Notes on Dynamic Delegation

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Motivation

- Dynamic delegation to multiple heterogeneously specialized agents
- one principal v.s. many agents. Misalignment of interests.
  - manager v.s. workers
  - head quarters(overall profits) v.s. division managers(their own division's profits)
  - A military v.s. elite units (a prestigious operation)
- 各期に一つ task が降ってくる．毎期最も適している人に割り当てたい．
- Monetary transfer, commitment to future allocation decisions が不可能な状況下での dynamic mechanism design.
  - monetary transfer による動機付けが難しいのは organizational decision making ではよくありそう？
  - commitment が難しいってどういう状況？この仮定は重要なの？

Model

- Players $i \in \{0, 1, 2\}$. principal: 0, agents: 1,2
- $\omega_t \in \{A, B\}$: the type of period-t job, observable from everyone.
- $\theta_{it} \in \{\alpha, \beta\}$: agent i's type in period t, private info.
- $\phi^i \in [0, 1]$: i's specialization, common knowledge.
  - $\Pr(\theta_{it} = \alpha) = \phi^i$, independent across periods and agents.
  - ϕ^i の値が大きいほど，task A に向いている．
  - Assume for simplicity that $\phi^1 := \phi$, $\phi^2 := 1 - \phi \quad (\phi \in [1/2, 1))$.
- $y_t \in \{0, S, F\}$: outcome. $y_t = 0$ は，その気に誰にも task を割り振らなかったことを意味する．S, F はそれぞれ，success と failure を表す．
- $\Pr(y_t = S \mid (\theta_{it}, \omega_t) \in \{(\alpha, A), (\beta, B)\}, i \text{ is assigned}) = \overline{\mu}$

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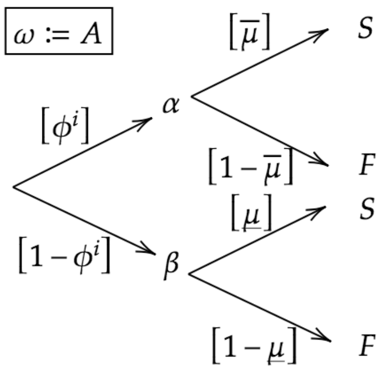
- $\Pr(y_t = S \mid (\theta_{it}, \omega_t) \in \{(\alpha, B), (\beta, A)\}, i \text{ is assigned}) = \underline{\mu}, (\underline{\mu} < \overline{\mu})$



- Repeated games model.
- Each period-t stage game proceeds as follows:
  1. $\omega_t \in \{A, B\}$ realizes with equal probability. Everyone observes the value of ω_t.
  2. Each agent i send a message $m_{it} \in \{0, 1\}$ to the principal simultaneously.
  3. The principal allocate the task to one of the agents $j_t = i \in \{1, 2\}$, or throw it away ($j_t = 0$).
  4. Whether the agent succeeds or not, y_t, is observed. Payoff realizes.
  5. (Public Randomization)
- Adverse selection + Imperfect monitering
- period-t history: $((m_{1t}, m_{2t}), \omega_t, j_t, y_t)$
  - m_{it}: the message that the agent send in period t
  - $\omega_t \in \{A, B\}$: the type of period-t task
  - $y_t \in \{0, S, F\}$: success or failure.
  - $j_t \in \{0, 1, 2\}$: who is assigned
  - $j_t = 0$ は，どちらの agent にも割り振らないことを表す．(その場合，全員の利得 0．)
  - $j_t = 0$ (どちらの agent にも振らない) をモデルに入れる必要があるか？元論文では入れているが，不必要なようにも思える．要検討．(feasible payoff にも影響？)
- public history: $h^t := ((m_{1\tau}, m_{2\tau}), \omega_\tau, j_\tau, y_\tau)_{\tau=1}^{t-1}$
- Utility functions:
  - $U_0 := (1-\delta)\sum_t \delta^t \mathbb{1}_{\{y_t=S\}}, \quad U_i := (1-\delta)\sum_t \delta^t \pi \mathbb{1}_{\{j_t=i\}} \mathbb{1}_{\{y_t=S\}}\ (i \in \{1, 2\})$
  - principal は，仕事が成功すると利得 1．agent は，仕事が自分に振られた上で仕事に成功すると利得 π をその期に得る．
  - myopically には,「俺が俺が」といって仕事を振られた方が agent としては得.
- 均衡概念としては，ex-post public perfect equilibrium(XPPE) を考える．
  - public perfect equilibrium: public strategy(public history にのみ依存した戦略) による均衡.
  - σ : PPE $\overset{\Delta}{\iff} \forall i; \sigma_i$ is public, $\forall t \forall h^{t-1}; \sigma :$ NE from that time on.
  - "ex-post": 各 agent は，その期の他のプレイヤーのタイプに関する belief に対して無関係に最適となるような戦略をとる．
  - principal は普通に期待効用 (agents のタイプに対して期待値をとる) に基づいて戦略を決めている．(principal については ex-post ではない？)

