

# 图像处理与分析

## — 图像多分尺度处理技术

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授课教师：孙剑

[jiansun@mail.xjtu.edu.cn](mailto:jiansun@mail.xjtu.edu.cn)

<http://jiansun.gr.xjtu.edu.cn>

西安交通大学 数学与统计学院

# 目录

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- 图像多尺度表达
- 图像的多尺度金子塔
- 图像的多尺度小波分解
- 图像多尺度技术的应用例子（图像去噪、增强、融合）

# 图像多尺度表达

图像多尺度表达：将图像分解为一系列不同尺度下的图像集合，分别对图像在不同尺度下进行处理，最后将处理后的多尺度图像重构得到处理后的图像。

- 多尺度分析有利于提取图像在不同尺度下的不同特征，针对不同特征进行有针对性的处理



# 图像多尺度表达

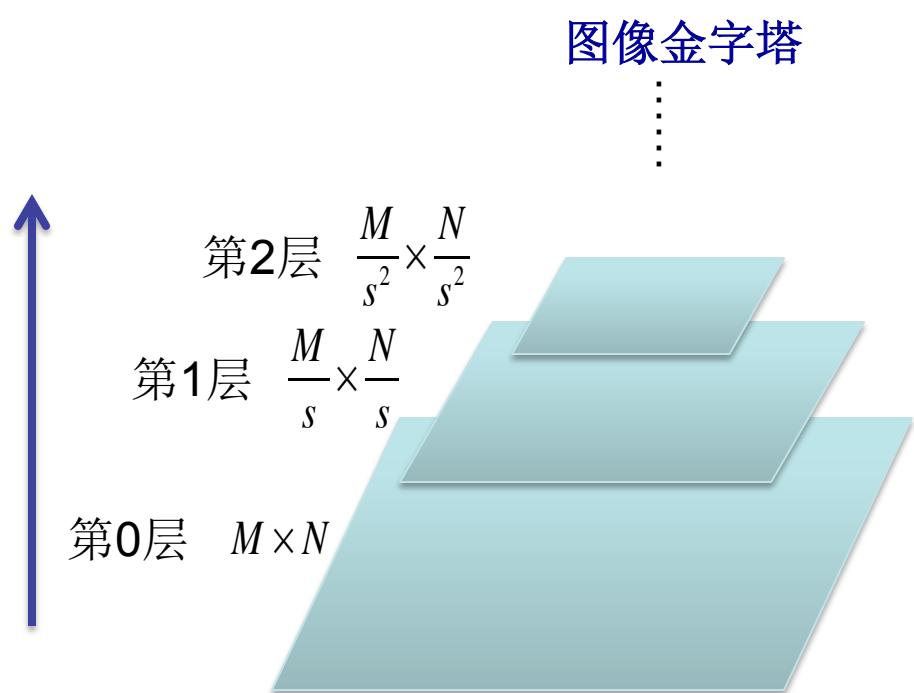
- 图像金字塔：将图像表达为一组不同分辨率下的图像集合。

第0层为原始图像分辨率  $M \times N$ ， 第*i*层图像分辨率  $\frac{M}{s^i} \times \frac{N}{s^i}$

图像金字塔的总像素个数

$$MN\left(1 + \frac{1}{s^2} + \frac{1}{s^4} + \dots\right) < \frac{s^2}{s^2 - 1} MN$$

当  $s=2$  时候， 总像素个数  
 $< 4MN/3$



# 图像多尺度表达

- 尺度空间：对图像数据引入一个新的尺度参数 $\mathbf{s}$ 。

$$f(x, y) \longrightarrow f(x, y, s)$$

↓ 尺度参数

尺度空间：  $\{f(x, y, s) : s \geq 0\}$

典型例子：由偏微分方程导出的尺度空间

## PM 扩散方程（非线性）

$$\begin{cases} \partial_t u = \nabla \cdot (c(|\nabla u|) \nabla u), & u(x, y, 0) = I(x, y) \\ c(|\nabla u|) = \frac{1}{1 + |\nabla u|^2 / k^2} \end{cases} \longrightarrow u(x, y, s)$$

# 图像多尺度金字塔

- 高斯金字塔：对图像不断施加高斯滤波并进行降采样，或者一组不同分辨率的图像金字塔。



$s = 0$



$s = 1$



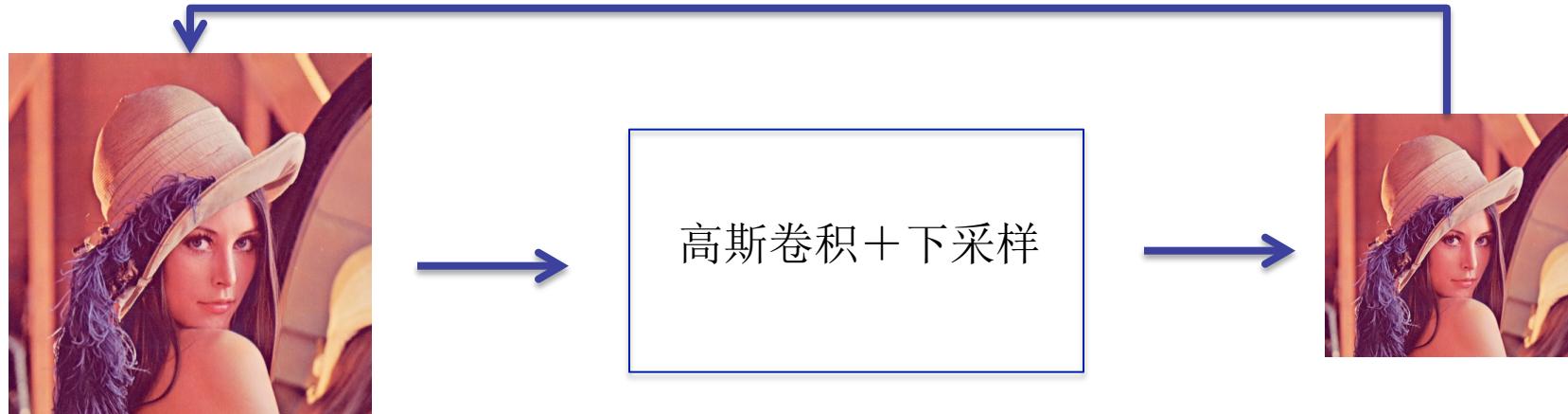
$s = 2$



$s = 3$

$$f^{(s+1)} = (G_{\sigma} * f^{(s)}) \downarrow$$

# 图像多尺度金字塔



$$f^s(x, y)$$

```
im = imread('../Figs/lena.jpg');
s = 2;
nlvs = 5;
filt = fspecial('gauss', 7, 3);
pyramid{1} = im;
figure, imshow(im);
for k = 2 : nlvs
    factor = s^(k-1);
    pyramid{k} = imresize(imfilter(im, filt), 1 / factor, 'bilinear');
    figure, imshow(uint8(pyramid{k}))
end
```

