Project: Classical motion in artificial magnetic monopole fields

Background

Berry demonstrated that a general quantum system undergoing adiabatic evolution may pick up a nontrivial phase of purely geometric origin [1]. For a spin in a slowly rotating magnetic field this phase turns out to be proportional to the flux of an artificial magnetic monopole sitting at the energy crossing at the origin (corresponding to vanishing magnetic field). Due to the elusiveness of magnetic charges in standard electromagnetism, this result has triggered numerous studies in adiabatic evolution of spin systems [2].

Real-space realizations of artificial magnetic monopoles have been demonstrated in spin ice [3] and Bose-Einstein condensates [4]. Similarly, Berry artificial monopoles in parameter space can be mapped nontrivially to physical space via a nonuniform magnetic field that drives the adiabatic spin system. This opens up for the experimental study of classical trajectories in the presence of magnetic monopoles.

Scope and purpose

The aim of the project is to derive classical equations of motion for the centre of mass (CM) of a particle with spin moving in an arbitrary inhomogeneous magnetic field. A key ingredient is to examine the influence of internal spin-spin interaction on the trajectories of the CM. The spin-spin interactions are known to modify the distribution of artificial monopoles in the parameter space, for example splitting magnetic charges yielding monopoles located at points where the magnetic field is nonzero [5].

The key objectives of the project are:

- to derive and (numerically) simulate classical CM equations of motion of a composite system (such as an atom or molecule) consisting of pairs of interacting spins;
- to estimate parameter ranges needed to see effects of the internal spin-spin interaction on the CM trajectories.
- to discuss the plausibility of realising a system within those parameter ranges.

The form of spin-spin interaction is restricted to an Ising model interaction. Spin-$\frac{1}{2}$ constituents only are considered.

The overall purpose is to develop a physical setting that allows for direct experimental studies of artificial magnetic monopoles in real space.

Method

Through the Born-Oppenheimer approximation an effective Hamiltonian for the adiabatic evolution of a spin-system in an external magnetic field can be found. This is dependent on a scalar potential akin to an electric potential, as well as a vector potential analogous to a magnetic field[6].This vector potential can be seen as an artificial magnetic field giving rise to the geometric Berry phase, and it is the dynamics arising from this field as well as the scalar potential that are to be simulated. Evaluating these potentials for a Hamiltonian containing the interactions of the system, the effective Hamiltonian is used as a basis for classical Hamiltonian dynamics.

Numerically simulating the resultant differential equations of motion for select parameters is carried out by development of a suitable Python-script. Auxiliary tools such as graphical illustration and evaluation of the external magnetic field are also necessary.

With these tools in place the effect of different parameter choices on the dynamical evolution can be explored and analysed. Of certain importance is the choice of external magnetic field, as some properties (e.g. points of zero magnitude) are suspect to yield interesting results but must be found within the context of realisable fields (e.g. zero divergence).

Thesis outline

As a rough outline the following sections will be included in the report:

Abstract

This will be a regular abstract.

Sammanfattning

This will be a Swedish version of the abstract.

Introduction

This will be a short summary of the background and a thorough statement of the problem and aim of the thesis. In particular some example realisation of the simulated system can be sketched, and some context of magnetic monopoles. Some summary over the principle of geometric phase could also fit in here.

Background

This will be mostly a theoretical background considering the work done previously on geometric phase and its peculiarities (for example the dependence of artificial monopole positions on field and system characteristics).

Theory

A section taking a more in depth look at the theory necessary for this specific problem, culminating in the equations of motion. One area of interest is specifying the approximations necessary.

Simulation

Here details of the implementation of the numerical simulation can be described. Complete code will follow in an appendix.

Results

As results mainly the exploration of the parameter dependence of the resultant dynamics should here be considered. Comparisons of different systems would be of interest as well as less comparative estimations of say the orders of magnitude of parameters necessary for appreciable effects. All of this is preferably done through presenting select simulations and their parameters.

Discussion

Here a summary and discussion of the relevance of the above results can be carried out. In particular the plausibility to realise systems with the required parameters, one of the key objectives, should be considered. This is also the somewhat appropriate space for reflecting on encountered hindrances and their solutions or negative effects on the final result, for example related to the numerical simulation.

Outlook

This will be a discussion of unresolved issues and areas of improvement for this project, but perhaps also possible further projects, experimental or theoretical, related to or suggested by the results achieved.

Conclusions

This will be a short summary of the results and discussion above, so as to conveniently summarise the thesis when combined with the abstract and introduction.

Time plan

The project is carried out during spring 2022. A draft of the time plan with notable milestones is as follows:

- Literature study, 10 Jan. - 23 Jan.
- Formal start of course 25 Jan.
- Deriving equations of motion, construction of model (external magnetic field configuration), 24 Jan. - 13 Feb.
- Development of numerical code for classical trajectories, 14 Feb. - 13 Mar.
- Deadline for complete project plan 25 Feb.
- Numerical simulations and interpretation, 14 Mar. - 10 Apr.
- ”Mittavstämning”, a five minute presentation on project status, 8 Apr.
- Writing report, 28 Mar. - 5 Jun. First draft has deadline 13 May, a first version has deadline 20 May and the final report has deadline 5 June.
- Feedback on report 27 May.
- Preparation and presentation, 16 May - 5 Jun.
- Symposium with poster and presentation, 30 and 31 May

Supervision

Weekly consultations with the supervisor, to a large extent by means of zoom meetings, will take place predominantly Thursdays. The aim is to provide if applicable the week's results, say drafts of documents or problems encountered, some time in advance so that the meetings remain effective and their subjects clear.

References

[1] M. V. Berry, Quantal Phase Factors Accompanying Adiabatic Changes, Proc. R. Soc. London Ser. A **392**, 45 (1984).

[2] A. Shapere and F. Wilczek, *Geometric phases in physics* (1989).

[3] C. Castelnovo, R. Moessner, and S. L. Sondhi, Magnetic monopoles in spin ice, Nature (London) **451**, 42 (2008).

[4] M. W. Ray, E. Ruokokoski, K. Tiurev, M. Möttönen, and D. S. Hall, Observation of isolated monopoles in a quantum field, Science **348**, 544 (2015).

[5] A. Eriksson and E. Sjöqvist, Monopole field textures in interacting spin systems, Phys. Rev. A **101**, 050101(R) (2020).

[6] M. V. Berry, R. Lim, The Born-Oppenheimer electric gauge force is repulsive near degeneracies, J. Phys. A: Math. Gen. **23**, L655 (1990).