Computational physics: Buckling beam

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I Task 1

The equation we want to rewrite is

$$\gamma \frac{d^2u}{dx^2} = -Fu \tag{1}$$

Using that $\hat{x} = x/L$ we rewrite the derivative.

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\left(\frac{d\hat{x}}{dx}\frac{du}{d\hat{x}}\right) = \frac{1}{L}\frac{d}{dx}\left(\frac{du}{d\hat{x}}\right) = \frac{1}{L}\frac{d\hat{x}}{dx}\frac{d}{d\hat{x}}\left(\frac{du}{d\hat{x}}\right) = \frac{1}{L^2}\frac{d}{d\hat{x}}\left(\frac{du}{d\hat{x}}\right) = \frac{1}{L^2}\frac{d^2u}{d\hat{x}^2} \tag{2}$$

We can then write

$$\frac{\gamma}{L^2}\frac{d^2u}{d\hat{x}^2} = -Fu \Rightarrow \frac{d^2u}{d\hat{x}^2} = -\frac{FL^2}{\gamma}u \to -\lambda u \tag{3}$$

II Task 2, 3ab, 4

You can find these exercises in our GitHub repository (`https://github.com/olamaa/Fys4150`).

III Task 5a

In figure 1 we see how the number of iterations increases when N increases. We see that the number of interations increases exponentially. We therefore expect the scaling to evolve as $\propto e^N$.

IV Task 5b

If A were dense, there would be more iterations required to make A diagonal. We therefor assume that the scaling would be steeper, perhaps $\propto e^{2N}$ or something like that.

Table 1: Number of iterations calculated for different values of N

N	**Iterations**
2	2
3	27
4	24
5	155
6	210
7	483
8	736
9	1.080
10	1.530
19	11.704
29	41.238
60	215.600

V Task 6a

In figure 1 we see the three first solutions to the buckling beam problem. The numerical solutions seem to lie close to the analytical solutions at the points where the solutions have been calculated. This suggests that the algorithm produces accurate results.

VI Task 6b

In figure 2 we again see the three first solutions to the buckling beam problem. The numerical approximations match very well with the analytical result. We therefore conclude that the Jacobi method we used provides good approximations to the solutions of the analytical problem known as the buckling beam.