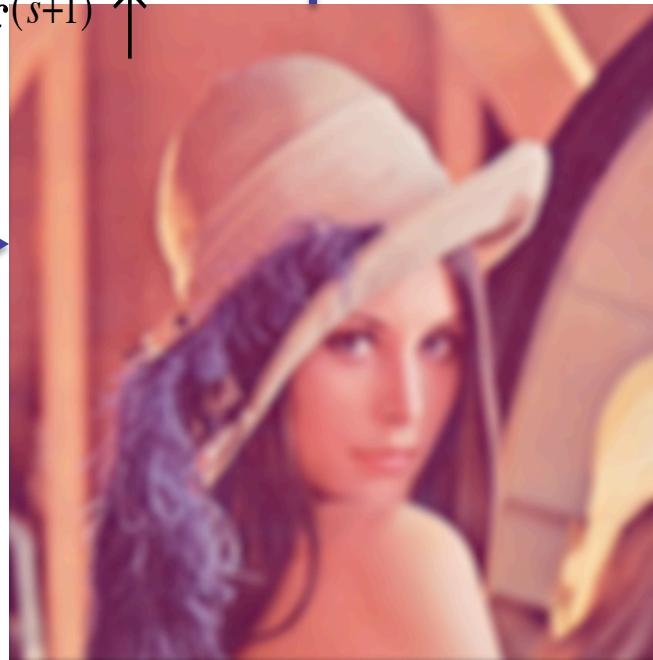
# 图像多尺度金字塔

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- 拉普拉斯金字塔：将图像分解为金字塔，金字塔中的每一层包含一定带宽频率的图像信息，原始图像可以由该金字塔完全重构。

图像分解为金字塔过程（由高斯金字塔生成）：

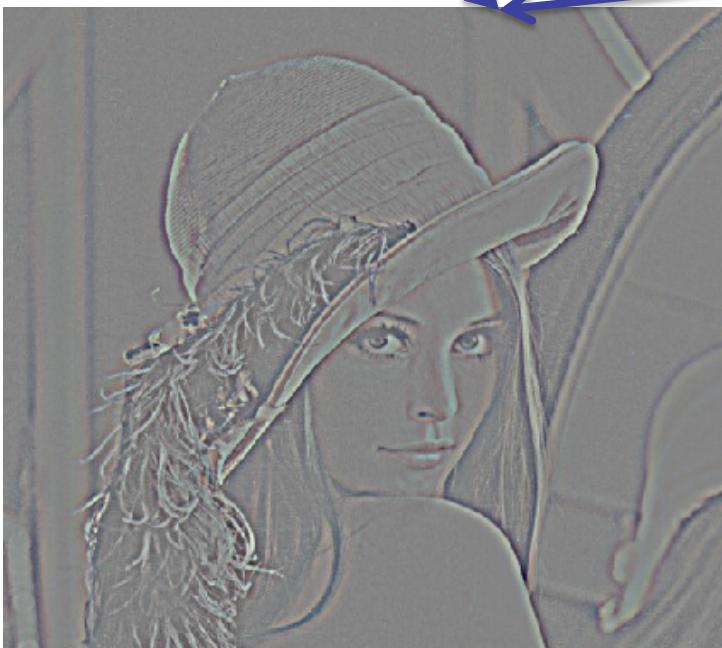
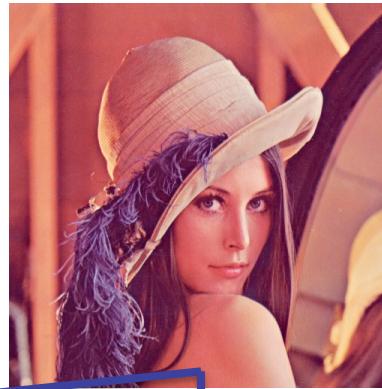
$$L^{(s)} = f^{(s)} - f^{(s+1)} \uparrow$$

 $f^{(s)}$  $f^{(s+1)} \uparrow$  $f^{(s+1)}$ 

$$L^{(s)} = f^{(s)} - f^{(s+1)} \uparrow$$



高斯金字塔



拉普拉斯金字塔



```
im = double(imread('..../Figs/lena.jpg'));
```

---

```
% Gaussian Pyramid
```

```
s = 2;  
nlvs = 5;  
filt = fspecial('gauss', 11, 5);  
pyramid{1} = im;  
figure, imshow(uint8(im));  
for k = 2 : nlvs  
    factor = s^(k-1);  
    pyramid{k} = imresize(imfilter(im, filt), 1 / factor, 'bilinear');  
    figure, imshow(uint8(pyramid{k}))  
end
```

```
% Laplacian Pyramid
```

```
for k = 1 : nlvs-1  
    factor = s^(k-1);  
    Lap_pyramid{k} = pyramid{k} - imresize(pyramid{k+1}, s, 'bilinear');  
    im = (Lap_pyramid{k} - min(Lap_pyramid{k}(:))) / (max(Lap_pyramid{k}(:)) - min(Lap_pyramid{k}(:)));  
    figure, imagesc(im);  
end  
Lap_pyramid{nlvs} = pyramid{nlvs};  
figure, imshow(uint8(Lap_pyramid{nlvs}))
```

