Question:

Over all real numbers, find the minimum value of a positive real number, y such that

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

Answer:

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

differentiating the equation, to get the minimum input

$$\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{((x+6)^2 + 25)} + \sqrt{((x-6)^2 + 121)} \right)$$
$$\frac{dy}{dx} = \frac{x+6}{\sqrt{(x+6)^2 + 25}} + \frac{x-6}{\sqrt{(x-6)^2 + 121}} = 0$$

Minimum value of y is.

$$y = \sqrt{(6)^2 + 25} + \sqrt{(-6)^2 + 121}$$
$$y = \sqrt{61} + \sqrt{157}$$
$$y = 20.34$$



As shown in the above plot.