Optimization Techniques

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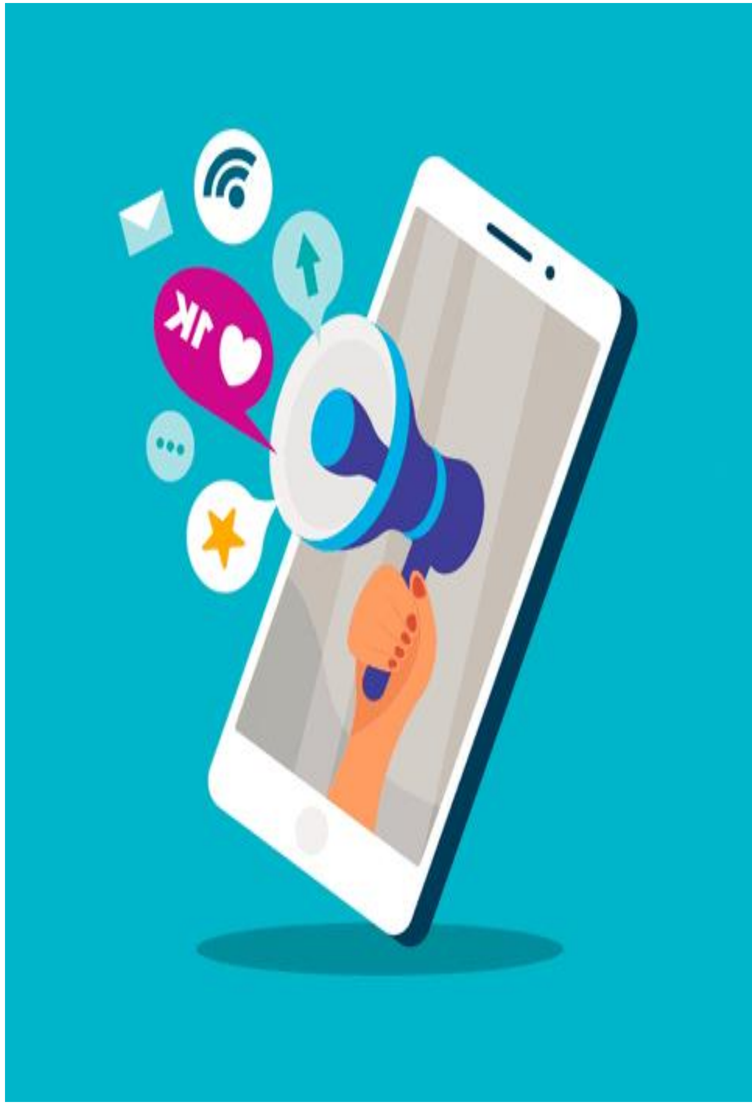
Course Objectives

Upon completion of this course, students should be able to:

1. Formulate and solve linear and non-linear optimization problems.
2. Analyze and solve transportation and assignment problems.
3. Apply network analysis tools to real-world projects.
4. Understand the principles of game theory and apply them in strategic decision-making.
5. Model and analyze queuing systems.
6. Schedule and manage projects effectively.
7. Design and implement effective inventory control systems.

Outline

- Definition of Optimization
- Importance of Optimization
- Types of Optimization Problems
- Applications of Optimization



Linear Programming

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What is Linear Programming?

Linear Programming (LP) is a mathematical technique used for optimizing a linear objective function, subject to a set of linear equality and/or inequality constraints. The goal is to find the best values for the decision variables that maximize or minimize the objective function.

Key Elements of an LP Problem:

- Decision Variables: The unknowns we want to solve for.
- Objective Function: A linear function representing what we want to optimize (maximize profit or minimize cost).
- Constraints: Linear equations or inequalities that define the limits within which the solution must lie.
- Feasibility Region: The set of all possible solutions that satisfy the constraints.

Assumptions of Linear Programming

1. Linearity: Relationships among variables must be linear.
2. Additivity: The total effect is the sum of individual effects.
3. Divisibility: Decision variables can take fractional values (unless it's an integer programming problem).
4. Certainty: All coefficients in functions and constraints are known with certainty.
5. Non-negativity: Decision variables cannot be negative.

Formulation of a Linear Programming Model

Steps:

1. Identify the decision variables.
2. Formulate the objective function.
3. Write the constraints based on the problem context.
4. Add non-negativity restrictions.

Example

A company produces two products (x and y). Each unit of x requires 2 hours of labor and 1 unit of material. Each unit of y requires 1 hour of labor and 2 units of material. The company has 100 labor hours and 80 units of material available. The profit is N40 per unit of x and N50 per unit of y. How should the company produce to maximize profit?

Solution:

Identify the decision variables

Let x = units of product x

Let y = units of product y

Objective Function:

Maximize $Z = 40x + 50y$

Constraints

Labour: $2x + y \leq 100$

Material: $x + 2y \leq 80$

$x, y \geq 0$

Graphical Method (for Two-Variable Problems)

Used when the LP problem has only two decision variables.

Steps:

1. Plot each constraint on a graph.
2. Identify the feasible region.
3. Find corner points (vertices) of the feasible region.
4. Evaluate the objective function at each corner point.
5. Choose the point that gives the best (maximum or minimum) value.

Linear Programming:

https://www.youtube.com/watch?v=Bzzqx1F23a8

Graphical Method:

https://www.youtube.com/watch?v=qQFAvPF2OSI

Simplex Method (for Larger Problems)

The Simplex method is an iterative algebraic approach used to find the optimal solution for LP problems with more than two variables.

Steps:

1. Convert inequalities into equalities using slack, surplus, and artificial variables.
2. Construct the initial simplex tableau.
3. Identify the pivot element.
4. Perform row operations to update the tableau.
5. Repeat until there are no more positive indicators in the objective row (for maximization).

Simplex Method:

https://www.youtube.com/watch?v=M8POtpPtQZc

Duality in Linear Programming

Every linear programming problem (Primal) has an associated Dual Problem.

Key Points:

•If the primal is a maximization, the dual is a minimization.

•The number of constraints in the primal equals the number of variables in the dual.

•Solving the dual gives insights into the shadow prices (marginal worth of resources).

Duality helps in understanding resource valuation.

Duality in Linear Programming

https://www.youtube.com/watch?v=H0sBZdjuego

Sensitivity Analysis

Also known as post-optimality analysis, this examines how changes in the coefficients of the objective function or right-hand side values of constraints affect the optimal solution.

Aim:

- Understand stability of the solution.
- Determine how much a coefficient can change before the optimal solution changes.

Linear Programming Sensitivity Analysis

https://www.youtube.com/watch?v=5Pgxo_7bNa8

Application of Linear Programming

Linear programming is widely used in various fields:

Field	Application
Operations	Production scheduling, workforce allocation
Finance	Investment portfolio selection
Transportation	Cost-effective shipping routes
Agriculture	Crop planning and land allocation
Manufacturing	Resource utilization and cost minimization
Marketing	Media planning and advertising mix


Thank You!