- なぜ ex-post なのを考えるの？計算が楽だから？
    - ∗ 「decision のタイミングを遅らせるなどして他の agent のタイプを学んでも意味がない.」という意味で robust.
    - ∗ Athey and Miller (2007), Bergemann and Valimaki (2010) などで導入？
    - ∗ PPE との関係が §5 で若干触れられている．(PPE まで許すと，効率的な均衡利得の集合が厳密に増加)
- $\sigma := (\chi_t, M_{1t}, M_{2t})_t : \chi_t(h_t) : (\omega_t, m_{1t}, m_{2t}) \mapsto j_t \in \{1, 2\}, M_{it}(h_t) : (\omega_t, \theta_{it}) \mapsto m_{it} \in \{0, 1\}$

- σ : efficient XPPE $\overset{\Delta}{\iff}$ $\forall t$; (i) The principal do not throw the task away, and (ii) if there is a suited agent, the task is allocated to him/her.

§3 Efficient Delegation

When can efficiency be attained? – When the degree of specialization ϕ is not too high.

$$\mathcal{E}^* \neq \emptyset \iff \phi(1-\phi) \geq \frac{\underline{\mu}(1-\delta\overline{\mu})}{\delta(\overline{\mu}-\underline{\mu})(1-\underline{\mu})} \quad \left(\phi \in \left[\frac{1}{2}, 1\right)\right)$$
$$\therefore \quad \exists \phi^* \; \forall \phi; \quad [\phi \geq \phi^* \iff \mathcal{E}^* \neq \emptyset]$$

§4 Rules for efficient delegation

What form of dynamic incentive provision should take? – Dynamic favoritism

Markov Priority Rules(MPR)

- Whenever $\mathcal{E}^* \neq \emptyset$, efficiency is attainable using a rule with in this family.

Def. 1 (MPR). *A Markov priority rule is characterized by* $(f_1, \mathcal{X}, \psi)$*:*

1. $f_1 \in \{1, 2\}$*: favored agent in period 1.*
2. $\mathcal{X}(\omega, f, m) \in \{(0,0), (1,0), (0,1)\}$*: allocation*
3. $\psi(\omega, f, j, y) \in \Delta(1, 2)$*: transition of* f

- $\psi^f(\omega, j, y) := \Pr(f_{t+1} = f_t \mid \omega, j, y)$

Def. 2 (failure-driven). *A MPR* $(f_1, \mathcal{X}, \psi)$ *is failure-driven* $\overset{\Delta}{\iff} \psi^f(\omega, j, y) = \mathbb{1}_{\{(j,y) \neq (f,F)\}}$

- In particular, the following simple failure-driven MPR, named *maximal-priority* can always attain efficiency if possible.

