# Akbarpour and Li (2018, EC18)
# Credible Mechanisms

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## 1 Motivation

- メカニズムを実際に動かすとき，CP が本当に当初の約束通りの手順で実施するかは怪しい．
  - 例えば，入札者間でのコミュニケーションがない場合，sealed bid SPA において auctioneer は二位価格をこっそり吊り上げて勝者に伝えることで得をできる．
  - 実際にそういった過去の事例もある．(切手販売・オンライン広告であったらしい．)
  - 原因の一例: auctioneer の収入が歩合制 etc.
- 入札者間のコミュニケーションがないオークションは現実でも多い．
  - 電話・手紙・オンラインによる入札: 実際に bid があったのか他者にはわからない．
  - 電波オークションでは，入札者間のコミュニケーションを明示的に禁止．
  - CP が入札額を公表したくない理由: bid の数字を用いて collusion が可能．
  - agents が入札額を公表したくない理由: valuation が他人にバレると後々悪用されるかも．
  - 誰が財を得たのかも公表されないことが多い: 長期的な collusion の可能性．身元保護 (?)
- 2 つの仮定:
  1. Auctioneer and bidders communicate privately. (No communication among bidders.)
  2. No watches or stopwatches. (Bidders do not know how many calls the auctioneer made to other bidders.)
- そのような状況で，個々の参加者の誰にもバレないように悪いことをしようとする CP を考える．
- Credible mechanisms: auctioneer がルールから逸脱する気にならないような制度．
- 特に，有名な 3 つの auctions (FPA, SPA, AA) について考えてみる．
  - FPA: static, AA: strategy-proof, SPA: static and strategy-proof
  - SPA is seemingly a great mechanism... Why do all three formats persist?
- credible, static, strategy-proof の関係は？

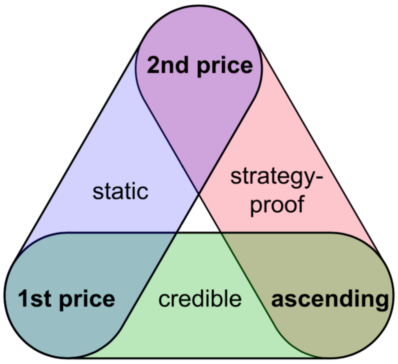
## 2 Approach

- メカニズム (展開形ゲームの木) と，それに対応する message games というものを考える．
- message game: CP が各参加者と一対一でコミュニケーションを取る．実際のメカニズムの動作を模倣しながらゲームを進める．
- Auctioneer にとって妥当な逸脱の方法と，メカニズムの信頼性 (credibility) を定義: 各参加者がメカニズムの動作中に観測した出来事が，ありえるメカニズムの動作と矛盾しないか．

## 3 Contribution

### 3.1 Main results

**Ass. 1.** *Regular and i.i.d. values; Winner-paying; Auctioneer maximizes revenue.*




### 3.2 Related Literature

**Almost the same concept, but restricted to direct (static) mechanisms**

- Dequiedt and Martimort (2015, AER), Vertical contracting with informational opportunism.
- This paper: Extensive forms.

**Commit to today's auction, not tomorrow's auction**

- Milgrom 1987, mcAfee and Vincent 1997, Skreta 2015, Liu et al. 2017
- This paper: Not a repeated game.

**Auctions as bargaining games**

- McAdams and Schwarz 2007, Vartiainen 2013, Lobel and Paes Leme 2017
- This paper: No 'red-handed' rule-breaking.

## 4 Model

- Agents: $i \in N$. A mechanism: an extensive game tree $G$. Outcomes: $X$.
- $\theta_N \sim \mathcal{D} \in \Delta(\Theta_1, \ldots, \Theta_N)$. Each type space is finite: $\Theta_i := \{\theta_i^1, \ldots, \theta_i^{K_i}\}$.
- $S_i(I_i, \theta_i) \in A(I_i)$: agent $i$'s strategy. At each info. set, the set of possible action is finite.
- A protocol: $(G, S_N)$. Utiliry: $u_i : X \times \Theta_N \to \mathbb{R}$.
- agent $i$'s partition of the outcome space: $\mathcal{X}_i$.
  - e.g.) Each agent can observe only whether he makes a payment and receives the object.
- $(G, S_N)$: BIC $\overset{\Delta}{\iff}$ ... (as usual)
- Assume that $G$ is pruned. (This is w.l.o.g. when we consider credible and BIC mechanisms.)

