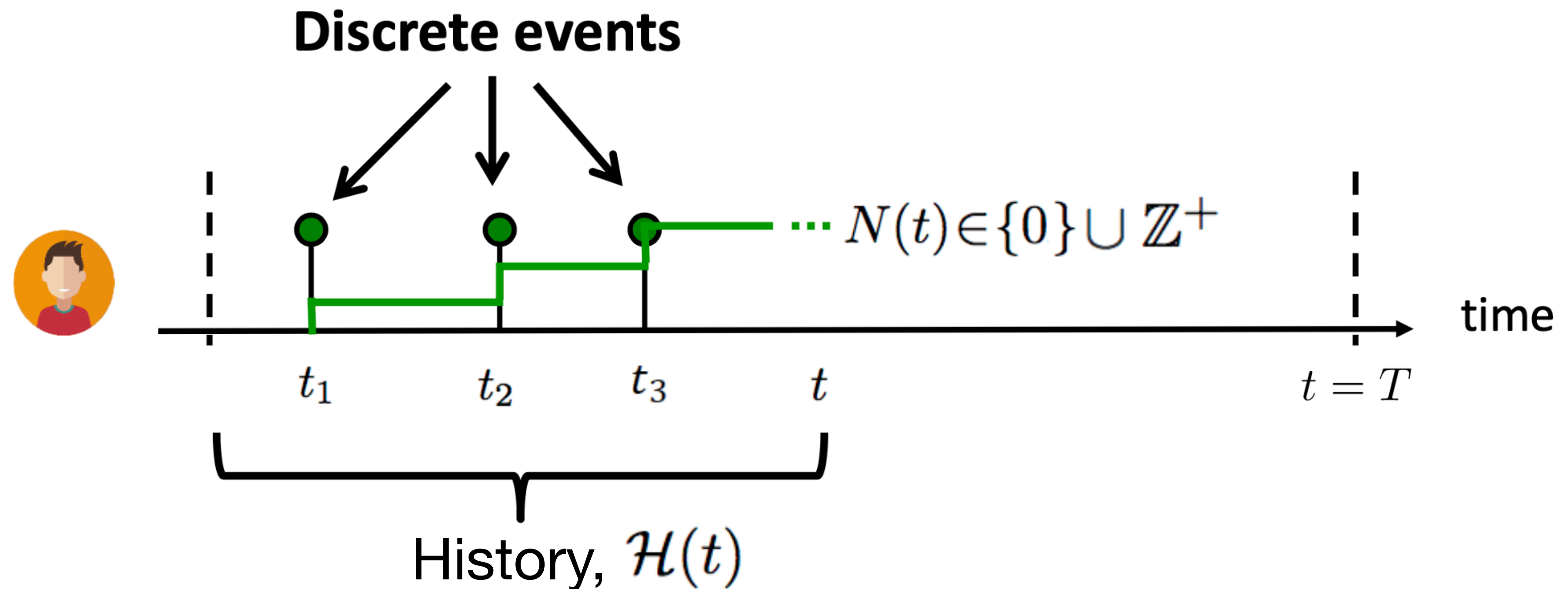


# Hawke's neural process

# Temporal Point Processes: Recap

# Temporal point processes

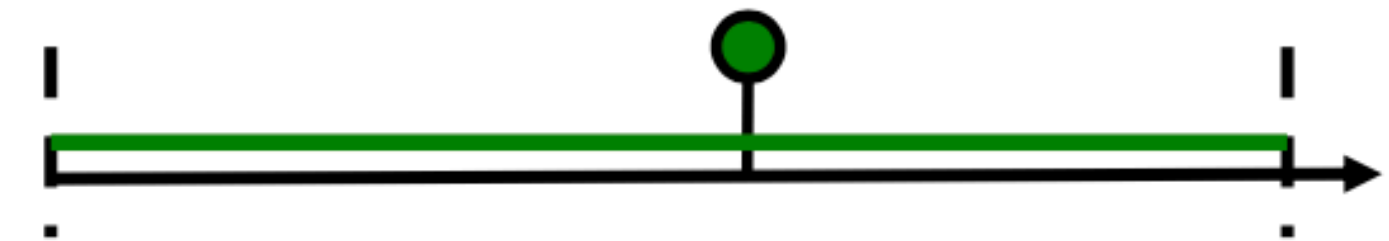
- **Temporal point process** — a random process whose realization consists of discrete events localized in time  $\mathcal{H} = \{t_i\}$



