Name Surname: **Signature:** 18-04-2013
Number:

FORMAL LANGUAGES & AUTOMATA
QUIZ-3

Consider the regular expression given below

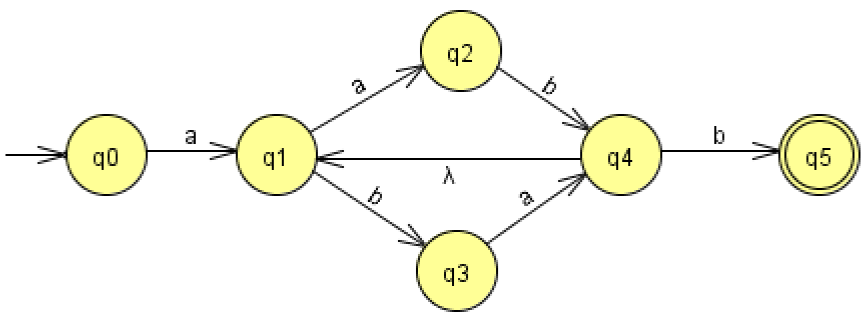
$$a(ab \vee ba)^{+}b$$

a) Produce the NFA that recognizes the language defined by this regular expression.
b) Produce an equivalent DFA to the NFA you have just designed.
c) Systematically produce the regular expression for the language defined by the DFA you have designed. Show that it is equivalent to the original regular expression.

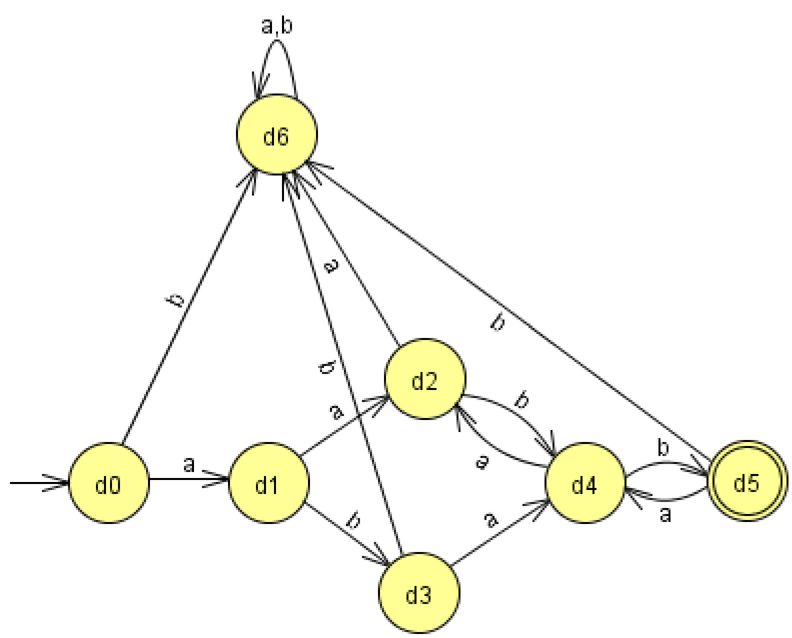
Duration: 30 mins

Solution:

a)



b) $S = q_0 \rightarrow d_0$
$\delta(d_0,a) = \delta(\{q_0\},a) = \{q_1\} = d_1$
$\delta(d_0,b) = \delta(\{q_0\},b) = \emptyset$
$\delta(d_1,a) = \delta(\{q_1\},a) = \{q_2\} = d_2$
$\delta(d_1,b) = \delta(\{q_1\},b) = \{q_3\} = d_3$
$\delta(d_2,a) = \delta(\{q_2\},a) = \emptyset$
$\delta(d_2,b) = \delta(\{q_2\},b) = \{q_4\} = d_4$
$\delta(d_3,a) = \delta(\{q_3\},a) = \{q_4\} = d_4$
$\delta(d_3,b) = \delta(\{q_3\},b) = \emptyset$
$\delta(d_4,a) = \delta(\{q_4\},a) = \{q_2\} = d_2$
$\delta(d_4,b) = \delta(\{q_4\},b) = \{q_3,q_5\} = d_5$
$\delta(d_5,a) = \delta(\{q_3,q_5\},a) = \{q_4\} = d_4$
$\delta(d_5,b) = \delta(\{q_3,q_5\},b) = \emptyset$
$\delta(\emptyset,a) = \delta(\emptyset,b) = \emptyset = d_6$



c) **Theorem:** The one and only solution to the equation $X = XA \cup B \ \wedge \ \Lambda \notin A$ is X = BA*.
The same statement using regular expressions:

$$x = xa \vee b \ \wedge \ \Lambda \notin A \ \Rightarrow x = ba^{*}$$

$L(M) = d_5$

$d_0 = \Lambda$

$d_1 = d_0 a$

$d_2 = d_1 a \vee d_4 a$

$d_3 = d_1 b$

$d_4 = d_2 b \vee d_3 a \vee d_5 a$

$d_5 = d_4 b$

d_6 → dead state

Using $d_0 = \Lambda$ → $d_1 = d_0 a = \Lambda\text{a} = \text{a}$

Using $d_1 = \text{a}$ → $d_2 = d_1 a \vee d_4 a = aa \vee d_4 a$

Using $d_1 = \text{a}$ → $d_3 = d_1 b = \text{ab}$

Using $d_2 = aa \vee d_4 a$, $d_3 = \text{ab}$ and $d_5 = d_4 b$

→ $d_4 = d_2 b \vee d_3 a \vee d_5 a$

$d_4 = (aa \vee d_4 a)b \vee aba \vee d_4 ba$

Rearranging in the form $x = xa \vee b \wedge \Lambda \notin A$ and using the theorem ($x = ba^*$)

→ $d_4 = d_4(ab \vee ba) \vee (aab \vee aba) = (aab \vee aba)(ab \vee ba)^* = a(ab \vee ba)(ab \vee ba)^*$

$d_4 = a(ab \vee ba)^+$

Using $d_4 = a(ab \vee ba)^+$ → $d_5 = d_4 b = a(ab \vee ba)^+ b$

$L(M) = d_5 = a(ab \vee ba)^+ b$ → the same as the original regular expression given in the question