age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high ✓	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes ✓
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes ✓
<=30	medium	no	fair	no
<=30	low	yes	fair	yes ✓
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high ✓	yes	fair	yes
>40	medium	no	excellent	no

Ofts 01

① Normal

Known Information : age > 40 , income = low .

Asked : P(Buy ? | age > 40 , income = low)

Sol^n :

→ $P(C_i)$ ⇒ $P(\text{buy computer} = \text{"yes"}) = \frac{9}{14}$

$P(\text{buy computer} = \text{"no"}) = \frac{5}{14}$

→ $P(X \mid C_i)$

$P(\text{age} > 40 \mid \text{buy} = \text{yes}) = \frac{3}{9}$

$P(\text{age} > 40 \mid \text{buy} = \text{no}) = \frac{2}{5}$

$P(\text{low} \mid \text{buy} = \text{yes}) = \frac{3}{9}$

$P(\text{low} \mid \text{buy} = \text{no}) = \frac{1}{5}$

$P(X \mid \text{buy} = \text{yes}) = \frac{3}{9} \cdot \frac{3}{9} = \frac{9}{81} = \frac{1}{9}$

$P(X \mid \text{buy} = \text{no}) = \frac{2}{5} \cdot \frac{1}{5} = \frac{2}{25}$

$P(X \mid C_i) * P(C_i) = P(X \mid \text{buy} = \text{yes}) \cdot P(\text{buy} = \text{yes})$

$= \frac{1}{9} \cdot \frac{9}{14} = \frac{1}{14}$

$P(X \mid C_i) * P(C_i) = P(X \mid \text{buy} = \text{no}) * P(\text{buy} = \text{no})$

$= \frac{2}{25} \cdot \frac{5}{14} = \frac{2}{70} = \frac{1}{35}$

Since $P(X \mid \text{buy} = \text{yes}) P(\text{buy} = \text{yes}) > P(X \mid \text{buy} = \text{no}) P(\text{buy} = \text{no})$. then the customer is buying the computer.

P(buy ? | age > 40, income = high)

→ $P(X \mid C_i)$

$P(\text{age} > 40 \mid \text{buy} = \text{yes}) = \frac{3}{9}$

$P(\text{age} > 40 \mid \text{buy} = \text{no}) = \frac{2}{5}$

$P(\text{high} \mid \text{buy} = \text{yes}) = \frac{2}{9}$

$P(\text{high} \mid \text{buy} = \text{no}) = \frac{2}{5}$

$P(X \mid \text{buy} = \text{yes}) = \frac{3}{9} \cdot \frac{2}{9} = \frac{2}{27}$

$P(X \mid \text{buy} = \text{no}) = \frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

$P(X \mid C_i) * P(C_i) = P(X \mid \text{buy} = \text{yes}) \cdot P(\text{buy} = \text{yes})$

$= \frac{2}{27} \cdot \frac{9}{14} = \frac{2}{42} = \frac{1}{21}$

$P(X \mid C_i) * P(C_i) = P(X \mid \text{buy} = \text{no}) * P(\text{buy} = \text{no})$

$= \frac{4}{25} \cdot \frac{5}{14} = \frac{2}{35}$

P (Buy ? | age 31-40, income high)

2. Laplacian Correction

- P(Ci) ⇒ P(buy computer = "yes") = $\frac{9}{14}$

  P(buy computer = "no") = $\frac{5}{14}$

- P(X | Ci)

  P(age 31-40 | buy = yes) = $\frac{4}{9}$ ⇒ $\frac{5}{9+3} = \frac{5}{12}$

  P(age 31-40 | buy = no) = $\frac{0}{5}$ ⇒ $\frac{1}{5+3} = \frac{1}{8}$

  P(high | buy = yes) = $\frac{2}{9}$

  P(high | buy = no) = $\frac{2}{5}$

$P(X \mid buy = yes) = \frac{5}{\cancel{12}\,6} \cdot \frac{\cancel{2}}{9} = \frac{5}{54}$

$P(X \mid buy = no) = \frac{1}{8} \cdot \frac{2}{5} = \frac{1}{20}$

$P(X \mid C_i) * P(C_i) = P(X \mid buy = yes) \cdot P(buy = yes)$

$= \frac{5}{54} \cdot \frac{9}{14} = \frac{5}{84} = 0.0595$

$P(X \mid C_i) * P(C_i) = P(X \mid buy = no) * P(buy = no)$

$= \frac{1}{20} \cdot \frac{5}{14} = \frac{1}{56} = 0.0178$

So, the customer is buying the computer