# Building blocks to represent different dynamic processes

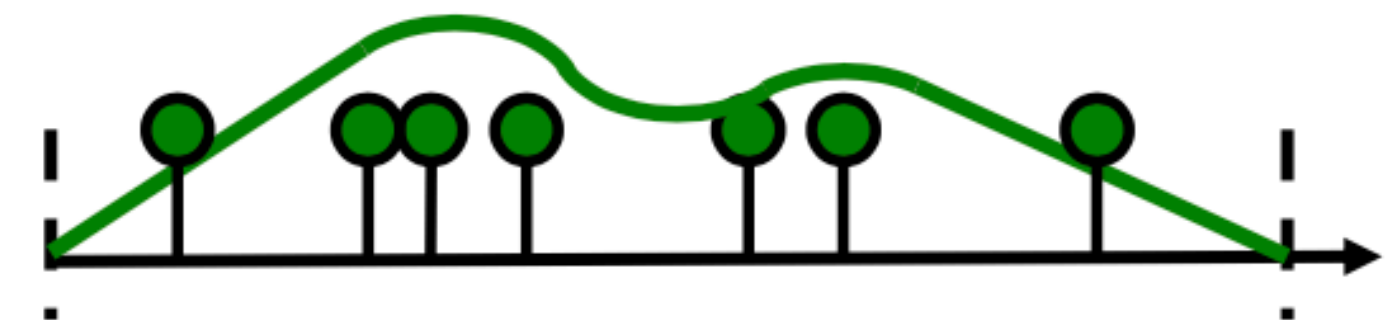
Poisson processes:

$$\lambda^*(t) = \lambda$$



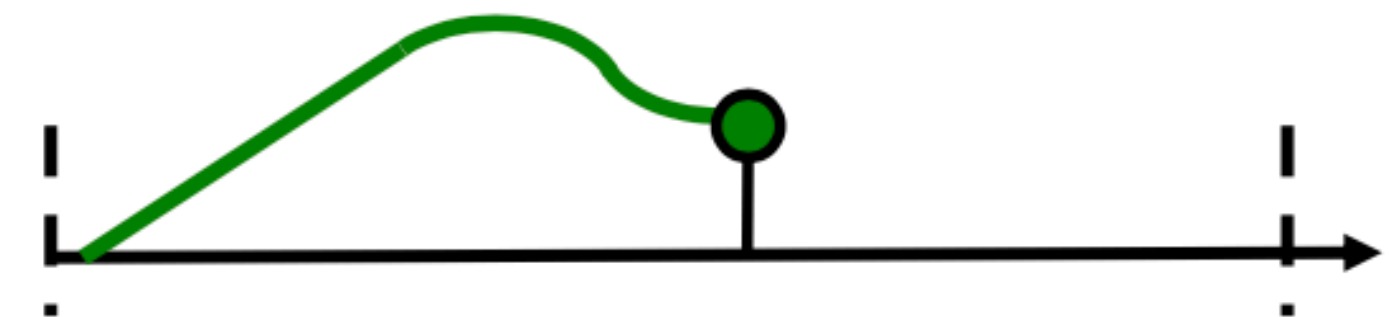
Inhomogeneous Poisson processes:

$$\lambda^*(t) = g(t)$$



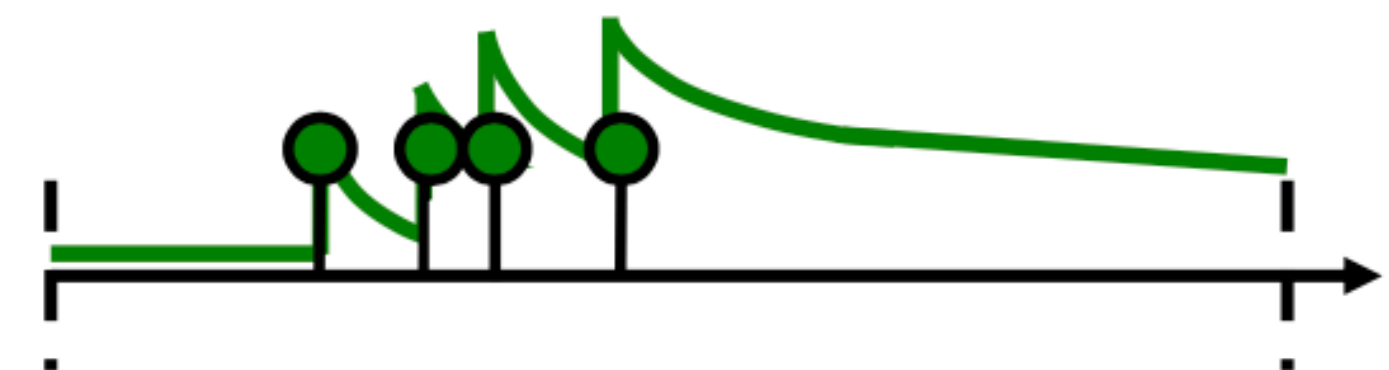
Terminating point processes:

