Practice Session for week 7

Determine whether the following function is a quadratic spline:

$$Q(x) = \begin{cases} x^2 & (-10 \le x \le 0) \\ -x^2 & (0 \le x \le 1) \\ 1-2x & (1 \le x \le 20) \end{cases}$$

Whether Q and Q' are continuous an interior knots can be determined as follows:

$$\lim_{x\to 0^-} Q(x) = \lim_{x\to 0^-} x^2 = 0 \quad , \quad \lim_{x\to 0^+} Q(x) = \lim_{x\to 0^+} (-x^2) = 0$$

$$\lim_{x\to 1^-} Q(x) = \lim_{x\to 1^-} (-x^2) = -1 \quad , \quad \lim_{x\to 1^+} Q(x) = \lim_{x\to 1^+} (1-2x) = -1$$

Derivative of transaction points can be determined in below:

$$\lim_{x\to 0^-} Q'(x) = 2x = 0 \quad , \quad \lim_{x\to 0^+} Q'(x) = -2x = 0$$

$$\lim_{x\to 1^-} Q'(x) = -2x = -2 \quad , \quad \lim_{x\to 1^+} Q'(x) = -2$$

Consequently $Q(x)$ is a quadratic spline.

Interpolation using splines,

Temp (C)	Pressure (atmos.)
0	200
5	300
10	340
20	380
30	420

Write spline functions as form;

$$F(t) = F(t_0) + \frac{F(t_1) - F(t_0)}{t_1 - t_0}\left(t - t_0\right)$$

Range 0-5 $\quad F(t) = F(0) + \frac{F(5) - F(0)}{5-0}\left(t-0\right) = 200 + 20t$

Range 5-10 $\quad F(t) = F(5) + \frac{F(10) - F(5)}{10-5}\left(t-5\right) = 260 + 8t$

Range 10-20 $\quad F(t) = F(10) + \frac{F(20) - F(10)}{20-10}\left(t-10\right) = 300 + 4t$

Range 20-30 $$F(t) = F(20) + \frac{F(30) - F(20)}{30 - 20}\left(t - 20\right) = ...$$

Practice Session for week 8

Linear regression with least square method

Minimize error; $\sum_{i=1}^{n}(y_i - (ax_i + b))^2$ n=number of data points

$$\frac{\partial_{err}}{\partial_a} = -2\sum_{i=1}^{n} x_i(y_i - ax_i - b) = 0$$

$$\frac{\partial_{err}}{\partial_b} = -2\sum_{i=1}^{n}(y_i - ax_i - b) = 0$$

Rewrite,

$$a\sum x_i^2 + b\sum x_i = \sum (x_i y_i)$$

$$a\sum x_i + (b * n) = \sum y_i$$

Matrix form,

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix}$$

x	y=f(x)
0	8,4121
1	7,4882
2	6,4038
3	7,0530
4	6,6072
5	5,3039
6	5,9597
7	5,4933
8	5,7356
9	5,9598

$$n = 10$$

$$\sum x_i = 45$$

$$\sum x_i^2 = 285$$

$$\sum y_i = 64,4166$$

$$\sum (x_i y_i) = 268,1374$$

$$\begin{bmatrix} 10 & 45 \\ 45 & 285 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 64,4166 \\ 268,1374 \end{bmatrix}$$

$b = 7,6275$, $a = -0,2635$

$$y = ax + b = -0,2635x + 7,6273$$

1)

x	0	1.12	1.96	2.38	2.80	3.46	4.25	6.74	8
y=F(x)	4.45	9.37	14.78	13.26	23.98	17.64	25.88	69.51	66.15

a) For exponential regression model general formula notation;

$$y = ae^{bx}$$

$$\ln(y) = \ln(ae^{bx})$$
$$\ln(y) = \ln(a) + \ln(e^{bx})$$
$$\ln(y) = \ln(a) + bx$$

$$z = \ln(y) \ , \ a_0 = \ln(a)$$
$$z = a_0 + bx \text{ (Linear model)}$$

$$f(x) = ax + b$$
$$A = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \qquad X = \begin{bmatrix} b \\ a \end{bmatrix} \qquad B = \begin{bmatrix} \sum y_i \\ \sum (x_i y_i) \end{bmatrix} \qquad AX = B$$

