ISTANBUL TECHNICAL UNIVERSITY

COMPUTER ENGINERING DEPARTMENT

BLG 527E MACHINE LEARNING

CRN: 13817

Instructor: Zehra Çataltepe

Homework #1

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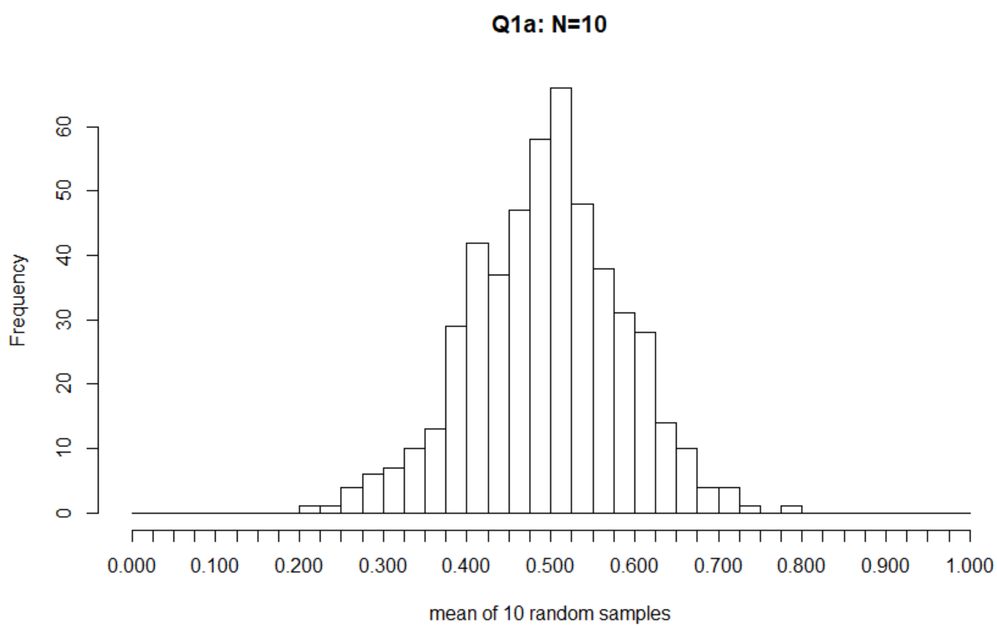
Answers

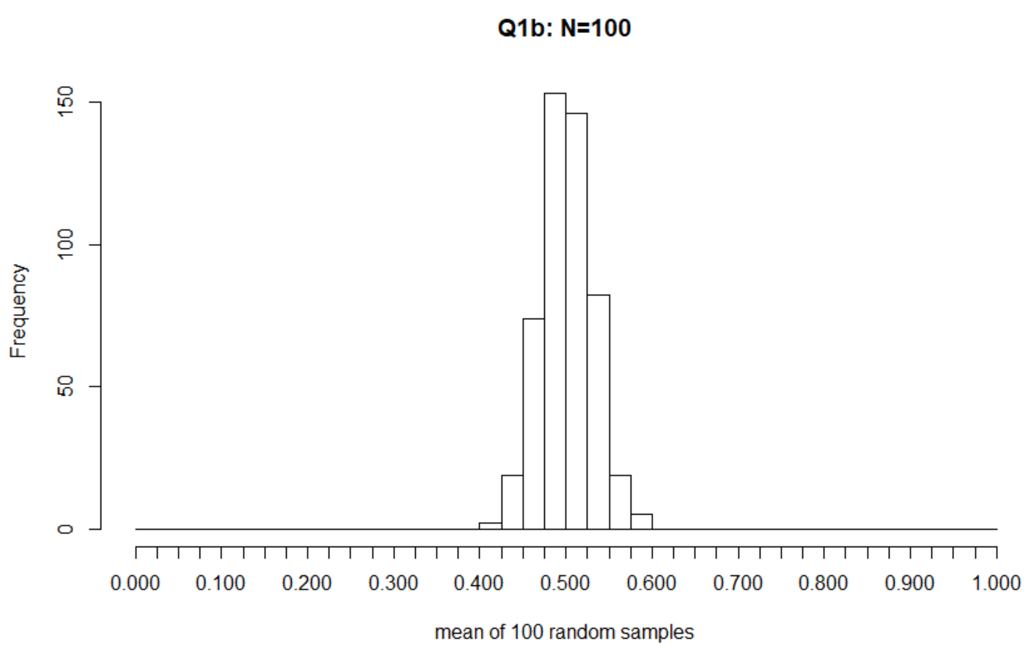
Q1a)

