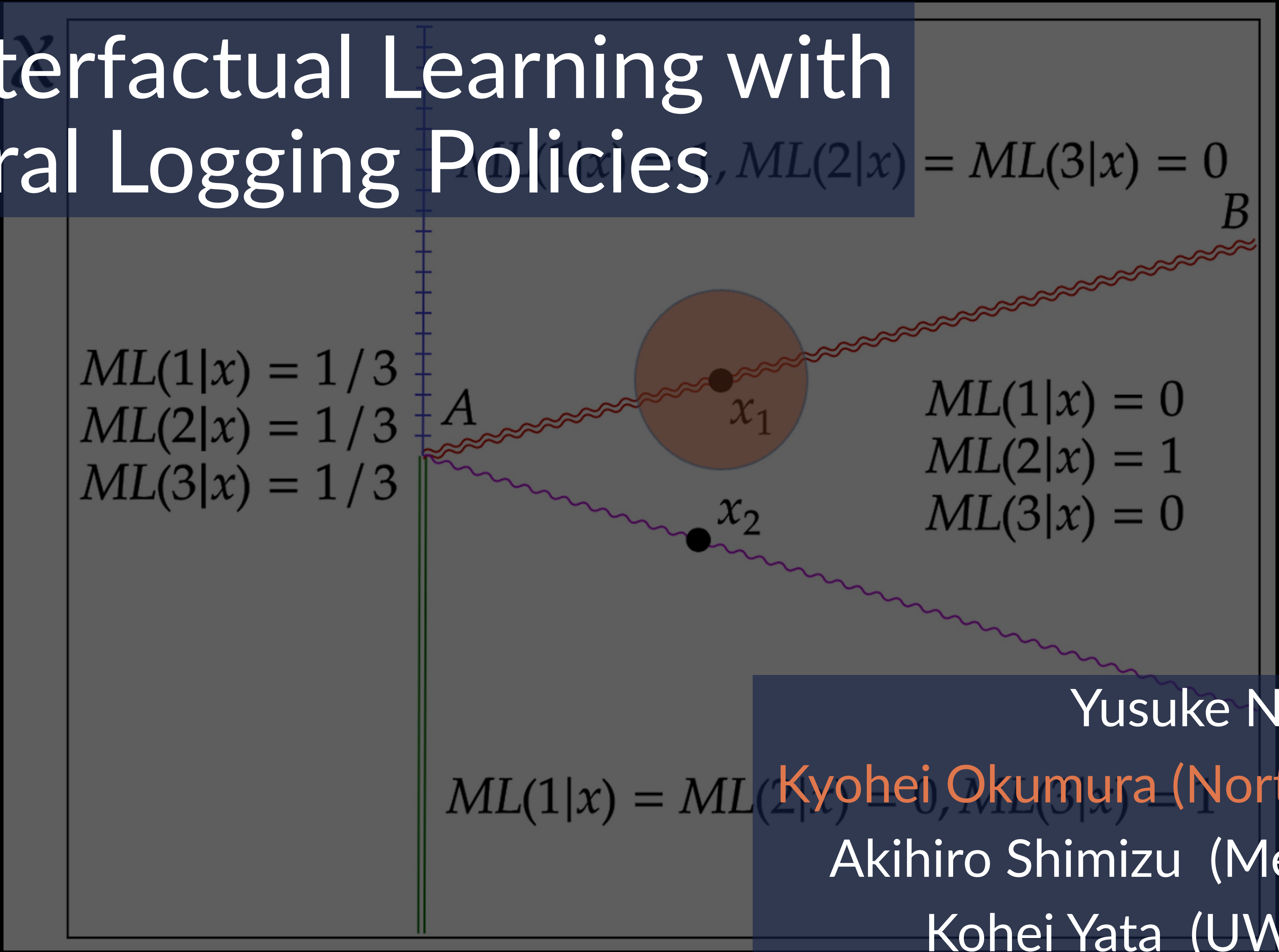
Counterfactual Learning with General Logging Policies



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Algorithms are eating the world

Decision making by algorithms are everywhere in the world.

