In [1]:
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%%html
<style>.container { width:95% !important; }</style>
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Media Mix Optimization Report

Executive Summary

Our media mix modeling analysis reveals significant opportunities to improve marketing efficiency. The current media budget allocation ($50,375 monthly) is vastly overallocated across all channels, with each operating well beyond their optimal efficiency points.

Key Findings:

- **All channels are currently operating in unprofitable regions**, with spend levels exceeding their ROI=1 breakeven points
- **Maximum ROI opportunity**: By reducing total budget to $20,150 (40% of current) and reallocating across channels, we could increase ROI from 3.11 to 6.63 (+113%)
- **Maximum sales opportunity**: By maintaining current budget but optimizing allocation, we could increase sales impact by 18.6%
- **Digital and TV** are our most effective channels, though both are significantly overfunded

Primary Recommendations:

1. Implement a strategic budget reallocation to align with each channel's unique response curve
2. Reduce TV spend by 64% and Radio spend by 98% regardless of strategic approach
3. Digital should remain the highest-funded channel under any scenario
4. Choose either the ROI-maximizing approach (higher efficiency) or sales-maximizing approach (higher volume) based on current business priorities
5. Continuously monitor and adjust as campaign effects and competitive dynamics evolve

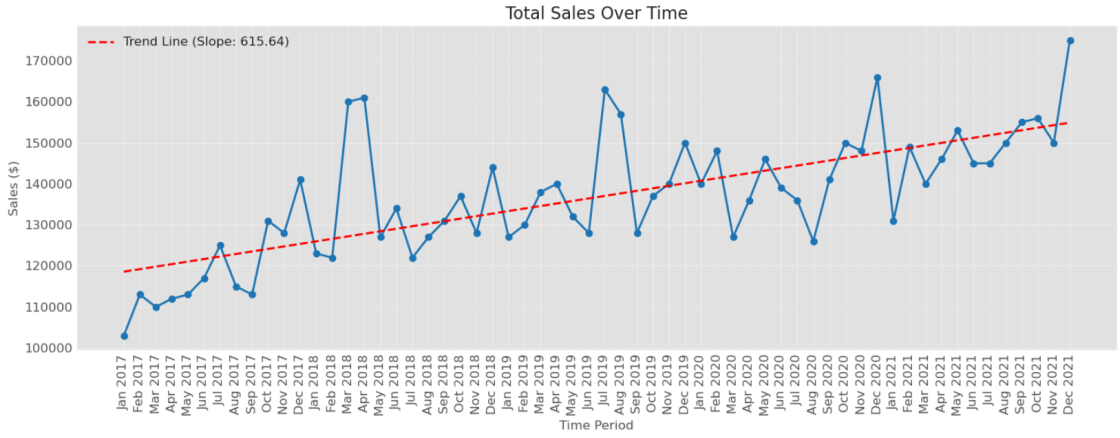
Understanding Our Marketing Model

Our media mix model (MMM) captures three critical aspects of marketing effectiveness:

1. **Base sales vs. incremental impact**: How much of our sales would occur without advertising
2. **Carryover effects**: How long the impact of each channel persists over time
3. **Diminishing returns**: How efficiency declines as spending increases