$$\lambda^*(t) = g^*(t)(1 - N(t))$$



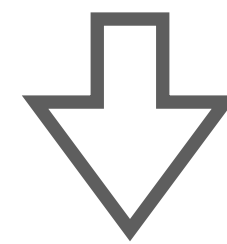
Self-exciting point processes:

$$\lambda^*(t) = \mu + \alpha \sum_{t_i \in \mathcal{H}(t)} \kappa_\omega(t - t_i)$$



# Vanilla Hawkes process drawbacks

- Past events can temporarily raise the probability of future events
- *Real world application*: the flexibility of vanilla Hawkes process is very limited to approximate many sections of the problems



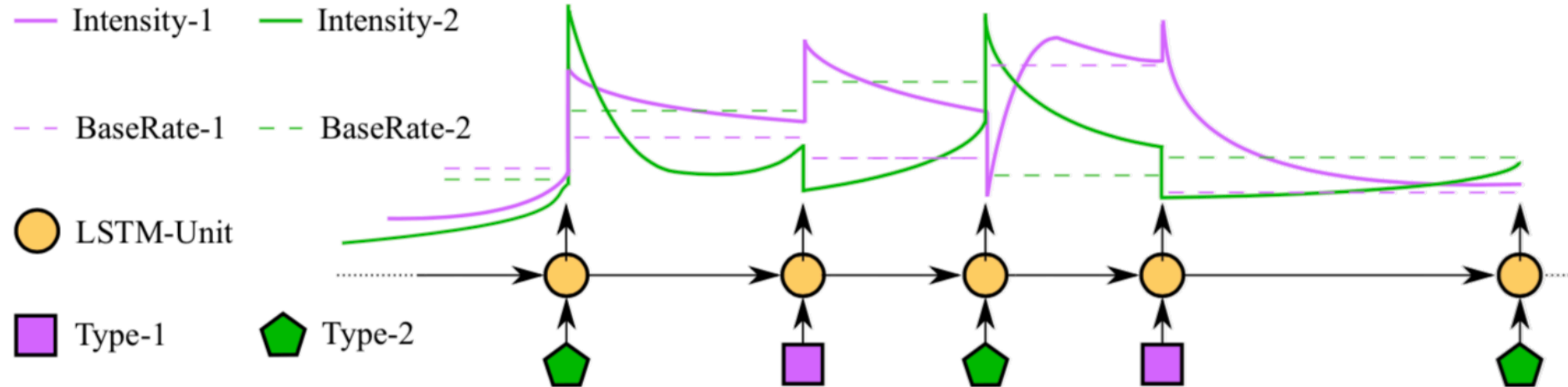
- Need a better model with least assumptions to approximate the conditional intensity function

**Solution:** Neural Hawkes process

# Neural Hawkes process

# Neural Hawkes process advantages

- History effect does not need to be additive
- Allows for complex memory effects (such as delays)