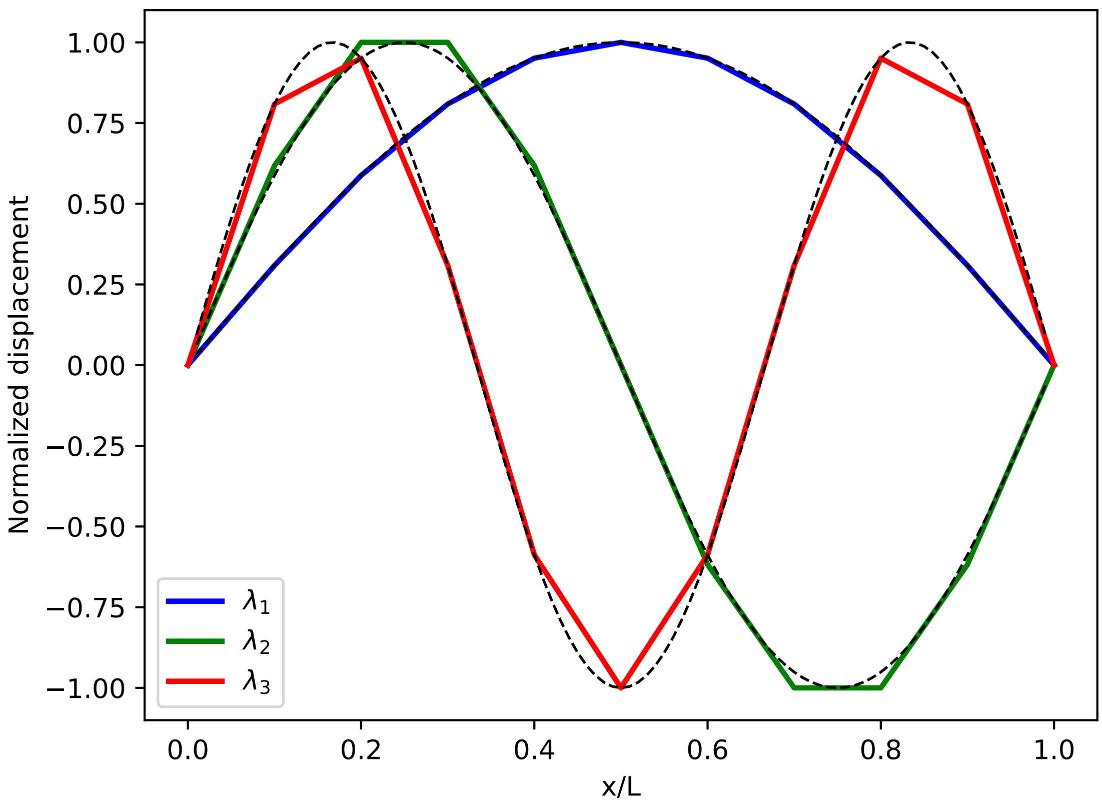

Figure 1: The solutions corresponding to the three lowest eigenvalues have been plotted together with the analytical solutions for n = 10. Solutions are normalized to one.

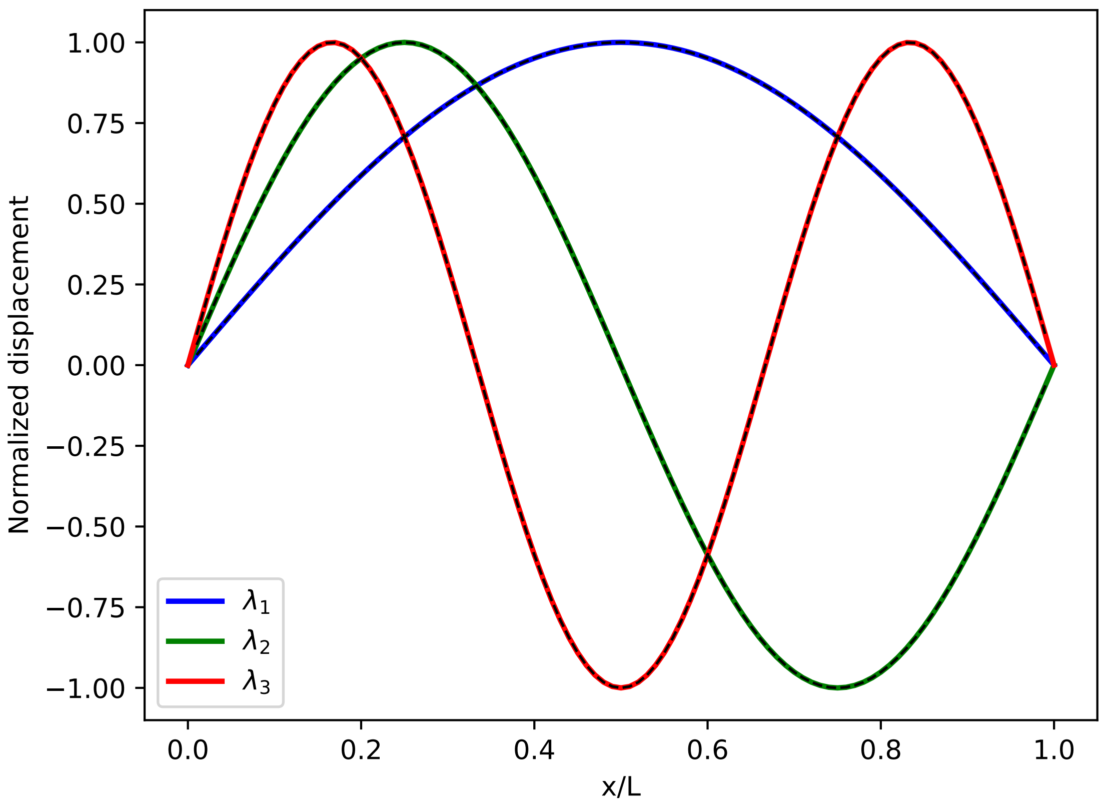

Figure 2: The solutions corresponding to the three lowest eigenvalues have been plotted together with the analytical solutions for n = 100. Solutions are normalized to one.