**Message games**

- In each turn, CP either ends the game, or chooses some agent and send him a pair of a message and a set of acceptable replies $(m, R)$; then, the chosen agent sends CP a reply $r \in R$.
- Each agent's strategy: {what they observed, current $R$} $\to r \in R$.
- Agent $i$'s observation $o_i(S_0, S_N, \theta_N)$: communication sequence + cell of outcome partition.

**The relationship between message games and mechanisms**

- CP can run a protocol $(G, S_N)$ as a message game: $m := I_i$, $R := A(I_i)$.
- $S_0^G$: CP's strategy that runs the protocol $(G, S_N)$.

**Def. 4.1** (Safe deviations). *G: given. An observation $o_i(S_0, S_N, \theta_N)$ has an **innocent explanation** if*

$$\exists \theta'_{-i};\ o_i(S_0, S_N, \theta_N) = o_i(S_0^G, S_N, (\theta_i, \theta'_{-i}))$$

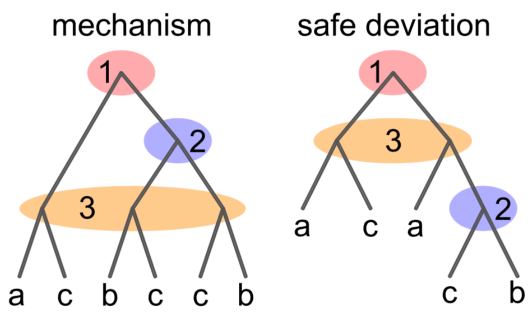
*A CP's strategy $S_0$ is **safe** if $o_i(S_0, S_N, \theta_N)$ has an innocent explanation for all $i$ and $\theta_N$.*

- $\mathcal{S}_0^*(S_0^G, S_N)$: the set of safe deviations.
- As long as CP chooses a safe strategy, each agent cannot detect by himself that CP is cheating.

**Credible mechanisms**

- The protocol $(G, S_N)$ is credible if there is no profitable safe deviation for CP.

**e.g. 4.1.** *The trees below illustrates the mechanism that is not credible:*




*Assume that $\Omega_1 := \{\{a, b\}, \{c\}\}$, $\Omega_2 = \Omega_3 = \{\{a\}, \{b\}, \{c\}\}$, and $\theta_N := \{l_1, r_2, r_3\}$. If CP loves the outcome $a$, CP has a profitable safe deviation.*

# 5 Preliminary

- To use extensive forms, discretize Myerson 1981.
- In discrete settings, the analogue of continuous settings holds: the expected revenue is closely connected to virtual valuations.
- $u_i^{G,S_N}(k,k')$: the expected payoff of agent $i$ with his true type $\theta_i^k$ when he behaves as if his type is $\theta_i^{k'}$ under the protocol $(G, S_N)$.

**Def. 5.1** (Virtual valuation, Regular distribution).

$$\eta_i(\theta_i^k) := \left(\theta_i^k - \frac{1-F_i(\theta_i^k)}{f_i(\theta_i^k)}\right)(\theta_i^{k+1} - \theta_i^k).$$

- *$F_N := (F_i)_i$ is regular if $\eta_i$ is strictly increasing for all $i$.*

**Def. 5.2** (optimal mechanisms). *The mechanism $(G, S_N)$ is optimal if it maximizes the expected revenue subject to the following conditions:*

- *IC: $(G, S_N)$ is BIC.*
- *Voluntary participation: $\forall i \exists S_i$; $i$ does not win and has a zero transfer.*

**Prop. 5.1** (Elkind(2007)). *$(G, S_N)$ is optimal iff*

1. *PCs bind for the lowest type: $\forall i; u_i^{G,S_N}(1,1) = 0$.*
2. *ICs bind locally downward: $\forall i\ \forall k \geq 2$; $u_i^{G,S_N}(k,k) = u_i^{G,S_N}(k,k-1)$.*
3. *The allocation maximizes virtual value: $\forall \theta_N$;*
   - $\max_i \eta_i(\theta_i) > 0 \implies y^{G,S_N}(\theta_N) \in \operatorname{argmax}_i \eta_i(\theta_i)$.
   - $\eta_i(\theta_i) < 0 \implies i \neq y^{G,S_N}(\theta_N)$.

