Formal Logic: Handout 2

Introduction to Logic

Yılmaz Kılıçaslan March
March 01, 2023

Key Concepts: *Reasoning, Argument, Argument Schema, Declarative (Sentence), Premise, Conclusion, Syllogism, Modus Ponens, Modus Tollens, Deductive, Inductive, Abductive, Logical Certainty, Validity, Counterexample, Truth Value, Soundness, Factual, Hypothetical, Logical Constant / Operator.*

1. What is Logic About?

Logic can be defined as the science of **reasoning**. When used as a term of logic, 'reasoning' refers to an idealized system of thinking. That is, logic studies how an ideal individual (i.e., Man) thinks, not a concrete individual's way of thinking.[1]

The processes of reasoning studied in logic are called **arguments**, or **argument schemata**. An argument can be conveniently seen as a finite sequence of **declarative sentence**s in some language, with the last being distinguished as the **conclusion** and the rest (if any) being the **premises**:

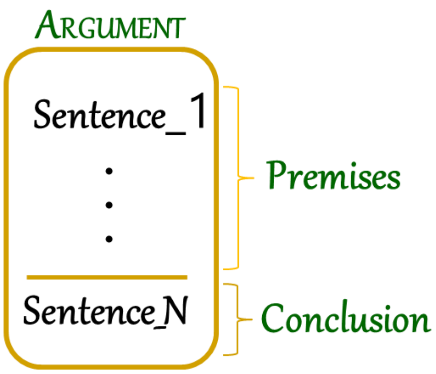

Figure 1. Arguments as a finite sequence of declarative sentences.

Below is the most famous argument schema, ***Modus (Ponendo) Ponens***:[2]

P implies Q (Major Premise)
P (Minor Premise)

Q (Conclusion)

Argument forms like this consisting of two premises and a conclusion are called **syllogism**s. They constitute the theory of logic developed by Aristotle, who laid down the foundations of

[1] The latter falls in the scope of Cognitive Psychology, which is a field of study much younger that Logic.

[2] Latin for "the way that affirms by affirming."

(rule-based) logic around twenty-two centuries ago. Another famous syllogism is ***Modus (Tollendo) Tollens***, which can be schematized as below:[3]

P implies Q
not Q
----
not P

As we will see today and in the subsequent lectures, arguments expressed concretely in a particular language need not conform to syllogistic schemas: they can also have more or less than two premises, which can all be general or specific statements. However, it is interesting to note that (neglecting one further constraint on syllogisms that we have not mentioned yet[4]) all logic can be reduced to and defined in terms of Modus Ponens and Modus Tollens.[5]

Let us be more specific. We can discern **three** types of logical reasoning:

1. **Deductive** reasoning
2. **Inductive** reasoning
3. **Abductive** reasoning

Deductive reasoning is a process of reasoning ending up with a **logically certain** conclusion. In this type of reasoning, we proceed from a general rule (e.g., P implies Q) and the antecedent of that rule (e.g., P) and necessarily conclude with the consequent of the same rule (e.g., Q). Certainty disappears in the other two types of reasoning. In inductive reasoning, we proceed from observations to a generalization. The observations are considered as supplying strong evidence for the truth of the generalization. That is, the truth of the generalization is taken to be highly probable on the basis of the evidence observed. In abductive reasoning, we start with a generalization and a specific observation and end up with an explanation for that observation. Again, the certainty that deductive reasoning enjoys is lacking. Making use of the frame underlying Modus Ponens, we can schematically describe this three-way partitioning of logical reasoning as follows:

	Deductive	**Inductive**	**Abductive**
P implies Q	✓	**?**	✓
P	✓	✓	**?**

Q	**?**	✓	✓

[3] Latin for "the way that denies by denying."
[4] Another constraint on Arristotle's syllogisms is that sentences have to be of a subject-predicate form. We will, later on, see that modern logic has done away with this constraint and, in this way, become more powerfull.
[5] If we equate negation with the failure to demonstrate something, as is done in logic programming, we can even get rid of Modus Tollens.