Def. 3 (Maximal-priority). • *Choose any agents as the favored agent at the beginning.*

- *(For* $t \geq 1$*)*
- *The favored agent* i *gets the task iff* $(m_i, m_{-i}) \neq (0, 1)$.
- *If* $(m_i, m_{-i}) = (0, 1)$, $-i$ *gets the task.*
- *If the favored agent* i *fails,* j *becomes the favored agent instead; otherwise* i *keeps being favored.*

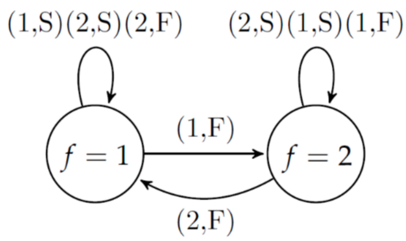


Figure 5: Transitions determining the identity of the favored agent, f, under maximal-priority.

Equivalence between ex-post and performance-based equilibria

- Performance-based: 過去の message に依存しない戦略

Preliminaries

- In §3, we characterize the efficient XPPE payoffs $\mathcal{E}^*$.
- Before entering §3, we check some basic properties of $\mathcal{E}^*$: the characterization of $\mathcal{E}^*$ is reduced to some fixed point problem.
- $\mathcal{E}$: the set of XPPE payoffs.
- $\mathcal{O} := \mathrm{co}\{(0,0,0),(v^*,\pi v^*,0),(v^*,0,\pi v^*)\}$: the set of feasible payoffs.
  - 本当に？(0,0,0) は入るのか？ $j=0$ をモデルに入れるかどうかとも関係？
  - $\mathcal{O} = \{(v_0,v_1,v_2) \in \mathbb{R}^3 \mid v_1+v_2 = \pi v_0, v \geq 0, v_0 \leq v^*\}$
- $\mathcal{E}^* := \{v \in \mathcal{E} \mid v_1+v_2 = \pi v^*\}$, where $v^* := \phi(1-\phi)\underline{\mu} + (1-\phi(1-\phi))\overline{\mu}$
- $\mathcal{W}(V) := \{v \in \mathbb{R}^3_+ \mid v \text{ is decomposable on } V\}$
  - Definition of "decomposability"
  - principal's strategy: $\chi(\omega,(m_1,m_2)) = (\chi_1(\omega,(m_1,m_2)),\chi_2(\omega,(m_1,m_2))) \in \{(0,0),(1,0),(0,1)\}$
  - agent's strategy: $M_i(\omega,\theta_i) \in \{0,1\}$
  - continuation payoffs: $\mathcal{V}(\omega,m,j,y) \in \mathbb{R}^3_+$, $\mathcal{V}(\omega,m,j,y) \in \mathcal{O}$ とすべきな気がする．後の証明をみてもそうでないと辻褄が合わない箇所がある．
  - policy $\mathcal{Z} := (\chi, M, \mathcal{V})$
  - (interim) utility functions
    - $*$ agent $i \in \{1,2\}$ については，ex-post incentive compatibility を考えるので，他人の type が決まっているときについての utility を考える．

$$U_i(m_i,\theta_i;\omega,\theta_{-i},M_{-i},\chi,\mathcal{V}) := \dots$$

    - $*$ principal については，普通に期待利得を考える．

$$U_0(\chi;\omega,M,\mathcal{V}) := E_\theta[\sum_i \dots]$$

  - ex-post incentive compatibility (XIC): $(\chi, M, \mathcal{V})$ satisfies *XIC* if
    - $*$ agents $i \in \{1,2\}$:

$$\forall(\omega,i,\theta,m_i);\quad U_i(M_i(\omega,\theta_i),\theta_i;\omega,\theta_{-i},M_{-i},\chi,\mathcal{V}) \geq U_i(m_i,\theta_i;\omega,\theta_{-i},M_{-i},\chi,\mathcal{V})$$

    - $*$ principal:

$$\forall(\omega,\tilde{\chi});\quad U_0(\chi;\omega,M,\mathcal{V}) \geq U_0(\tilde{\chi};\omega,M,\mathcal{V})$$

