

MAT 271E Probability and Statistics

Homework 5

Assigned: March 23, 2012

Due: March 28, 2012 (in class, before class starts)

No late homework will be accepted!

Do not copy from solutions from your classmates. All work must be your own!

Show all your steps! Just writing a number as a result is not enough. Make sure you answer everything that is asked (subquestions, etc.). This homework includes **5 problems** all of which must be answered!

Read: “Probability and Stochastic Processes”, Yates and Goodman, Ch. 3

1) The cumulative distribution function of random variable X is

$$F_X(x) = \begin{cases} 0 & , x < -3 \\ \frac{(x+3)^2}{8} & , -3 \leq x < -1 \\ \frac{1}{2} & , -1 \leq x < 1 \\ \frac{(x+1)}{4} & , 1 \leq x < 3 \\ 1 & , x \geq 3 \end{cases}$$

- a) Find $P[X \leq 0]$.
- b) Find $P[-1/3 < X \leq 1/5]$.
- c) Find $P[X > 1]$.
- d) Find the value of c such that $P[X \leq c] = 3/4$.

2) The random variable X has probability density function

$$f_X(x) = \begin{cases} c(x^2 + 4) & , -4 \leq x \leq 3 \\ 0 & , \text{otherwise} \end{cases}$$

- a) Find the value of the constant c .
- b) Find $P[-2 \leq X \leq 2]$.
- c) Find $P[-6 \leq X \leq -3]$.
- d) Find the CDF $F_X(x)$.

- 3) Let X be a continuous random variable that is uniformly distributed between 5 and 10. Let $Y = g(X) = X^2 - X$.

- a) Find $E[X]$ and $\text{Var}[X]$.
- b) Find $g(E[X])$ and $E[g(X)]$.
- c) Find $E[Y]$ and $\text{Var}[Y]$.

- 4) Let X have the following PDF:

$$f_X(x) = \begin{cases} \frac{1}{4}e^{-x/4} & , x > 0 \\ 0 & , \text{otherwise} \end{cases}$$

- a) Find the CDF of Y .
- b) Find $E[X^2 + X]$.
- c) Find $P[X < 4]$.

- 5) K is a Gaussian random variable with a variance of 25. Its expected value is 2.

(**Note:** Tables similar to Table 3.1 and Table 3.2 on pages 123 and 124 of your textbook are given at the end of this homework assignment. You may use these tables or the ones in your textbook.)

- a) Find $P[|K - 4| > 3]$.

