Jpn. J. Appl. Phys. Vol. 34 (1995) pp. 4508–4510
Part 1, No. 8B, August 1995

Photon Emission from a False Vacuum of Semiconductors

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(Received March 8, 1995; accepted for publication March 29, 1995)

We consider photon creation through the dynamical Casimir effect in a semiconductor material. Unlike previous studies, we evaluate the number of created photons using a microscopic model in which polarization degree of freedom is included in the theory as a microscopic variable, under realistic situations in which material parameters vary by a finite magnitude within a finite time, in a material which exhibits strong dispersions in the dielectric constant. Our results differ strikingly from previous results.

KEYWORDS: Casimir effect, dynamical Casimir effect, photon, exciton, polariton, photonic crystal, microcavity, quantum well

1. Introduction

In 1948 Casimir[1] pointed out that mirrors in vacuum attract each other. This phenomenon, called the Casimir effect, was demonstrated experimentally by Sparnaay in 1958.[2] Since mirrors are electrically neutral, the Casimir force can never be explained by classical electrodynamics, and is a manifestation of quantum-mechanical vacuum fluctuations. A more interesting phenomenon, called the dynamical Casimir effect (DCE), occurs when the mirrors are moved rapidly:[3,4] photons are created from the vacuum because the vacuum becomes a "false vacuum" as the positions of the mirrors are changed. This phenomenon can be understood as a general property of quantum field theory: particles are created form vacuum when the equations of motion or boundary conditions of the field change as a function of time.[5] We hereafter use the term DCE to refer to this general phenomenon. Unlike the static case, the DCE has not been observed experimentally. This is probably because rather unrealistic situations, such as the movement of mirrors at half the velocity of light, would be required to observe the DCE.

On the other hand, the DCE is expected also in condensed matter. Yablonovitch[6] and Schwinger[7] pointed out that a sudden change of dielectric constant in semiconductors or in liquids induces photon radiation. Since we have a wide choice of materials and structures, it seems possible to achieve a higher efficiency of photon creation in condensed matter than in an empty space. However, the previous theoretical studies[6,7] employed rather unrealistic assumptions: (i) The change of dielectric constant was assumed to be either instantaneous[6-8] or a slowly varying limit.[6] (ii) The magnitude of the change of dielectric constant was assumed to be infinitesimal.[7] (iii) Dispersions in dielectric constant were neglected.[6,7] Therefore, it has not yet been clarified whether we can actually observe non-negligible photon radiations in realistic situations.

In this work, we evaluate the number of created particles (which are polaritons in our case) using more realistic models. We consider the case when material parameters vary by a finite magnitude within a finite time, in a material which exhibits strong dispersions in the dielectric constant.

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2. Formulation

The previous studies[6,7] were based on a simple quantization procedure: the macroscopic electromagnetic fields obeying the macroscopic Maxwell equations are regarded as basic dynamical variables and quantized. However, we do not know any justification for such a phenomenological formulation in treating the DCE. Possible problems include the following. (i) Although the dielectric constant shows strong dispersion in most cases of interest (in view of the DCE), the formulation cannot treat such a case. (ii) We do not know which of the fields appearing in the macroscopic Maxwell equations should be continuous under a sudden change of the dielectric constant. Unfortunately, the number of created particles strongly depends on the choice of the continuous fields. (iii) The formulation would run into difficulties when it is applied to the DCE in microcavities or photonic crystals, because in such a case the dielectric constant varies both temporally and spatially. This leads to a time-dependent gauge-fixing condition, which is rather troublesome.

We develop here a microscopic formulation which is free from these problems. Moreover, we do not employ the unrealistic assumptions of the previous studies, such as an instantaneous change of the dielectric constant.[6-8]

For simplicity, we assume here a uniform crystal. This would be a good approximation even of non-uniform materials (such as a multiple quantum well (MQW) structure) when the characteristic scale of the non-uniformity (the period of the MQW structure) is much smaller than the photon wavelengths of interest. The generalization to other non-uniform materials, such as photonic crystals, will be described elsewhere. To avoid introducing the phenomenological, macroscopic parameter—dielectric constant—into our microscopic theory, we take into account the microscopic degrees of freedom of polarization which determine the dielectric response. Thus the dielectric constant is not a basic parameter in our formulation but only a property of the system that can be calculated from the microscopic parameters.