N=10



Q1b)

N=100



Q1c)

Both sampling distribution of the mean approaches a normal distribution. But, as N grows, the shape of the histogram resembles a Normal distribution more closely. N is the sample size for each mean.

Q2)

$g_i(x) = \ln(p(x|C_i)) + \ln(P(C_i))$

$p(x|C1) = N(0,1)$

$p(x|C2) = N(1,2)$

Q2a)

$P(C1) = P(C2) = 0.5$

$g_1(x) = \ln(p(x|C_1)) + \ln(P(C_1)) = \ln(N(0,1)) + \ln(0.5)$

$g_2(x) = \ln(p(x|C_2)) + \ln(P(C_2)) = \ln(N(1,2)) + \ln(0.5)$

Gaussian distribution is:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

So;

$N(0,1) = \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-x^2}{2}}$ $\quad\quad$ $\ln(N(0,1)) = \ln(\frac{1}{\sqrt{2\pi}}) - \frac{x^2}{2}$

$N(1,2) = \frac{1}{2\sqrt{2\pi}} \cdot e^{\frac{-(x-1)^2}{8}}$ $\quad\quad$ $\ln(N(1,2)) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{(x-1)^2}{8}$

Then;

$g_1(x) = \ln(\frac{1}{\sqrt{2\pi}}) - \frac{x^2}{2} + \ln(0.5) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{x^2}{2}$

$g_2(x) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{(x-1)^2}{8} + \ln(0.5) = \ln(\frac{1}{4\sqrt{2\pi}}) - \frac{(x-1)^2}{8}$

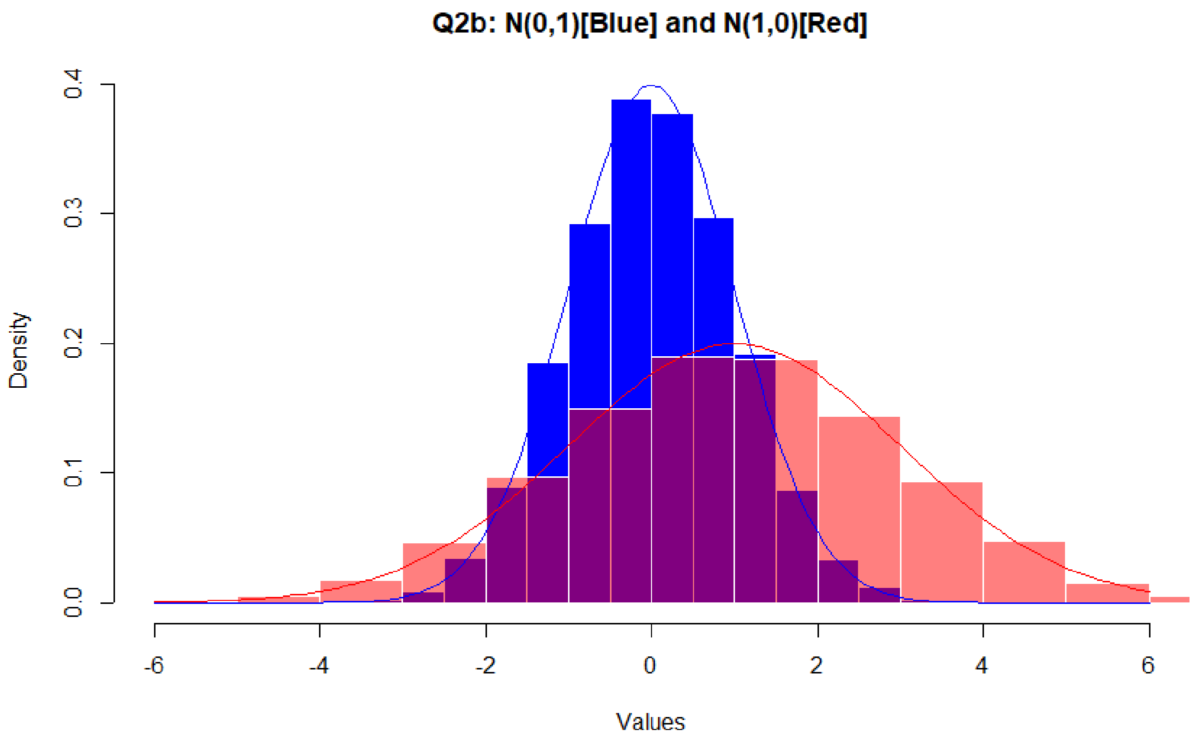
Q2b)

