In a non-multiprogrammed environment
- 106 operations: 88% `contains()`, 10% `add()`, 2% `remove()`

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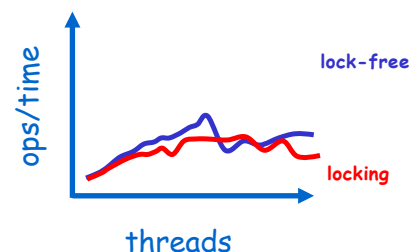
## Work = 0



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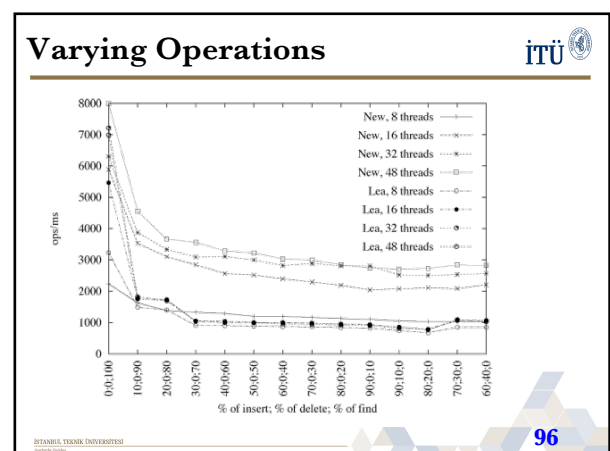
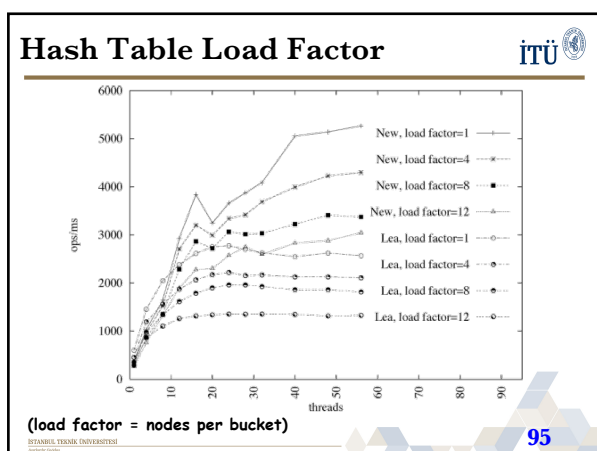
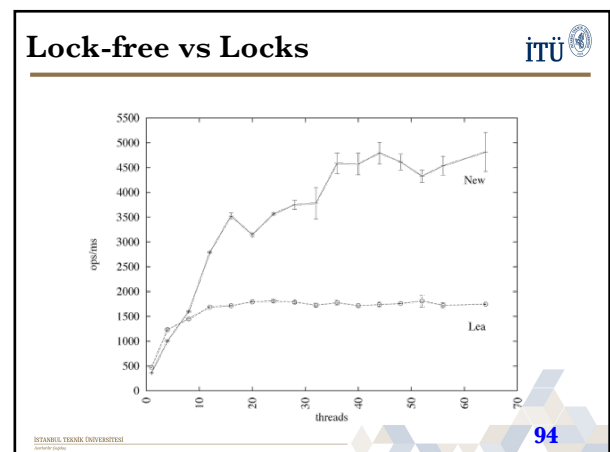
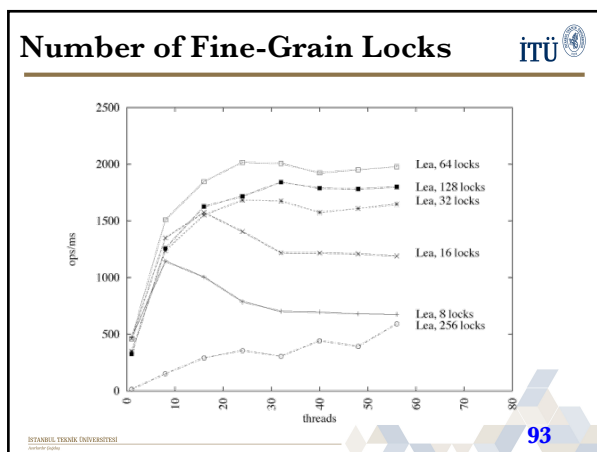
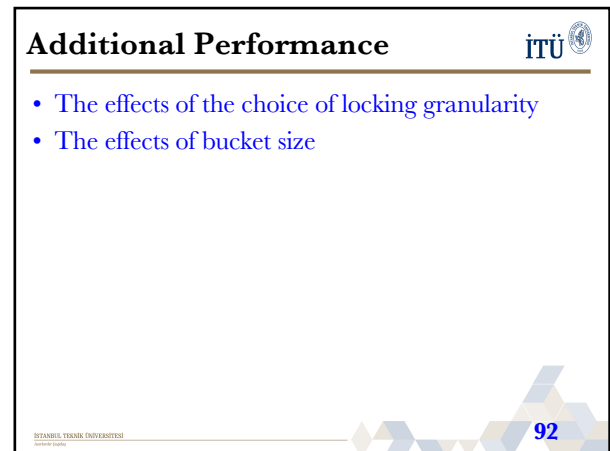
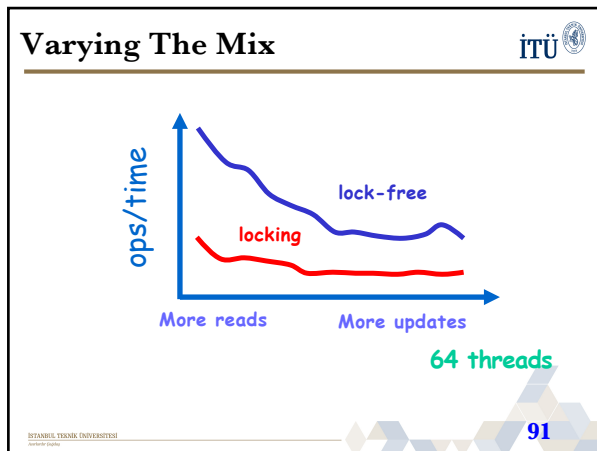
89

## Work = 500



İSTANBUL TEKNİK ÜNİVERSİTESİ

90





## Summary



- Lock-based
  - Fine-grained
  - Read/write locks
  - Optimistic
- Lock-free
  - Builds on lock-free list



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