Standard Normal CDF $\Phi(z)$

| z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ | z | $\Phi(z)$ |
|----------|-----------------------------|----------|-----------------------------|----------|-----------------------------|----------|-----------------------------|----------|-----------------------------|----------|-----------------------------|
| 0.00 | 0.5000 | 0.50 | 0.6915 | 1.00 | 0.8413 | 1.50 | 0.9332 | 2.00 | 0.9772 | 2.50 | 0.9938 |
| 0.01 | 0.5040 | 0.51 | 0.6950 | 1.01 | 0.8438 | 1.51 | 0.9345 | 2.01 | 0.9778 | 2.51 | 0.9940 |
| 0.02 | 0.5080 | 0.52 | 0.6985 | 1.02 | 0.8461 | 1.52 | 0.9357 | 2.02 | 0.9783 | 2.52 | 0.9941 |
| 0.03 | 0.5120 | 0.53 | 0.7019 | 1.03 | 0.8485 | 1.53 | 0.9370 | 2.03 | 0.9788 | 2.53 | 0.9943 |
| 0.04 | 0.5160 | 0.54 | 0.7054 | 1.04 | 0.8508 | 1.54 | 0.9382 | 2.04 | 0.9793 | 2.54 | 0.9945 |
| 0.05 | 0.5199 | 0.55 | 0.7088 | 1.05 | 0.8531 | 1.55 | 0.9394 | 2.05 | 0.9798 | 2.55 | 0.9946 |
| 0.06 | 0.5239 | 0.56 | 0.7123 | 1.06 | 0.8554 | 1.56 | 0.9406 | 2.06 | 0.9803 | 2.56 | 0.9948 |
| 0.07 | 0.5279 | 0.57 | 0.7157 | 1.07 | 0.8577 | 1.57 | 0.9418 | 2.07 | 0.9808 | 2.57 | 0.9949 |
| 0.08 | 0.5319 | 0.58 | 0.7190 | 1.08 | 0.8599 | 1.58 | 0.9429 | 2.08 | 0.9812 | 2.58 | 0.9951 |
| 0.09 | 0.5359 | 0.59 | 0.7224 | 1.09 | 0.8621 | 1.59 | 0.9441 | 2.09 | 0.9817 | 2.59 | 0.9952 |
| 0.10 | 0.5398 | 0.60 | 0.7257 | 1.10 | 0.8643 | 1.60 | 0.9452 | 2.10 | 0.9821 | 2.60 | 0.9953 |
| 0.11 | 0.5438 | 0.61 | 0.7291 | 1.11 | 0.8665 | 1.61 | 0.9463 | 2.11 | 0.9826 | 2.61 | 0.9955 |
| 0.12 | 0.5478 | 0.62 | 0.7324 | 1.12 | 0.8686 | 1.62 | 0.9474 | 2.12 | 0.9830 | 2.62 | 0.9956 |
| 0.13 | 0.5517 | 0.63 | 0.7357 | 1.13 | 0.8708 | 1.63 | 0.9484 | 2.13 | 0.9834 | 2.63 | 0.9957 |
| 0.14 | 0.5557 | 0.64 | 0.7389 | 1.14 | 0.8729 | 1.64 | 0.9495 | 2.14 | 0.9838 | 2.64 | 0.9959 |
| 0.15 | 0.5596 | 0.65 | 0.7422 | 1.15 | 0.8749 | 1.65 | 0.9505 | 2.15 | 0.9842 | 2.65 | 0.9960 |
| 0.16 | 0.5636 | 0.66 | 0.7454 | 1.16 | 0.8770 | 1.66 | 0.9515 | 2.16 | 0.9846 | 2.66 | 0.9961 |
| 0.17 | 0.5675 | 0.67 | 0.7486 | 1.17 | 0.8790 | 1.67 | 0.9525 | 2.17 | 0.9850 | 2.67 | 0.9962 |
| 0.18 | 0.5714 | 0.68 | 0.7517 | 1.18 | 0.8810 | 1.68 | 0.9535 | 2.18 | 0.9854 | 2.68 | 0.9963 |
| 0.19 | 0.5753 | 0.69 | 0.7549 | 1.19 | 0.8830 | 1.69 | 0.9545 | 2.19 | 0.9857 | 2.69 | 0.9964 |
| 0.20 | 0.5793 | 0.70 | 0.7580 | 1.20 | 0.8849 | 1.70 | 0.9554 | 2.20 | 0.9861 | 2.70 | 0.9965 |
| 0.21 | 0.5832 | 0.71 | 0.7611 | 1.21 | 0.8869 | 1.71 | 0.9564 | 2.21 | 0.9864 | 2.71 | 0.9966 |
| 0.22 | 0.5871 | 0.72 | 0.7642 | 1.22 | 0.8888 | 1.72 | 0.9573 | 2.22 | 0.9868 | 2.72 | 0.9967 |