Computer science consists of two theories: computing theory and information theory. Deductive logic serves as the basis of computing theory whereas inductive logic lays the ground for information theory. Abductive logic has not (yet) been officially accepted as a scientific way of thinking. It is deductive reasoning that we will be concerned with in this course. Inductive logic will be discussed in courses like Artificial Intelligence and Machine Learning.

2. VALID ARGUMENTS AND DEDUCTIVE LOGIC

It would not be too much to say that the single most important property of an argument is that of validity. An argument is **valid** if and only if it is not possible for the premises all to be true while the conclusion is false. In other words, if the premises were all true, then the conclusion would have to be true. So a valid argument has no possible **counterexample**. Below are a few examples of valid arguments:[6]

(1) *John will come to the party, or Mary will come to the party.*
John will not come to the party.

Mary will come to the party.

(2) *John will come to the party, or Mary will come to the party.*
If John has not found a baby sitter, he will not come to the party.
John has not found a baby sitter.

Mary will come to the party.

(3) *All airplanes can crash.*
All DC-10s are airplanes.

All DC-10s can crash.

(4) *John is a teacher.*
John is friendly.

Not all teachers are unfriendly.

(5) *All fish are mammals.*
Moby Dick is a fish.

Moby Dick is a mammal.

[6] The examples used in this handout are adopted from Gamut (1991).

In these arguments, the **truth** of the premises involves the truth of the conclusion. Therefore, in each case any reasonable person who considers the premises to be true will have to consider the conclusion as true, too.

The task of (deductive) logic is to find out what makes a valid argument valid. Let us start our quest to answer this question by posing its opposite: what is not relevant for the validity of an argument?

3. What does not Make a Valid Argument Valid?

In order to know the validity of an argument, one does not have to know the truth values of its premises and its conclusion. Take, for instance, argument (1). One need not know whether Mary and John did really attend the party, even who they are, in order to see that this argument is valid.

Besides, the premises of a valid argument can even be plainly false. This is apparent from example (5). Fish are not mammals and Moby Dick is a whale, not a fish. That is, both premises of this argument are false. But, that does not stop the argument as a whole from being valid: if one were to accept that the premises were true, then one would also have to accept the conclusion. It is worth noting *en passant* that the truth of the premises is only necessary for an argument to be **sound**: a sound argument is a valid argument with true premises. We will not go into the issue of soundness in this course.

The short discussion above simply says that the validity of an argument does not depend on the truth of its premises: the premises of a valid argument can be false or be unknown as to their truth values. Furthermore, the truth of the premises does not assure the validity of the argument, either. That is to say, we can have an invalid argument with all the premises being perfectly true, as is the case in the example below:

(6) All horses are mammals.
All horses are vertebrates.

All mammals are vertebrates.

This argument is invalid simply because the truth of the premises does not oblige one to accept the truth of the conclusion. As long as it is possible for one to imagine a counterexample, i.e., a situation where the premises are true while the conclusion, an argument fails to be valid. Notice that this is a matter of reasonable imagination, not one bounded by empirical facts. There is no logical constraint on all horses being mammals and vertebrates while all mammals being vertebrates. Thus, (6) is not valid.

What follows from the discussion in this section is that the validity of an argument does not depend on the **factual** truth of its premises or that of its conclusion.

4. WHAT MAKES A VALID ARGUMENT VALID?

Logic is not an empirical science. That is, it does not rest its subject matter on empirical evidence, on evidence acquired by means of the senses. It is (or pretends to be) an abstract science in the sense that everything and anything relating to empirical observations falls outside its scope of investigation. Therefore, it should not be surprising but expected for the factual truth of the premises or of the conclusion to be irrelevant for the validity of an argument. It is not factual truth but hypothetical truth that comes into play when determining the validity of arguments. In other words, when deciding upon the validity of an argument, we do not care about the actual correspondence between the statements in an argument and a real state of affairs but are concerned with whether the conclusion has to be true just in case the premises happen to be true.

