# Formal Logic: Handout 3

## Propositional Logic (Propositions and Logical Operators)

Yılmaz Kılıçaslan March
March 08, 2023

**<u>Key Concepts:</u>** *Proposition, Propositional Logic, Declarative (Sentence), Semantic (Content), Syntactic (Structure), Illocutionary (Act), Truth-Functionality, Operator, Operand, Conjunction, Conjunct.*

## 1 INTRODUCTION

Last week we made an introduction to logic and saw that validity is a central notion to this field of study. We concluded the lecture by pointing out that the validity of an argument is solely dependent on logical constants and that it is by means of these logical constants that a logical system is defined. Remember that the set of logical constants characterizing propositional logic is given as {*and, or, if (... then), if and only if, not*}. From the argument schemata that we used in that lecture we also observed that such logical constants co-occur with some variables standing for some declarative sentences. Such declarative sentences are considered by some to refer to **proposition**s, which is the reason why the system in question is called *propositional logic*.

The aim of the lecture today is to make a conceptual introduction to propositional logic. More specifically, we will try to understand what kind of things propositions are and how the logical constants of propositional logic are determined.

## 2 PROPOSITIONS

Propositions are not the sorts of things that we talk about in everyday life. As they are abstract things, some pondering is needed to get some understanding of what they are.

Propositions seem to be an artifact of theoretical studies of meaning. Many semanticists (i.e., those studying linguistic meaning) take the meaning of a declarative sentence to be a proposition and some semanticists maintain that “[t]o know the meaning of a (declarative) sentence is to know what the world would have to be like for the sentence to be true”. (Dowty et al., 1981) Rather than delving further into this philosophically deep matter, let us keep things simple and state that a proposition is a truth bearer. In other words, a proposition is what can be **true** or **false**. Being true or also false is not only a sufficient condition for being a proposition; but furthermore, there is no property other than a truth-value to be predicated of a proposition.

Sometimes a declarative sentence or a statement is said to be a proposition. Such are not appropriate descriptions. A proposition can be encoded with different sentences. For example, the sentences below are taken to express one and the same identical proposition; they are considered to be about the same factual situation:

(1) The dog chased the cat.

(2) The cat was chased by the dog.

The same proposition could also be expressed in a different language. (3) and (4) can be utterances of exactly what is stated in (1) and (2) in Turkish and in French, respectively:

(3) Köpek kediyi kovaladı.

(4) Le chien a chassé le chat.

In short, declarative sentences are 'containers' for propositions, not propositions themselves. Broadly speaking, propositions are *semantic* contents of sentences, which have a *syntactic* structure.

As for statements, these are illocutionary acts made for asserting that a proposition is true. For instance, the following sentence will be a statement when uttered on a possible occasion:

(5) John owns a donkey.

However, the same sentence can also be used in a linguistic context where the proposition conveyed is left unasserted and, hence, without its truth value being determined, as is the case below:

(6) If *John owns a donkey*, he beats it.

In brief, statements are pragmatic means of determining the truth value of a proposition. Propositions are objects that are either true or false but they can be left unspecified as to whether they are actually true or false.

## 3 From Grammatical Words to Logical Operators

### 3. 1 TRUTH-FUNCTIONALITY

Logical constants are recruited from among grammatical words of natural language. Hence, the next question to be discussed will be that of which linguistic expressions can be adapted as a logical operator of propositional logic.

The set of logical constants of propositional logic comprises some connectives and the negation operator. The connectives are binary operators: they take two propositions as inputs and yield a single compound proposition. The negation operator is unary: it takes a single proposition as input and outputs the negation of it.

Propositional logic recruits its connectives from among conjunctions of natural language, which link two sentences into a new one. However, not every conjunction from natural language can serve as a logical connective. Note that a logical connective is a function from binary-tuples of propositions to propositions. Semantically, a proposition can be reduced just to a truth-value. To put it another way, a proposition refers to a truth-value. Therefore, in order for a conjunction word to be eligible as a logical constant, it has to be truth-functional, i.e., semantically definable as a function on truth-values. Consider the following sentences:

(7) John has bumped his head and he is crying.

(8) John is crying because he has bumped his head.

(9) John is crying.

(10) John has bumped his head.

The sentences in (7) and (8) are formed by conjoining the sentences in (9) and (10) with albeit two different conjunction words: 'and' and 'because', respectively. Of these two words, only 'and' serves as a truth-functional operator. The truth-functional behavior of this word is evident from the interpretation of sentence (7): it is true if both of its conjuncts, i.e., (9) and (10), are true, and false if either is false. That is, the truth-value of this compound sentence depends only on the truth-values of the smaller sentences conjoined by 'and'. Hence, we say that the conjunction word 'and' behaves as a function that takes only two truth-values and outputs one single truth-value: it is truth-functional. But, such truth-functional interpretation does not apply to sentence (8). It might be the case that John has in fact bumped his head and is indeed crying. But, this ensures neither the truth nor the falsity of sentence (8): his bumping his head might be the real reason behind John's crying, making (8) true; or John might be crying for another reason (e.g., because it is raining), which makes (8) false. Apparently, the truth value of a 'because' sentence (i.e., a sentence formed by conjoining two smaller sentences with 'because') depends on more than just the truth values of the two parts constituting it. So, we say that 'because' is not a truth-functional connective. To conclude, the word 'and' but not the word 'because' can be adapted as a logical constant of propositional logic.

Among other conjunctions that serve as logical constants in propositional logic are 'or', 'if … (then)', and 'if and only if'.

## 3.2 ABSTRACTING AWAY FROM IRRELEVANCIES

Note that we use the word 'adapt' for the process of borrowing a logical connective from natural language. This is because a conjunction word does not carry with itself all its possible interpretations in natural language when it is accepted as a logical constant. It is assigned one single and fixed interpretation. For example, sometimes the order of the conjuncts matters in the use of the word and in natural language:

(11) John shaved and he got out.

(12) John got out and he shaved.

The sentences above are not truth-conditionally equivalent: (11) might be true while (12) is false; or vice versa. However, ‘and’ serves as a commutative operator in logic: the order of its operands does not matter. It follows that any component of the interpretation of the word ‘and’ that imposes an ordering constraint on its conjuncts is abstracted away when this word is adopted from natural language to be used as a logical constant. The last statement of the preceding lecture generalizes well the principle underlying such adoption processes: a logical system is characterized by its logical constants together with the interpretations placed upon them.

# REFERENCES


- *Introduction to Logic*: Logic, Language, and Meaning (Volume 1), Gamut, L.T.F. Chicago: The University of Chicago Press, 1991.
- Introduction to Montague Semantics. Dowty D. R., Wall R. E. and Peters S. Dordrecht, The Netherlands: Kluwer Academic Publishers, 1981.