Question – 1

(a) $f(n) = 100n^3 + 8n^2 + 4n$ is $O(n^4)$. I will show this is true by specifying appropriate c and n_0 values in Big-Oh definition.

Assigning $c = 100$, and $n_0 = 1.4$, we see that:

$c.f(n) = 100(100n^3 + 8n^2 + 4n) = O(n^4)$ for $n >= n_0 = 1.4$

(b) $T(n) = 8T(n/2) + n^3$
$T(n) = 16T(n/4) + 2n^3$
$T(n) = 32T(n/8) + 3n^3$
$T(n) = 64T(n/16) + 4n^3$

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- If we assume that we have $n = 2^{10} = 1024$, then,
  $T(1024) = 8T(512) + 1024^3$
  $T(512) = 8T(256) + 512^3$
  $T(256) = 8T(128) + 256^3$
  $T(128) = 8T(64) + 128^3$ and as we know that $T(n) = 1$ for all $n <= 100$, we have,
  $T(64) = 1$.
- By using backwards substitution, we will obtain,
  $T(1024) = T(2^{10}) = 4\ (2^{10}) + 25/8\ (2^{10})^3$, we can generalize this as follows:
  $T(n) = 4n + (25/8)n^3 = O(n^3)$ for $n >= 100$.

(c)