  - v is *ex-post decomposable (enforceable)* by $\mathcal{Z}$ on $V \subseteq \mathbb{R}^3_+$ if
    1. $\mathcal{Z}$ satisfies XIC.
    2. $v = \Lambda(\mathcal{Z})$, where $\Lambda(\mathcal{Z})$ denotes the ex-ante payoff under $\mathcal{Z}$.
    3. $\forall(\omega,m,j,y);\ \mathcal{V}(\omega,m,j,y) \in V$
  - v is *ex-post E-decomposable* by $\mathcal{Z}$ on $V \subseteq \mathbb{R}^3_+$ if
    1. v is ex-post decomposable by $\mathcal{Z}$ on $V \subseteq \mathbb{R}^3_+$.
    2. (continuation payoffs attain efficiency)
       $\forall(\omega,j,y,\theta);\ \mathcal{V}_1(\omega,M_1(\theta_1),M_2(\theta_2),j,y) + \mathcal{V}_2(\omega,M_1(\theta_1),M_2(\theta_2),j,y) = \pi v^*$.
    3. (current allocation is efficient)
       (i) The task is allocated to at least one agent, and (ii) if there is a suited agent, the task is allocated to him/her.

- $V \subseteq \mathbb{R}^3_+$: ex-post self-generating $\overset{\Delta}{\iff} V \subseteq \mathcal{W}(V)$

Lem. 1 (The property of $\mathcal{E}^*$).
(i) $v \in \mathcal{E}^* \implies v$ *is E-decomposed by some* $\mathcal{Z}$ *on* $\mathcal{E}^*$
(ii) $\mathcal{E}^* = \emptyset$, *or* $\mathcal{E}^* = co\{\widehat{v}, \widetilde{v}\}$ *for some* $\widehat{v}, \widetilde{v} \in \{v \in \mathbb{R}^2_+ \mid v_1 + v_2 = \pi v^*\}$.

Proof. (i) follows from APS self-generation argument. (ii) follows from the existence of public randomization device. □

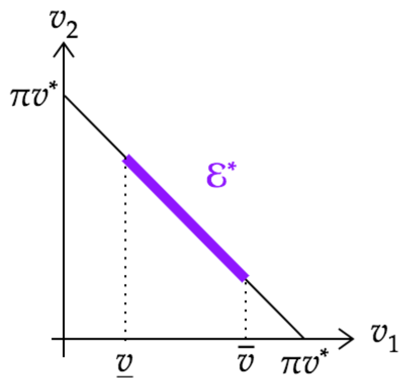
- If $\mathcal{E}^*$ is nonempty, any vector $v^* \in \mathcal{E}^*$ can be decomposed by a policy with efficient allocation: there is at least one profile of XPPE strategies that attains v^* as its average payoff.

- When is $\mathcal{E}^*$ empty?

- Can we characterize $\mathcal{E}$, that is, the values of $\widehat{v}, \widetilde{v}$, in case $\mathcal{E} \neq \emptyset$.

- Fix the primitives $(\delta, \phi, \overline{\mu}, \underline{\mu}, \pi)$.

- $\underline{v} := \min\{v \in \mathbb{R}_+ \mid (v^*, v, \pi v^* - v) \in \mathcal{E}^*\}$. $\overline{v} := \pi v^* - \underline{v}$.



- $\Psi : [\underline{v}, \frac{1}{2}\pi v^*] \to [\underline{v}, \overline{v}]$.