As the microscopic polarization field, we employ here an exciton field. The exciton energy ω_x and its coupling α to the optical field may be changed temporally by various methods including the plasma creation by a laser pulse (as suggested in ref. 6), ultrasonic (phonon) excitation (as suggested in ref. 7), the optical Stark ef-

fect[9,10] and the quantum confined Stark effect.[11-13] We may employ the following model Lagrangian to describe the system of the exciton field $\boldsymbol{X}$ and the photon field $\boldsymbol{A}$:

$$\mathcal{L} = \frac{\epsilon_0}{2}E^2 - \frac{1}{2\mu_0}B^2 + \frac{\rho}{2}\frac{\partial X^2}{\partial t} - \frac{\rho\omega_x(t)^2}{2}X^2 - \alpha(t)(\boldsymbol{A}\cdot\dot{\boldsymbol{X}} + U\nabla\cdot\boldsymbol{X}) \quad (1)$$

where $\boldsymbol{E} = -\dot{\boldsymbol{A}} - \nabla U$, $\boldsymbol{B} = \nabla\times\boldsymbol{A}$ and ρ is the density of $\boldsymbol{X}$. This Lagrangian is a generalization to the time-dependent ω_x and α of the Lagrangian proposed by Huttner *et al.*[14] Their quantization procedure can be applied also to our t-dependent case because the quantization relies on the equal-time commutation relations. For the t-dependence of the parameters, we assume here the following forms:

$$\omega_x(t) = \begin{cases} \omega_1 & (t\le 0) \\ \omega_1 - \dfrac{\omega_1-\omega_2}{2\pi}\left[\dfrac{2\pi t}{T} - \sin\left(\dfrac{2\pi t}{T}\right)\right] & (0<t<T) \\ \omega_2 & (t\ge T) \end{cases} \quad (2)$$

$$\alpha(t) = \begin{cases} \alpha_1 & (t\le 0) \\ \alpha_1 - \dfrac{\alpha_1-\alpha_2}{2\pi}\left[\dfrac{2\pi t}{T} - \sin\left(\dfrac{2\pi}{t}T\right)\right] & (0<t<T). \\ \alpha_2 & (t\ge T) \end{cases} \quad (3)$$

These are continuous up to the second derivative.

Following ref. 14, we focus on the transverse fields only. We then obtain the Heisenberg equations of motion for the transverse fields. The equations have t-dependent parameters. We solve the equations numerically. The number of created particles (polaritons) can then be evaluated in a similar manner as that in the previous studies.[2-7]

Note that we are considering the inside of the crystal and evaluating the number of created polaritons, whereas the most interesting quantity is the number of photons which are emitted *outside* the crystal. At the crystal surfaces, a polariton will be transformed into a photon with a finite probability. The probability is higher for a polariton which has larger photon amplitude. For the lower-branch (LB) polariton near the band gap, the probability is typically 0.1–1. Therefore, for the present order-of-magnitude estimation, we can approximate the number of emitted photons by the number of created LB polaritons.

3. Results and Discussion

The creation spectra of LB polaritons are shown in Fig. 1. This figure plots $\langle n_{k\lambda}\rangle$—the number of LB polaritons per mode—as a function of k (in units of ω_1/c) for T ranging from 0.1 to 4 (in units of $1/\omega_1$), $\omega_2 = 0.99\,\omega_1$, $\alpha_1 = 0.04$ and $\alpha_2 = 0.03$ (in units of $\omega_1/\sqrt{\epsilon_0\rho}$). The total number of LB polaritons between k and $k+\Delta k$ is obtained by integrating $\langle n_{k\lambda}\rangle$:

$$N(k,\Delta k) = 2\cdot\left(\frac{L}{2\pi}\right)^3\int_k^{k+\Delta k} 4\pi k^2\langle n_{k\lambda}\rangle dk \simeq 8\pi\left(\frac{L}{\lambda_2}\right)^3\frac{\Delta\lambda_2}{\lambda_2}\langle n_{k\lambda}\rangle \quad (4)$$

where the second equality holds when $\Delta k \ll k$, and $\lambda_2 \equiv 2\pi c/\omega_2$. For example, $N(k,\Delta k) \sim 10^{13}\langle n_{k\lambda}\rangle$ for $L =$ 1 cm, $\lambda_2 = 1$ μm and $\Delta\lambda_2 = \lambda_2/2$.

Figure 1 shows that the spectra are much different from those of the previous studies.[6,7] More importantly, it is found that $\langle n_{k\lambda}\rangle$ decreases very rapidly as T is increased. To understand the decrease, let us investigate the large-k part. (Although the small-k part can be understood in a similar manner, the large-k part is easier to understand.) We note that the exciton-photon coupling becomes less important at large k, and that the large-k part of the LB polariton is mostly the exciton. Therefore, the behavior of the spectra in Fig. 1 at large k may be described by a simplified model in which the exciton-photon coupling is absent; $\alpha = 0$. (We have confirmed that this is indeed the case.) In this case, pure excitons will be created whereas no photons are created, and $\langle n_{k\lambda}\rangle$ is independent of k. In Fig. 2 we have plotted $\langle n_{k\lambda}\rangle$ as a function of T (again, T is in units of $1/\omega_1$) by bold lines. We find that $\langle n_{k\lambda}\rangle$ decreases exponentially for large T. We also find small oscillations for large T. This is due to the fact that our $\omega_x(t)$ has singularities