Key Marketing Dynamics Revealed

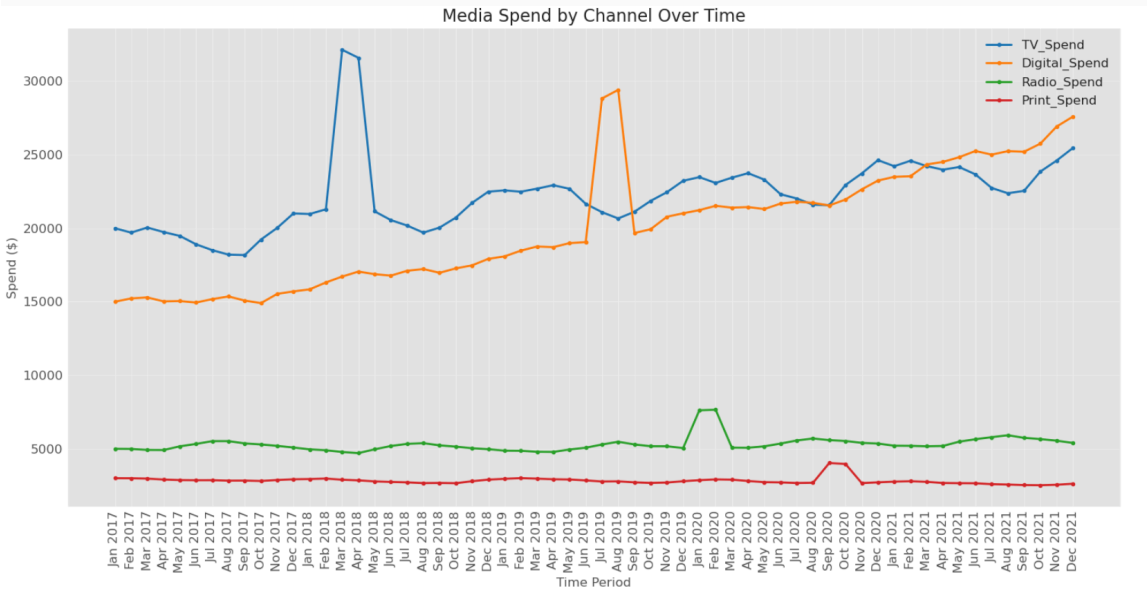
Media Spend Trends (2017-2021)



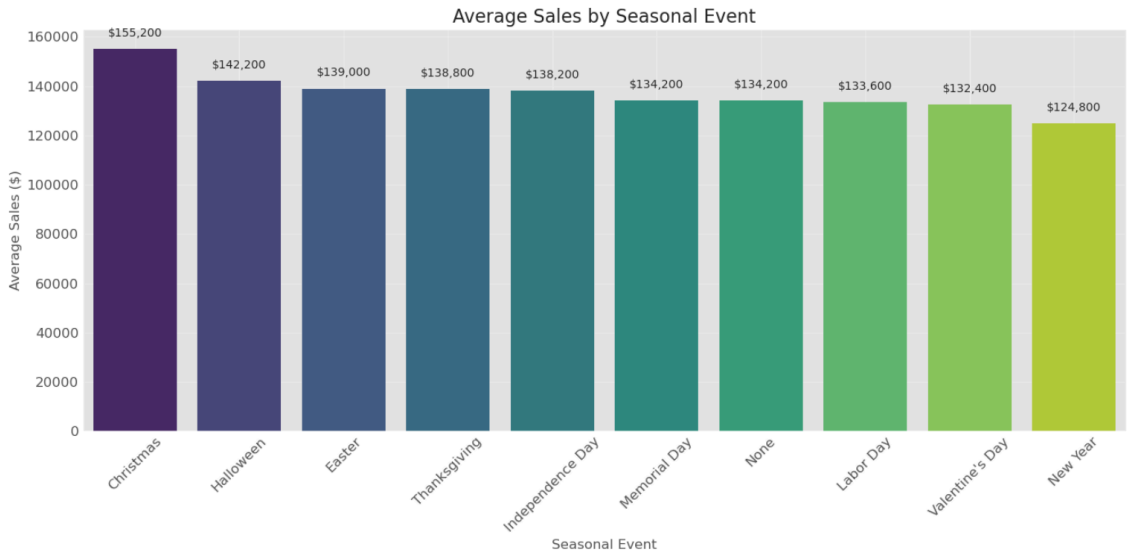
- **Digital spend** has grown steadily, overtaking TV in late 2020
- **TV spend** has fluctuated with occasional spikes
- **Radio and Print** remain relatively stable at lower levels
- The current monthly allocation is approximately: TV $(22,219)$, $Digital$(20,010), Radio ($5,314)$, $and Print$(2,832)

External Factors Impacting Sales

Our analysis identified several non-media factors that significantly influence sales:



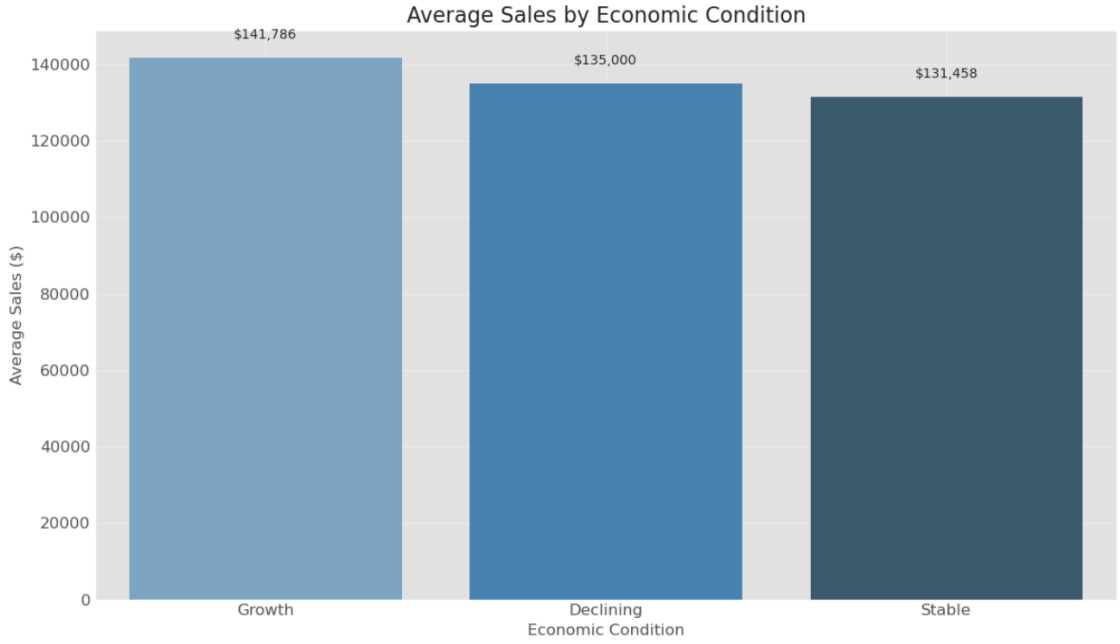
- **Seasonal events** drive substantial sales variation, with Christmas generating the highest average ($155,200)
- Halloween, Easter, and Thanksgiving also create notable sales increases



- **Economic conditions** significantly affect baseline sales performance
- Growth periods show a 7.9% increase over sales during economic decline

Channel Performance Analysis

Carryover Effects (Adstock Modeling)



Our analysis reveals distinct carryover patterns across channels:

- **TV** shows moderate persistence (decay rate = 0.60), with effects lasting several months
- **Digital** demonstrates rapid decay (0.20), with minimal impact beyond the first month
- **Print** has the strongest persistence (0.80), creating substantial long-term impact
- **Radio** shows moderate carryover (0.50)

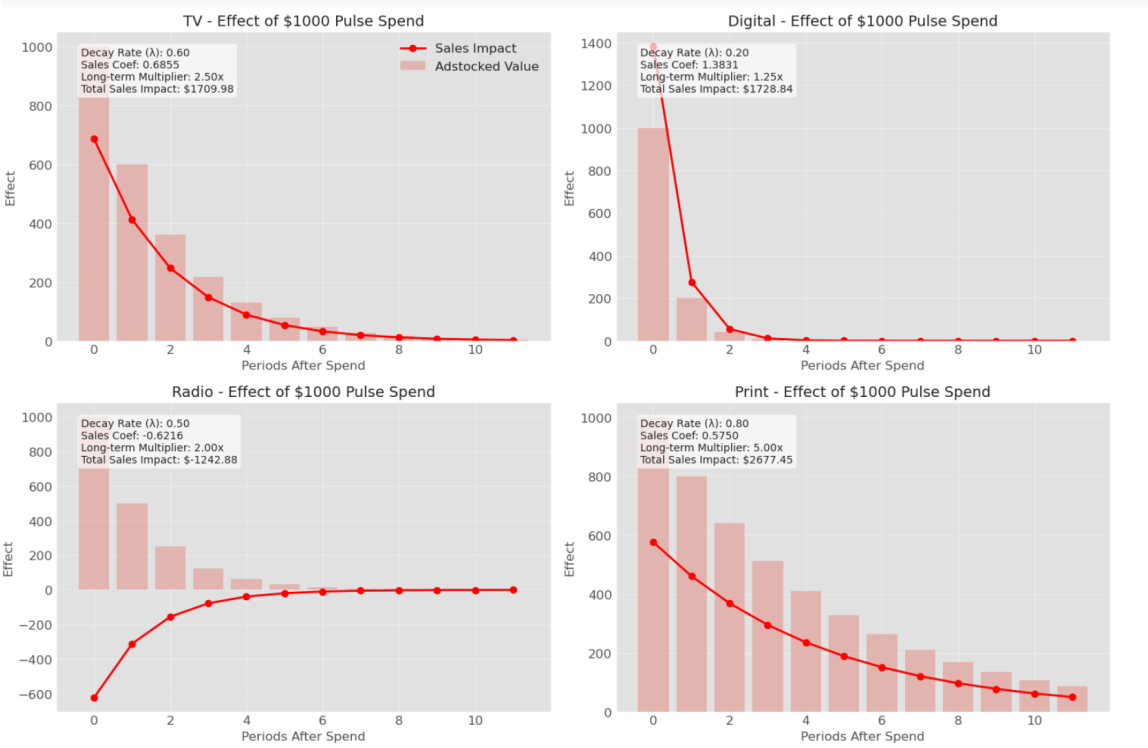
```
Adstock Ridge Model Performance:
R²: 0.6675, RMSE: $8753.12
Improvement over simple model: -7.29 percentage points

Media Mix Model Parameters and Effects:
=======================================
Channel  Adstock Decay Rate (λ)  Sales Effect (per $1 adstocked)  Long-term Multiplier  Long-term Sales Impact (per $1 spent)
     TV                  0.6000                           0.6855                2.5000                                 1.7137
Digital                  0.2000                           1.3831                1.2500                                 1.7288
  Radio                  0.5000                          -0.6216                2.0000                                -1.2432
  Print                  0.8000                           0.5750                5.0000                                 2.8750
```