*If $(G, S_N)$ is optimal, its expected revenue coincides with the expected virtual valuation of the winner: [要確認]*

$$\pi(G, S_N) = \mathbb{E}_{\theta_n}\left[\sum_{i\in N} y^{G,S_N}(\theta_N)\eta_i(\theta_i)\right]$$

**Prop. 5.2** (the expected revenue $\simeq$ the expected virtual value). *If $(G, S_N)$ is BIC, then*

$$0 \leq \mathbb{E}_{\theta_n}\left[\sum_{i\in N} y^{G,S_N}(\theta_N)\eta_i(\theta_i)\right] - \pi(G, S_N) - \sum_{i\in N} u_i^{G,S_N}(1,1) \leq \max_i \max_{2\leq k\leq K_i}\{\theta_i^k - \theta_i^{k-1}\}$$

# 6 Results

**Ass. 2.** *$F_N$: regular, symmetric. Auctions are winner-paying. $\theta_1 \leq 0$. $\theta^1 < \cdots < \theta^K$.*

## 6.1 Quasi-FPAs and credible static auctions.

**Def. 6.1** (quasi-FPA). *A quasi-FPA is a static mechanism that can have at most one special agent. If there is no special agent, it is a normal FPA. If there is a special agent $i^*$ with a posted price $p^*$, he can surely win if he bids $p^*$. (NB: Even if $p^*$ is not the highest bid among all bidders, CP should allocate the object to $i^*$.)*

**Thm. 6.1** (The characterization of credible and static auctions.). *Suppose the auction $(G, S_N)$ is BIC and winner-paying. Then,*

$$(G, S_N) \text{ is credible and static} \iff (G, S_N) \text{ is a quasi FPA.}$$

*Proof.* $\Leftarrow$): Easy.

$\Rightarrow$) First, observe that if the mechanism is credible and agent $i$ have chances to win when he chooses an action $a$, the possible payment is uniquely determined; otherwise CP can improve his payoff safely. Hence, each action corresponds to a unique payment $b_i(a)$, and it can be regarded as agent $i$'s bid.

**Case (i): There is a special agent $i^*$ with a posted price $p^*$, and $i^*$ bids $p^*$.** If there is a special agent $i^*$, who can surely obtain the object if he bids $p^*$. Suppose $i^*$ bids $p^*$. CP should allocate the object to $i^*$; there is no profitable safe deviation for CP. This is a quasi-FPA. (NB: Since the protocol is BIC, $p^* := \max_{a \in A(I_i)} b_i(a)$.)

**Case (ii): There is no special agent, or $i^*$ does not bid $p^*$.** In this case, it is best for CP to allocate the object to the highest bidder. This is also a (quasi-)FPA. □

**Prop. 6.1** (There is an almost-optimal FPA. $p^*$ cannot be very low.). *Let* $\varepsilon := \max_{2 \le k}\{\theta^k - \theta^{k-1}\}$.

- *There is an $\varepsilon$-optimal FPA with reserve* $\rho^* := \min_k\{\theta^k \mid \eta_i(\theta^k) > 0\}$.
- *If a quasi-FPA is BIC and maximizes virtual value, then $p^*$ (if it exists) is at least* $\max_i b_i(S_i(I_i, \theta^{K-2}))$.

*Proof.* Note that we need to construct a set of feasible actions in a quasi-FPA with reserve $\rho^*$ so that $(G, S_N)$ is $\varepsilon$-optimal.

First, consider the SPA with reserve $\rho^*$. It is best for each agent to bid truthfully. Let $\bar{b}_i(\theta_i)$ be the payment for type $\theta_i$ agent conditional on winning. (If agent $i$ never wins with type $\theta_i$, $\bar{b}_i(\theta_i) := -1$.)

Next, we construct the desirable FPA $G$ with reserve $\rho^*$. Let $\bar{b}_i(\theta_i) \in A(I_i)$. Suppose that every agent $i$ bid $\bar{b}_i(\theta_i)$. In this case, the strategy profile $S_N := (\bar{b}_i(\theta_i))_{i \in N, \theta_i \in \Theta_i}$ is BIC under $G$; otherwise, in a SPA with a sufficiently fine action space, there is a profitable deviation that allows an agent to obtain strictly better payoff compared to the payoff he can obtain under truthful bidding. Note that, in $(G, S_N)$, PCs for the lowest types bind.

