Simulating classical motion in synthetic monopole fields

Bachelor's thesis 15 credits

Ola Carlsson
Supervisor: Erik Sjöqvist
Subject reviewer: Patrik Thunström

Department for Physics and Astronomy, division of Materials Theory

June 1, 2022

Magnetic monopoles

- The Maxwellian M-field does not display monopoles.
- Non-fundamental or non-Maxwellian monopoles do however appear in nature.
- A particular example are the synthetic magnetic monopoles generating the geometrical phase.
- These synthetic monopoles sit in parameter space, presently the space of external magnetic fields, at points of energy degeneration.

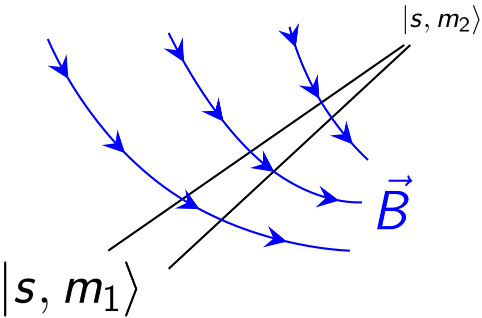
Stranger field textures

- Multipartite spin systems complicate the situation first explored by Berry, splitting the monopoles to positions away from the origin.[1]
- In addition the synthetic field texture then displays nonzero curl, i.e. we can define a synthetic "current" to complete the analogy with Maxwellian fields.
- How does such an exotic magnetic field structure affect our world, i.e. what dynamics result?
- To this end a specific bipartite system displaying desired properties was modelled and simulated.

[1] A. Eriksson and E. Sjöqvist, "Monopole field textures in interacting spin systems", Physical Review A 101 (2020).

The dumbbell system

- Two masses each carrying spin-$\frac{1}{2}$ with fixed distance to another were considered in translation and rotation —the "dumbbell".
- An external magnetic field is present.
- $\mathcal{H} = \sum_{i=1}^{5} \frac{\vec{p_i}^2}{2m_i} + \frac{4J}{\hbar} S_\mu^{(1)} S_\mu^{(2)} - \gamma \vec{B} \cdot \vec{S}.$



- Effective masses: $m_i = \begin{cases} m, & i = 1, 2, 3, \\ \frac{ml^2}{4}, & i = 4, 5. \end{cases}$
- Matrix forms of the spin operators must be rotated according to the dumbbell and external field, $\mathcal{U}_{m'm''} = \langle s, m' |\, e^{\frac{-iS_\mu \alpha}{\hbar}} e^{\frac{-iS_y \beta}{\hbar}} e^{\frac{-iS_\mu \delta}{\hbar}} \,| s, m'' \rangle$

$$\frac{\vec{B}\cdot\vec{S}}{B\hbar} = \begin{pmatrix} 0 & & & \\ & -1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\cos(\vartheta_B) & \frac{e^{-i\varphi_B}}{\sqrt{2}}\sin(\vartheta_B) & 0 \\ 0 & \frac{e^{i\varphi_B}}{\sqrt{2}}\sin(\vartheta_B) & 0 & \frac{e^{-i\varphi_B}}{\sqrt{2}}\sin(\vartheta_B) \\ 0 & 0 & \frac{e^{i\varphi_B}}{\sqrt{2}}\sin(\vartheta_B) & \cos(\vartheta_B) \end{pmatrix}$$

$$4\frac{S_\mu^{(1)} S_\mu^{(2)}}{\hbar^2} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \cos^2(\vartheta_r) & -\frac{e^{i\varphi_r}}{\sqrt{2}}\sin(2\vartheta_r) & e^{-2i\varphi_r}\sin^2(\vartheta_r) \\ 0 & -\frac{e^{-i\varphi_r}}{\sqrt{2}}\sin(2\vartheta_r) & -\cos(2\vartheta_r) & \frac{e^{-i\varphi_r}}{\sqrt{2}}\sin(2\vartheta_r) \\ 0 & e^{2i\varphi_r}\sin^2(\vartheta_r) & \frac{e^{i\varphi_r}}{\sqrt{2}}\sin(2\vartheta_r) & \cos^2(\vartheta_r) \end{pmatrix}$$