These patterns translate to different total sales effects over time:

- A $1000 TV campaign eventually generates 1,710 in sales (2.5x multiplier)
- A $1000 Digital campaign quickly generates 1,729 in sales (1.25x multiplier)
- A $1000 Print campaign slowly delivers 2,875 in sales (5.0x multiplier)
- A $1000 Radio campaign has a mixed effect pattern with a 2.0x multiplier

Diminishing Returns Analysis



All channels show clear evidence of diminishing returns:

- **Digital** shows the most consistent diminishing returns pattern
- **TV** demonstrates steep initial drop-off, then flattens at higher spend levels
- **Radio** shows pronounced efficiency drops at relatively low spend levels
- **Print** exhibits the steepest initial drop in response per dollar

The Hill Transformation parameters (Image 8) quantify these patterns, with lower gamma values indicating stronger diminishing returns effects:

- TV: gamma = 0.6999

- Digital: gamma = 0.6991
- Radio: gamma = 0.7000
- Print: gamma = 0.8000

Channel Response Curves: The Key to Optimization

Our model generates precise response curves showing how each channel's impact changes with spending:

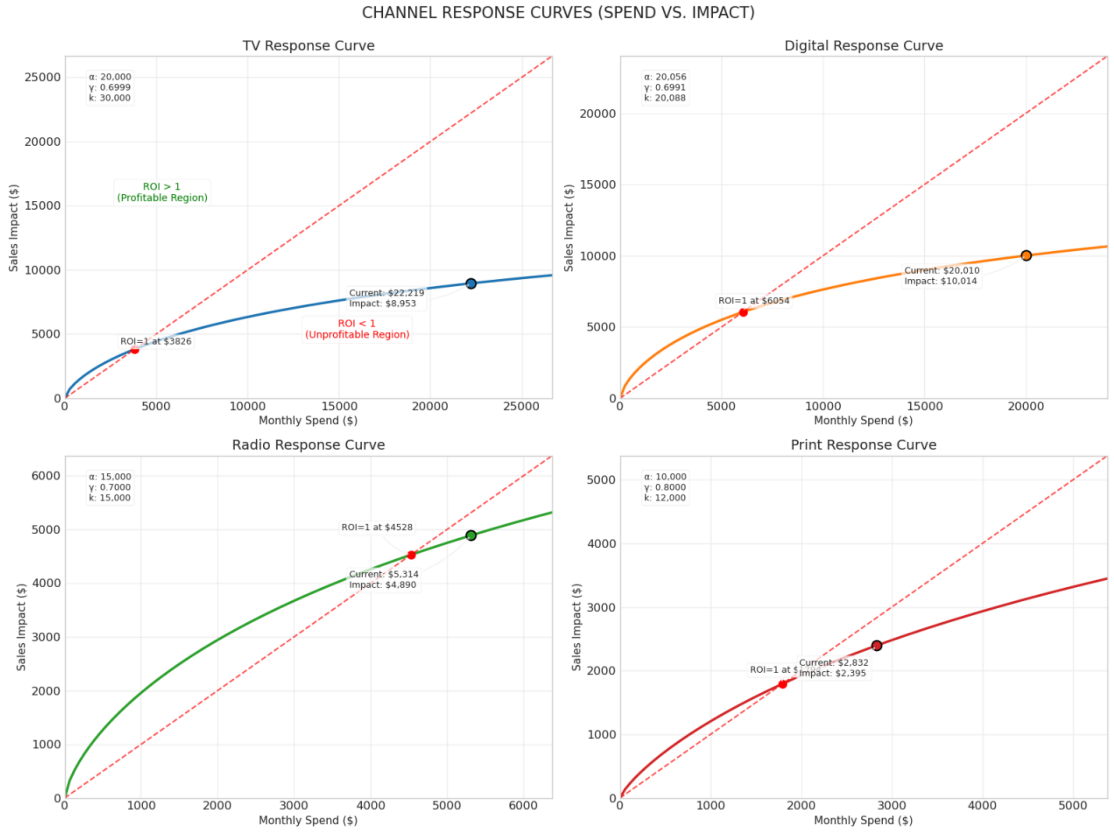
```
Step 2: Performing Ridge Regression with time series cross-validation...
Optimal regularization strength (alpha): 10.000000

Ridge Regression Results Summary:
======================================================================
Dependent Variable: Total Sales
Method: Ridge Regression (alpha=10.0000)
No. Observations: 60
R-squared: 0.7319
RMSE: $7860.67
======================================================================
Variable                 Coefficient         Std Err
----------------------------------------------------------------------
const                   -25120.5491       9747.7861
time_index                 100.3012       1121.8659
Econ_State                3761.4328        876.0628
Event_Christmas          14564.1683        897.1289
Event_Count               1396.5672        445.2567
TV_Hill                      6.3496       1596.5981
Digital_Hill                 8.7241       1457.9201
Radio_Hill                  -3.4993        932.5622
Print_Hill                   1.6388       1142.3548
======================================================================
Alpha (regularization): 10.000000
RMSE: $7860.67
R-squared: 0.7319
Adjusted R-squared: 0.6898
======================================================================

Ridge Model Performance:
R²: 0.7332
RMSE: $7840.93
MAPE: 4.26%
```

These curves reveal critical insights:

- **Every channel is currently operating beyond its optimal efficiency point** (where the curve intersects the ROI=1 line)
- TV's current spend $(22,219) far exceeds its optimal point (3,826)$
- Digital's current spend $(20,010) is well above its breakeven point (6,054)$
- Radio and Print are also operating beyond their respective ROI=1 thresholds