# 图像多尺度金字塔

由拉普拉斯金字塔重构原始图像

$$f^{(s)} = L^{(s)} - f^{(s+1)} \uparrow$$

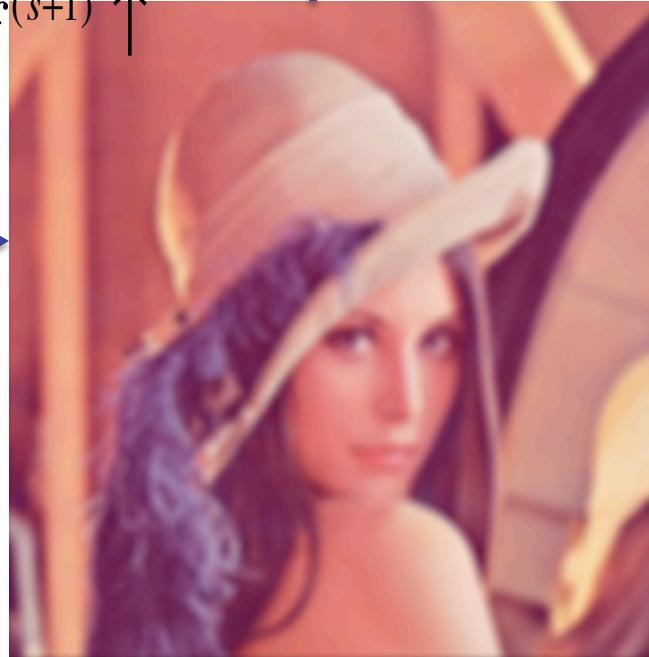
图像重构：拉普拉斯金字塔分解逆过程

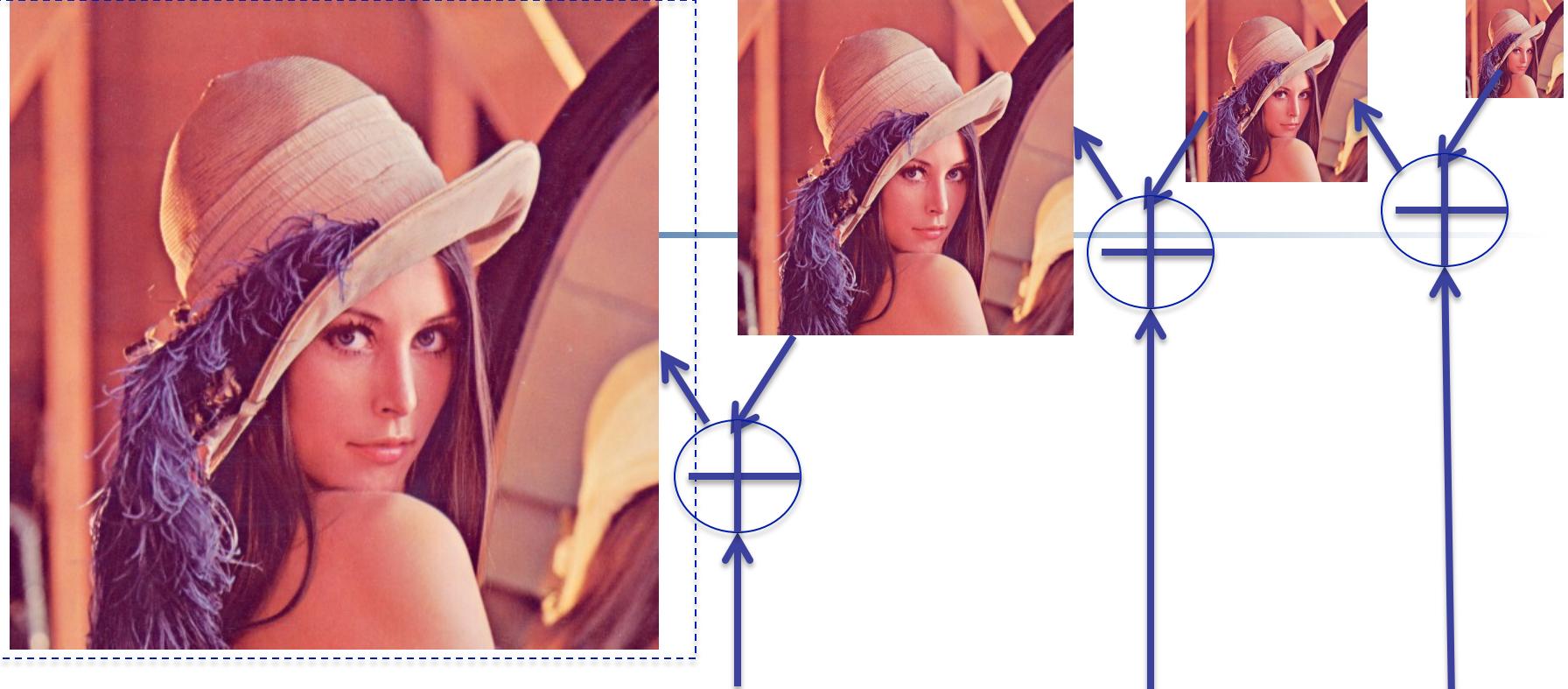
for  $s = L : 0$

$$f^{(s)} = L^{(s)} + f^{(s+1)} \uparrow \quad \longrightarrow \quad f^{(0)} \text{ 为重构图像}$$

end

[可由拉普拉斯金字塔完全重构原始图像]

 $L^{(s)}$  $f^{(s+1)} \uparrow$  $L^{(s)} + f^{(s+1)} \uparrow = f^{(s)}$  $f^{(s+1)}$ 



由拉普拉斯金字塔重构原始  
图像

# 图像二维离散小波变换

- 二维离散小波分解将二维图像分解为多尺度表达。原图像可以由多尺度小波系数精确重建。
- 二维离散小波分解设计主要在于如何设计二维滤波器。

encode low  
resolution info     $\varphi(t)$        $\psi(t)$       encode details or  
high resolution info

$$f(t) = \sum_k c_k \varphi(t - k) + \sum_k \sum_j d_{jk} \psi(2^j t - k)$$

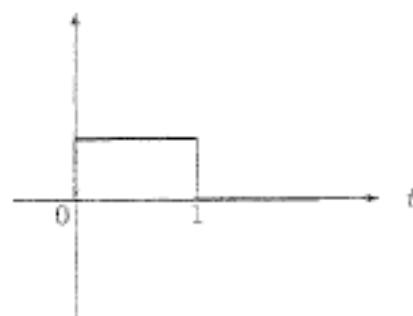
scaling function

wavelet function

# 图像二维离散小波变换

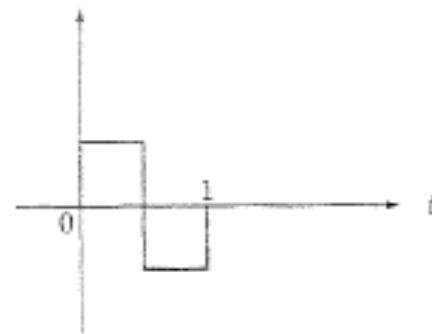
- 1D-Haar小波: scaling and wavelet functions

$$\varphi(t)$$



(a)  $\phi(t)$

$$\psi(t)$$



(b)  $\psi(t)$

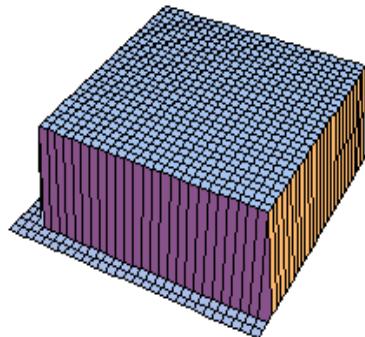
computes average

computes details

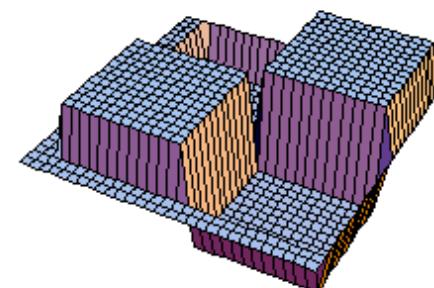
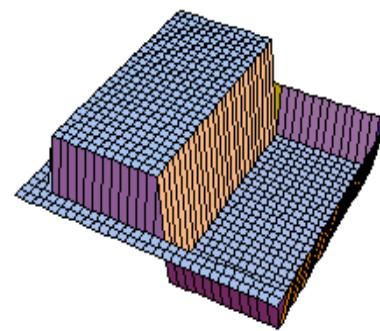
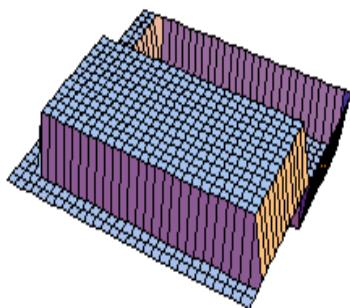
# 图像二维离散小波变换