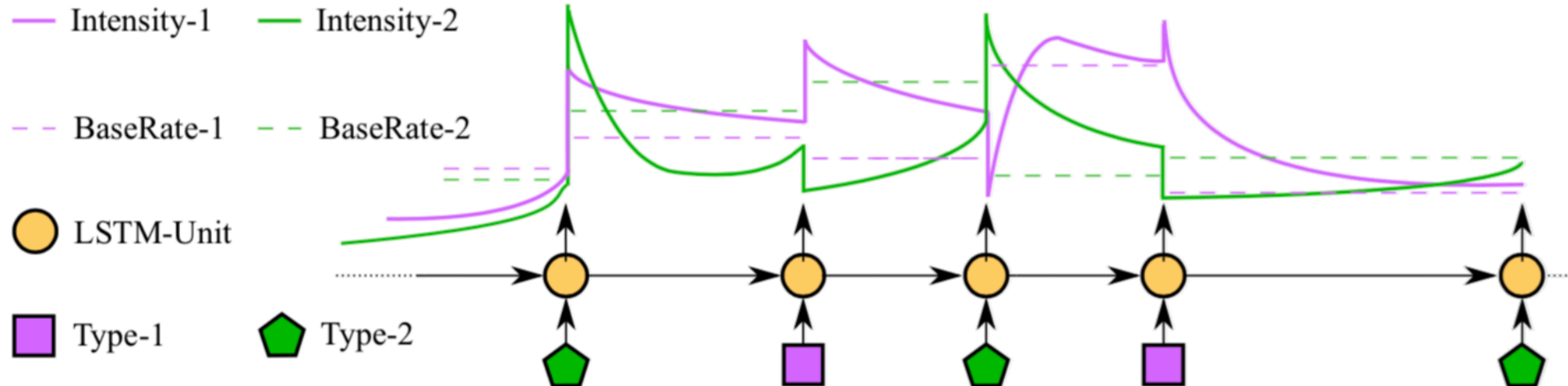


Source: Mei & Eisner, NIPS 2017



# Neural Hawkes process: RNNs

- Memory via the continuous-time LSTM
  - Excitation & inhibition via activation function
- $$\mathbf{h}(t) = \text{RNN}(\mathcal{H}(t))$$
- $$\lambda_u(t) = f_u(\mathbf{w}_u^\top \mathbf{h}(t))$$



Source: Mei & Eisner, NIPS 2017



# Event stream data

- Financial transactions
- Medical notes
- ...



