Week 5 Tutorial 5

(5.1) Suppose a company manufactures two products, A and B, using three inputs, labor, material R, and materials S. To make one unit of product A requires 6 pounds of R, 7.5 pounds of S, and 9 person-hours of labor; to make one unit of product B requires 12 pounds of R, 4.5 pounds of S, and 6 person-hours of labor. The demands for the products are such that the company can sell as much of each product as it can produce and earn a profit of 3 per unit of A and 4 per unit of B. However, only 900 pounds of R, 675 pounds of S, and 1200 person-hours of labor are available to the company each day.

1. Formulate the company's problem as a linear program to maximize profit.
2. Graph the feasible region for this problem.
3. Solve the problem graphically by finding the best extreme point.

(5.2) Consider the following linear program

$$\max_{(x_1,x_2)} x_1 + x_2 \tag{0.1a}$$
$$\text{subject to } -3x_1 + 2x_2 \leq -1 \tag{0.1b}$$
$$x_1 - x_2 \leq 2 \tag{0.1c}$$
$$x_1, x_2 \geq 0 \tag{0.1d}$$

Show that

1. Show that the solution to it is unbounded.
2. State the dual problem and show that it is infeasible.
3. Explain how this relates to the duality results discussed in class.

(5.3) Calculate the solution of the following equality constrained minimisation problem:

$$\min_{(x_1,x_2)} 2x_1^2 + x_2^2 \text{ subject to } x_1 + x_2 = 1.$$

Given a graphic interpretation of the solution, that is sketch the contour linear of the objective and the constraint.

(5.4) Calculate the solution to the following constrained optimisation problem:

$$\begin{aligned}\max_{(x_1,x_2)} & -(x_1-2)^2 - 2(x_2-1)^2 \\ \text{subject to } & x_1 + 4x_2 \leq 3 \\ & x_1 \geq x_2\end{aligned}$$