Solution To The Maths Exercise

September 24, 2020

Over all real numbers, find the minimum value of a positive real number, y such that:

$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$

$y = ((x+6)^2 + 25)^{1/2} + ((x-6)^2 + 121)^{1/2}$

1 Differntiate w.r.t x

$dy/dx = \frac{((x+6)^2+25)^{-1/2} * (2(x+6))}{2} + \frac{((x-6)^2+121)^{-1/2} * (2(x-6))}{2}$

$dy/dx = \frac{x+6}{((x+6)^2+25)^{1/2}} + \frac{x-6}{((x-6)^2+121)^{1/2}}$

for the minimum value of y, $dy/dx = 0$

$\frac{x+6}{((x+6)^2+25)^{1/2}} + \frac{x-6}{((x-6)^2+121)^{1/2}} = 0$

$\frac{x+6}{((x+6)^2+25)^{1/2}} = -\frac{x-6}{((x-6)^2+121)^{1/2}}$

2 Find the square of both sides

$\frac{(x+6)^2}{((x+6)^2+25)} = \frac{(x-6)^2}{((x-6)^2+121)}$

3 Cross multiply both sides

$(x+6)^2(x-6)^2 + 121(x+6)^2 = (x-6)^2(x+6)^2 + 25(x-6)^2$

$121(x^2 + 12x + 36) = 25(x^2 - 12x + 36)$

$121x^2 + 1452x + 4356 = 25x^2 - 300x + 900$

$96x^2 + 1752x + 3456 = 0$

4 Divide through by 24

$4x^2 + 73x + 144 = 0$

5 Solve for x using the quadratic formular

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 4, b = 73, c = 144$

$\sqrt{b^2 - 4ac} = \sqrt{73^2 - 4(4)(144)} = 55$

$2a = 8$

$x = \frac{-73 \pm 55}{8}$

$x_1 = -16$

$x_2 = -2.25$

at $x_1 = -16$; $y = \sqrt{(-16 + 6)^2 + 25} + \sqrt{(-16 - 6)^2 + 121} = 35.77$

at $x_2 = -2.25$; $y = \sqrt{(-2.25 + 6)^2 + 25} + \sqrt{(-2.25 - 6)^2 + 121} = 20$

Over all real numbers, $y \geq 20$

$\therefore$ the minimum value of y is 20.