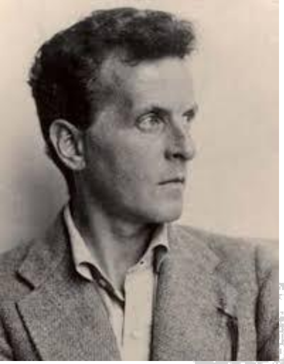


"The limits of my language mean the limits of my world. All I have is what I have words for."

Ludwig Wittgenstein

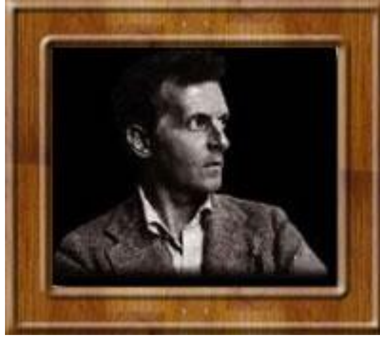
Analysis of Propositions

Yılmaz Kılıçaslan
Aydın Adnan Menderes University

Presentation Plan

- Language and Logic
- Argumentation in L_1
- Problems with L_1
- Need for new languages
- A new formal language: L_2
- Need for quantifiers
- The *universal* and *existential* quantifiers
- A new logical system containing the universal and existential quantifiers: First-Order Predicate Logic
- A language for First-Order Predicate Logic: L_3

WITTGENSTEIN'S VIEW OF LANGUAGE



- **The boundary of my language represents the boundary of my world.**

  Logic spreads throughout the world, so the boundaries of the world are also its boundaries. One cannot logically say that the world has one thing in it but not the other. This exclusion would mean one can go beyond the boundaries of the world. That, after all, is the only way to view those boundaries from the other side. What we cannot think, we cannot think; so also, we cannot *say* what we cannot think.

  This remark provides the key to deciding how much truth there is to solipsism. What the solipsist *means* to say is quite correct; only it cannot be *said*, but proves to be so. That the world is my world is shown by the fact that the limits of language (the only language I understand) comprise the limits of my world. The world and life are one. (Tractatus Logico-Philosophicus 5.6 to 5.621)

LANGUAGE ⟷ (INTERNAL) WORLD ⟷ LOGIC

Translation into L_1 (a Retrospective)

- Now that we have a formal language at our disposal, namely $\mathbf{L_1}$, we can translate (indicative) sentences of English into that language.

- The following are, respectively, the translations of two arguments introduced in one of our early lectures:

1. $p \vee q$

$\neg p$

----

q

← Ali will come to the party or Ayşe will come to the party.

Ali will not come to the party.

Ayşe will come to the party.

2. $p \vee q$

$\neg r \rightarrow \neg p$

$\neg r$

----

q

← Ali will come to the party or Ayşe will come to the party.

If Ali has not found a baby sitter, he will not come to the party.

Ali has not found a baby sitter.

Ayşe will come to the party.

Capturing Validity in L_1

- Notice that under the guidance of one or both of the following schemas we can formally capture the validity of argument 1 and argument 2.

3. *A or B*
   Not A

   B

4. *If A then B*
   A

   B

MISSING VALIDITY IN L_1

- However, not all arguments can be shown to be valid in L_1. For example, translating the arguments below (again introduced earlier) into L_1 would result in the loss of some parts of their meanings upon which a valid argumentation would be based.

All airplanes can crash.
All F-16s are airplanes.

All F-16s can crash.

Ali is a teacher.
Ali is friendly.

Not all teachers are unfriendly.

All fish are mammals.
Moby Dick is a fish.

Moby Dick is a mammal.

MISSING VALIDITY IN L_1

- Let us look at another case of a valid argument whose translation into L_1 will again bring about loss in meaning intolerable for validity:

5. Muhammad Ali is taller than Richard Nixon.
   Richard Nixon is taller than Noam Chomsky.

   Muhammad Ali is taller than Noam Chomsky.

- This is a valid argument. However, that cannot be shown if we translate the premises and conclusion into propositional letters:

6. p
   q
   -----
   r

?

Ali is happy.
Ayşe loves Murat.

Fido is chasing a cat.

MISSING VALIDITY IN L_1

- The problem with L_1 when dealing with arguments like 5 is that propositions are treated as the most detailed level of analysis.
- We, at least , need to see the relation stated to hold between the individuals. If we replace the relation 'taller than' with 'love', for example, the argument ceases to be valid:

7. Muhammad Ali loves Richard Nixon.
   Richard Nixon loves Noam Chomsky.

   Muhammad Ali loves Noam Chomsky.

- Hence, to capture the validity of 5 in formal terms, it is necessary to analyze the propositions constituting it into a structure where the individuals and their properties or the relations between them (i.e., the predicate) can be discerned.

TOWARDS A NEW LANGUAGE

- What we need is a language that allows us to go into the internal structure of propositions.
- More specifically, it is necessary (though not sufficient) to discern the predicates and the individuals bearing these predicates.
- Technically, we will discard propositional variables and introduce variables for predicates and individuals instead.
- Therefore, we can represent argument 5 as below:

8. *T(m, r)*
   T(r, n)

   T(m, n)

where T, m, r, and n stand for 'taller than', 'Muhammad Ali ', 'Richard Nixon', and 'Noam Chomsky', respectively.