p(x|C1) = N(0,1)

p(x|C2) = N(1,2)

Histograms represent two Gaussian distribution with mean=0,1 and standard deviation=1,2

Lines represent density distribution of two Gaussian distribution with mean=0,1, standard deviation=1,2



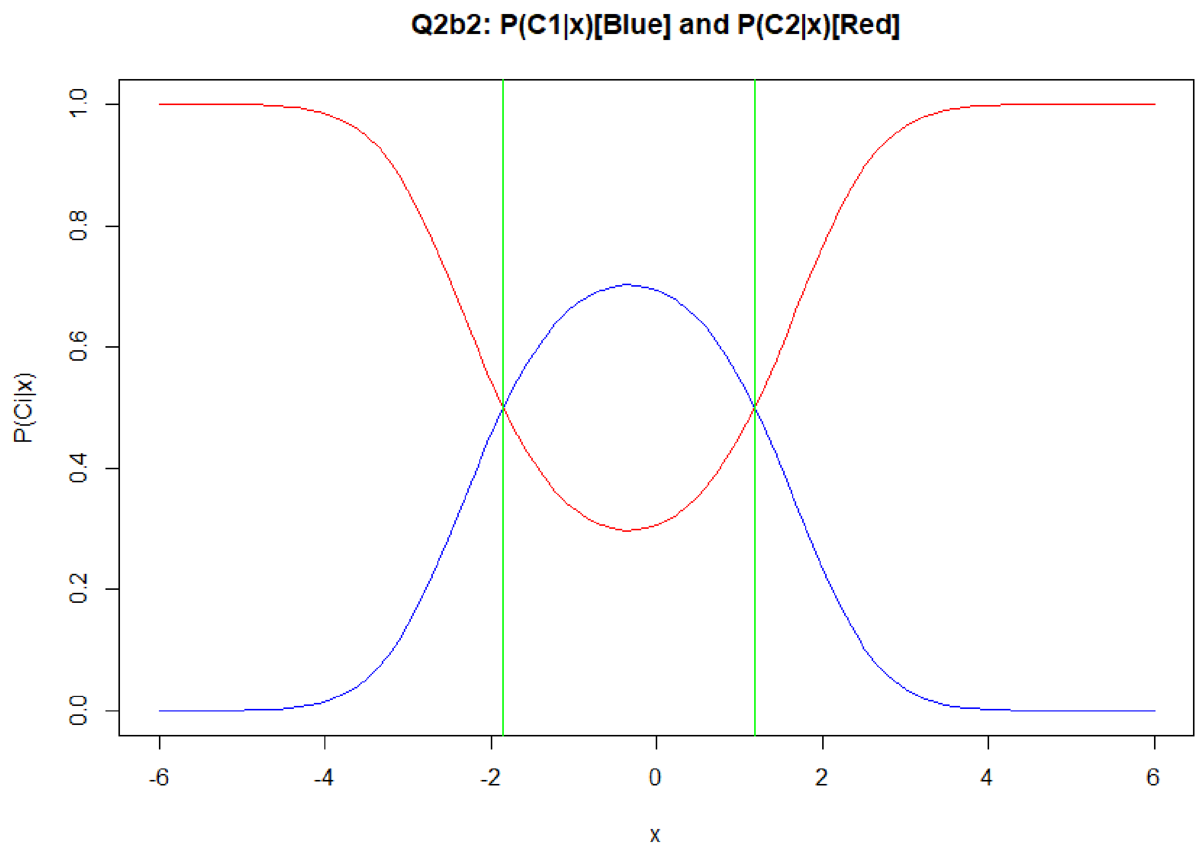
$$P(C1|x) = \frac{P(x|C1)\,.\,P(C1)}{P(x)} = \frac{P(x|C1)\,.\,P(C1)}{P(x|C1)\,.\,P(C1) + P(x|C2)\,.\,P(C2)} = \frac{N(0,1)\,.\,P(C1)}{N(0,1).\,P(C1) + N(1,2)\,.\,P(C2)}$$

