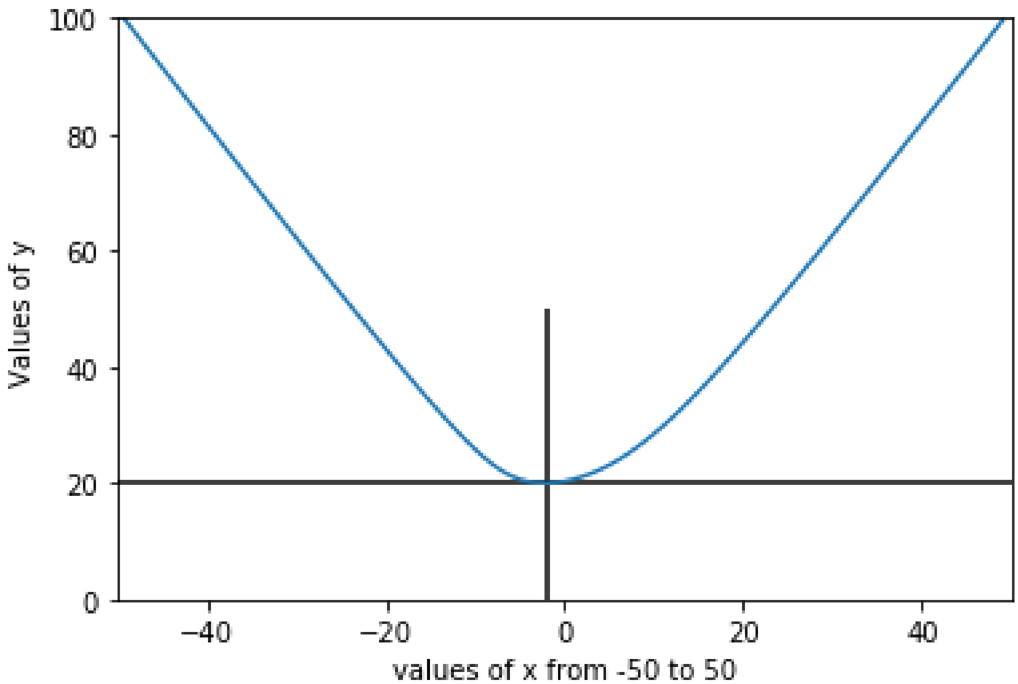
Question:

Over all real numbers, find the minimum value of a positive real number, y such that

y = $\sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$

Answer:

Plotting the graph of y, yields the below



From the graph, the minimum value of y is 20.00459, approximately 20.

So, to calculate the value of x at y minimum, we can equate the y=20

Y = $\sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$ = 20

$\sqrt{(x-6)^2 + 121}$ = 20 -$\sqrt{(x+6)^2 + 25}$

Squaring both sides

$$\left(\sqrt{(x-6)^2 + 121}\right)^2 = \left(20 - \sqrt{(x+6)^2 + 25}\right) \text{^}2$$

$$(x-6)^2 + 121 = 400 - 40\sqrt{(x+6)^2 + 25} + (x+6)^2 + 25$$

$$(x-6)^2-(x+6)^2+121 = 400+25-40\sqrt{(x+6)^2 + 25}$$

$$x^2-12x+36-x^2-12x-36 = 425-121-40\sqrt{(x+6)^2 + 25}$$

$$-24x = 304-40\sqrt{(x+6)^2 + 25}$$

Dividing both sides by 8,

$$-3x = 38-5\sqrt{(x+6)^2 + 25}$$

$$5\sqrt{(x+6)^2 + 25} = 38+3x$$

Squaring both sides,

$$\left(5\sqrt{(x+6)^2 + 25}\right)^2 = (38+3x)^2$$

$$25(x^2+12x+61) = 9x^2 + 228x+1444$$

$$25x^2+300x+1525 = 9x^2 + 228x+1444$$

$$25x^2-9x^2+300x- 228x+1525-1444 = 0$$

$$16x^2+72x+81 = 0$$

Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Where a=16, b =72, c= 81

$$x = \frac{-72 \pm \sqrt{72^2-4(16)(81)}}{2*16}$$

$$x = \frac{-72}{32}$$

$$x = \frac{-9}{4}\ or - 2.25$$