MAT 271E Probability and Statistics

Homework 7

Assigned: April 6, 2012

Due: April 11, 2012 (in class, before class starts)

No late homework will be accepted!

Do not copy from solutions from your classmates. All work must be your own!

Show all your steps! Just writing a number as a result is not enough. Make sure you answer everything that is asked (subquestions, etc.). This homework includes **5 problems** all of which must be answered!

Read: "Probability and Stochastic Processes", Yates and Goodman, Ch. 6.

1) Assume that $\phi_X(s)$ is an MGF.

- **a)** Is $\phi_X(s)\phi_X(5s)$ also an MGF? Explain in detail.
- **b)** Is $2\phi_X(s)$ also an MGF? Explain in detail.
- **c)** Is $e^{-s}\phi_X(s)$ also an MGF? Explain in detail.

2) Assume that K has MGF

$$\phi_K(s) = \frac{1}{6}e^{-2s} + \frac{1}{3}e^{-s} + \frac{1}{4}e^{s} + \frac{1}{4}e^{2s}$$

- **a)** Find $P[|K| \leq 1]$.

3) A network has three routers, X, Y, and Z. The number of packets dropped by the three routers are independent. The moment generating functions for the number of packets dropped by the routers are

$$\phi_X(s) = (1-2s)^{-3}, \quad \phi_Y(s) = (1-2s)^{-2.5}, \quad \phi_Z(s) = (1-2s)^{-4.5}$$

Let T represent the total number of packets dropped by the three routers.

- **a)** Calculate $E[T^3]$.

4) Data packets are sent between Computer *A* and Computer *B*. The size of each packet is independent and identically distributed according to a Gaussian distribution with mean 90 and variance 8. In a day, the probability of *B* receiving *n* packets is $1/(en!)$.

- **a)** Find the expectation of the size of the data *B* receives in a day.
- **b)** Find the variance of the size of the data *B* receives in a day.

5) A large building has three water tanks. At any given moment, each one of them is "out of order" (i.e., not available) with probability 1/2, independently of the others. The amount of water in each tank (X) is unknown and is uniformly distributed between 0 and 1000 liters.

a) Find the moment generating function of the total amount of water available in the water tanks that are <u>not</u> "out of order".