3.) $A(AB)^{-1}B$

Firstly, $(AB)^{-1} = B^{-1}A^{-1}$

Proving:

Since matrices A and B are invertible,

$$(B^{-1}A^{-1})(AB) = I$$

Matrices are associative in property

$$(B^{-1}A^{-1})(AB)$$
$$=((B^{-1}A^{-1})A)B$$
$$= (B^{-1}(A^{-1}A))B$$
$$A^{-1}A = I$$

This is because A^{-1} is the inverse of A

$$=(B^{-1}I)B$$
$$= B^{-1}B$$

I is the identity

$$\therefore B^{-1}B = I$$
$$\therefore (AB)^{-1} = B^{-1}A^{-1}$$

From the question, $A(AB^{-1})B$

$$= A(B^{-1}A^{-1})B$$
$$= AB^{-1}A^{-1}B$$

Answer $= AB^{-1}A^{-1}B$

4.) $\lim_{x\to\infty} \frac{ax^2}{c-bx^2}$

Since the degree of the numerator equals that of the denominator,

divide both the numerator and denominator by x^2

$$\lim_{x\to\infty} \frac{ax^2 \div x^2}{c-bx^2 \div x^2}$$

$$= \lim_{x\to\infty} \frac{a}{\frac{c}{x^2} - b}$$

$$= \frac{a}{0-b}$$

$$= -\frac{a}{b}$$

5.) Minimum value of $x^2 - \sqrt{5}x - 2 = 0$

The minimum value of a quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{2a}$

From the equation given,

$a = 1$, $b = -\sqrt{5}$, $c = -2$

$$\therefore \frac{-b}{2a} = \frac{-(-\sqrt{5})}{2 \times 1}$$

$$= \frac{\sqrt{5}}{2}$$

$$= 1.118$$

$$\simeq 1.12$$

6) $5p^2 - 4p - 3 = 0$

Quadratic formula = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

where $a = 5$, $b = -4$, $c = -3$

Substituting the numbers:

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 5 \times -3}}{2 \times 5}$$

$$= \frac{4 \pm \sqrt{16 + 60}}{10}$$

$$= \frac{4 \pm \sqrt{76}}{10}$$

$$= \frac{4 + 8.72}{10}$$

$$= \underline{\underline{1.27}}$$

or

$$= \frac{4 - 8.72}{10}$$

$$= -0.47$$

$P = 1.27$ or -0.47

7a) $5, 8, 13, 21, \ldots$

Prove that it is <u>neither</u>

For arithemetric progression, common difference

$= T_2 - T_1 = T_3 - T_2$

But for the sequence;

$\Rightarrow 8 - 5 \neq 13 - 8$

For geometric progression, common ratio;

$= \frac{r_2}{r_1} = \frac{r_3}{r_2}$

But for the sequence;

$\Rightarrow \frac{8}{5} \neq \frac{13}{8}$