**Stock trading**



**Flu spreadin**



**Reviews and sales in Amazon**



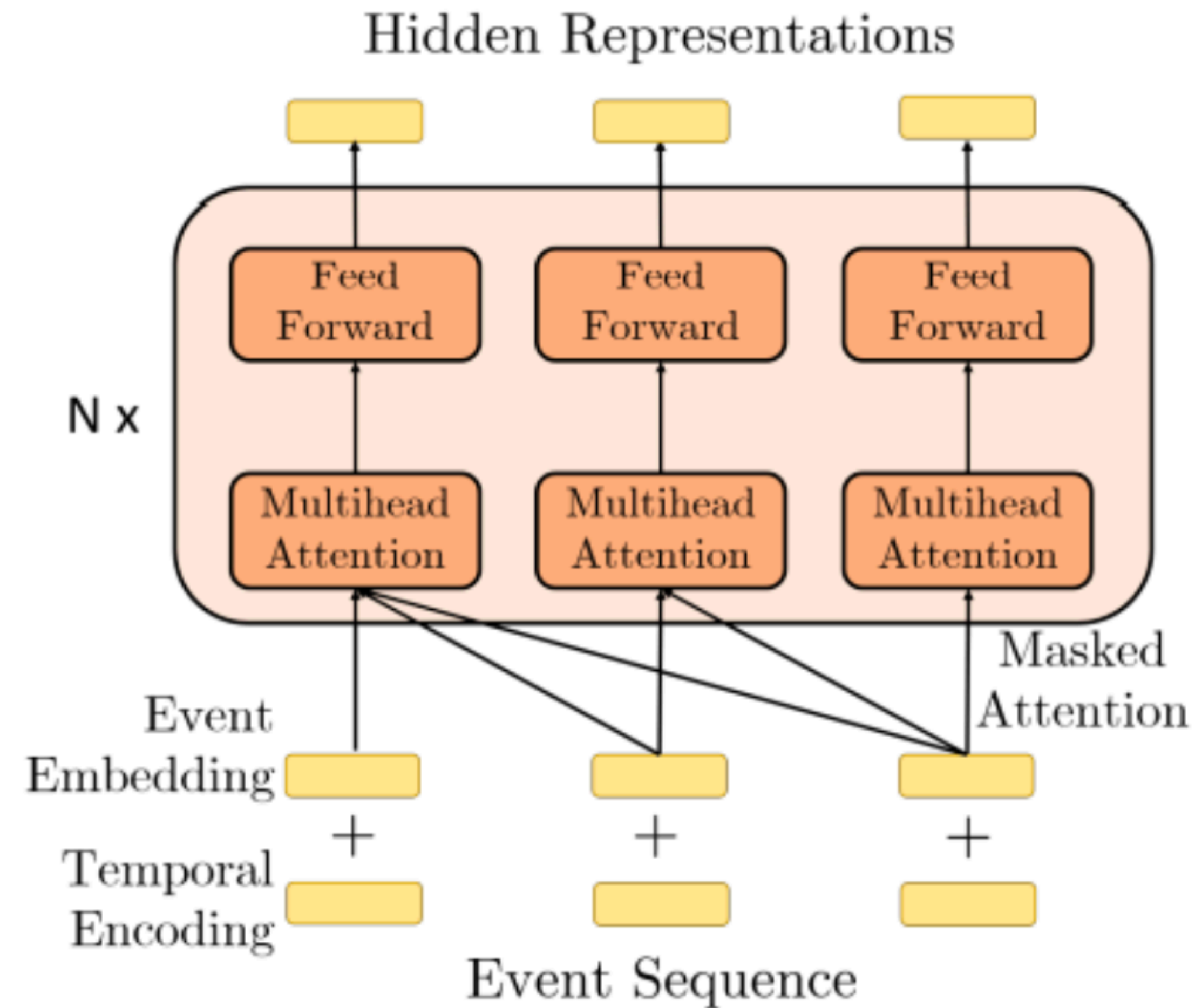
**News spread in Twitter**

**Skoltech**

**Long-term dependencies issues**

- Financial transactions: policy issues
- Medical notes: chronic diseases

# Neural Hawkes process: Transformers



Source: Zuo, Simiao, et al. "Transformer Hawkes process." ICML. 2020

# Neural Hawkes process: Transformers

$$\lambda_k(t|\mathcal{H}_t) = f_k \left( \underbrace{\alpha_k \frac{t - t_j}{t_j}}_{\text{current}} + \underbrace{\mathbf{w}_k^\top \mathbf{h}(t_j)}_{\text{history}} + \underbrace{b_k}_{\text{base}} \right)$$

History up to  
time  $t$

Interpolation  
between two  
observed time  
stamps

$$p(t|\mathcal{H}_t) = \lambda(t|\mathcal{H}_t) \exp \left( - \int_{t_j}^t \lambda(\tau|\mathcal{H}_\tau) d\tau \right),$$