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Decision making by algorithms

Input	Action (Decision)	Outcome
X	A	Y

$$X \xrightarrow{\text{policy } \pi} A$$

e.g.	Individual Characteristics (Age, Gender, etc.)	Give a coupon or not	Purchase Value

How can we choose policy π that achieves better outcomes？

Off-Policy Evaluation

- **Goal:** Estimate the performance of a new counterfactual policy
- **Solution 1:** A/B Testing
  - Problem: Costly and/or risky
- **Solution 2:** Off-Policy Evaluation
  - Use the log data generated by the existing policy
  - Improve the system without conducting A/B tests

Full Support vs. Deficient Support Policies

- **Full support policy**: for all covariates, all actions are chosen w.p. >0

$$\forall x \forall a, \pi(a \mid x) > 0$$

- Real-world decision-making often uses deficient support policies
  - e.g. Give a coupon iff one's covariate is in some region

- **Problem:** Hard to conduct estimation using the log data generated by deficient support policies

Our Contribution

- Propose an OPE estimator applicable to data generated by a broad class of policies including deficient support policies.
  - **Theory**: the estimator has consistency:
    the prediction converges in probability to the true performance of a counterfactual policy as the sample size increases.
  - **Real-world Application:** evaluate coupon targeting policies by a major online platform, Mercari.

    - How much more do people spend when they get a coupon?
    - Should the company allocate more coupons or not?

Overview

Framework

Key Concept: APS

Results

Conclusion

Framework

- Action $a \in \mathcal{A} := \{1, \ldots, m\}$.
- Potential rewards $\left(Y(a)\right)_{a \in \mathcal{A}}$
  - Action a is chosen → Reward $Y(a)$ is observed
- Context X. $\mathcal{X} := \mathrm{supp}(X) \subseteq \mathbb{R}^p$
- Logging policy $ML: \mathcal{X} \to \Delta(\mathcal{A})$
  - $ML(a \mid x)$ = proba of taking action a for indiv with context x.
  - The researcher can simulate ML (i.e., knows $ML(a \mid x)$).

DGP of log data $(Y_i, X_i, A_i)_{i=1}^n$

- Logging policy ML generates log data $(Y_i, X_i, A_i)_{i=1}^n$
- For each i,
  1. $\left(\left(Y_i(a)\right)_a, X_i\right)$ is i.i.d.-drawn from an unknown distribution
  2. Action A_i is chosen w.p. $ML(A_i \mid X_i)$
  3. Reward $Y_i := Y_i(A_i)$ is recorded.

Input | Action (Decision) | Outcome

$$\boxed{X_i \xrightarrow{\;ML\;} A_i} \longrightarrow Y_i = Y_i(A_i)$$

Goal

Using log data $(Y_i, X_i, A_i)_{i=1}^{n}$,
estimate the performance of a counterfactual policy π