L_2 (Syntax)

Syntax of L_2:

A. *Basic Expressions*:

Category	***Basic Expressions***
Names	*d*, *n*, *j*, and *m*
One-place predicates	*M*, *B*
Two-place predicates	*K*, *L*

B. *Formation Rules:*

1. If δ is a one-place predicate and α is a name, then $\delta(\alpha)$ is a sentence.

2. If γ is a two-place predicate and α and β are names, then $\gamma(\alpha, \beta)$ is a sentence.

3. If φ is a sentence, then $\neg\varphi$ is a sentence.

4. If φ and ψ are sentences, then $[\varphi \wedge \psi]$ is a sentence.

5. If φ and ψ are sentences, then $[\varphi \vee \psi]$ is a sentence.

6. If φ and ψ are sentences, then $[\varphi \rightarrow \psi]$ is a sentence.

7. If φ and ψ are sentences, then $[\varphi \leftrightarrow \psi]$ is a sentence.

L_2 (SEMANTICS)

SEMANTICS OF L_2:

A. Basic Expressions:

[d] = Richard Nixon; [n] = Noam Chomsky; [j] = Jacque Chirac; [m] = Muhammad Ali

[M] = the set of all living people with moustaches; [B] = the set of all living people who are bald

[K] = the set of all pairs of living people such that the first knows the second;

[L] = the set of all pairs of living people such that the first loves the second.

B. Semantic Rules:

1. If δ is a one-place predicate and α is a name, then $\delta(\alpha)$ is true iff $[\alpha] \in [\delta]$.
2. If γ is a two-place predicate and α and β are names, then $\gamma(\alpha, \beta)$ is true iff $<[\alpha], [\beta]> \in [\delta]$.
3. If φ is a sentence, then $\neg\varphi$ is true iff φ is not true.
4. If φ and ψ are sentences, then $[\varphi \wedge \psi]$ is true iff φ and ψ are true.
5. If φ and ψ are sentences, then $[\varphi \vee \psi]$ is true iff either φ or ψ is true.
6. If φ and ψ are sentences, then $[\varphi \rightarrow \psi]$ is true iff either φ is false or ψ is true.
7. If φ and ψ are sentences, then $[\varphi \leftrightarrow \psi]$ is true iff either φ and ψ are both true or else φ and ψ are both false.

Invalidity of Argument 5 in L_1 and L_2

- Unfortunately, like L_1, L_2 is not capable of capturing the validity of the argument which was previously introduced as argument (5):

Argument 5

Muhammad Ali is taller than Richard Nixon.
Richard Nixon is taller than Noam Chomsky.
___
Muhammad Ali is taller than Noam Chomsky.

Translation into L_1

$$\begin{array}{l} p \\ q \\ \hline r \end{array}$$

Translation into L_2

$$\begin{array}{l} T(m, r) \\ T(r, n) \\ \hline T(m, n) \end{array}$$

Argument 6

Muhammad Ali loves Richard Nixon.
Richard Nixon loves Noam Chomsky.
___
Muhammad Ali loves Noam Chomsky.

ARITY

- The number of arguments that a predicate can take is referred to as the arity of that predicate.
- In $\mathbf{L_2}$, the arity of a predicate can be either **1** or **2**.
  - The arity of *M* and *B* is 1. That is, they are unary predicates.
  - The arity of *K* and *L* is 2. That is, they are binary predicates.
- However, there is no theoretical upper limit on arities. In general, n-ary predicates may be introduced for any whole number.
- In natural language, there seem to be even nullary (0-ary) predicates: ***rain*****,** ***snow*****,** ***cold*****,** etc.
- Yet, natural languages seem to have an upper limit for the arities of their predicates. In addition to nullary natural-language predicates, we have:
  - Unary (intransitive) predicates: ***sleep*****,** ***snore*****,** ***in*****,** ***from*****, etc.**
  - Binary (transitive) predicates: ***cut*****,** ***see*****, etc.**
  - Ternary (ditransitive) predicates: ***give*****,** ***buy*****,** ***show*****, etc.**

Arguments and Roles

- In general an atomic formula is obtained by writing n names or variables (in brackets) after an n-ary predicate letter.
- These names or variables denote the *arguments* of the predicates.
- Note that the term *argument* here is different from the term *argument* which refers to a sequence of premises and a conclusion. They are just homonyms.
- The arguments of a predicate need not necessarily be different.
- However, each argument of a predicate bears a different role.
- In many (albeit not all) languages, including L_2, the role an argument plays is specified by the order of the arguments.

  Exercise: Compare English and Turkish in this respect.
- Therefore, the order of the arguments can make a difference for some relations in these languages: if Muhammad Ali is taller than Richard Nixon, then Richard Nixon is not taller than Muhammad Ali.

KEYS AND VARIABLES

- When writing keys to translations of natural language sentences, variables can be used to specify the roles of the arguments.
- Variables include x, y, and z and their subscripted versions.
- For example, the key to the translation of "Muhammad Ali is taller than Richard Nixon" into L_2 can be as follows:

  T(x, y): x is taller than **y;**

  m: Muhammed Ali;

  n: Richard Nixon.
- Needless to say, translations need be as explicit as possible in order not to lose anything that is logically important.

REFERENCES

- Introduction to Logic: Logic, Language, and Meaning (Volume 1), Gamut, L.T.F. Chicago: The University of Chicago Press, 1991.
- Dowty D.R., Wall R. E. and Peters S. (1981) Introduction to Montague Semantics. Dordrecht, The Netherlands: Kluwer Academic Publishers.