$$\hat{t}_{j+1} = \int_{t_j}^{\infty} t \cdot p(t|\mathcal{H}_t) dt,$$

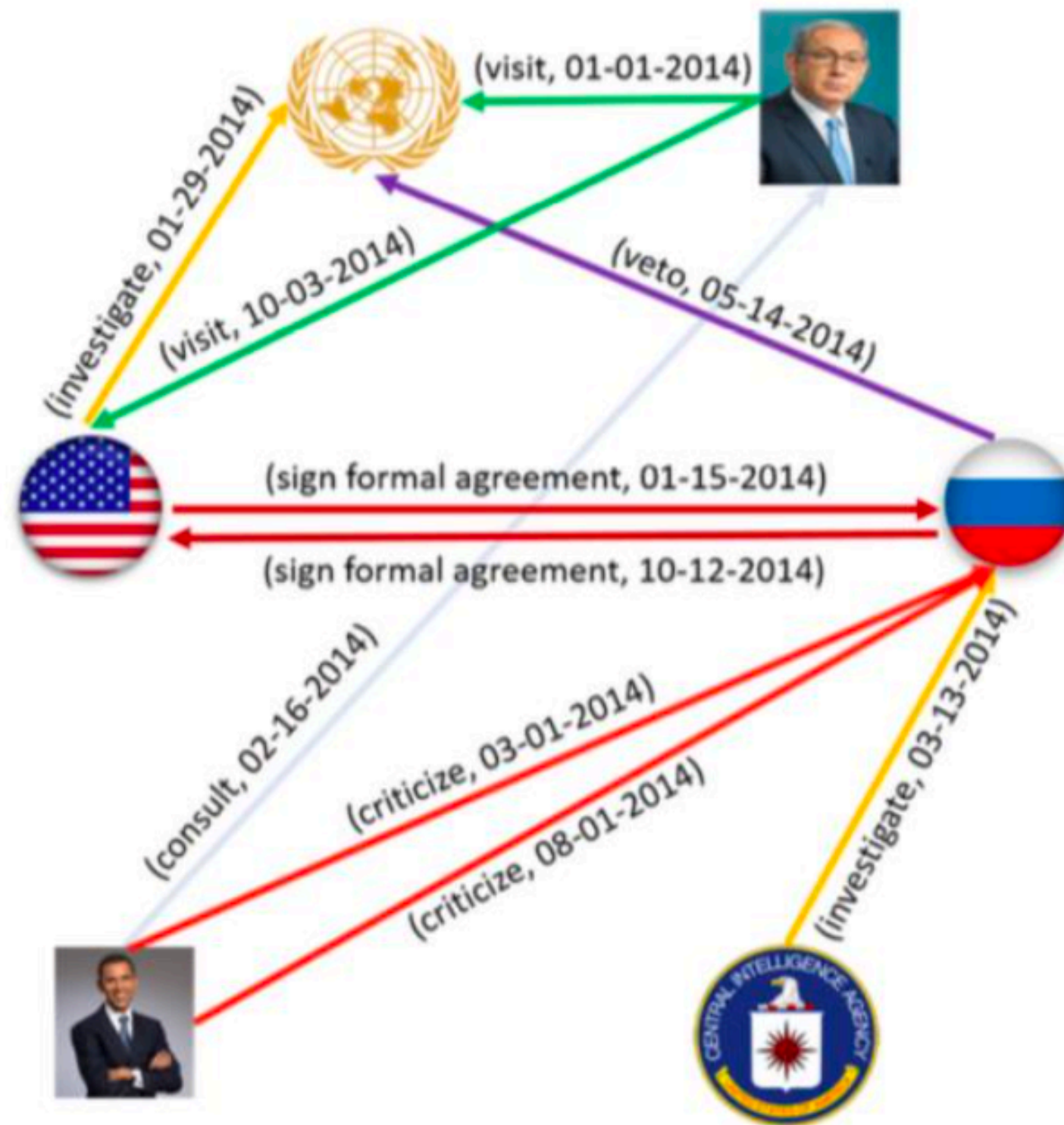
$$\hat{k}_{j+1} = \operatorname{argmax}_k \frac{\lambda_k(t_{j+1}|\mathcal{H}_{j+1})}{\lambda(t_{j+1}|\mathcal{H}_{j+1})}.$$

