**The linear mixed effects model (LMM)**

Denote $\mathbf{Y\_i} = (y\_{m\_ii}, \cdots, y\_{m\_i, i})^T$ be an $m\_i$-dimensional vector of repeated responses for the $i^{th}$ (independent) individual, $\mathbf{X\_i}$ and $\mathbf{Z\_i}$ be $m\_i \times p$ and $m\_i \times q$-dimensional vector of covariates, respectively, and $\mathbf{\beta}$ be a $p$-dimensional vector of fixed effects. The standard LMM is then **[Laird and Ware 1982].**

\begin{align}

\mathbf{Y\_i} &= \mathbf{\beta}\mathbf{X\_i} + \mathbf{b\_i}\mathbf{Z\_i} + \mathbf{\epsilon\_i}, \; (i = 1, \cdots, n)\\

\epsilon & \sim N\_m\_i(0, \mathbf{\Sigma\_i})

\label{eq:eqn-one}

\end{align}

where $\mathbf{\epsilon\_i}$ is the residual vector and $\mathbf{\Sigma\_i}$ is $m\_i \times m\_i$ positive definite (p.d) covariance matrix. The $q \times 1$ vector of random effects, $\mathbf{b\_i \sim N\_q(0, \mathbf{D})$ where $\mathbf{D}$ denotes the $q \times q$ p.d covariance matrix. The classical LMM represented by Equation (\ref{eq:eqn-one}) combines fixed effects $\mathbf{\beta}$ with the subject specific random effects $\mathbf{b\_i}, \cdots, \mathbf{b\_n}$ and inference might be focused on the regression coefficients, $\mathbf{\beta}$, the subject-specific coefficients, $\mathbf{b\_i}$, or the variance-covariance structure, $\mathbf{\Sigma\_i}$ and $\mathbf{D}$. This is in contrast to a linear model defined as in Equation (\ref{eq:eqn-two}) that assumes the subject-specific coefficients $\mathbf{b\_i}$.

\begin{equation}

\mathbf{Y\_i} &= \mathbf{\beta}\mathbf{X\_i} + \mathbf{\epsilon\_i}, \; (i = 1, \cdots, n)\\

\epsilon & \sim N\_m\_i(0, \mathbf{\Sigma\_i})

\label{eq:eqn-two}

\end{equation}

**The robust linear mixed effects model**

RLMMs, or robust linear mixed models, are a type of linear mixed models (LMMs) that can handle outliers and influential observations in the data. They are implemented in R using the \*\*robustlmm\*\* package, which provides the function \*\*rlmer()\*\* to fit RLMMs ¹. This function is similar to the \*\*lmer()\*\* function from the \*\*lme4\*\* package, which is used to fit LMMs, but it uses different methods to estimate the model parameters and the variance components. Specifically, \*\*rlmer()\*\* uses the \*\*M-estimation\*\* method, which minimizes a robust loss function, such as Huber's loss or Tukey's biweight loss, instead of the usual least squares loss ¹². This makes the estimates more resistant to outliers and influential observations, and improves the accuracy and interpretation of the fixed effects.

The main difference between RLMMs and LMMs is that RLMMs can account for both within- and between-cluster variability, and handle outliers and influential observations in the data, while LMMs assume that the data are normally distributed and homogeneous. RLMMs can also provide more reliable inference and prediction than LMMs, especially when the data are skewed or heavy-tailed ¹². However, RLMMs are more computationally intensive and require more tuning than LMMs, and they may not perform well when the data are sparse or have complex correlation structures ¹². Therefore, RLMMs and LMMs have different strengths and limitations, and the choice of the best model depends on the characteristics and objectives of the data analysis.