$$V(\pi) := \mathbb{E}\left[\sum_{a \in \mathcal{A}} Y(a)\pi(a \mid X)\right]$$

1. Is it possible to estimate $V(\pi)$? → **Identification**

In the ideal world where we could have an infinite amount of data

2. How can we estimate $V(\pi)$ given finite data? → **Estimation**

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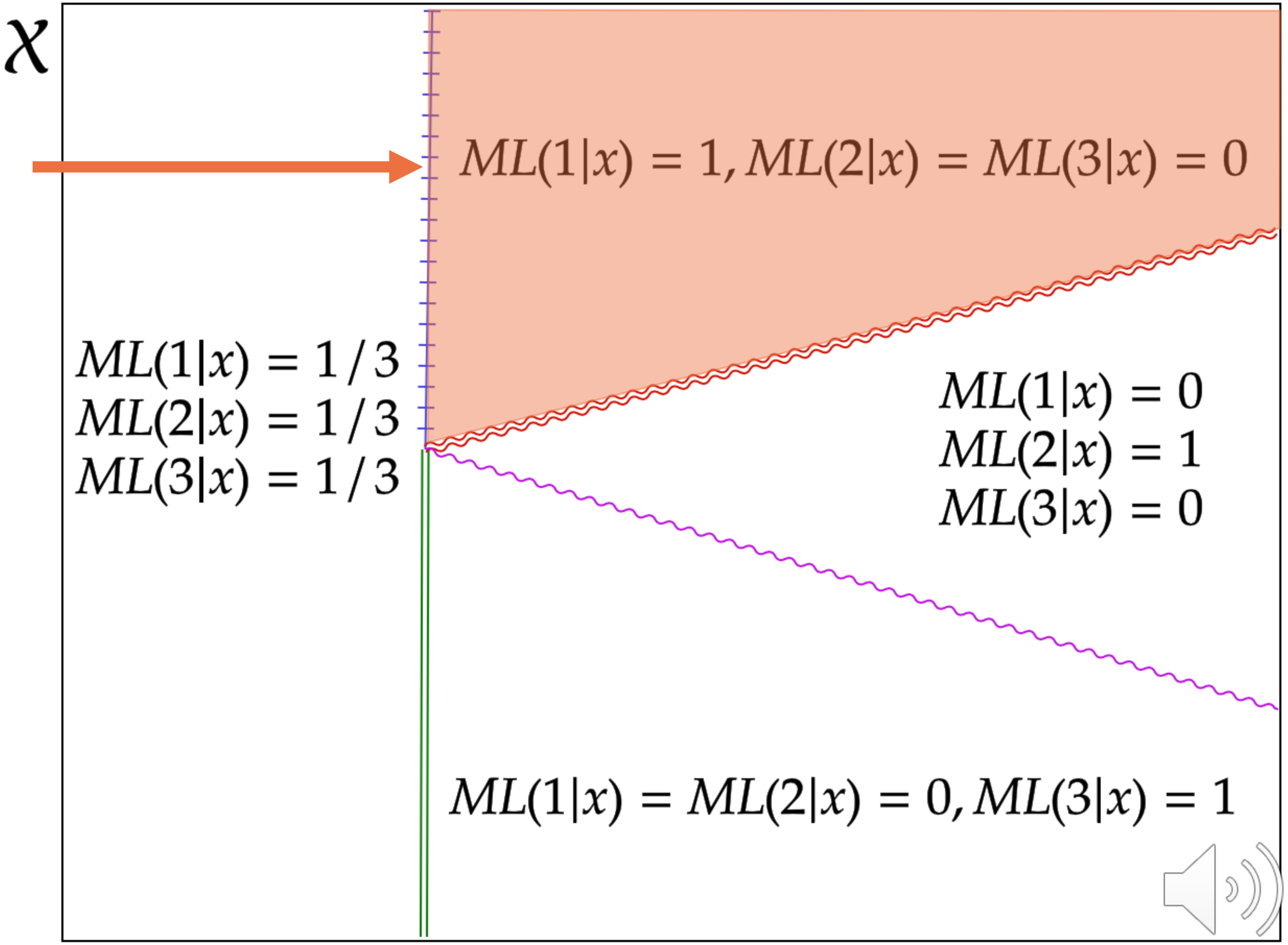
Estimation with deficient support policy is hard

- **Full-support logging policy**: For any x and a, $ML(a \mid x) \in (0,1)$
  - In this case, $V(\pi)$ is identified using propensity score.
- **Deficient support logging policy**: For some $\overline{a}, \overline{x}$, the logging policy produce no data on $Y(\overline{a}) \mid X = \overline{x}$
  - $\mathbb{E}[Y(\overline{a}) \mid X = \overline{x}]$ is not directly identified.

Ex. Deficient support $\mathcal{X} \subseteq \mathbb{R}^2, \mathcal{A} = \{1,2,3\}$

- Logging policy ML only chooses $a = 1$ in this region.

- Since there is no data, $\mathbb{E}[Y(2) \mid x], \mathbb{E}[Y(3) \mid x]$ are not easily identified.



Q: Can we estimate some causal effects using log data generated by a deficient support policy?

A: Yes, we can.

