(メモ) 商の微分

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

(1) $$f(x) = \frac{x^2+1}{x-1}$$

$$f'(x) = \frac{(x^2+1)'(x-1) - (x^2+1)(x-1)'}{(x-1)^2}$$

$$= \frac{2x(x-1) - (x^2+1)}{(x-1)^2} = \frac{2x^2-2x-x^2-1}{(x-1)^2}$$

$$= \frac{x^2-2x-1}{(x-1)^2}$$ //

(2) $$f(x) = x + \sqrt{2x^2+1}$$

$$f'(x) = \left(x + \sqrt{2x^2+1}\right)' = x' + \left(\sqrt{2x^2+1}\right)'$$

$$= 1 + \left(\sqrt{2x^2+1}\right)'$$

合成関数の微分

ここで $$\left(\sqrt{2x^2+1}\right)' = \frac{d}{dx}\left(\left(\underbrace{\boxed{2x^2+1}}_{u}\right)^{\frac{1}{2}}\right) = \frac{d}{dx}u^{\frac{1}{2}} = \frac{du^{\frac{1}{2}}}{du}\frac{du}{dx}$$

$u = 2x^2+1$ とおいた

$$= \frac{1}{2}u^{-\frac{1}{2}}(2x^2+1)' = \frac{1}{\cancel{2}}u^{-\frac{1}{2}}\cancel{4}^{2}x = 2x(2x^2+1)^{-\frac{1}{2}}$$

よって

$$f'(x) = 1 + \frac{2x}{\sqrt{2x^2+1}}$$ //

合成関数の微分

(3) $f(x) = \cos x^{\frac{3}{2}}$

$$f'(x) = \frac{d}{dx} \cos \underset{\substack{\parallel \\ u\text{とおく}}}{\boxed{x^{\frac{3}{2}}}} = \frac{d}{dx} \cos u = \frac{d\cos u}{du} \frac{du}{dx}$$

$$= -\sin u \, (x^{\frac{3}{2}})' = -\sin x^{\frac{3}{2}} \times \frac{3}{2} x^{\frac{3}{2}-1}$$

$$= \underline{-\frac{3}{2} x^{\frac{1}{2}} \sin x^{\frac{3}{2}}} \,/\!/$$

合成関数の微分

(4) $f(x) = \log\left(e^{2x} + e^{-2x}\right)$

$$f'(x) = \frac{d}{dx} \log \left(\underset{\substack{\parallel \\ u\text{とおく}}}{\boxed{e^{2x} + e^{-2x}}}\right) = \frac{d}{dx} \log u = \frac{d\log u}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \cdot \left(e^{2x} + e^{-2x}\right)' = \underline{\frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}}} \,/\!/$$

(5) $f(x) = \sin^{-1} \sqrt{1-2x^2}$

$$f'(x) = \frac{d}{dx} \sin^{-1} \underset{\substack{\parallel \\ u\text{とおく}}}{\boxed{\sqrt{1-2x^2}}} = \frac{d}{dx} \sin^{-1} u = \frac{d}{du} \sin^{-1} u \cdot \frac{du}{dx}$$

$$= \frac{1}{\sqrt{1-u^2}} \left(\sqrt{1-2x^2}\right)' \quad \cdots\cdots (*)$$

(メモ)
$$\frac{d}{du} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}}$$

ここで

$$\sqrt{1-u^2} = \sqrt{1-\left(\sqrt{1-2x^2}\right)^2} = \sqrt{1-(1-2x^2)} = \sqrt{2x^2}$$

$$\left(\sqrt{1-2x^2}\right)' = \left((1-2x^2)^{\frac{1}{2}}\right)' = \frac{1}{2}(1-2x^2)^{\frac{1}{2}-1}(1-2x^2)'$$

$$= \frac{1}{2}(1-2x^2)^{-\frac{1}{2}}(-4x) = \frac{-2x}{\sqrt{1-2x^2}}$$

より

$$(*) = \frac{1}{\sqrt{2x^2}} \frac{-2x}{\sqrt{1-2x^2}} = \underline{\frac{-2x}{\sqrt{2x^2}\sqrt{1-2x^2}}} \,/\!/$$