# Applications

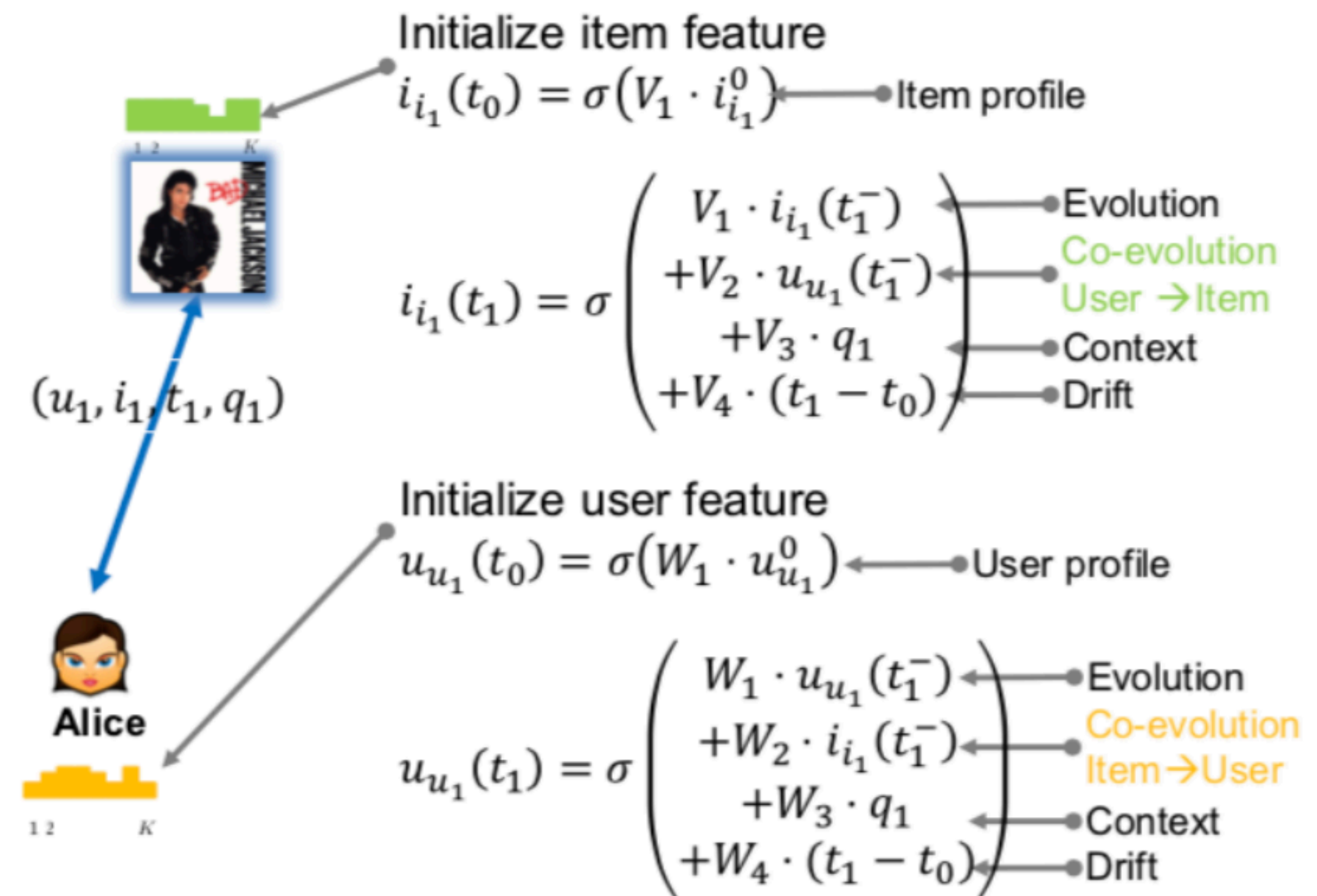


# Predictive Models

**Know-Evolve, Trivedi et al. (2017)**



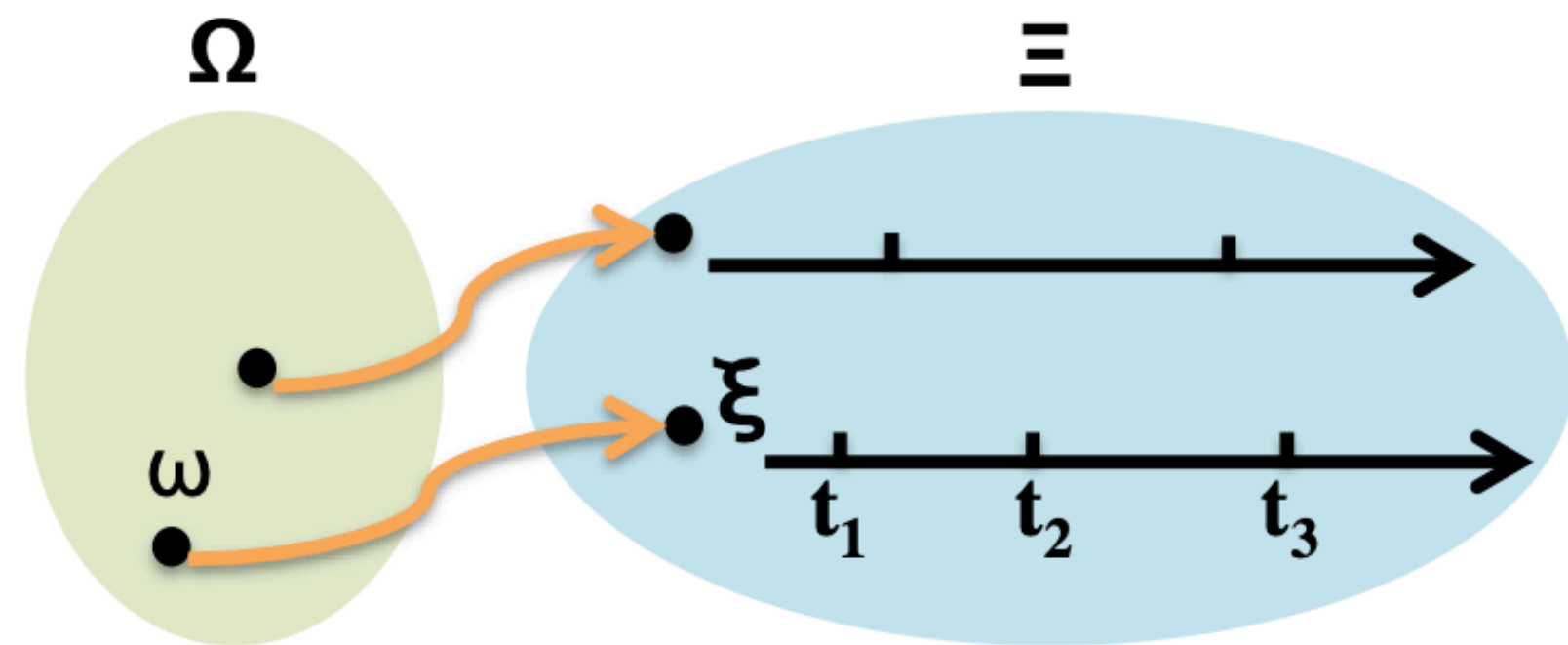
**Coevolutionary Embedding, Dai et al. (2017)**