- 2D-Haar小波:

- 2D Haar scaling:



- 2D Haar wavelets:

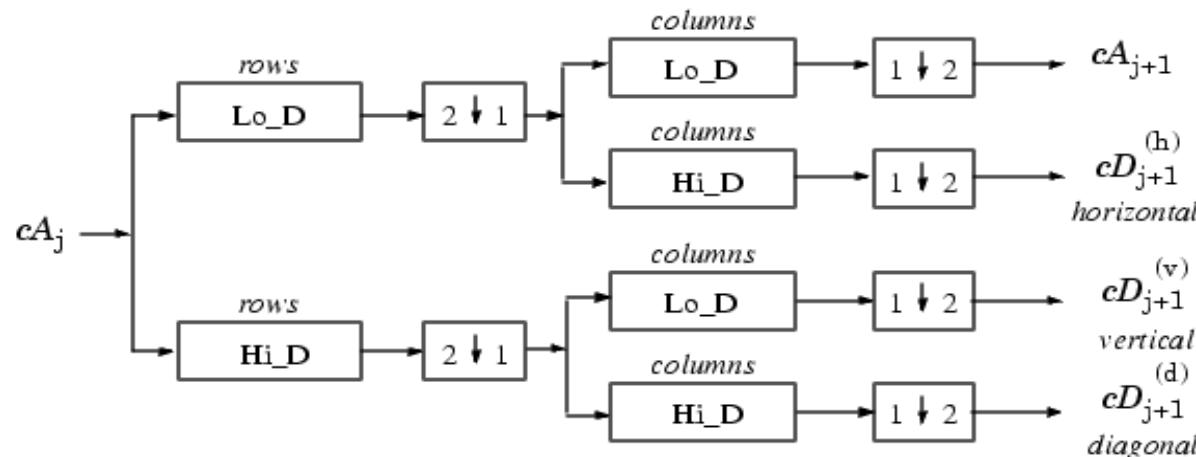


# 图像二维离散小波变换

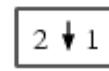
## ● 2D—小波正变换：

### Two-Dimensional DWT

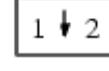
#### Decomposition Step



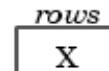
where



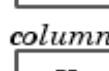
Downsample columns: keep the even indexed columns.



Downsample rows: keep the even indexed rows.



Convolve with filter X the rows of the entry.



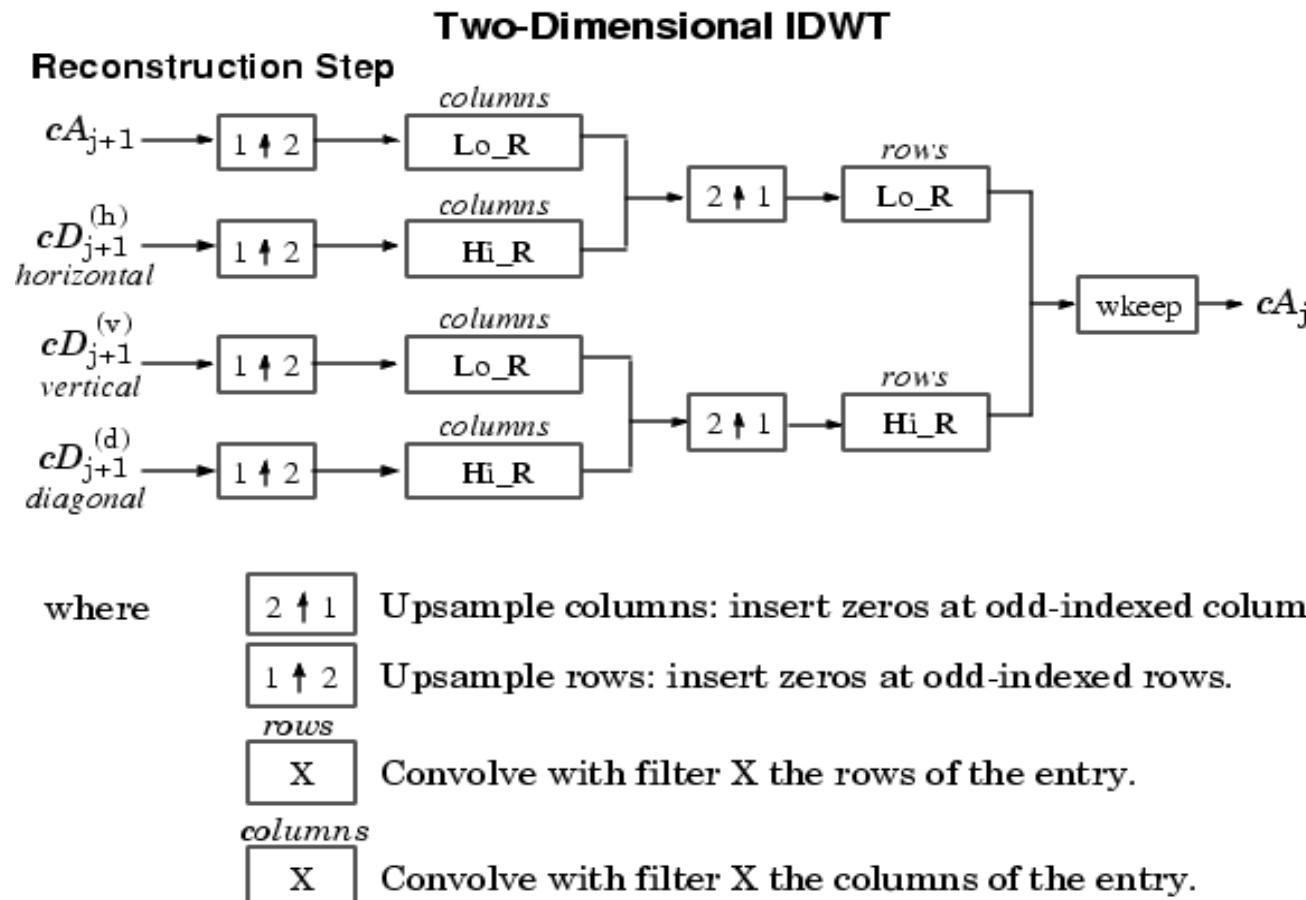
Convolve with filter X the columns of the entry.

#### Initialization

$CA_0 = s$  for the decomposition initialization.