$$P(C2|x) = \frac{P(x|C2)\,.\,P(C2)}{P(x)} = \frac{P(x|C2)\,.\,P(C2)}{P(x|C1)\,.\,P(C1) + P(x|C2)\,.\,P(C2)} = \frac{N(1,2)\,.\,P(C2)}{N(0,1).\,P(C1) + N(1,2)\,.\,P(C2)}$$

If we assume P(C1)=P(C2)=0.5, then;

$$P(C1|x) = \frac{N(0,1)\,.\,P(C1)}{N(0,1).\,P(C1) + N(1,2)\,.\,P(C2)} = \frac{N(0,1)}{N(0,1) + N(1,2)} = \frac{\frac{1}{\sqrt{2\pi}}.e^{\frac{-x^2}{2}}}{\frac{1}{\sqrt{2\pi}}.e^{\frac{-x^2}{2}} + \frac{1}{2\sqrt{2\pi}}.e^{\frac{-(x-1)^2}{8}}} = \frac{e^{\frac{-x^2}{2}}}{e^{\frac{-x^2}{2}} + \frac{1}{2}.e^{\frac{-(x-1)^2}{8}}}$$

$$P(C2|x) = \frac{N(1,2)\,.\,P(C2)}{N(0,1).\,P(C1) + N(1,2)\,.\,P(C2)} = \frac{N(1,2)}{N(0,1) + N(1,2)} = \frac{\frac{1}{2\sqrt{2\pi}}.e^{\frac{-(x-1)^2}{8}}}{\frac{1}{\sqrt{2\pi}}.e^{\frac{-x^2}{2}} + \frac{1}{2\sqrt{2\pi}}.e^{\frac{-(x-1)^2}{8}}} = \frac{\frac{1}{2}.e^{\frac{-(x-1)^2}{8}}}{e^{\frac{-x^2}{2}} + \frac{1}{2}.e^{\frac{-(x-1)^2}{8}}}$$