| 0.23 | 0.5910 | 0.73 | 0.7673 | 1.23 | 0.8907 | 1.73 | 0.9582 | 2.23 | 0.9871 | 2.73 | 0.9968 |
| 0.24 | 0.5948 | 0.74 | 0.7704 | 1.24 | 0.8925 | 1.74 | 0.9591 | 2.24 | 0.9875 | 2.74 | 0.9969 |
| 0.25 | 0.5987 | 0.75 | 0.7734 | 1.25 | 0.8944 | 1.75 | 0.9599 | 2.25 | 0.9878 | 2.75 | 0.9970 |
| 0.26 | 0.6026 | 0.76 | 0.7764 | 1.26 | 0.8962 | 1.76 | 0.9608 | 2.26 | 0.9881 | 2.76 | 0.9971 |
| 0.27 | 0.6064 | 0.77 | 0.7794 | 1.27 | 0.8980 | 1.77 | 0.9616 | 2.27 | 0.9884 | 2.77 | 0.9972 |
| 0.28 | 0.6103 | 0.78 | 0.7823 | 1.28 | 0.8997 | 1.78 | 0.9625 | 2.28 | 0.9887 | 2.78 | 0.9973 |
| 0.29 | 0.6141 | 0.79 | 0.7852 | 1.29 | 0.9015 | 1.79 | 0.9633 | 2.29 | 0.9890 | 2.79 | 0.9974 |
| 0.30 | 0.6179 | 0.80 | 0.7881 | 1.30 | 0.9032 | 1.80 | 0.9641 | 2.30 | 0.9893 | 2.80 | 0.9974 |
| 0.31 | 0.6217 | 0.81 | 0.7910 | 1.31 | 0.9049 | 1.81 | 0.9649 | 2.31 | 0.9896 | 2.81 | 0.9975 |
| 0.32 | 0.6255 | 0.82 | 0.7939 | 1.32 | 0.9066 | 1.82 | 0.9656 | 2.32 | 0.9898 | 2.82 | 0.9976 |
| 0.33 | 0.6293 | 0.83 | 0.7967 | 1.33 | 0.9082 | 1.83 | 0.9664 | 2.33 | 0.9901 | 2.83 | 0.9977 |
| 0.34 | 0.6331 | 0.84 | 0.7995 | 1.34 | 0.9099 | 1.84 | 0.9671 | 2.34 | 0.9904 | 2.84 | 0.9977 |
| 0.35 | 0.6368 | 0.85 | 0.8023 | 1.35 | 0.9115 | 1.85 | 0.9678 | 2.35 | 0.9906 | 2.85 | 0.9978 |
| 0.36 | 0.6406 | 0.86 | 0.8051 | 1.36 | 0.9131 | 1.86 | 0.9686 | 2.36 | 0.9909 | 2.86 | 0.9979 |
| 0.37 | 0.6443 | 0.87 | 0.8078 | 1.37 | 0.9147 | 1.87 | 0.9693 | 2.37 | 0.9911 | 2.87 | 0.9979 |
| 0.38 | 0.6480 | 0.88 | 0.8106 | 1.38 | 0.9162 | 1.88 | 0.9699 | 2.38 | 0.9913 | 2.88 | 0.9980 |
| 0.39 | 0.6517 | 0.89 | 0.8133 | 1.39 | 0.9177 | 1.89 | 0.9706 | 2.39 | 0.9916 | 2.89 | 0.9981 |
| 0.40 | 0.6554 | 0.90 | 0.8159 | 1.40 | 0.9192 | 1.90 | 0.9713 | 2.40 | 0.9918 | 2.90 | 0.9981 |
| 0.41 | 0.6591 | 0.91 | 0.8186 | 1.41 | 0.9207 | 1.91 | 0.9719 | 2.41 | 0.9920 | 2.91 | 0.9982 |
| 0.42 | 0.6628 | 0.92 | 0.8212 | 1.42 | 0.9222 | 1.92 | 0.9726 | 2.42 | 0.9922 | 2.92 | 0.9982 |
| 0.43 | 0.6664 | 0.93 | 0.8238 | 1.43 | 0.9236 | 1.93 | 0.9732 | 2.43 | 0.9925 | 2.93 | 0.9983 |
| 0.44 | 0.6700 | 0.94 | 0.8264 | 1.44 | 0.9251 | 1.94 | 0.9738 | 2.44 | 0.9927 | 2.94 | 0.9984 |
| 0.45 | 0.6736 | 0.95 | 0.8289 | 1.45 | 0.9265 | 1.95 | 0.9744 | 2.45 | 0.9929 | 2.95 | 0.9984 |
| 0.46 | 0.6772 | 0.96 | 0.8315 | 1.46 | 0.9279 | 1.96 | 0.9750 | 2.46 | 0.9931 | 2.96 | 0.9985 |
| 0.47 | 0.6808 | 0.97 | 0.8340 | 1.47 | 0.9292 | 1.97 | 0.9756 | 2.47 | 0.9932 | 2.97 | 0.9985 |
| 0.48 | 0.6844 | 0.98 | 0.8365 | 1.48 | 0.9306 | 1.98 | 0.9761 | 2.48 | 0.9934 | 2.98 | 0.9986 |
| 0.49 | 0.6879 | 0.99 | 0.8389 | 1.49 | 0.9319 | 1.99 | 0.9767 | 2.49 | 0.9936 | 2.99 | 0.9986 |

