

Calculation of mass of Z boson

We all know Einstein's equation:

$$E = mc^2$$

The more complete equation is:

$$E = \sqrt{(\mathbf{p} \cdot \mathbf{c})^2 + (m_0 \cdot c)^2}$$

where E is the energy of the particle, \mathbf{p} is its momentum and m_0 is the mass of the particle at rest.

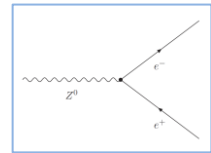
Or in natural units ($c=1$): $E = \sqrt{\mathbf{p}^2 + m_0^2}$

Rearranging this gives:

$$m_0 = \sqrt{E^2 - \mathbf{p}^2}$$

These quantities are conserved in a particle collision or decay. This means we can use them to infer the energy of a decaying ("mother") particle. For example, for a Z boson decaying to an electron (e^-) and positron (e^+) we have:

$$m_0(Z) = \sqrt{(E_{e^-} + E_{e^+})^2 - (\mathbf{p}_{e^-} + \mathbf{p}_{e^+})^2}$$



So from the energy and momentum of the decay particles (e^+ and e^-) which we see in our detector we can calculate the mass of the decaying particle (Z)!!

You may have noticed that the momentum (\mathbf{p}) is in bold. This is because it is a vector, it has magnitude AND direction. When we perform mathematical operations with vectors we have to treat them with care. We have to decompose the vector into how much it points in some chosen directions, its components in x, y and z. When we multiply two momenta we get a scalar (a normal number with no direction):

$$\mathbf{p}_1 \cdot \mathbf{p}_2 = \begin{pmatrix} p_1^x \\ p_1^y \\ p_1^z \end{pmatrix} \cdot \begin{pmatrix} p_2^x \\ p_2^y \\ p_2^z \end{pmatrix} = p_1^x \cdot p_2^x + p_1^y \cdot p_2^y + p_1^z \cdot p_2^z$$

So to calculate the mass of the Z we have:

$$m_0(Z) = \sqrt{(E_{e^-} + E_{e^+})^2 - (p_{e^-}^x + p_{e^+}^x)^2 - (p_{e^-}^y + p_{e^+}^y)^2 - (p_{e^-}^z + p_{e^+}^z)^2}$$

where: $E_{e^-} = \sqrt{(p_{e^-}^x)^2 + (p_{e^-}^y)^2 + (p_{e^-}^z)^2 + m_{e^-}^2}$

(we can ignore m_{e^-} as it is small)

Now look at some Z decays and calculate the mass. You can use "pick" on the tracks to get p^x , p^y and p^z for the electrons or muons. Do you think your events are real Z's??