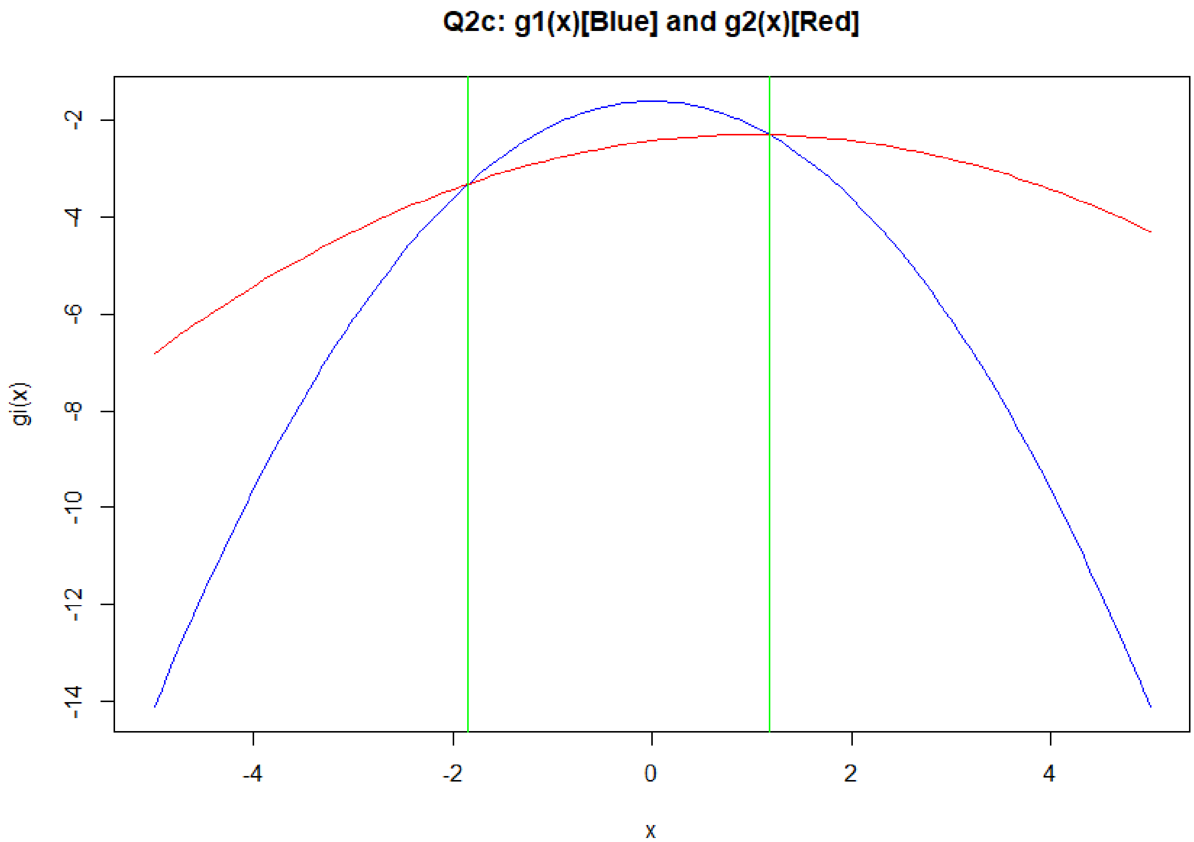


Regions where P(C1|x) is greater than P(C2|x) is class C1, Regions where P(C2|x) is greater than P(C1|x) is class C2.

Q2c)

g1(x) = $\ln(\frac{1}{\sqrt{2\pi}}) - \frac{x^2}{2} + \ln(0.5) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{x^2}{2}$

g2(x) = $\ln(\frac{1}{2\sqrt{2\pi}}) - \frac{(x-1)^2}{8} + \ln(0.5) = \ln(\frac{1}{4\sqrt{2\pi}}) - \frac{(x-1)^2}{8}$



Regions where g1(x) is greater than g2(x) is class C1, Regions where g2(x) is greater than g2(x) is class C2.

