

$$u = u_0 - \frac{1}{2\pi} \left[\frac{\rho x}{y^2 + x^2} - \frac{(x+L)}{y^2 + (x+L)^2} \right]$$

$$v = \frac{1}{2\pi} \left[\frac{y}{y^2 + x^2} \right]$$

$$\frac{Q_2}{Q_1} = \rho$$

$$u = u_0 + \frac{1}{2\pi} \left[\frac{\rho x}{y^2 + x^2} - \frac{\rho(x-L)}{y^2 + (x-L)^2} \right]$$

$$v = \frac{1}{2\pi} \left[\frac{y}{y^2 + x^2} - \frac{\rho y}{y^2 + (x-L)^2} \right]$$

This is the velocity field vector of a source and sink pair. With the source at (0,0) and the sink at (L,0). The ratio of the sink to the source is $\rho = \frac{Q_2}{Q_1}$.

We wish to find the location of the particle for the velocity to be unity in both components:

$$\frac{x}{2\pi(y^2 + x^2)} = 1, \quad \frac{y}{2\pi(y^2 + x^2)} = 1$$

$$y^2 + x^2 = \frac{x}{2\pi} \quad \Rightarrow \quad y^2 + x^2 = \frac{y}{2\pi}$$

$$\frac{x}{2\pi} - x^2 + x^2 = \frac{\left[\frac{x}{2\pi} - x^2 \right]}{2\pi} \Rightarrow \frac{x}{2\pi} = \frac{\left[\frac{x}{2\pi} - x^2 \right]}{2\pi}$$

$$\frac{x^2}{4\pi^2} = \frac{\frac{x}{2\pi} - x^2}{4\pi^2} \Rightarrow 2x^2 - \frac{x}{2\pi} = 0 \Rightarrow x \left(x - \frac{1}{2\pi} \right) = 0$$

$$x = 0 \quad \text{or} \quad x = \frac{1}{4\pi} \Rightarrow y = \sqrt{\frac{1}{8\pi^2} - \frac{1}{16\pi^2}}$$

$$y = \frac{1}{2\pi} \sqrt{\frac{1}{2} - \frac{1}{4}} = \frac{1}{2\pi} \sqrt{\frac{1}{4}} = \frac{1}{4\pi}$$

At the point $\left(\frac{1}{4\pi}, \frac{1}{4\pi} \right)$, the source is of unity velocity.

The distance from the origin to $(\frac{1}{4u}, \frac{1}{4u})$ is

$$r = \sqrt{\left(\frac{1}{4u}\right)^2 + \left(\frac{1}{4u}\right)^2} = \sqrt{\frac{2}{16u^2}} = \frac{1}{4u}\sqrt{2}$$

$y_0 = r \sin \theta$
 $x_0 = r \cos \theta$ } For any angle θ , the distance of the point (x_0, y_0) from the origin is $r = \frac{\sqrt{2}}{4u}$, and this ensures the magnitude of the velocity is unity.