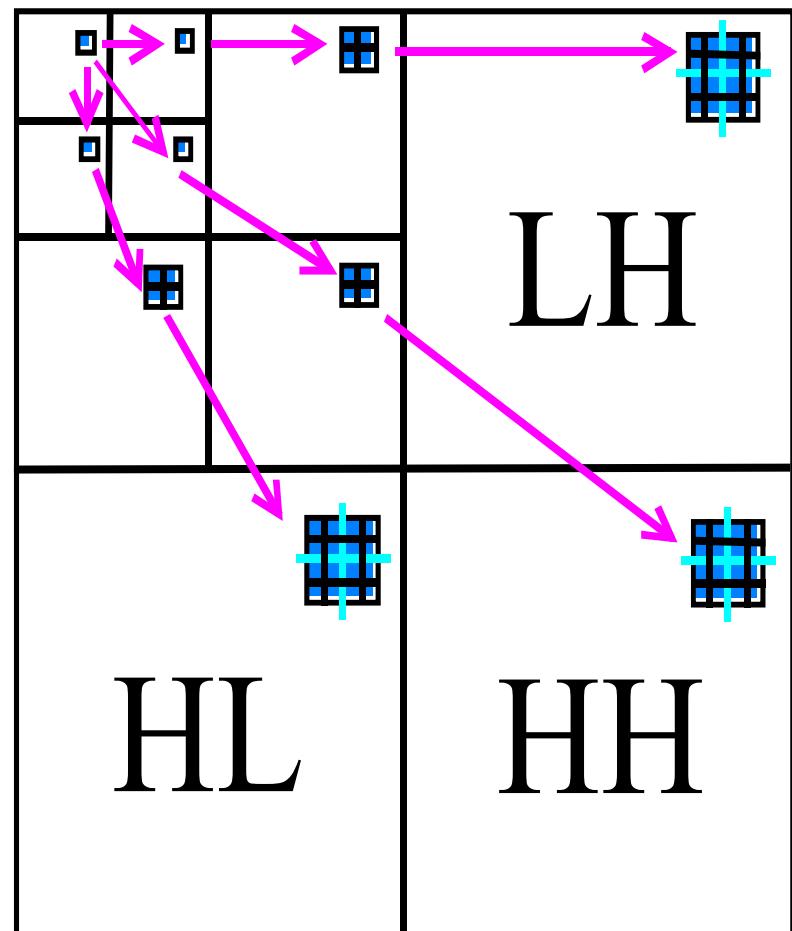
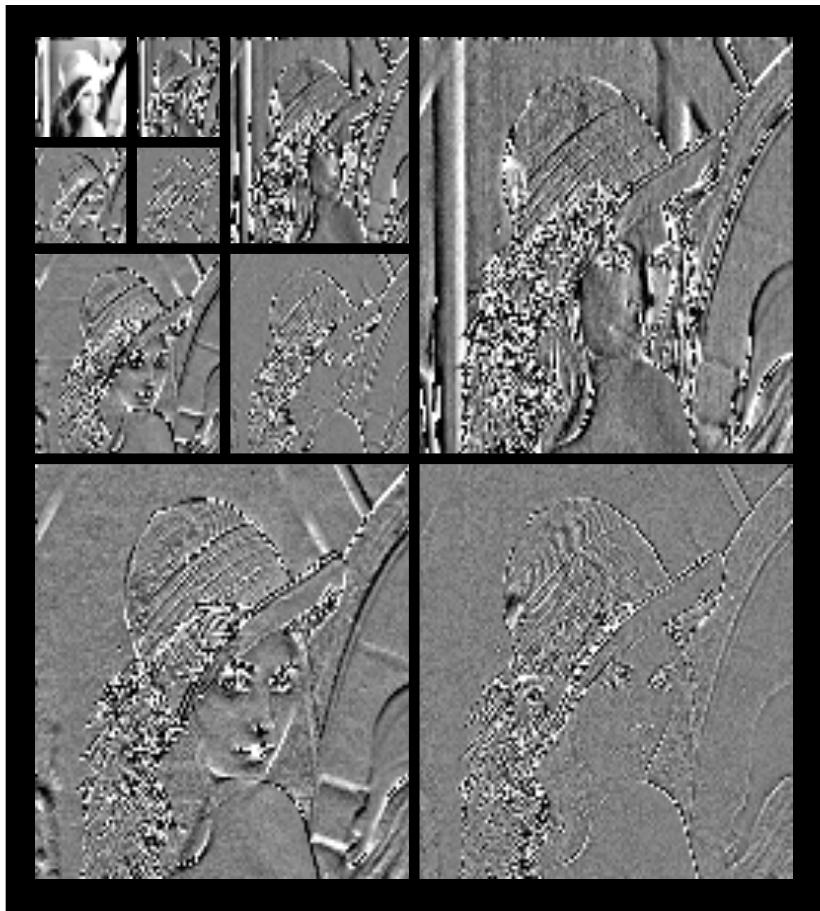
# 图像二维离散小波变换

## ● 2D—小波逆变换：



# 图像二维离散小波变换

Lena图

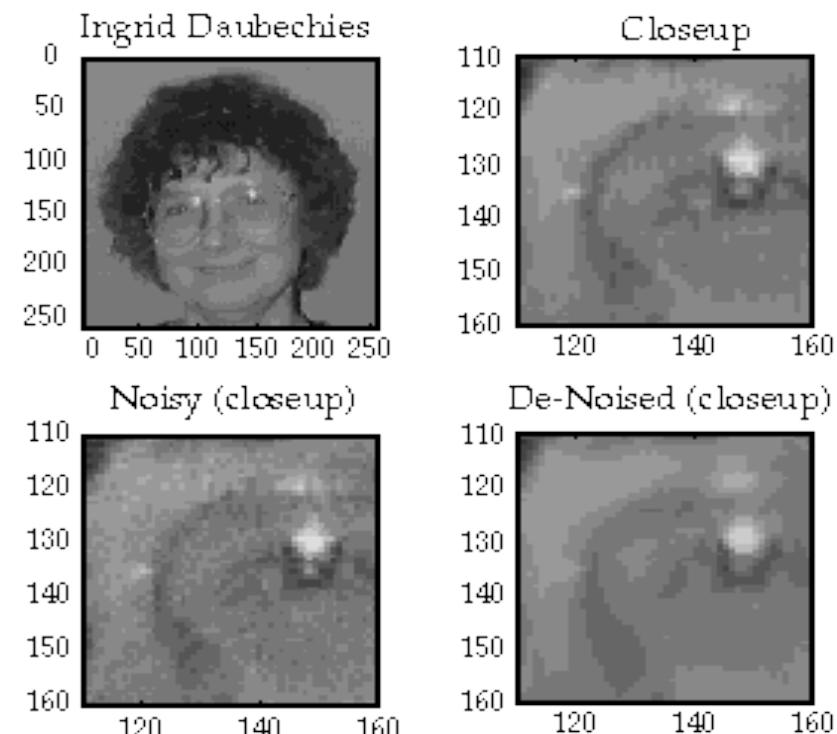


# 应用: 基于小波分解的图像去噪

- 去噪流程:

- 将图像进行多尺度变换
- 对变换域内的各个尺度小波系数进行去噪操作
- 对去噪后的多尺度小波系数进行逆变换，得到去噪后的图像

$$c(x, y) = W^{-1}[T(W[f(x, y)])]$$



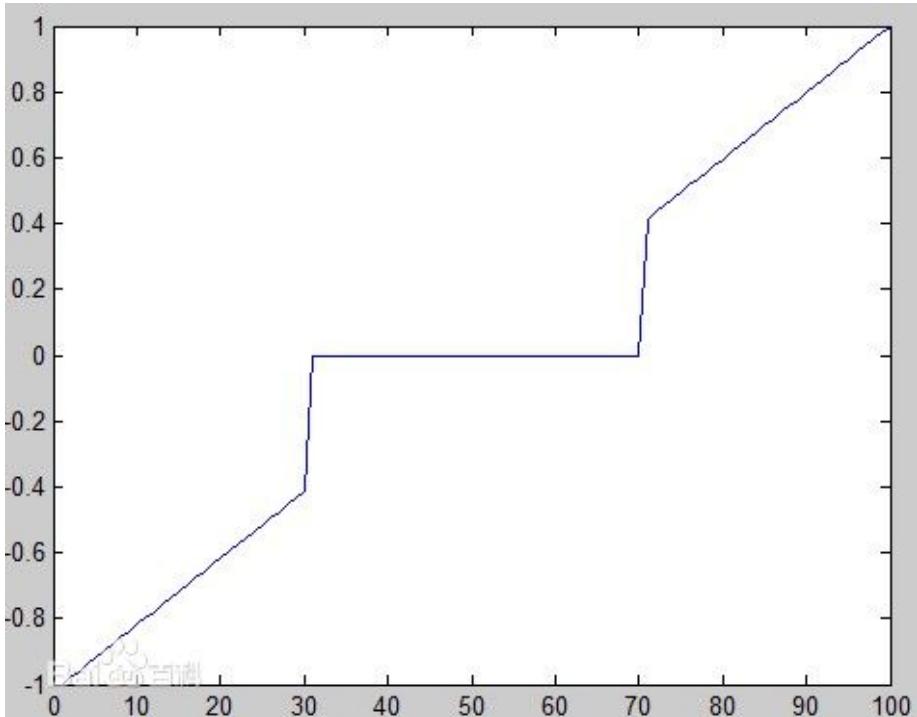
# 应用: 基于小波分解的图像去噪

- 小波变换系数的去噪操作

Hard Thresholding (硬阈值函数):

$$t_{hard}(x) = \begin{cases} x & \text{if } |x| < \tau \\ 0 & \text{else} \end{cases}$$

$$t_{hard}(x) = \arg \min_y \{ \|y - x\|^2 + \lambda \|x\|_0 \}$$



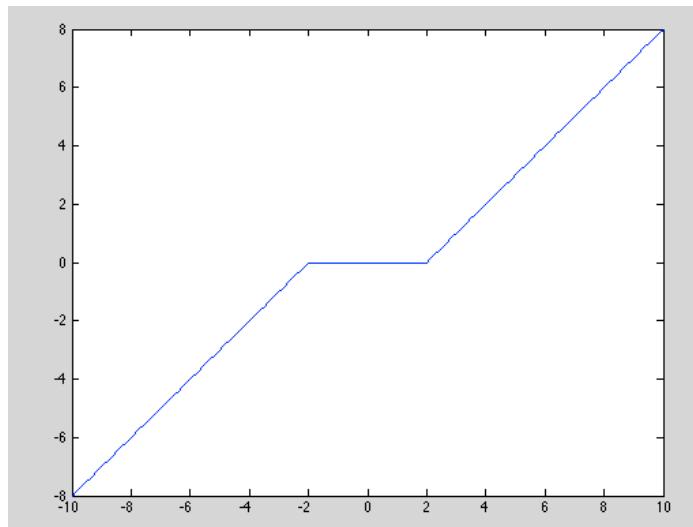
# 应用: 基于小波分解的图像去噪

Soft Thresholding (软阈值函数):

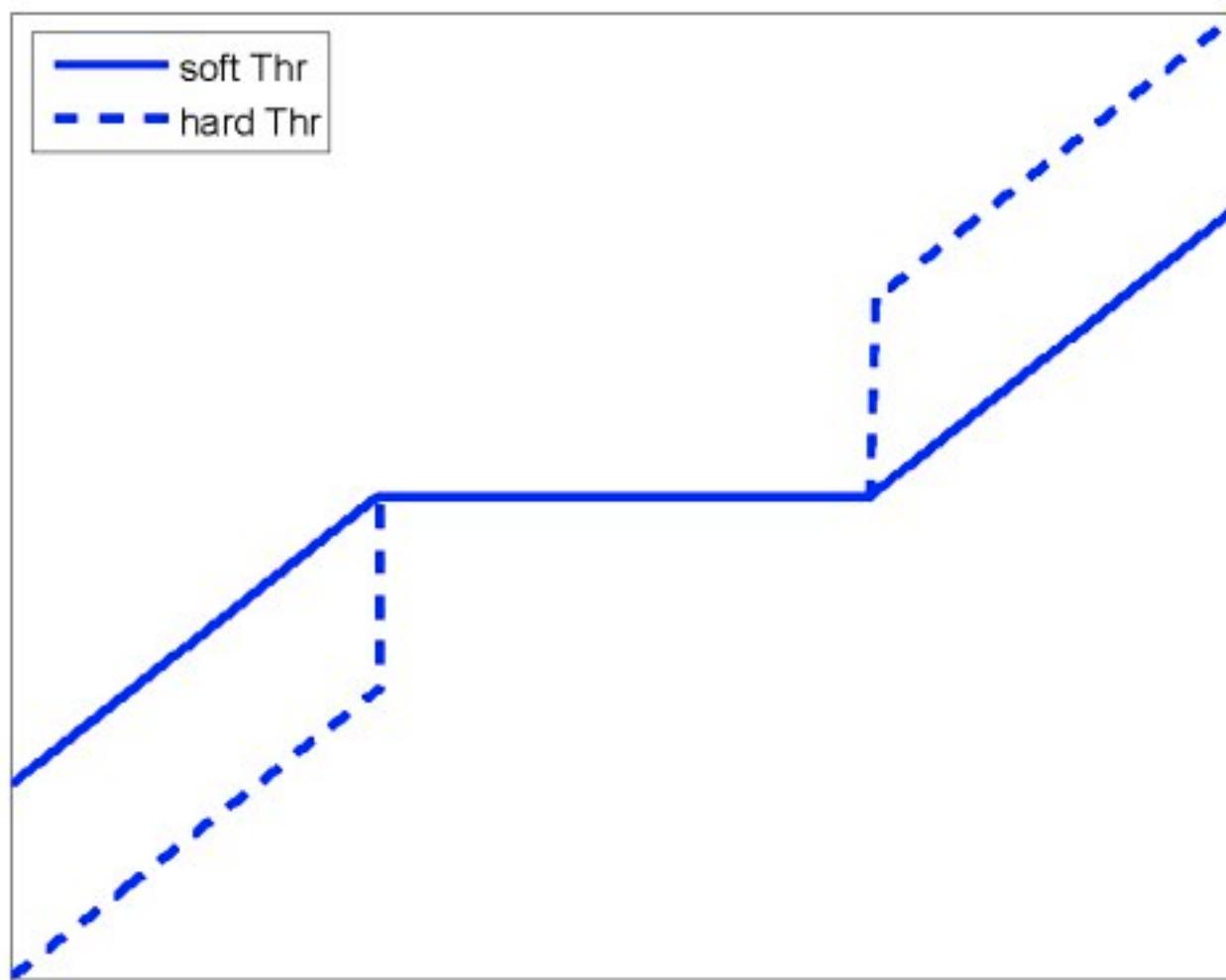
$$t_{soft}(x) = \begin{cases} x - \tau & \text{if } x > \tau \\ x + \tau & \text{if } x < -\tau \\ 0 & \text{else} \end{cases}$$

$$t_{soft}(x) = \arg \min_y \{ \|y - x\|^2 + \lambda \|x\|_1 \}$$

$$\tau = \sqrt{2\lambda}$$



DL Donoho, [De-noising by soft-thresholding](#), Information Theory, IEEE Transactions on, 1995