Key: Approximate Propensity Score (APS)

Approximate Propensity Score

Ball w/ center $x \in \mathcal{X}$, radius $\delta > 0$

$$p_\delta^{ML}(a \mid x) := \frac{\int_{\boxed{B(x,\delta)}} ML(a \mid x^*) dx^*}{\int_{B(x,\delta)} dx^*}$$

Average probability that action $a \in \mathcal{A}$ is chosen by ML in the neighborhood of point $x \in \mathcal{X}$

Approximate Propensity Score

Approximate Propensity Score (APS)

$$p^{ML}(a \mid x) := \lim_{\delta \downarrow 0} p_{\delta}^{ML}(a \mid x)$$

Average probability that action $a \in \mathcal{A}$ is chosen by ML
in the neighborhood of point $x \in \mathcal{X}$

Ex. APS

$$\mathcal{X} \subseteq \mathbb{R}^2, \mathcal{A} = \{1,2,3\}$$

$$p^{ML}(\, a \mid x \,) := \lim_{\delta \downarrow 0} p_{\delta}^{ML}(a \mid x)$$

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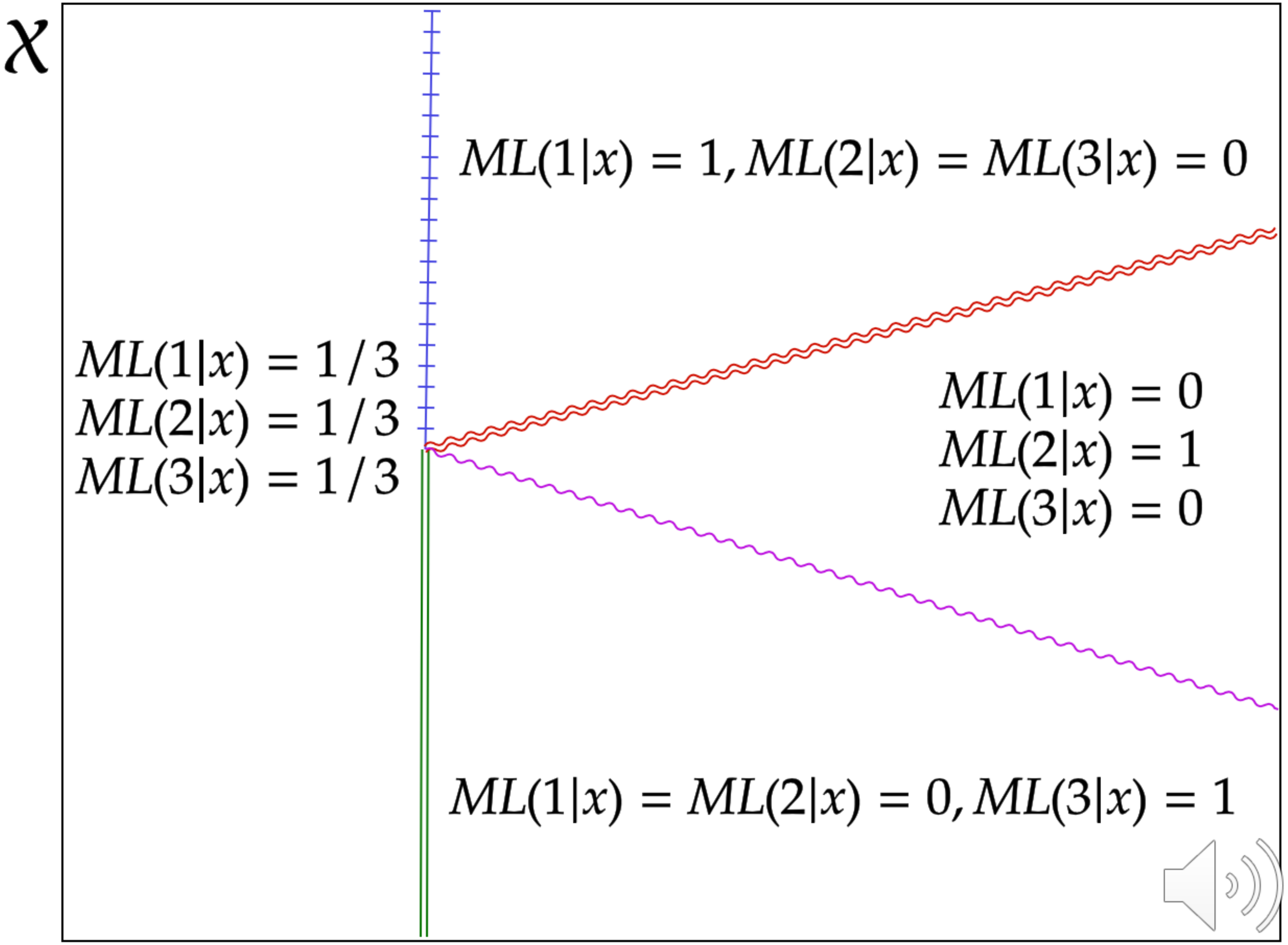