(6)

(メモ) ロピタルの定理　$\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = 0$ のとき

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0} \frac{5x + \sin 3x}{2x}$$

$$\left(\begin{array}{l} \lim_{x\to 0} 分子 = \lim_{x\to 0} 5x + \sin 3x = 0 + 0 = 0 \\ \lim_{x\to 0} 分母 = \lim_{x\to 0} 2x = 2\times 0 = 0 \end{array} \right) より$$

$$= \lim_{x\to 0} \frac{(5x+\sin 3x)'}{(2x)'} = \lim_{x\to 0} \frac{5 + 3\cos 3x}{2}$$

$$= \frac{5 + 3\cos 0}{2} = \frac{5+3}{2} = \underline{4}_{\,//}$$

(7) $f(x) = \sqrt{1 - x + \frac{x^2}{2}}$ の マクローリン展開 2次まで求めよ.

(メモ) マクローリン展開 より.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots \quad ——①$$

$f'(x)$, $f''(x)$ を求めて $f(0)$, $f'(0)$, $f''(0)$ を求めて ① を計算する.

$$f'(x) = \left(\left(1 - x + \frac{x^2}{2}\right)^{\frac{1}{2}}\right)' = \frac{1}{2}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{1}{2}}\left(1 - x + \frac{x^2}{2}\right)'$$

$$= \frac{1}{2}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{1}{2}}(-1 + x)$$

$$f''(x) = -\frac{1}{4}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{3}{2}}\underbrace{\left(1 - x + \frac{x^2}{2}\right)'}_{-1+x}(-1 + x) + \frac{1}{2}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{1}{2}}\underbrace{(-1+x)'}_{1}$$

$$= -\frac{1}{4}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{3}{2}}(-1+x)^2 + \frac{1}{2}\left(1 - x + \frac{x^2}{2}\right)^{-\frac{1}{2}}$$

よって

$$f(0) = \sqrt{1 - 0 + 0} = 1, \quad f'(0) = \frac{1}{2} 1^{-\frac{1}{2}}(-1) = -\frac{1}{2}$$

$$f''(0) = -\frac{1}{4}(-1)^2 + \frac{1}{2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

① に代入すると

$$f(x) = 1 + \left(-\frac{1}{2}\right)x + \frac{1}{2}\left(\frac{1}{4}\right)x^2 = \underline{1 - \frac{x}{2} + \frac{x^2}{8}}_{\,//}$$

(8) $\displaystyle\int x \underbrace{\sin 2x}_{\left(-\frac{\cos 2x}{2}\right)'} dx$

(メモ) 部分積分法
$$\int fg'dx = fg - \int f'g\,dx$$

$$= \int x\left(-\frac{\cos 2x}{2}\right)' dx = x\left(-\frac{\cos 2x}{2}\right) - \int \underbrace{x'}_{1}\left(-\frac{\cos 2x}{2}\right)dx$$

$$= -\frac{x\cos 2x}{2} + \int \frac{\cos 2x}{2}\,dx$$

$$= -\frac{x\cos 2x}{2} + \frac{\sin 2x}{2\times 2} + C$$

$$= -\frac{x\cos 2x}{2} + \frac{\sin 2x}{4} + C$$ //

(9) $\displaystyle\int \frac{x}{x^2+x-2}\,dx$　　$\dfrac{x}{x^2+x-2}$ を部分分数に分ける

$x^2+x-2 = (x-1)(x+2)$ より

$$\frac{x}{x^2+x-2} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2)+B(x-1)}{(x-1)(x+2)} = \frac{(A+B)x+2A-B}{(x-1)(x+2)}$$

$$\begin{cases} A+B=1 \\ 2A-B=0 \end{cases} \Rightarrow \quad 3A=1,\ A=\frac{1}{3},\quad B=1-A=\frac{2}{3}$$

よって

$$\int \frac{x}{x^2+x-2}\,dx = \int \frac{\frac{1}{3}}{x-1} + \frac{\frac{2}{3}}{x+2}\,dx$$

$$= \frac{1}{3}\underbrace{\int \frac{dx}{x-1}}_{\log|x-1|} + \frac{2}{3}\underbrace{\int \frac{dx}{x+2}}_{\log|x+2|} = \frac{1}{3}\log|x-1| + \frac{2}{3}\log|x+2| + C$$ //