$\Psi(v) := \inf\{\widetilde{v} \in \mathbb{R}_+ \mid (v^*, \widetilde{v}, \pi v^* - \widetilde{v})$ can be E-decomposable on $\mathrm{co}\{(v^*, v, \pi v^* - v), (v^*, \pi v^* - v, v)\}\}$

- inf の中身は $\emptyset$ になることもありうる．(その場合 $\inf\emptyset = \infty$ としておく．)

- $n \geq 3$ のときも同様な議論ができるか？ $\pi v_0 = \sum_i v_i, v \geq 0, v_0 \leq v^*$, v^* の定義は要検討

Lem. 2 (The characterization of $\mathcal{E}$ is reduced to search for the fixed point of Ψ.).
If $\mathcal{E}^* \neq \emptyset$, *then* $\underline{v}$ *is the fixed point of* Ψ, *i.e.,* $\Psi(\underline{v}) = \underline{v}$.

Proof. Note that $\mathcal{E}^* = \mathrm{co}\{(v^*, \underline{v}, \pi v^* - \underline{v}), (v^*, \pi v^* - \underline{v}, \underline{v})\}$ and $\underline{v} \in \mathcal{E}^*$. Since $\mathcal{E}^* \subseteq \mathcal{W}(\mathcal{E}^*)$ by Lem.1, $\underline{v}$ is E-decomposable on $\mathrm{co}\{(v^*, \underline{v}, \pi v^* - \underline{v}), (v^*, \pi v^* - \underline{v}, \underline{v})\}$. Thus, $\Psi(\underline{v}) \leq \underline{v}$.

We want to show that $\Psi(\underline{v}) = \underline{v}$. Suppose toward contradiction that $\Psi(\underline{v}) < \underline{v}$. Then, there exists $\widetilde{v}$ such that $\Psi(\underline{v}) < \widetilde{v} < \underline{v}$. By definition of Ψ, $(v^*, \widetilde{v}, \pi v^* - \widetilde{v})$ can be E-decomposed on $\mathrm{co}\{(v^*, \underline{v}, \pi v^* - \underline{v}), (v^*, \pi v^* - \underline{v}, \underline{v})\} = \mathcal{E}^* \subseteq \mathcal{E}$. In particular, $(v^*, \widetilde{v}, \pi v^* - \widetilde{v}) \in \mathcal{W}(\mathcal{E}) = \mathcal{E}$. Note that, since $\widetilde{v} + (\pi v^* - \widetilde{v}) = \pi v^*$, if $(v^*, \widetilde{v}, \pi v^* - \widetilde{v}) \in \mathcal{E}$, then $\widetilde{v} \in \mathcal{E}^*$. By definition of $\underline{v}$, $\widetilde{v} \notin \mathcal{E}^*$, and thus $\widetilde{v} \notin \mathcal{E}$. A contradiction. □

Technical Problems

- DP with uncertainty. (One-shot deviation principles with uncertainty)

- Self-generation (compactness of the set of equilibrium payoffs)

- public randomization (convexity of the set of equilibrium payoffs)

Proofs for Section 3

- Characterize the set of efficient XPPE payoffs $\mathcal{E}^*$.
- Note that if $v \in \mathcal{O}$ is E-decomposed by $\mathcal{Z} := (\chi, M, \mathcal{V})$ on $V \subseteq \mathcal{O}$, then the principal has no incentive to deviate from χ: By the definition of E-decomposition, both present and continuation payoffs of the principal are maximized.

Lem. 3. 元論文の *lemma* の主張はよくわからない. *Revelation Principle* のようなもの？
Suppose a policy $\mathcal{Z}' := (\chi', M', \mathcal{V}')$ E-decompose $v \in \mathcal{O}$ on $V \subseteq \mathcal{O}$. Then, we can construct $\mathcal{Z} := (\chi, M, \mathcal{V})$ that decomposes v on V with the following properties:

- *Both agents tell their type truthfully: $\forall i \forall (\omega, \theta_i);\ M_i(\omega, \theta_i) = 1$ iff $(\omega, \theta_i) \in \{(A, \alpha), (B, \beta)\}$*

Proof. (Incomplete) Suppose $\mathcal{Z}' := (\chi', M', \mathcal{V}')$ E-decompose $v \in \mathcal{O}$ on $V \subseteq \mathcal{O}$.