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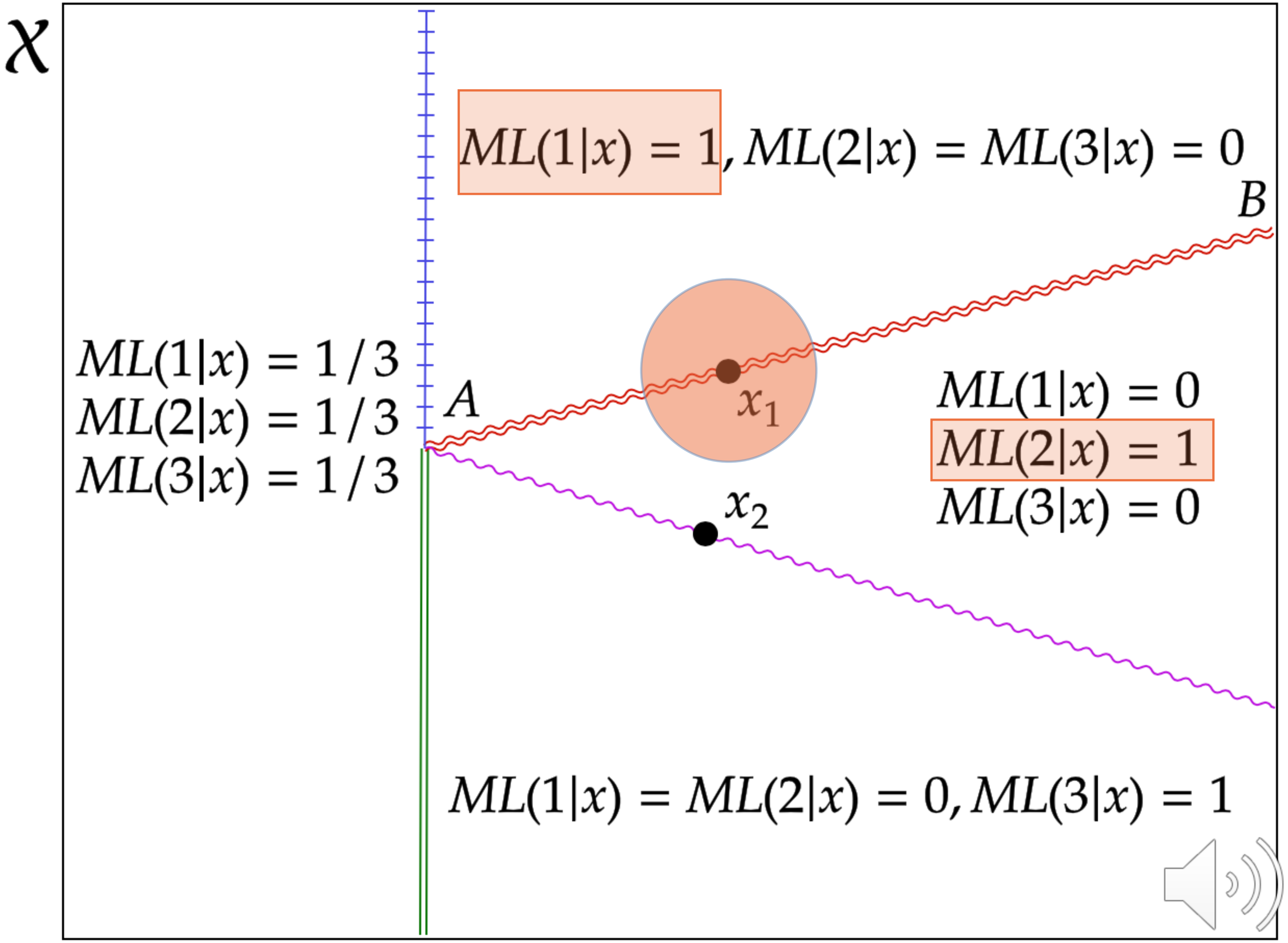