For CPU-1 linear regression model;

$$\sum x_i = (0) + (1.12) + (1.96) + (2.38) + (2.80) + (3.46) + (4.25) + (6.74) + (8)$$
$$\sum x_i = 30.71$$

$$\sum x_i^2 = (0)^2 + (1.12)^2 + (1.96)^2 + (2.38)^2 + (2.80)^2 + (3.46)^2 + (4.25)^2 + (6.74)^2 + (8)^2$$
$$\sum x_i^2 = 158.0621$$

$$\sum y_i = \ln(4.45) + \ln(9.37) + \ln(14.78) + \ln(13.26) + \ln(23.98) + \ln(17.64) + \ln(25.88) + \ln(69.51) + \ln(66.15)$$
$$\sum y_i = 26.7427$$

$$\sum x_i y_i = 108.7137$$

$$A = \begin{bmatrix} 9 & 30.71 \\ 30.71 & 158.0621 \end{bmatrix} \qquad X = \begin{bmatrix} b \\ a \end{bmatrix} \qquad B = \begin{bmatrix} 26.7427 \\ 108.7137 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 30.71 \\ 30.71 & 158.0621 \end{bmatrix} \cdot \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 26.7427 \\ 108.7137 \end{bmatrix} \qquad \begin{matrix} b = 1.8530 \\ a = 0.3278 \end{matrix}$$

$b = 1.8530$
$a = 0.3278$

$$f(x) = (0.3278)x + 1.8530$$

For $z = a_0 + bx$ linear model;

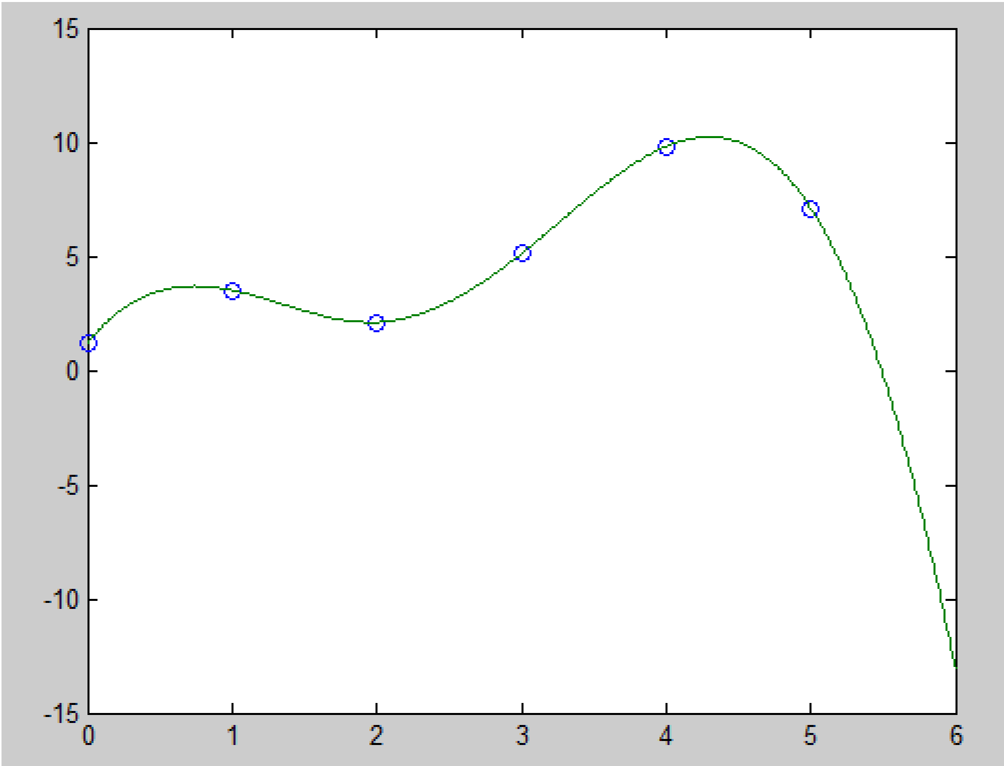
$a_0 = 1.8530$ $\quad a_0 = \ln(a)$ $\quad a = e^{a_0}$ $\quad a = e^{1.8530} = 6.3789$
$b = 0.3278$

$y = ae^{bx}$

$y = (6.3789) \cdot e^{0.3278x}$

MATLAB EXERCISES

```
x=[0 1 2 3 4 5];
y=[1.2 3.5 2.1 5.1 9.8 7.1];
xx=0:0.01:6;
yy=interp1(x,y,xx,'spline');
plot(x,y,'o',xx,yy)
```



```
x=[0 1 2 3 4 5];
y=[1.2 3.5 2.1 5.1 9.8 7.1];
ab=polyfit(x,y,1); %Linear regression
xx=0:0.01:6;
a=ab(1);
b=ab(2);
yy=a*xx+b;
plot(x,y,'o',xx,yy)
```