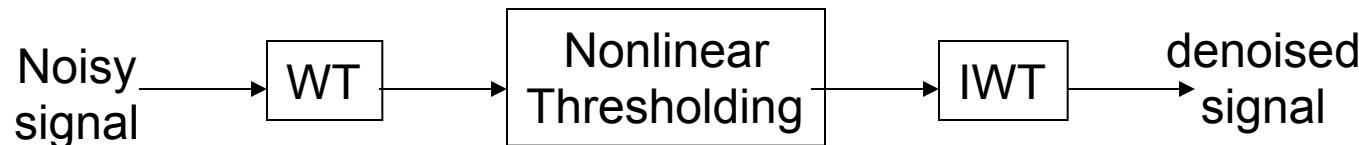


# 应用: 基于小波分解的图像去噪

- 如何选取阈值

$$\tau = \sigma \sqrt{2 \log M}$$

- $M$ 为像素个数,  $\sigma$ 为噪声方差.
- 阈值越高, 图像越光滑.



去噪流程图

## Hard Thresholding



## Soft Thresholding



# 应用: 基于小波分解的图像去噪

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SUREShrink: 在不同的小波域带（sub-band）中设置不同阈值

- 通过极小化风险无偏估计选取阈值

$$SURE(t; X) = d - 2 \# \{i : |X_i| \leq t\} + \sum_{i=1}^d \min(|X_i|, t)^2$$

- 去噪流程: 对图像进行小波变换, 在不同子带中估计阈值并进行小波系数阈值化处理(硬 / 软阈值化), 对变换后的小波系数进行逆变换得到去噪后的图像。

# 应用: 基于小波分解的图像去噪

- 基于小波系数统计建模的图像去噪算法
  - 将图像的小波系数的分布建模为高斯尺度混合函数(GSM: Gaussian Scale Mixture model)

$$\begin{aligned} p_{\mathbf{x}}(\mathbf{x}) &= \int p(\mathbf{x}|z)p_z(z)dz \\ &= \int \frac{\exp\left(\frac{-\mathbf{x}^T(z\mathbf{C}_u)^{-1}\mathbf{x}}{2}\right)}{(2\pi)^{N/2}|z\mathbf{C}_u|^{1/2}} p_z(z)dz \end{aligned}$$

[参考文献] :

IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 12, NO. 11, NOVEMBER 2003

## Image Denoising Using Scale Mixtures of Gaussians in the Wavelet Domain

Javier Portilla, Vasily Strela, Martin J. Wainwright, and Eero P. Simoncelli

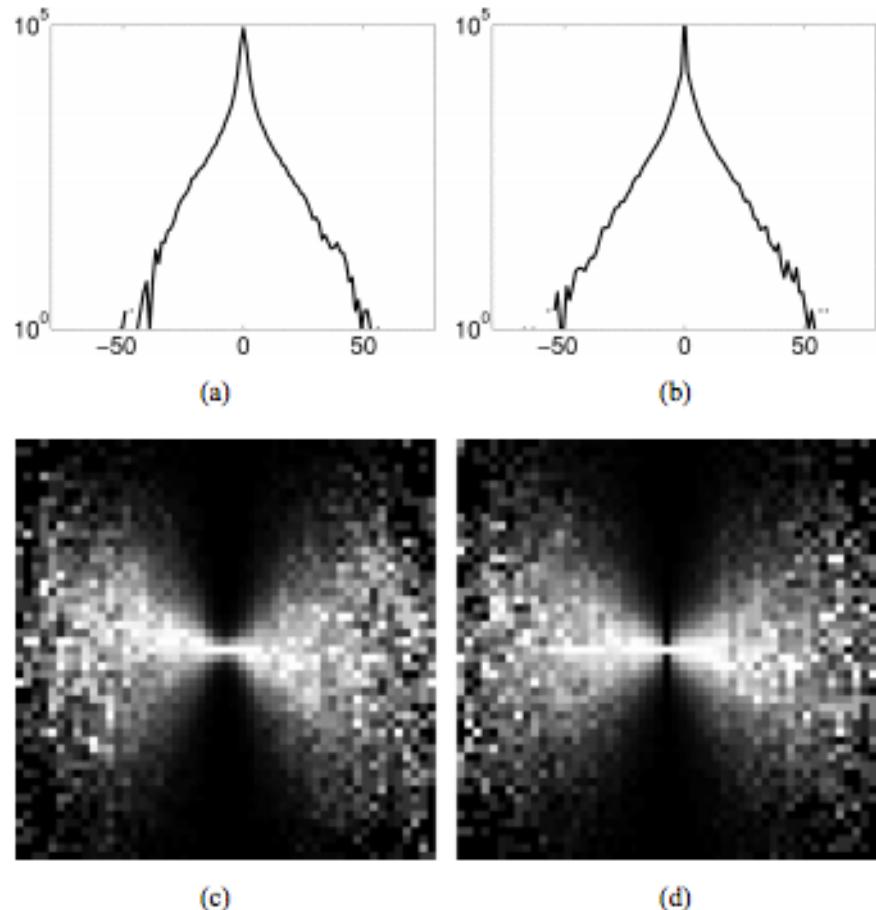


Fig. 1. Comparison of coefficient statistics from an example image subband (a vertical subband of the *Boats* image, left panels) with those arising from simulation of a local GSM model (right panels). Model parameters (covariance matrix and the multiplier prior density) are estimated by maximizing the likelihood of the observed set of wavelet coefficients. (a,b) Log marginal histograms. (c,d) Conditional histograms of two spatially adjacent coefficients. Brightness corresponds to probability, except that each column has been independently rescaled to fill the range of display intensities.