Ex. APS

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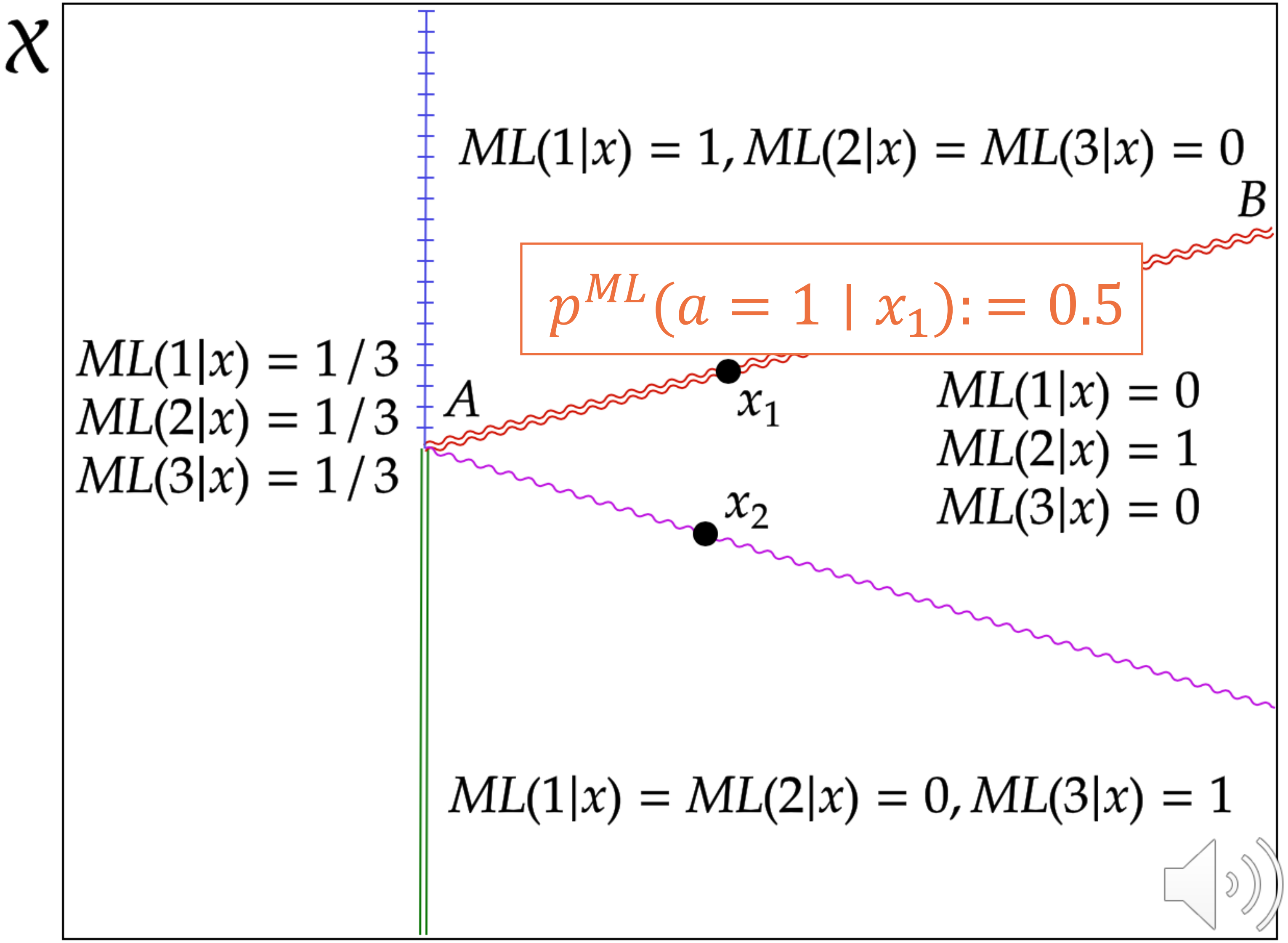


