Q_4: $\max \; -(x_1-2)^2 - 2(x_2-1)^2$

s.t.

$x_1 + 4x_2 \leq 3$

$x_1 \geq x_2 \Rightarrow x_2 - x_1 \leq 0$

Soln:

$\mathcal{L} = -(x_1-2)^2 - 2(x_2-1)^2 + \lambda_1(x_1+4x_2-3) +$
$\lambda_2(x_2-x_1)$

$$\nabla_{x_i}\mathcal{L} = 0 \Rightarrow \begin{cases} \text{(i)} & -2(x_1-2)+\lambda_1-\lambda_2=0 \\ \text{(ii)} & -4(x_2-1)+4\lambda_1+\lambda_2=0 \end{cases}$$

and

$$\nabla_{\lambda_i}\mathcal{L} = 0 \Rightarrow \begin{array}{ll} \text{(iii)} & x_1+4x_2-3 = 0 \\ \text{(iv)} & x_2-x_1 = 0 \end{array}$$

if $\lambda_1 = \lambda_2 = 0$

(i) $\rightarrow$ $x_1 = 2$, (ii) $x_2 = 1$ # Since eqn (iii) & (iv) will not exist.

if $\lambda_1 = 0$:

(i) $\rightarrow -2(x_1-2) - \lambda_2 = 0$
(ii) $\rightarrow -4(x_2-1) + \lambda_2 = 0$ $\Big\}$ $+$: $2x_1 - 4 + 4x_2 - 4 = 0$
(iv) $\rightarrow$ $x_2 - x_1 = 0 \Rightarrow x_2 = x_1$ $\qquad \Downarrow$ $x_1(2+4)-8=0$

$\lambda_2 = -2(4/3-2) = 4/3 \Leftarrow x_2 = x_1 = 8/6 = 4/3$

if $\lambda_2 = 0$:

(i) $\rightarrow -2(x_1-2) + \lambda_1 = 0 \times -4$
(ii) $\rightarrow -4(x_2-1) + 4\lambda_1 = 0$ $\Big\}$ $+$ $\quad 8x_1 - 4x_2 = 12$
(iii) $\rightarrow$ $x_1 + 4x_2 - 3 = 0 \Rightarrow x_1 + 4x_2 = 3$ $\quad +$

$\lambda_1 = -\frac{2}{3} \Leftarrow x_2 = \frac{1}{3} \Leftarrow x_1 = \frac{5}{3} \Leftarrow 9x_1 = 15$

if $\lambda_1, \lambda_2 \neq 0$

$-2(x_1-2)+\lambda_1-\lambda_2=0$

$-4(x_2-1)+4\lambda_1+\lambda_2=0$

$\Big\}$ +: $-2x_1+5\lambda_1+8=0$

$x_1+4x_2-3=0 \Rightarrow x_1+4x_2-3=0$

$x_2-x_1=0 \Rightarrow x_2=x_1$

$\boxed{x_1=x_2=\frac{3}{5}}$

$$\lambda_1=\frac{-8+2x_1}{5}=\boxed{\frac{-34}{25}}$$