INPUTTING OF FUNCTION

The design of the program is heavily tied to the design of the input for the expressions in the function. The expressions allowed to be inputted for the function are very limited. The production rule for the function f (contained in class partialDiff) is given by

$$< \mathrm{f} > \coloneqq < \mathrm{expression} > \; + \; < \mathrm{expression} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{expression} > \; * \; < \mathrm{expression} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{Algebraic} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{cos} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{sin} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{tan} >$$

$$< \mathrm{expression} > \coloneqq < \mathrm{exp} >$$

where their individual definitions are given by

$$< \mathrm{Algebraic} > \coloneqq \mathrm{k}_1.\mathrm{x}^{\mathrm{k}_2}.\mathrm{y}^{\mathrm{k}_3}$$

$$< \cos > \coloneqq \mathrm{k}_1.\cos(\mathrm{k}_2.\mathrm{x}^{\mathrm{k}_3}.\mathrm{y}^{\mathrm{k}_4})$$

$$< \sin > \coloneqq \mathrm{k}_1.\sin(\mathrm{k}_2.\mathrm{x}^{\mathrm{k}_3}.\mathrm{y}^{\mathrm{k}_4})$$

$$< \tan > \coloneqq \mathrm{k}_1.\tan(\mathrm{k}_2.\mathrm{x}^{\mathrm{k}_3}.\mathrm{y}^{\mathrm{k}_4})$$

$$< \exp > \coloneqq \mathrm{k}_1.\exp(\mathrm{k}_2.\mathrm{x}^{\mathrm{k}_3}.\mathrm{y}^{\mathrm{k}_4})$$

To input an expression into the function, the constants (k_i) need to be inputted appropriately after the expressions the expression type (Algebraic, cos, sin, tan, exp) has been chosen, then the next operator ('+' or '*') will be selected, and the entire process repeats until each expression in the function has been inputted. The input for a function is terminated entering the character '.' as the next operator.

Note: the above can be extended to functions having an input vector of 3 co-ordinates.

Example

1. $f(x,y) = x^2 - \sin(y)$

   Inputs

   - Choice of expression is **1** (Algebraic)
   - Constants are **1 2 0** ($\mathbf{1}.x^2y^0 = 1.x^2.1 = x^2$)
   - Next operator is '**+**'
   - Next choice of expression is **3** (sin)
   - Next set of constants are **-1 1 0 1** ($\mathbf{-1}.\sin[\mathbf{1}.x^0y^1] = -\sin[y]$)
   - Finally, we terminate the function by entering '**.**' as the next operator

2. $f(x,y,z) = 2y^2z.\cos(x) - \frac{2}{5}$

   Inputs

   - Choice of expression is **1** (Algebraic)
   - Constants are **2 0 2 1** ($\mathbf{2}.x^0y^2z^1 = \mathbf{2}.1.y^2z = \mathbf{2}y^2z$)
   - Next operator is '*****' (to multiply in $\sin(x)$)
   - Next choice of expression is **2** (cos)
   - Next set of Constants are **1 1 1 0 0** ($\mathbf{1}.\cos(\mathbf{1}.x^1y^0z^0) = \cos(x)$)
   - Next operator is '**+**'
   - Next choice of expression is **1** (Algebraic)
   - Next set of constants are **-0.4 0 0 0** ($\mathbf{-0.4}.x^0y^0z^0 = \mathbf{-0.4}.1.1.1 = \mathbf{-0.4}$). Note that $\frac{2}{5} = 0.4$
   - Finally, we terminate the function by entering '**.**' as the next operator

3.

$$3x_1 - \cos(x_2x_3) - \frac{1}{2}$$

Inputs

- Choice of expression is **1** (Algebraic)
- Constants are **3 1 0 0** (**$3.X_1^1.X_2^0.X_3^0 = 3X_1$**)
- Next operator is '**+**'
- Next choice of expression is **2** (cos)
- Next set of Constants are **-1 1 0 0 1** (**$-1.\cos(1.X_1^0.X_2^1.X_3^0) = -\cos(X_2.X_3)$**)
- Next operator is '**+**'
- Next choice of expression is **1** (Algebraic)
- Next set of constants are **-0.5 0 0 0** (**$-0.5.X_1^0.X_2^0.X_3^0 = -0.5$**). Note that **½= 0.5**
- Finally, we terminate the function by entering '**.**' as the next operator

