

Question

Buycoins Inc is listed on the LSE with a stock price of £12.75 - the company is not known to pay dividends. We need to price a put option with a strike of \$10 maturing in 6 months. The continuously-compounded risk-free rate is 2.75%/year, the mean return on the stock is 6%/year, and the standard deviation of the stock return is 20%/year. What is the Black-Scholes put price?

Solution:

To calculate the Black-Scholes put price

$$P = Ke^{-rt} * N(-d_2) - S_0 N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}, \quad d_2 = d_1 - \sigma\sqrt{t}$$

Where:

S_0 = Current stock price

K = Strike stock price

σ = represents the underlying volatility

r = risk-free interest rate

t = duration in years

$N(d_1)$ and $N(d_2)$ are cumulative distribution functions for a standard normal distribution

Given:

$S_0 = £12.75$, $K = £10$, $r = 2.75\%$, $\sigma = 20\%$, $T = 6\text{months}(1/2 \text{ years})$

Calculating the d_1 and d_2

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}} \\ &= \frac{\ln\left(\frac{12.75}{10}\right) + \left(0.0275 + \frac{0.2^2}{2}\right)0.5}{0.4\sqrt{0.5}} \\ &= \frac{0.2429 + 0.02375}{0.1414} = 1.8858 \end{aligned}$$

$$\begin{aligned}
 d_2 &= d_1 - \sigma\sqrt{t} \\
 &= 1.8858 - 0.2\sqrt{0.5} \\
 &= 1.7414
 \end{aligned}$$

Checking the normal distribution table for $N(-d_1)$ and $N(-d_2)$

$$N(d_1) = 0.97062, \quad N(d_2) = 0.95907$$

$$\begin{aligned}
 N(-d_1) &= 1 - N(d_1) \\
 &= 1 - 0.97062 \\
 &= 0.02938
 \end{aligned}$$

$$\begin{aligned}
 N(-d_2) &= 1 - N(d_2) \\
 &= 1 - 0.95907 \\
 &= 0.04093
 \end{aligned}$$

Calculating the put price

$$\begin{aligned}
 P &= Ke^{-rt} * N(-d_2) - S_0 N(-d_1) \\
 &= 10e^{-0.0275*0.5}(0.04093) - 12.75(0.02938) \\
 &= 0.4037 - 0.3746 \\
 &= 0.0291
 \end{aligned}$$

The Put Price is **£ 0.0291**