(10) $\int_0^1 x(x-1)^9\,dx$ 　置換積分する

$t = x-1$ とおくと

$\frac{dt}{dx} = (x-1)' = 1$ より $dt = dx$

$$\begin{array}{c|ccc} x & 0 & \longrightarrow & 1 \\ \hline t & 0-1 & & 1-1 \\ & = & & = \\ & -1 & \longrightarrow & 0 \end{array}$$

$x = t+1$ より 積分は、

$$\int_{-1}^{0} (t+1)t^9\,dt = \int_{-1}^{0} t^{10} + t^9\,dt$$

$$= \left[\frac{t^{11}}{11} + \frac{t^{10}}{10}\right]_{-1}^{0} = 0 - \left(\frac{(-1)^{11}}{11} + \frac{(-1)^{10}}{10}\right)$$

$$= -\frac{-1}{11} - \frac{1}{10} = \frac{1}{11} - \frac{1}{10} = \frac{10-11}{11\times 10} = \underline{\frac{-1}{110}}\ _{//}$$

(11) $\int_0^{\pi/2} \sin 3x \sin 2x\,dx$ 　三角関数の積和公式を用いて.

(メモ) $\sin\alpha\sin\beta = -\frac{1}{2}\left(\cos(\alpha+\beta) - \cos(\alpha-\beta)\right)$

$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$

$$= \int_0^{\pi/2} -\frac{1}{2}\left(\cos(3x+2x) - \cos(3x-2x)\right)dx$$

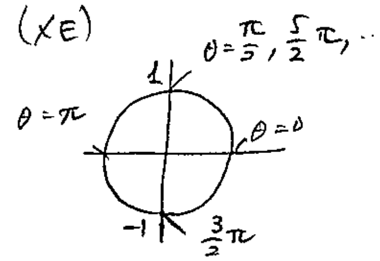
$$= -\frac{1}{2}\int_0^{\pi/2} \cos 5x - \cos x\,dx$$

$$= -\frac{1}{2}\left[\frac{\sin 5x}{5} - \sin x\right]_0^{\pi/2}$$

$$= -\frac{1}{2}\left\{\left(\frac{\sin\frac{5}{2}\pi}{5} - \sin\frac{\pi}{2}\right) - \underbrace{\left(\frac{\sin 0}{5} - \sin 0\right)}_{0}\right\}$$

$$= -\frac{1}{2}\left(\frac{1}{5} - 1\right) = -\frac{1}{2}\cdot\frac{-4}{5} = \underline{\frac{2}{5}}\ _{//}$$

(メモ)



(12) $\displaystyle\int_0^{\infty} e^{-x}(2x+1)\,dx$

$$e^{-x} = \left(\frac{e^{-x}}{-1}\right)'$$

$$= \int_0^{\infty} \left(\frac{e^{-x}}{-1}\right)'(2x+1)\,dx$$

部分積分法 $\displaystyle\int_a^b f'g\,dx = [fg]_a^b - \int_a^b fg'\,dx$

$$= \left[\frac{e^{-x}}{-1}(2x+1)\right]_0^{\infty} - \int_0^{\infty} \frac{e^{-x}}{-1}(2x+1)'\,dx$$

$$= \underbrace{\left(\lim_{x\to\infty} \frac{e^{-x}}{-1}(2x+1)\right)}_{0} - \frac{e^0}{-1} - \int_0^{\infty} \frac{e^{-x}}{-1}\cdot 2\,dx$$

$$= 1 + 2\int_0^{\infty} e^{-x}\,dx$$

$$= 1 + 2\left[\frac{e^{-x}}{-1}\right]_0^{\infty}$$

$$= 1 + 2\left\{\underbrace{\left(\lim_{x\to\infty} \frac{e^{-x}}{-1}\right)}_{0} - \frac{e^0}{-1}\right\}$$

$$= 1 + 2\times 1 = \underline{3}$$