Let us now try to find out what encodes the ‘abstract’ relation between the premises and the conclusion in a valid argument and what can be left out in this regard. If logic has nothing to do with particularities, proper names should be expected not to play a part in making an argument valid, as they denote particular individuals. We can immediately start to observe that this is really the case by exchanging one proper name with another in an argument. For example, when we write “Peter” instead of “John”, argument (1) remains valid:

(7) Peter will come to the party, or Mary will come to the party.
Peter will not come to the party.

Mary will come to the party.

By the same token, the particular activity engaged in should not be important in the question of validity. Again, we see from examples like the following that this expectation holds:

(8) Peter will come to the meeting, or Mary will come to the meeting.
Peter will not come to the meeting.

Mary will come to the meeting.

We can show that none of the expressions having a descriptive content, which can change from one particular context to another, bear any importance for the validity of an argument: they can simply be replaced by others without any effect on the validity of that argument. However, such replacements cannot be done so simply in the case of grammatical words, which are devoid of descriptive content. For instance, in the examples above, *or* and *not* cannot be exchanged for others while retaining the validity of the arguments. Such replacements would result in invalid arguments, as in (9) and (10).

(9) John will come to the party, or Mary will come to the party.
John will come to the party.

Mary will come to the party.

(10) John will come to the party if Mary will come to the party.
John will not come to the party.

Mary will come to the party.

What follows from this section is that what makes arguments (1), (7), and (8) valid is merely the fact that they have in common a form that can be described like this: each of the arguments comprises two premises; one premise consists of two sentences linked together by the conjunction *or*; the other premise is the denial of the first conjunct in that conjoined sentence; and the second conjunct is the conclusion.

5. ARGUMENT SCHEMATA

The common form ensuring the validity of (1), (7), and (8) can be schematically represented as follows:

(11) *A or B*
not A

B

In these schematic representations, which are referred to as *argument schemata*, the letters like *A* and *B* serve as variables which can take arbitrary **declarative** sentences as inputs. An argument schema turns into an actual argument when its variables are filled in with declarative sentences. (11) is said to be a *valid argument schema* because substituting the variables with actual sentences results in a valid argument.

Needless to say that what conjunction we are using when constructing argument schemas matters for the validity of these schemas. For example, replacing the conjunction *or* in (11) by another conjunction, say, *if* results in a schema which is not valid:

(12) *A if B*
not A

B

Considerations similar to those for (1) lead to the following argument schema for (5):

(13) *All P are Q*

a is P

----

a is Q

This is another valid schema. It differs from those in (11) and (12) in that the variables do not stand for sentences; P and Q stand for expressions referring to properties and the variable *a* stands for an expression referring to an individual or an entity; and, more importantly, in that its validity derives from, among other things, the meaning of the quantifying expression *all*. Other examples of quantifying expressions to be found in argument schemata are *some* and *no*.

A final remark to end this section. Formal logic investigates the validity of arguments (and, thereby, reasoning) by investigating the validity of argument schemata. Expressions on which argument schemata depend for their validity (e.g., *or, if, not*, *all*) are called **logical constants**.

6. Logical Constants and Logical Systems

What determines a logical system is the set of logical constants that it has:

Logical Constants	**Logical Systems**
and, or, if (... then), if and only if, not	Propositional Logic
all, some	Predicate Logic
believes that, knows that	Epistemic Logic
possibly, necessarily	Modal Logic
it was the case that, it will be the case that	Tense Logic
etc.	

It is also possible to develop new logical systems with the same set of logical constants considered under a new interpretation. Therefore, we say that a logical system is characterized by its logical constants together with the interpretations placed upon them.

REFERENCES

- ***Introduction to Logic*: Logic, Language, and Meaning (Volume 1), Gamut, L.T.F. Chicago: The University of Chicago Press, 1991.**