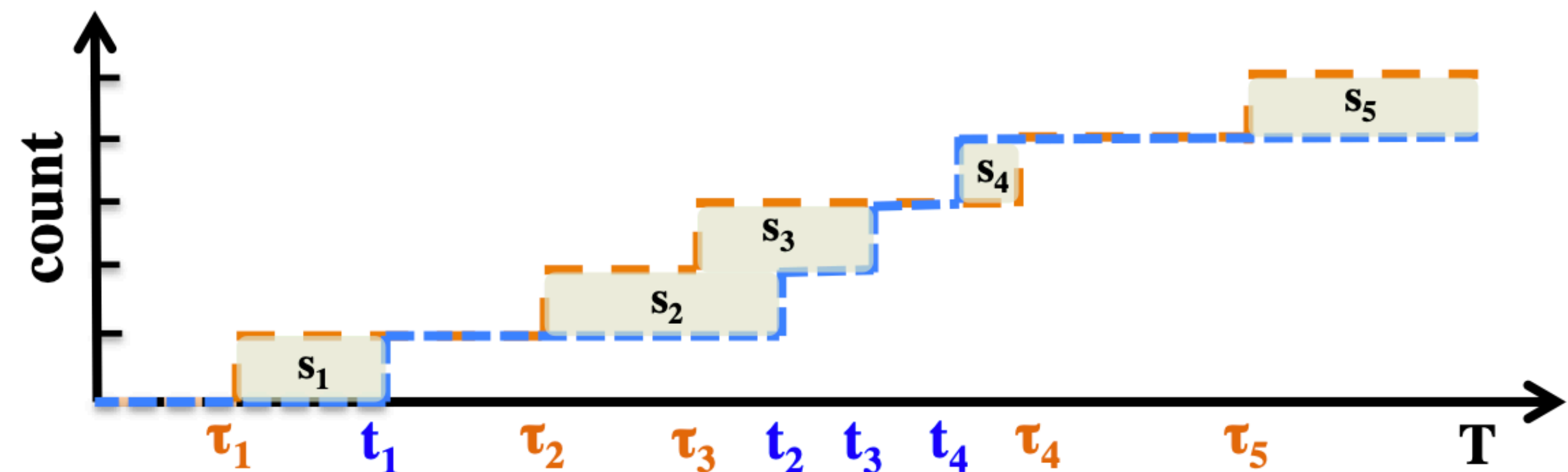
# Generative Models

- **Key idea:** Intensity- and likelihood-free models

- a) The outcome of the random experiment  $\omega$  is mapped to a point in space of count measures  $\xi$ ;  
 b) Distance between two sequences  $\xi = \{t_1, t_2, \dots\}$  and  $\rho = \{\tau_1, \tau_2, \dots\}$



a) Point process probability space



b)  $\|\cdot\|_*$  distance between sequences

Source: Shuai Xiao et al. Wasserstein Learning of Deep Generative Point Process Models

# Conclusion

- A type of sequential data observed in many applied problems:
  - Client's actions
  - Abnormal Events
  - Diseasespread
- Various models for intensity include LSTMs, Transformers