4.

$$e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3}$$

Inputs

- Choice of expression is **5** (exp)
- Constants are **1 -1 1 1 0** (**$1.\exp(-1.X_1^1.X_2^1.X_3^0) = \exp(-X_1.X_2)$**)Next operator is '**+**'
- Next choice of expression is **1** (Algebraic)
- Next set of Constants are **20 0 0 1** (**$20.X_1^0.X_2^0.X_3^1 = 20$**)
- Next operator is '**+**'
- Next choice of expression is **1** (Algebraic)
- Next set of constants are **-9.472 0 0 0** (**$-9.472.X_1^0.X_2^0.X_3^0 = -9.472$**). Note that **$(10\pi-3)/3 = -9.472$**
- Finally, we terminate the function by entering '**.**' as the next operator

5.

$$x_1{}^2 - 81(x_2 + 0.1)^2 + \sin(x_3) + 1.06$$

Inputs

- Choice of expression is **1** (Algebraic)
- Constants are **1 2 0 0** (**$1.X_1^2.X_2^0.X_3^0 = X_1^2$**)
- Next operator is '**+**'
- The expression **$-81(x_2 + 0.1)^2$** is first expanded so as to suit our predefined syntax. So we have the expanded form to be **$-81x_2^2 - 16.2x_2 - 0.81$** . The inputs are as follows
    - Next Choice of expression is **1** (Algebraic)
    - Next set of Constants are **-81 0 2 0** (**$-81.X_1^0.X_2^2.X_3^0 = -81X_2^2$**)
    - Next operator is '**+**'
    - Next Choice of expression is **1** (Algebraic)
    - Next set of Constants are **-16.2 0 1 0** (**$-16.2.X_1^0.X_2^1.X_3^0 = -16.2X_2$**)
    - Next operator is '**+**'
    - Next Choice of expression is **1** (Algebraic)
    - Next set of Constants are **-0.81 0 0 0** (**$-0.81.X_1^0.X_2^0.X_3^0 = -0.81$**)
    - Next operator is '**+**'

- Next Choice of expression is **3** (sin)
- Next set of Constants are **1 1 0 0 1** (**$1.\sin(1.X_1^0.X_2^0.X_3^1) = \sin(X_3)$**)
- Next operator is '**+**'
- Next Choice of expression is **1** (Algebraic)
- Next set of Constants are **1.06 0 0 0** (**$1.06.X_1^0.X_2^0.X_3^0 = 1.06$**)
- Next operator is '**+**'
- Finally, we terminate the function by entering '**.**' as the next operator

6. For the Van De Pol equation $y'' - 10(1-y^2)y' + y = 0$, first it is reduced to a system of first order ODE as given below

   Let **$y' = u$**

   $u' = 10(1-y^2)u - y = 10u - 10y^2u - y$

   From the above set of equations, it is obvious that each term is an algebraic expression of the form **k1.y^(k2).u^(k3)**. So we proceed to enter the inputs

   Inputs

   - For the first equation
     - Choice of expression is **1** (Algebraic)
     - Constants are **1 0 1** (**$1.y^0.u^1 = u$**)
     - Finally, we terminate the function by entering '**.**' as the next operator
   - For the second equation
     - Choice of expression is **1** (Algebraic)
     - Constants are **10 0 1** (**$10.y^0.u^1 = 10u$**)
     - Next operator is '**+**
     - Next choice of expression is **1** (Algebraic)
     - Next set of constants are **-10 2 1** (**$-10.y^2.u^1 = -10y^2u$**)
     - Next operator is '**+**`
     - Next choice of expression is **1** (Algebraic)
     - Next set of constants are **-1 1 0** (**$-1.y^1.u^0 = -y$**)
     - Finally, we terminate the function by entering '**.**' as the next operator