Observe that $(G, S_N)$ maximizes the expected virtual value, though $G$ may not satisfy the locally downward ICs condition. Then, by Prop.5.2,

$$0 \le \underbrace{\mathbb{E}_{\theta_n}\left[\sum_{i \in N} y^{G,S_N}(\theta_N)\eta_i(\theta_i)\right]}_{\text{the exp. rev. under the opt. auctions}} - \underbrace{\pi(G, S_N)}_{\text{the exp. rev. under } (G,S_N)} \le \max_{2 \le k \le K_i}\{\theta_i^k - \theta_i^{k-1}\}$$

The second part follows from the symmetry of $F_N$ and BIC. □

## 6.2 Ascending auctions and credible strategy-proof auctions.

**Def. 6.2** (Ascending auctions). *(omitted.)*

**Thm. 6.2** (credible + SP = AA, assuming the auction is optimal). *Assume the protocol $(G, S_N)$ is optimal. Then,*

$$(G, S_N) \text{ is credible and strategy-proof} \Longleftrightarrow (G, S_N) \text{ is an AA.}$$

*Sketch of the proof.* $\Leftarrow$) Assume $(G, S_N)$ is an AA. SP is ok. Suppose toward contradiction that $(G, S_N)$ is not credible. Then, there is a profitable safe deviation for CP $S_0'$. Observe that $S_N$ is a best reply even if CP announces that he will commit to $S_0'$, that is, for all agent $i$, the mechanism proceeds as if the other agents' types are different from the reported ones. The corresponding protocol $(G', S_N')$ has strictly higher expected revenue than $(G, S_N)$; this contradicts the optimality of $(G, S_N)$.

$\Rightarrow$) One major feature of AA: at each history, the action of all types' that might win in the future pools. If their action does not pool, CP has a profitable safe deviation. (To show this statement, they develop a smart algorithm.) $\square$

### 6.3 The Auction Trilemma

**Cor. 6.1** (The Auction Trilemma). *Assume $F_N$ is regular and symmetric, and $(G, S_N)$ is BIC and winner-paying.*

1. *Suppose there exists $i$ and $\theta_i, \theta_i'$ such that $t_i^{G,S_N}(\theta_i) > t_i^{G,S_N}(\theta_i') \geq 0$. If $(G, S_N)$ is optimal, it cannot be static, credible and strategy-proof at the same time.*
2. *Let $\varepsilon := \max_{k \geq 2}\{\theta^k - \theta^{k-1}\}$. There exist $\varepsilon$-optimal auctions with some reserve $\rho^*$ that satisfy one of the following properties:*
   - *static and strategy-proof (SPA)*
   - *static and credible (FPA)*
   - *strategy-proof and credible (AA)*

*Proof.* We can show the second part by the same argument as in Prop.6.1: Observe that, with reserve $\rho^* := \max_i\{\theta^k \mid \eta_i(\theta^k) > 0\}$, these auctions maximize the virtual value of winner, and PC for the lowest types bind.

**The first part:** By Thm.6.1, if the auction is static and credible, it is a quasi-FPA. By the assumption, $i$ has no less than two possible bids; each bid should win with positive probability because we consider pruned mechanisms. Therefore, for some type profile, $i$ can win by the lower bid; strategy-proofness cannot hold. $\square$

**Cor. 6.2** (Strategy-proofness holds only for one side.). *Assume $F_N$: regular and symmetric, and $(G, S_N)$ is orderly and optimal. Suppose the optimal reserve $\rho^* < \theta^{K-2}$. In the messaging game restricted to $\mathcal{S}_0^*(S_0^G, S_N)$, either of the followings holds:*

1. *$\exists S_N'$; $S_0^G$ is not a BR to $S_N'$.*
2. *$\exists i \in N\ \exists S'_{N\setminus\{i\}}$; $S_i$ is not BR to $(S_0^G, S'_{N\setminus\{i\}})$.*

*Proof.* **Case (i)** $(G, S_N)$ **is not credible:** For $S_N$, $S_0^G$ is not BR. (The first one holds.)
**Case (ii)** $(G, S_N)$ **is not strategy-proof:** The second one holds.
**Case (iii)** $(G, S_N)$ **is credible and strategy-proof:** Since it is optimal, it is an AA. Fix some $i$ and consider the strategy profile such that $i$ stays until the price hits $\theta^K$ and other agents quits before the reserve is met. CP apparently has a profitable safe deviation.

$\square$