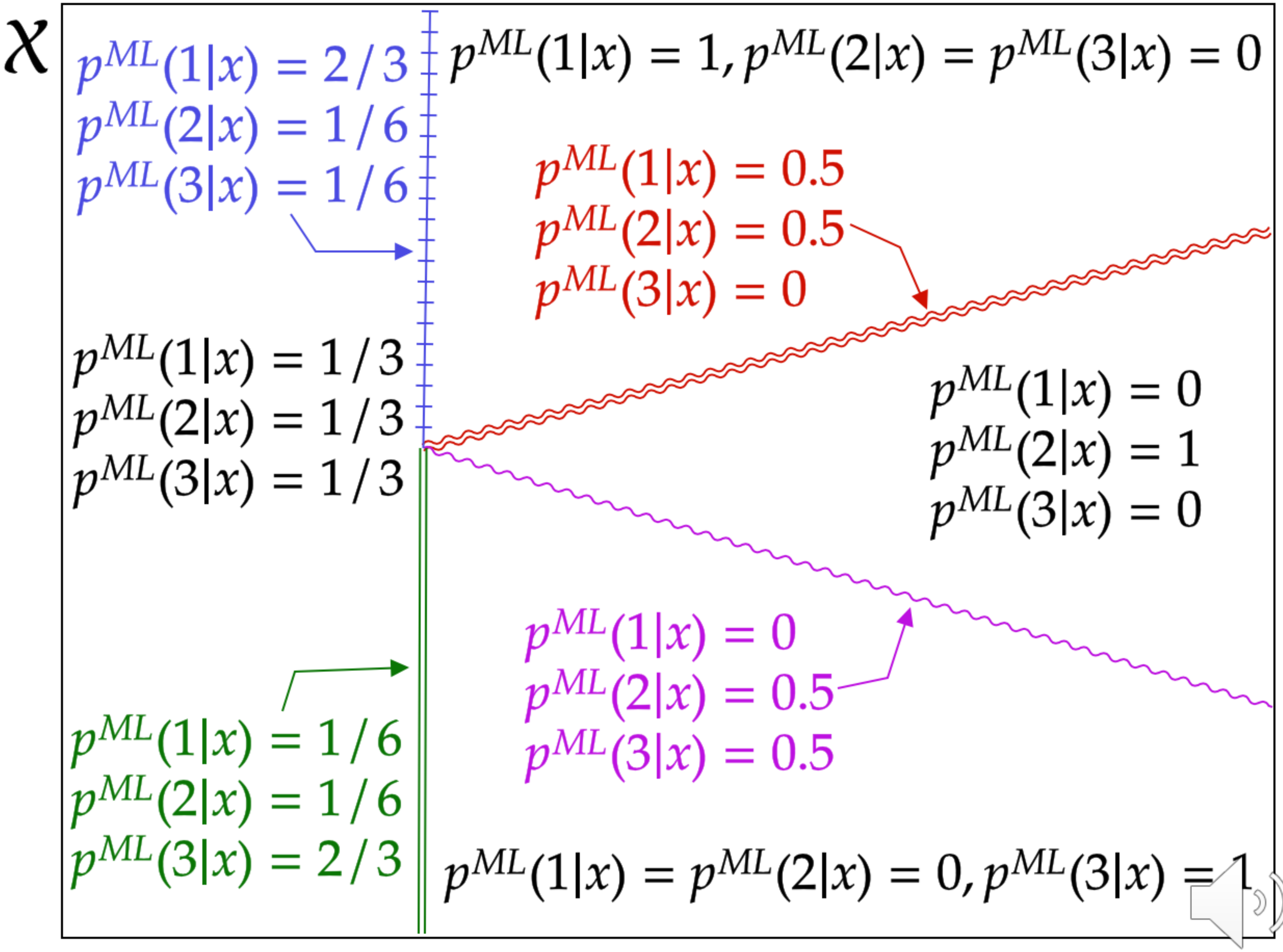
Ex. APS

$\mathcal{X} \subseteq \mathbb{R}^2, \mathcal{A} = \{1,2,3\}$

Under certain cond.'s,
For any a, x s.t.

$$p^{ML}(a \mid x) > 0,$$

$\mathbb{E}[\,Y(a) \mid x\,]$ is identified.