The Born-Oppenheimer approximation

- The Hamiltonian is heavily coupled and not easily solvable
- The Born-Oppenheimer approximation assumes parametrisation of "fast" subsystem by "slow" subsystem.
- $|\Psi_{full}\rangle = \Psi_s |n\rangle$
- $\mathcal{H}_f |n\rangle = E_n |n\rangle$
- $\mathcal{H}_s = \sum_{n=1}^{5} \frac{p_i^2}{2m_i}$
- Effective Hamiltonian for the slow subsystem (system coordinates) is governed by an effective Hamiltonian $\mathcal{H}_{eff}^{(n)} = \langle n| \mathcal{H}_s |n\rangle + E_n$

Synthetic fields

- Manipulation yields

$$\mathcal{H}_{eff}^{(n)} = \sum_{i=5}^{5} \frac{(p_i - A_i^{(n)})^2}{2m_i} + \Phi^{(n)} + E_n, \tag{1}$$

$$A_i^{(n)} = i\hbar \langle n | \partial_i n \rangle , \tag{2}$$

$$\Phi^{(n)} = \sum_{i=1}^{5} \frac{\hbar^2}{2m_i} \langle \partial_i n | \left(\mathbb{1} - |n\rangle \langle n| \right) | \partial_i n \rangle . \tag{3}$$

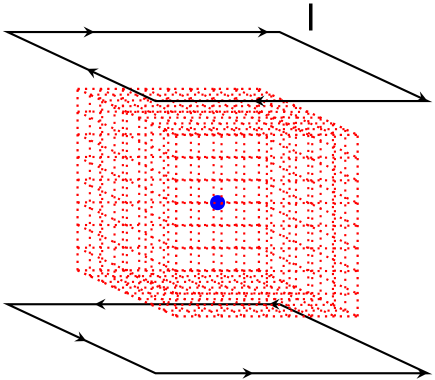
- $A_i^{(n)}$ is the synthetic magnetic field containing the monopoles.

Equations of motion

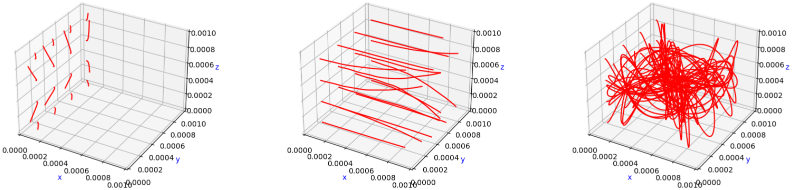
- It is now practical, and appropriate, to consider the slow system classical and find the dynamics per Hamilton's Canonical Equations:
- $m\frac{d\vec{r}}{dt^2} = \vec{F}^A - \frac{\partial \Phi^{(n)}}{\partial \vec{r}} - \frac{\partial E_n}{\partial \vec{r}}$
- $\frac{1}{i\hbar} F_i^A = 2i \sum_{j \neq i} \sum_{l \neq n} \frac{\frac{\partial r_j}{\partial t}}{(E_n - E_l)^2} \operatorname{Im}\left[\langle n| \partial_i \mathcal{H}_f |l\rangle \langle l| \partial_j \mathcal{H}_f |n\rangle\right]$
- $\Phi^{(n)} = \sum_{i=1}^{5} \sum_{l \neq n} \frac{\hbar^2}{2m_i} \frac{|\langle n|\partial_i \mathcal{H}_f|l\rangle|^2}{(E_n - E_l)^2}$
- The fast Hamiltonian is analytically differentiable.

Simulation strategy

- A "lab" region was discretized to allow for stored external field values.
- Two opposite square coils generate an external field zero at the centre.
- Numerical differentiation could benefit from finer lab division.



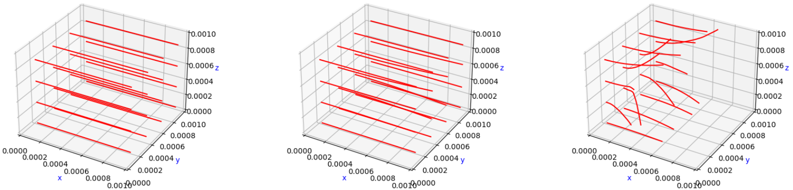
Simulations



(a) $n = 0$ (b) $n = 1$ (c) $n = 2$

- Trajectories with nonsynthetic behaviour for all three fast eigenstates.