First, we show at least one agent need to change his message according to his own type under $\mathcal{Z}'$:

$$\forall \omega \exists i;\ M_i(\omega, \alpha) \neq M_i(\omega, \beta)$$

Suppose toward contradiction that there is some ω such that both agents pool their information, i.e., $\exists \omega\ \forall i; M'_i(\omega, \alpha) = M'_i(\omega, \beta)$, then this leads to a contradiction: For such ω, we can show that $U_0^{\omega}(\chi', M', \mathcal{V}') = v_0 < v^*$, which contradicts the fact that v is E-decomposed by $\mathcal{Z}'$.

Next, we show that we can construct $\mathcal{Z}$ that E-decomposes v on V and $M_i(\omega, \theta_i)$ is truth-telling for all i. Fix ω.

Case (i): In case $\forall i;\ M_i(\omega, \alpha) \neq M_i(\omega, \beta)$: By relabeling, we can construct $\mathcal{Z}$.

Case (ii): In case $\exists i; M_i(\omega, \alpha) = M_i(\omega, \beta)$: Assume without loss of generality, $i := 1$. If agent 1 is not truth-telling, then relabeling is needed. As for agent 2, assume w.l.o.g that $M_2(\omega, \theta_2) \equiv 1$. Consider the following modification:

$$\chi_i^{\omega}(m_1, m_2) := \chi_i^{\omega'}(m_1, 1)$$
$$\mathcal{V}_i^{\omega}((m_1, m_2), j, y) := \mathcal{V}_i^{\omega'}((m_1, 1), j, y)$$

□

- XIC conditions for agents' truth-telling can be represented as a system of linear equations.
- (IC1) - (IC6)
- Agent 1's ex-ante payoff Λ_1 can also be explicitly written.

Lem. 4. *Suppose $\mathcal{E}^* \neq \emptyset$. Then, for any $v \in [\underline{v}, \frac{1}{2}\pi v^*]$,*

$$\Psi(v) = \left[\begin{array}{lc} \min\limits_{\chi, \mathcal{V}} & \Lambda_1 \\ \text{s.t.} & \text{XIC: (IC1)-(IC6)} \\ & \chi_1^{\omega}(m) + \chi_2^{\omega}(m) = 1 \\ & (m_i, m_{-i}) = (1, 0) \implies \chi_i^{\omega}(m) = 1 \\ & \mathcal{V}_i^{\omega}(m, j, y) \in [v, \pi v - v^*] \\ & \mathcal{V}_1^{\omega}(m, j, y) + \mathcal{V}_2^{\omega}(m, j, y) = \pi v^* \\ & \forall \omega \in \{A, B\}, m \in \{0,1\}^2, i, j \in \{1, 2\}, y \in \{S, F\} \end{array}\right.$$

Proof. By Lem.4. □

- 上の問題を頑張って解く. 一応頑張って式は追ってみたが, キチンと理解できていない.
- 線形計画問題を頑張って解いているとだけみなせる ⋯?

Lem. 5.

$$\mathcal{E}^* \neq \emptyset \iff \phi(1-\phi) \geq \frac{\underline{\mu}(1 - \delta\overline{\mu})}{\delta(\overline{\mu} - \underline{\mu})(1 - \underline{\mu})}$$

Proofs for Section 4

Prop.5

straight forward. XIC conditions and the definition of v^f and v^{-f}.

なにができそう？

- $n \geq 3$ で何が起こるか？
  - 効率的な均衡利得の特徴付けは，self-generation の議論のあとはただの線形計画問題な気がするので，$n \geq 3$ にしても拡張できそうな気もする．(Lem.10 の proof をもう少しきちんと理解する必要)
  - maximal-priority は人数が増えるとどうなるのか？
- principal がコミットできるとするとどうなるのか？(効率性が達成しやすくなる？)
- first-best が達成可能でない場合に何が起きているかをもうちょっと詳しくみる？
- learning – 片方についてのみ ϕ^i が未知とかにするとどうなる？