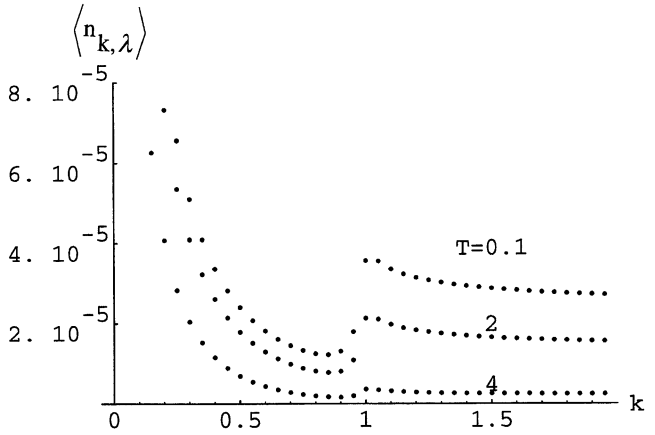

Fig. 1. The number of created lower-branch (LB) polaritons per mode is plotted against the wavenumber k (in units of ω_1/c) for T (in units of $1/\omega_1$) ranging from 0.1 to 4.

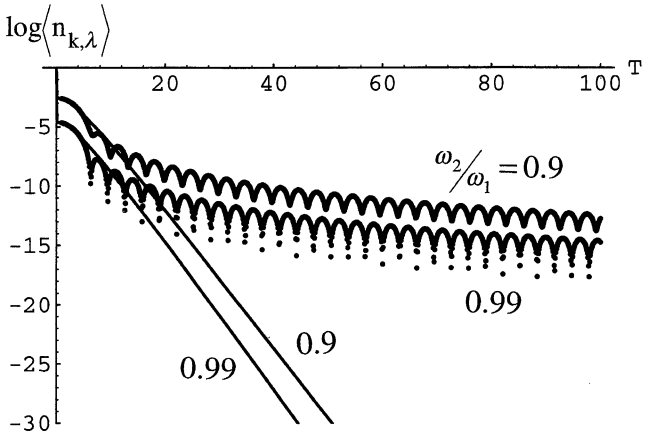

Fig. 2. The number of created excitons per mode is plotted against T (in units of $1/\omega_1$) for $\alpha = 0$. The bold and thin lines are the results for non-analytic and analytic $\omega_x(t)$, respectively. The ratio ω_2/ω_1 is assumed to be 0.99 or 0.9.

(third-order derivative is discontinuous) at $t = 0$ and T. To verify this let us consider the case of analytic $\omega_x(t)$ given by

$$\omega_x(t) = \sqrt{\omega_1^2 + \frac{(\omega_2^2 - \omega_1^2)}{1 + \exp(-t/\tau)}} \tag{5}$$

where $T = 2\tau \ln 8$. (The coefficient is chosen in such a way that a small-T behavior agrees with the case of the non-analytic ω_x.) For this $\omega_x(t)$ (and for $\alpha = 0$), we can obtain the exact solution.

$$\langle n_{k\lambda} \rangle = \frac{\sinh^2[\pi(\omega_1 - \omega_2)\tau]}{\sinh[2\pi\omega_1\tau]\sinh[2\pi\omega_2\tau]} \tag{6}$$

This is plotted in Fig. 2 by thin lines. It is seen that for large T non-analyticity in $\omega_x(t)$ not only induces small oscillations but also enhances $\langle n_{k\lambda} \rangle$ by many orders of magnitude. Since non-analytic change of $\omega_x(t)$ is unphysical, the result of analytic change of $\omega_x(t)$ should be more convincing. However, much of the previous studies[6-8] employed non-analytic forms for time-dependent parameters (such as the dielectric constant). The present result strongly suggests that it is only for relatively short T (say, $T < 10$) that the non-analytic forms would give physically correct results. Regarding Fig. 1, the result may be convincing because T is small enough.

In either case of analytic or non-analytic $\omega_x(t)$, it is found that $\langle n_{k\lambda} \rangle$, and thus $N(k, \Delta k)$, will be very small in a usual experimental setup for modulating ω_x. For example, in the usual units, $T = 0.1$ ps corresponds to $T\omega_1 = 150\hbar\omega_1$ where $\hbar\omega_1$ is measured in eV, and Fig. 2 predicts a very small number when $\hbar\omega_1 \sim 1$ eV. However, the values of various parameters used in the present calculation are typical values of a uniform crystal or MQW structure composed of GaAs/AlAs or similar materials. We must therefore explore other materials and/or structures to experimentally observe the DCE. A possible candidate is a photonic crystal or a microcavity, because an extremely large coupling between photons and polarizations can be obtained in such a system. Use of organic materials might also be promising. Another possibility is to introduce other t-dependent terms in the Lagrangian. These possibilities will be subjects of future studies.

Acknowledgement

This work has been supported by a Grant-in-Aid for Scientific Research on Priority Areas from the Ministry of Education, Science and Culture.

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