The middle state



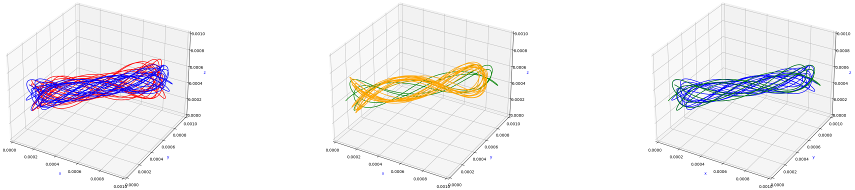
(d) $J = 0,\ n = 1,$ $m = 3.58 \times 10^{-25}$ kg

(e) $J = 0,\ n = 1,$ $m = 3.58 \times 10^{-28}$ kg

(f) $J = 0,\ n = 1,$ $m = 3.58 \times 10^{-29}$ kg

Figure: Three different masses for $n = 1,\ J = 0$

The high energy state I



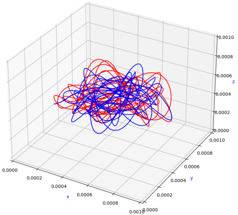
(a) With synthetic fields in red, without in blue

(b) Without synthetic magnetic field in green, without synthetic scalar field in yellow

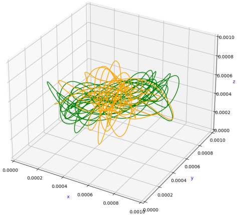
(c) Without any synthetic fields in blue, without synthetic magnetic field in green

- A direct comparison between different fields with $m = 3.58 \times 10^{-25}$ kg.

The high energy state II

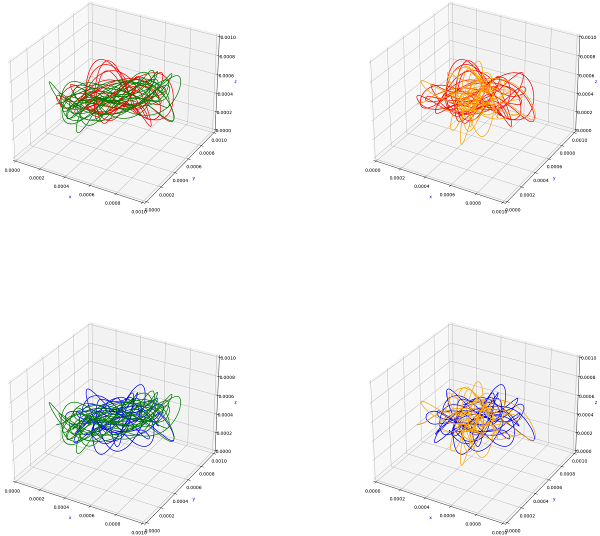


(d) With synthetic fields in red, without in blue



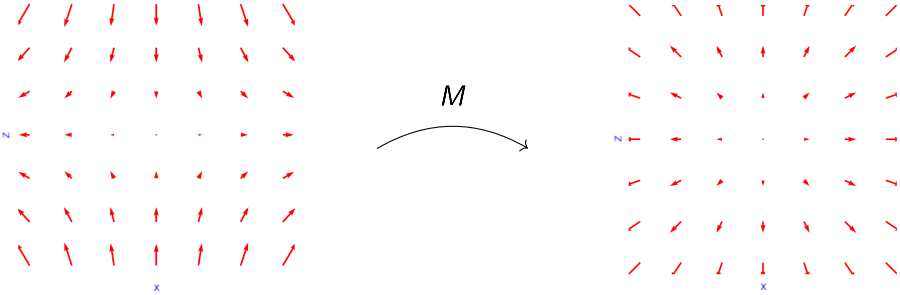
(e) Without synthetic magnetic field in green, without synthetic scalar field in yellow

- The same procedure for $m = 3.58 \times 10^{-27}$ kg and $\gamma = 10^8$ J$/\hbar$.


0.0000
0.0002
0.0004
0.0006
0.0008
0.0010
x
y
z

Conclusions

- EOM and a script for their evaluation have been developed.
- The repulsive nature of the synthetic scalar field has been demonstrated.
- Synthetic behaviour arising from monopolar fields play a significant role for the time evolution of the selected system, for reasonable masses.
- Mass is the main tool for increasing synthetic effects.
- Complication of the external field could also be viable but complicates interpretation of the result.



External field in real space. External field in parameter space.

Figure: An illustration of the map M between real space and parameter space, which must map every point of external field $\vec{b}$ to the parameter space point with the same external field $\vec{b}$. The planes shown are the xz-planes through the zero-field centre point of the lab and the origin, respectively.

The next steps

- How would we precisely separate and interpret the synthetic action as a monopolar one? Technical difficulties need be addressed.
- The performance of the simulations are a limiting factor, optimize or run on better hardware (UPPMAX).
- Realising the system could perhaps be done, how would such a setup look?

Questions?

Presentation template by *Frederic Haziza*, `http://www.it.uu.se/katalog/daz/uppsala_beamer`