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Identification(Learning w/ infinite data)

Prop. 1 (Identification of the performance)

Under A1-3, the performance $V(\pi)$ of counterfactual policy π is identified.

A2 (Constant Conditional Mean Differences)

There exists $\beta: \mathcal{A} \times \mathcal{A} \to \mathbb{R}$ s.t. $\mathbb{E}[Y(a) \mid X] - \mathbb{E}[Y(a') \mid X] = \beta(a, a')$

See the paper for discussion on A2
(and def. of A1&A3)

From Identification to Estimation

$$V(\pi) := \mathbb{E}\left[\sum_{a\in\mathcal{A}} Y(a)\pi(a \mid X)\right]$$

$$= V(ML) + \mathbb{E}\left[\sum_{a=2}^{m} \beta(a,1)(\pi(a \mid X) - ML(a \mid X))\right]$$

From Identification to Estimation

$$V(\pi) := \mathbb{E}\left[\sum_{a \in \mathcal{A}} Y(a)\pi(a \mid X)\right]$$

$$= V(ML) + \mathbb{E}\left[\sum_{a=2}^{m} \beta(a,1)\boxed{(\pi(a \mid X) - ML(a \mid X))}\right]$$

Known to the researcher

From Identification to Estimation

$$V(\pi) := \mathbb{E}\left[\sum_{a \in \mathcal{A}} Y(a)\pi(a \mid X)\right]$$

$$= \boxed{V(ML)} + \mathbb{E}\left[\sum_{a=2}^{m} \beta(a,1)(\pi(a \mid X) - ML(a \mid X))\right]$$

Can be estimated by $\frac{1}{n}\sum_{i=1}^{n} Y_i$

From Identification to Estimation

$$V(\pi) := \mathbb{E}\left[\sum_{a \in \mathcal{A}} Y(a)\pi(a \mid X)\right]$$

$$= V(ML) + \mathbb{E}\left[\sum_{a=2}^{m} \boxed{\beta(a,1)}(\pi(a \mid X) - ML(a \mid X))\right]$$

Can be estimated via OLS with APS control

(See the paper for more details)

Estimation (Learning w/ finite data)

$$V(\pi) = \boxed{V(ML)} + \mathbb{E}\left[\sum_{a=2}^{m} \boxed{\beta(a,1)}(\pi(a \mid X) - ML(a \mid X))\right]$$

$$\hat{V}(\pi) = \boxed{\frac{1}{n}\sum_{i=1}^{n} Y_i} + \frac{1}{n}\sum_{i=1}^{n}\left[\sum_{a=2}^{m} \boxed{\hat{\beta}_a}(\pi(a \mid X) - ML(a \mid X))\right]$$

Estimation (Learning w/ finite data)

Thm. 1 (Consistency)

Under certain cond.'s, $\hat{V}(\pi)$ converges in proba. to $V(\pi)$ as $n \to \infty$

If the sample size is sufficiently large,

the proposed method correctly estimates the performance of the counterfactual policy π

Application: Coupon Targeting Policy at Mercari

- Logging policy (current policy used by the company)
  - Use data from a past A/B test, train a prediction model τ
  - $\dim(\mathcal{X}) > 200$
  - Offer a coupon to those with a high predicted effect (top 80% of the distribution)

$$ML(a \mid x) = \boxed{1\{\tau(x) \geq q_{0.2}\}}$$

Deficient support

- **Result:** it would be profitable to expand the campaign.

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Summary

- This paper proposes an OPE method applicable to log data generated by a broad class of logging policies, including deficient support ones.
  - Based on approximated propensity score(APS)
  - The estimator has consistency.

- Apply the proposed method to the coupon targeting policy at Mercari.
  - An example of natural logging policies with deficient support
  - **Result**: it would be profitable to expand the campaign.