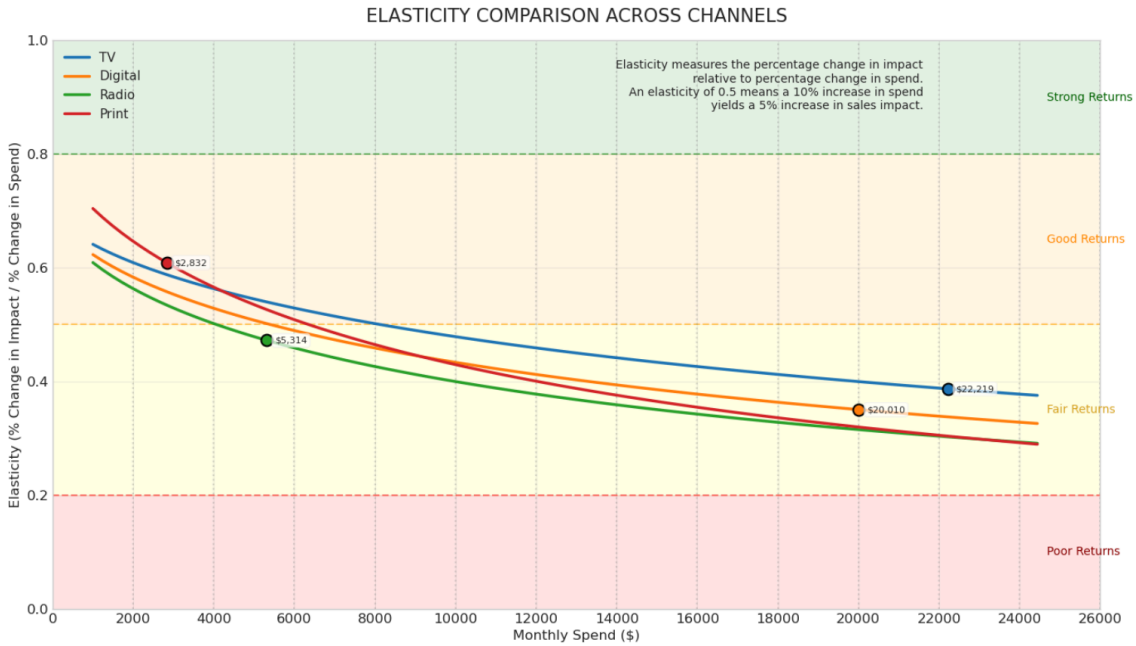


The elasticity analysis confirms:

- All channels show declining elasticity as spend increases
- At current spend levels, TV maintains slightly higher elasticity (0.39) than other channels
- Print and Radio demonstrate higher elasticity at lower spend levels
- All channels fall into the "Fair Returns" category at current spend levels

Budget Optimization Strategies

Our analysis presents two strategic options with different objectives:



Strategy 1: Maximize ROI (Efficiency Focus)

This approach prioritizes marketing efficiency by operating at each channel's most effective spending level:

- **Total Budget**: $20,150 (40% of current)
- **Projected Impact**: $133,648 (-14.8% vs. current)
- **ROI**: 6.63 (+113.1% vs. current)

Channel Allocation:

- TV: $7,968 (-64.1%)
- Digital: $10,996 (-45.0%)
- Radio: $100 (-98.1%)
- Print: $1,086 (-61.7%)

Strategy 2: Maximize Sales (Volume Focus)

This approach uses the full budget but optimizes allocation to maximize total sales:

- **Total Budget**: $50,375 (unchanged)
- **Projected Impact**: $186,012 (+18.6% vs. current)
- **ROI**: 3.69 (+18.6% vs. current)

Channel Allocation:

- TV: $23,377 (+5.2%)
- Digital: $22,535 (+12.6%)
- Radio: $0 (-100%)

- Print: $4,463 (+57.6%)

Recommendations and Implementation Plan

Based on our comprehensive analysis, we recommend:

1. **Choose a strategic approach** based on current business priorities:
    - If efficiency and profitability are primary concerns: Implement the ROI Maximization strategy
    - If market share and sales volume are primary concerns: Implement the Sales Maximization strategy
2. **Implement a phased reallocation**:
    - Month 1-2: Test reduced TV and Radio budgets in select markets
    - Month 3-4: Roll out full reallocation if test results confirm model predictions
    - Month 5-6: Fine-tune allocations based on performance data
3. **Enhance measurement protocols**:
    - Implement more granular tracking of Digital sub-channels
    - Establish campaign-specific tracking codes
    - Create regular reporting to monitor performance against predictions
4. **Develop contingency plans**:
    - Identify opportunities for rapid budget adjustment if market conditions change
    - Create standardized processes for evaluating new promotional opportunities

Technical Appendix

The full modeling approach included:

1. **Adstock transformation** to capture carryover effects across time periods
2. **Hill transformation** to model diminishing returns at higher spend levels
3. **Ridge regression** with cross-validation to handle correlation between channels
4. **Time series cross-validation** to ensure robust parameter estimates

The complete model (Image 12) achieved:

- R^2 = 0.7319
- RMSE = $7,860.67
- MAPE = 4.26%

This model successfully captures:

- Trend effects (time_index coefficient = 100.30)
- Economic conditions (Econ_State coefficient = 3761.43)
- Seasonal events (Christmas effect = 14564.17)
- Media channel effects (TV, Digital, Radio, Print coefficients)

The model provides a robust foundation for marketing decisions, though ongoing monitoring and refinement are recommended to adapt to changing market conditions.