Standard Normal Complementary CDF Q(z)

| z | Q(z) | z | Q(z) | z | Q(z) | z | Q(z) | z | Q(z) | z | Q(z) |
|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|-------|--------|
| -3.00 | 0.0013 | -2.50 | 0.0062 | -2.00 | 0.0228 | -1.50 | 0.0668 | -1.00 | 0.1587 | -0.50 | 0.3085 |
| -2.99 | 0.0014 | -2.49 | 0.0064 | -1.99 | 0.0233 | -1.49 | 0.0681 | -0.99 | 0.1611 | -0.49 | 0.3121 |
| -2.98 | 0.0014 | -2.48 | 0.0066 | -1.98 | 0.0239 | -1.48 | 0.0694 | -0.98 | 0.1635 | -0.48 | 0.3156 |
| -2.97 | 0.0015 | -2.47 | 0.0068 | -1.97 | 0.0244 | -1.47 | 0.0708 | -0.97 | 0.1660 | -0.47 | 0.3192 |
| -2.96 | 0.0015 | -2.46 | 0.0069 | -1.96 | 0.0250 | -1.46 | 0.0721 | -0.96 | 0.1685 | -0.46 | 0.3228 |
| -2.95 | 0.0016 | -2.45 | 0.0071 | -1.95 | 0.0256 | -1.45 | 0.0735 | -0.95 | 0.1711 | -0.45 | 0.3264 |
| -2.94 | 0.0016 | -2.44 | 0.0073 | -1.94 | 0.0262 | -1.44 | 0.0749 | -0.94 | 0.1736 | -0.44 | 0.3300 |
| -2.93 | 0.0017 | -2.43 | 0.0075 | -1.93 | 0.0268 | -1.43 | 0.0764 | -0.93 | 0.1762 | -0.43 | 0.3336 |
| -2.92 | 0.0018 | -2.42 | 0.0078 | -1.92 | 0.0274 | -1.42 | 0.0778 | -0.92 | 0.1788 | -0.42 | 0.3372 |
| -2.91 | 0.0018 | -2.41 | 0.0080 | -1.91 | 0.0281 | -1.41 | 0.0793 | -0.91 | 0.1814 | -0.41 | 0.3409 |
| -2.90 | 0.0019 | -2.40 | 0.0082 | -1.90 | 0.0287 | -1.40 | 0.0808 | -0.90 | 0.1841 | -0.40 | 0.3446 |
| -2.89 | 0.0019 | -2.39 | 0.0084 | -1.89 | 0.0294 | -1.39 | 0.0823 | -0.89 | 0.1867 | -0.39 | 0.3483 |
| -2.88 | 0.0020 | -2.38 | 0.0087 | -1.88 | 0.0301 | -1.38 | 0.0838 | -0.88 | 0.1894 | -0.38 | 0.3520 |
| -2.87 | 0.0021 | -2.37 | 0.0089 | -1.87 | 0.0307 | -1.37 | 0.0853 | -0.87 | 0.1922 | -0.37 | 0.3557 |
| -2.86 | 0.0021 | -2.36 | 0.0091 | -1.86 | 0.0314 | -1.36 | 0.0869 | -0.86 | 0.1949 | -0.36 | 0.3594 |
| -2.85 | 0.0022 | -2.35 | 0.0094 | -1.85 | 0.0322 | -1.35 | 0.0885 | -0.85 | 0.1977 | -0.35 | 0.3632 |
| -2.84 | 0.0023 | -2.34 | 0.0096 | -1.84 | 0.0329 | -1.34 | 0.0901 | -0.84 | 0.2005 | -0.34 | 0.3669 |
| -2.83 | 0.0023 | -2.33 | 0.0099 | -1.83 | 0.0336 | -1.33 | 0.0918 | -0.83 | 0.2033 | -0.33 | 0.3707 |
| -2.82 | 0.0024 | -2.32 | 0.0102 | -1.82 | 0.0344 | -1.32 | 0.0934 | -0.82 | 0.2061 | -0.32 | 0.3745 |
| -2.81 | 0.0025 | -2.31 | 0.0104 | -1.81 | 0.0351 | -1.31 | 0.0951 | -0.81 | 0.2090 | -0.31 | 0.3783 |
| -2.80 | 0.0026 | -2.30 | 0.0107 | -1.80 | 0.0359 | -1.30 | 0.0968 | -0.80 | 0.2119 | -0.30 | 0.3821 |
| -2.79 | 0.0026 | -2.29 | 0.0110 | -1.79 | 0.0367 | -1.29 | 0.0985 | -0.79 | 0.2148 | -0.29 | 0.3859 |
| -2.78 | 0.0027 | -2.28 | 0.0113 | -1.78 | 0.0375 | -1.28 | 0.1003 | -0.78 | 0.2177 | -0.28 | 0.3897 |
| -2.77 | 0.0028 | -2.27 | 0.0116 | -1.77 | 0.0384 | -1.27 | 0.1020 | -0.77 | 0.2206 | -0.27 | 0.3936 |