Q2d)

P(C1) = 0.2

P(C2) = 0.8

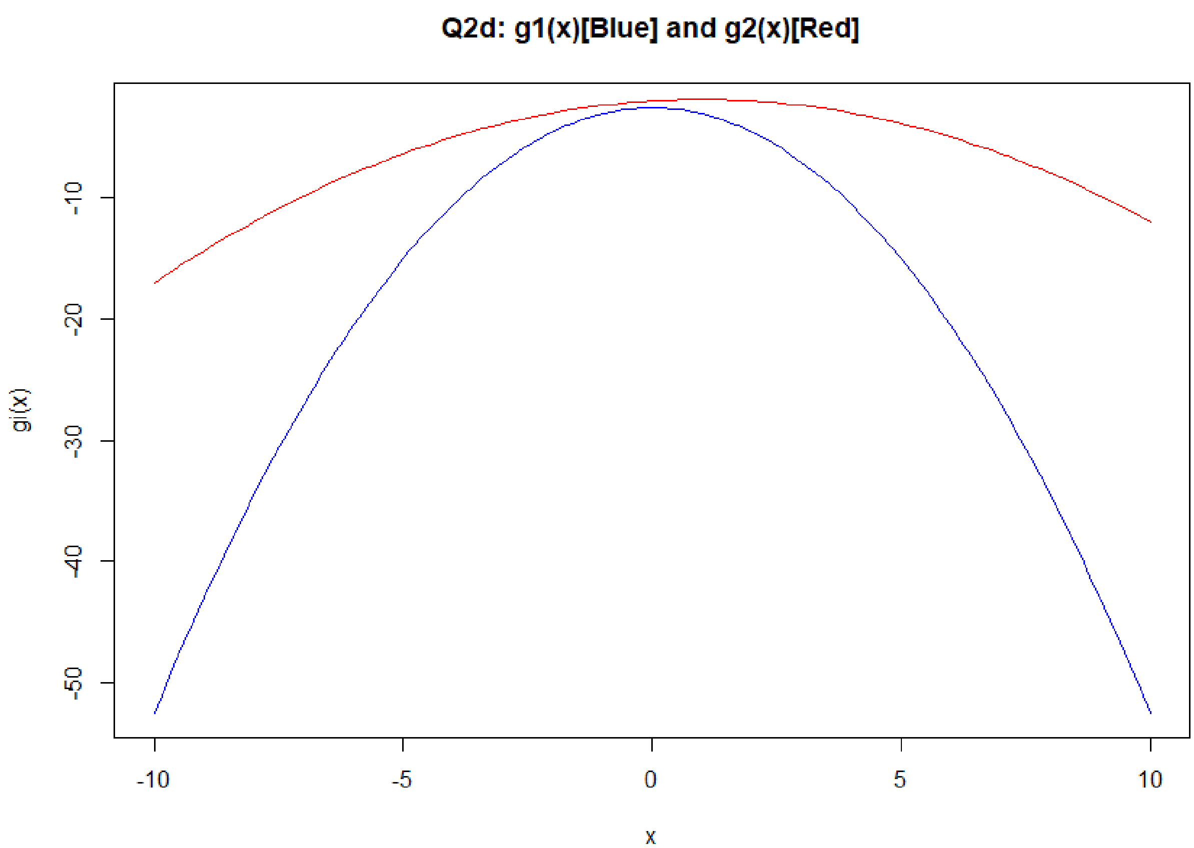
We know that from Q2a;

$\ln(N(0,1)) = \ln(\frac{1}{\sqrt{2\pi}}) - \frac{x^2}{2}$

$\ln(N(1,2)) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{(x-1)^2}{8}$

$g_1(x) = \ln(p(x|C_1)) + \ln(P(C_1)) = \ln(N(0,1)) + \ln(0.2) = \ln(\frac{1}{\sqrt{2\pi}}) - \frac{x^2}{2} + \ln(0.2)$

$g_2(x) = \ln(p(x|C_2)) + \ln(P(C_2)) = \ln(N(1,2)) + \ln(0.8) = \ln(\frac{1}{2\sqrt{2\pi}}) - \frac{(x-1)^2}{8} + \ln(0.8)$



g2(x) is always greater than g1(x) so C2 is always identified.