1 Introduction

1.1 On the occurrence of monopoles

Our theory of electromagnetism carries certain asymmetries between the magnetic and electric fields. Such a distinction well known and often discussed is the absence of magnetic monopoles, i.e. that any analogue of electric charge is absent for the magnetic field. It is an extension often suggested by theorists, initially by P.M. Dirac in 1931[3], to include magnetic charges and also potentially magnetic currents in Maxwell's equations, but there is as of yet no accepted empirical data to support this cause. Magnetic monopoles do however appear occasionally in a less fundamental sense, as emergent phenomena in many-body systems as for example in example spin ice [2] or Bose-Einstein condensates [5].

Another area in which magnetic monopoles has appeared is the study of so called *geometric* phases of quantum systems, first described by M. Berry in 1984 [1]. Roughly speaking this is the phase acquired by the wave function of a system which is not dynamic in origin. The dynamic phase of a system state is induced by the energy of that state, while the geometric phase is surprisingly enough *in*dependent of the energy values, and rather arise as a function of the path taken by the system through the relevant "parameter-space" parametrising the Hamiltonian. For certain systems this geometric phase evolution resembles the evolution of a charged system in a magnetic vector potential field that lives in parameter space. This field, henceforth the *synthetic* magnetic field, happens to exhibit non-zero divergence at certain points, the degeneracy points of the state energies, which thus correspond to monopoles of the synthetic field.

Geometrical phase is an interesting field of study in of itself, but it is here with the appearance of magnetic-type monopoles that the present body of work finds its premise. While the study of fundamental monopoles remains impossible, we can through construction of a suitable system study the effects of magnetic monopoles through their action on the system state in parameter space. The movement of charged matter through monopole fields becomes measurable, even though fundamental monopoles remain fictitious.

1.2 A select system

Berry's original 1984 article considers a simple spin-system with a single magnetic dipole moment in an external magnetic field $\vec{\mathbf{B}}$. The relevant parameter space is the space of all possible external fields, and the geometric phase contribution takes the form of a synthetic magnetic field purely generated by a monopole sitting at the origin, i.e. at $\vec{\mathbf{B}} = \vec{\mathbf{0}}$. This simple field is a most trivial example of a monopolar field, so to find more complex behaviour this starting point of a system can be extended to include multiple spin components, i.e. multiple magnetic dipoles, and interactions between those dipoles. The effects of introducing such interactions is roughly that of splitting the origin-centred dipole into smaller constituent parts whose positions in parameter space depend on the exact nature of the spin-spin interactions[4].

This splitting is desired, and so the system studied will be composed of two massive spin-$\frac{1}{2}$ components that interact though the so-called Ising interaction outlined below. This is to some extent the simplest spin-spin interaction. Movement through parameter space can easily be mapped to movement of the center of mass through real space given an external inhomogeneous field, and the movement of the spin components relative to one another is as a simplest case the rotation through polar and azimuthal angles with fixed inter-component distance. Such a setup is reminiscent of a dumbbell translating and rotating through space, with the added complication that each ”weight” of the dumbbell acts as a dipole (the spin) interacting with both the other weight and an external magnetic field.

It is the time evolution of this system, henceforth referred to as the dumbbell, with which this project is concerned. Approximate equations of motion will be derived and then be put to test in a numerical simulation in the hope of discerning effects of the synthetic magnetic monopoles.

1.3 Interpretations of the system

[Något om vad hanteln skulle kunna tänkas vara här, molekyl, atom et. c.]

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