

MAT 271E Probability and Statistics

Homework 6

Assigned: March 31, 2012

Due: April 4, 2012 (in class, before class starts)

No late homework will be accepted!

Do not copy from solutions from your classmates. All work must be your own!

Show all your steps! Just writing a number as a result is not enough. Make sure you answer everything that is asked (subquestions, etc.). This homework includes **5 problems** all of which must be answered!

Read: “Probability and Stochastic Processes”, Yates and Goodman, Ch. 4. Start reading Ch. 6.

- 1) X is an exponential random variable with parameter λ . Find the PDF of $Y = \sqrt{X}$.
- 2) X is a Gaussian random variable with expected value $\mu = 0$, and variance $\sigma^2 = 64$. Given the event $A = \{X < 0\}$,
 - a) Find the conditional PDF, $f_{X/A}(x)$.
 - b) Find the conditional expected value, $E[X/A]$.
 - c) Find the conditional variance, $\text{Var}[X/A]$.
- 3) Explain if the function below can be the joint CDF of random variables X and Y .

$$F_{X,Y}(x,y) = \begin{cases} 1 - e^{-(x+y)} & , x \geq 0, y \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

- 4) X and Y have the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 2 & , x + y \leq 1, x \geq 0, y \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

- a) What is the marginal PDF $f_X(x)$?
 - b) What is the marginal PDF $f_Y(y)$?
- 5) Suppose that X is Gaussian with $\mu_X = 1$ and $\text{Var}[X] = 4$. Let Y be another Gaussian with $\mu_Y = 2$ and $\text{Var}[Y] = 9$. Assume that X and Y are independent. Let

$$W_1 = 2X - Y \quad \text{and} \quad W_2 = \beta X + Y$$

where β is a real number.

- a)** Find the probability density function of W_1 .
- b)** Find a numerical value for β which makes W_1 and W_2 uncorrelated.
- c)** For the value of β that makes W_1 and W_2 uncorrelated, specify the joint PDF of W_1 and W_2 .
- d)** Find the conditional PDF of W_1 given X , i.e., $f_{W_1|X}(w_1|x)$.