| -2.76 | 0.0029 | -2.26 | 0.0119 | -1.76 | 0.0392 | -1.26 | 0.1038 | -0.76 | 0.2236 | -0.26 | 0.3974 |
| -2.75 | 0.0030 | -2.25 | 0.0122 | -1.75 | 0.0401 | -1.25 | 0.1056 | -0.75 | 0.2266 | -0.25 | 0.4013 |
| -2.74 | 0.0031 | -2.24 | 0.0125 | -1.74 | 0.0409 | -1.24 | 0.1075 | -0.74 | 0.2296 | -0.24 | 0.4052 |
| -2.73 | 0.0032 | -2.23 | 0.0129 | -1.73 | 0.0418 | -1.23 | 0.1093 | -0.73 | 0.2327 | -0.23 | 0.4090 |
| -2.72 | 0.0033 | -2.22 | 0.0132 | -1.72 | 0.0427 | -1.22 | 0.1112 | -0.72 | 0.2358 | -0.22 | 0.4129 |
| -2.71 | 0.0034 | -2.21 | 0.0136 | -1.71 | 0.0436 | -1.21 | 0.1131 | -0.71 | 0.2389 | -0.21 | 0.4168 |
| -2.70 | 0.0035 | -2.20 | 0.0139 | -1.70 | 0.0446 | -1.20 | 0.1151 | -0.70 | 0.2420 | -0.20 | 0.4207 |
| -2.69 | 0.0036 | -2.19 | 0.0143 | -1.69 | 0.0455 | -1.19 | 0.1170 | -0.69 | 0.2451 | -0.19 | 0.4247 |
| -2.68 | 0.0037 | -2.18 | 0.0146 | -1.68 | 0.0465 | -1.18 | 0.1190 | -0.68 | 0.2483 | -0.18 | 0.4286 |
| -2.67 | 0.0038 | -2.17 | 0.0150 | -1.67 | 0.0475 | -1.17 | 0.1210 | -0.67 | 0.2514 | -0.17 | 0.4325 |
| -2.66 | 0.0039 | -2.16 | 0.0154 | -1.66 | 0.0485 | -1.16 | 0.1230 | -0.66 | 0.2546 | -0.16 | 0.4364 |
| -2.65 | 0.0040 | -2.15 | 0.0158 | -1.65 | 0.0495 | -1.15 | 0.1251 | -0.65 | 0.2578 | -0.15 | 0.4404 |
| -2.64 | 0.0041 | -2.14 | 0.0162 | -1.64 | 0.0505 | -1.14 | 0.1271 | -0.64 | 0.2611 | -0.14 | 0.4443 |
| -2.63 | 0.0043 | -2.13 | 0.0166 | -1.63 | 0.0516 | -1.13 | 0.1292 | -0.63 | 0.2643 | -0.13 | 0.4483 |
| -2.62 | 0.0044 | -2.12 | 0.0170 | -1.62 | 0.0526 | -1.12 | 0.1314 | -0.62 | 0.2676 | -0.12 | 0.4522 |
| -2.61 | 0.0045 | -2.11 | 0.0174 | -1.61 | 0.0537 | -1.11 | 0.1335 | -0.61 | 0.2709 | -0.11 | 0.4562 |
| -2.60 | 0.0047 | -2.10 | 0.0179 | -1.60 | 0.0548 | -1.10 | 0.1357 | -0.60 | 0.2743 | -0.10 | 0.4602 |
| -2.59 | 0.0048 | -2.09 | 0.0183 | -1.59 | 0.0559 | -1.09 | 0.1379 | -0.59 | 0.2776 | -0.09 | 0.4641 |
| -2.58 | 0.0049 | -2.08 | 0.0188 | -1.58 | 0.0571 | -1.08 | 0.1401 | -0.58 | 0.2810 | -0.08 | 0.4681 |
| -2.57 | 0.0051 | -2.07 | 0.0192 | -1.57 | 0.0582 | -1.07 | 0.1423 | -0.57 | 0.2843 | -0.07 | 0.4721 |
| -2.56 | 0.0052 | -2.06 | 0.0197 | -1.56 | 0.0594 | -1.06 | 0.1446 | -0.56 | 0.2877 | -0.06 | 0.4761 |
| -2.55 | 0.0054 | -2.05 | 0.0202 | -1.55 | 0.0606 | -1.05 | 0.1469 | -0.55 | 0.2912 | -0.05 | 0.4801 |
| -2.54 | 0.0055 | -2.04 | 0.0207 | -1.54 | 0.0618 | -1.04 | 0.1492 | -0.54 | 0.2946 | -0.04 | 0.4840 |
| -2.53 | 0.0057 | -2.03 | 0.0212 | -1.53 | 0.0630 | -1.03 | 0.1515 | -0.53 | 0.2981 | -0.03 | 0.4880 |
| -2.52 | 0.0059 | -2.02 | 0.0217 | -1.52 | 0.0643 | -1.02 | 0.1539 | -0.52 | 0.3015 | -0.02 | 0.4920 |
| -2.51 | 0.0060 | -2.01 | 0.0222 | -1.51 | 0.0655 | -1.01 | 0.1562 | -0.51 | 0.3050 | -0.01 | 0.4960 |