Problem Sheet 2

Please turn in a solution by Monday on Week 5. The number of points attainable for each question is provided.

(2.1) For this problem we generalize the notion of convexity to function not necessarily defined on all of $\mathbb{R}^n$. Denote by $\mathrm{dom} f$ the *domain* of f, i.e., the set of $\boldsymbol{x}$ on which $f(\boldsymbol{x})$ attains a finite value. A function f is called *convex*, if $\mathrm{dom} f$ is a convex set and for all $\boldsymbol{x}, \boldsymbol{y} \in \mathrm{dom} f$ and $\lambda \in [0, 1]$,

$$f(\lambda \boldsymbol{x} + (1-\lambda)\boldsymbol{y}) \leq \lambda f(\boldsymbol{x}) + (1-\lambda) f(\boldsymbol{y}).$$

Which of the following functions are convex?

(a) $f(x) = \log(x)$ on $\mathbb{R}_{++}$ (the positive real numbers); **(2)**

(b) $f(\boldsymbol{x}) = x_1 x_2$ on $\mathbb{R}^2_{++}$; **(2)**

(c) $f(\boldsymbol{x}) = x_1/x_2$ on $\mathbb{R}^2_{++}$; **(2)**

(d) $f(\boldsymbol{x}) = \max_i x_i$ on $\mathbb{R}^n$. **(2)**

(2.2) Determine the order of convergence of each of the following sequences (if they converge at all).

$$\text{(a)}\;\; x_k = \frac{1}{k!}, \quad \text{(b)}\;\; x_k = 1 + (0.3)^{2^k}, \quad \text{(c)}\;\; x_k = 2^{-k}, \quad \text{(d)}\;\; x_k = 1/k$$

(4)

(2.3) Consider the function on $\mathbb{R}^2$, $f(\boldsymbol{x}) = (x_1^2 + x_2)^2$. Show that the direction $\boldsymbol{p} = (1, -1)^\top$ is a descent direction at $\boldsymbol{x}_0 = (0, 1)^\top$, and determine a step length α that minimizes $f(\boldsymbol{x}_0 + \alpha \boldsymbol{p})$. **(3)**

(2.4) Consider an optimisation problem

$$\begin{aligned} &\min f(x) \\ &\text{s.t.} \quad x \in \Omega \end{aligned}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is strictly convex on Ω and Ω is a convex set. Show that the optimal solution is unique (assuming it exists). **(5)**

(2.5) A local politician is budgeting for her media campaign. She will distribute her funds between TV ads and radio ads. She has been given the following advice by her campaign advisers:

- She should run at least 120 TV ads and at least 30 radio ads.
- The number of TV ads she runs should be at least twice the number of radio ads she runs but not more than three times the number of radio ads she runs.

The cost of a TV ad is $\pounds 8000$ and the cost of a radio ad is $\pounds 2000$.

Formulate the respective linear program, sketch the feasible set and explain the provided Python code. What do you observe for the solution.**(5)**