# 应用: 基于小波分解的图像去噪

- 小波系数的观测噪声模型 (图像)

$$\mathbf{y} = \mathbf{x} + \mathbf{w} = \sqrt{z}\mathbf{u} + \mathbf{w}, \quad p(\mathbf{y}|z) = \frac{\exp\left(\frac{-\mathbf{y}^T(z\mathbf{C}_u + \mathbf{C}_w)^{-1}\mathbf{y}}{2}\right)}{\sqrt{(2\pi)^N|z\mathbf{C}_u + \mathbf{C}_w|}}.$$

$$\mathbb{E}\{x_c|\mathbf{y}, z\} = \sum_{n=1}^N \frac{zm_{cn}\lambda_nv_n}{z\lambda_n + 1}$$

$$p(\mathbf{y}|z) = \frac{\exp\left(-\frac{1}{2} \sum_{n=1}^N \frac{v_n^2}{z\lambda_n + 1}\right)}{\sqrt{(2\pi)^N |\mathbf{C}_w| \prod_{n=1}^N (z\lambda_n + 1)}}.$$

## 算法流程:

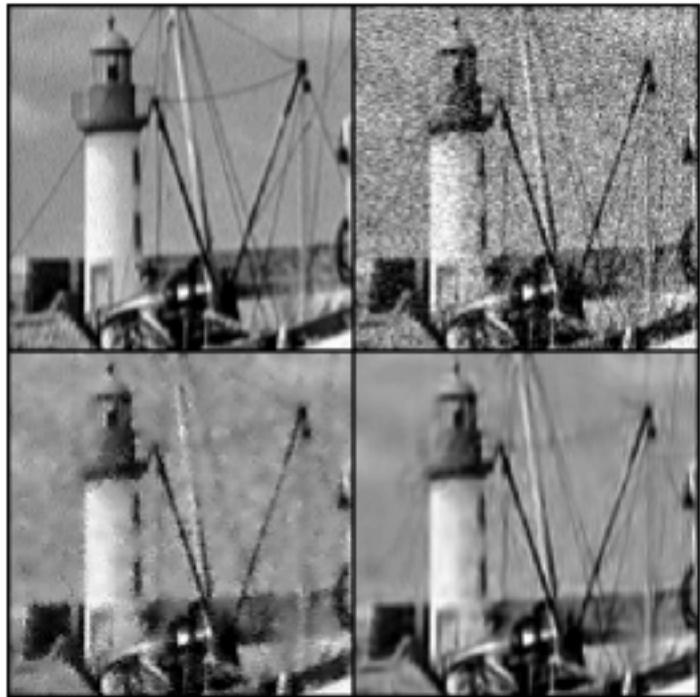
$$\mathbb{E}\{x_c|\mathbf{y}, z\} = \sum_{n=1}^N \frac{zm_{cn}\lambda_nv_n}{z\lambda_n + 1} \quad (12)$$

$$p(\mathbf{y}|z) = \frac{\exp\left(-\frac{1}{2} \sum_{n=1}^N \frac{v_n^2}{z\lambda_n + 1}\right)}{\sqrt{(2\pi)^N |\mathbf{C}_{wl}| \prod_{n=1}^N (z\lambda_n + 1)}}. \quad (14)$$

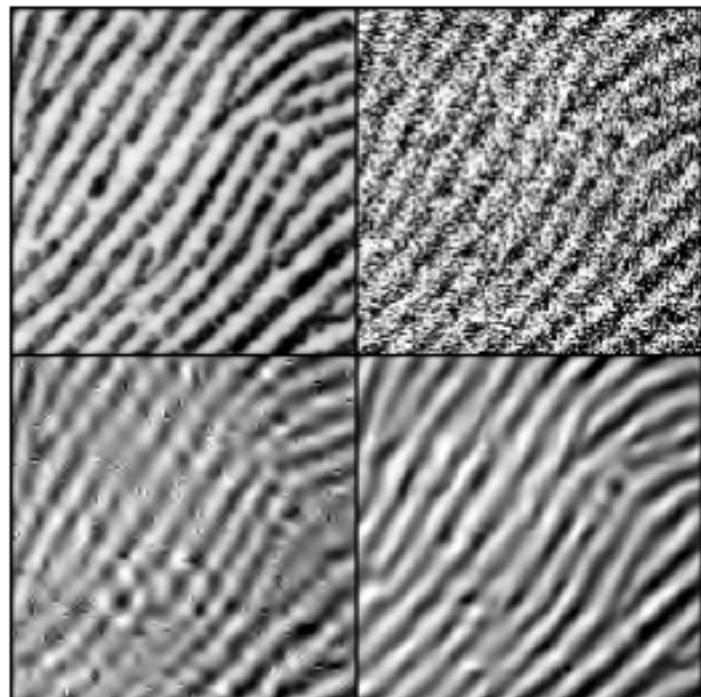
$$p(z|\mathbf{y}) = \frac{p(\mathbf{y}|z)p_z(z)}{\int_0^\infty p(\mathbf{y}|\alpha)p_z(\alpha) d\alpha}. \quad (13)$$

$$\begin{aligned} \mathbb{E}\{x_c|\mathbf{y}\} &= \int x_c p(x_c|\mathbf{y}) dx_c \\ &= \int \int_0^\infty x_c p(x_c, z|\mathbf{y}) dz dx_c \\ &= \int \int_0^\infty x_c p(x_c|\mathbf{y}, z) p(z|\mathbf{y}) dz dx_c \\ &= \int_0^\infty p(z|\mathbf{y}) \mathbb{E}\{x_c|\mathbf{y}, z\} dz \end{aligned} \quad (8)$$

- 1) Decompose the image into subbands.
- 2) For each subband (except the lowpass residual):
  - a) Compute neighborhood noise covariance,  $\mathbf{C}_w$ , from the image-domain noise covariance.
  - b) Estimate noisy neighborhood covariance,  $\mathbf{C}_y$ .
  - c) Estimate  $\mathbf{C}_u$  from  $\mathbf{C}_w$  and  $\mathbf{C}_y$  using (7).
  - d) Compute  $\mathbf{\Lambda}$  and  $\mathbf{M}$  (**Section III-B**).
  - e) For each neighborhood:
    - i) For each value  $z$  in the integration range:
      - A) Compute  $\mathbb{E}\{x_c|\mathbf{y}, z\}$  using (12).
      - B) Compute  $p(\mathbf{y}|z)$  using (14).
      - ii) Compute  $p(z|\mathbf{y})$  using (13) and (4).
      - iii) Compute  $\mathbb{E}\{x_c|\mathbf{y}\}$  numerically using (8).
  - 3) Reconstruct the denoised image from the processed subbands and the lowpass residual.



(a)

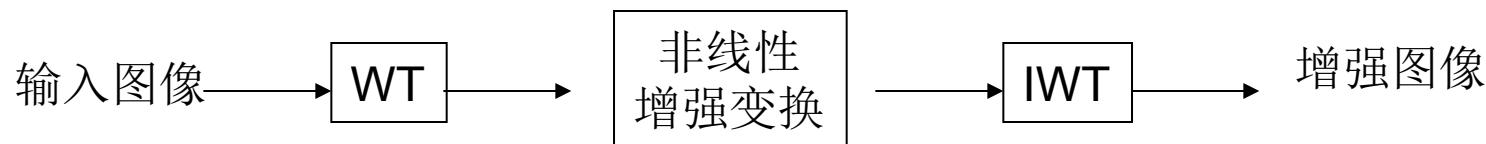


# 应用: 基于小波分解的图像去噪



# 应用: 基于小波分解的图像去噪

- 基于小波变换的图像增强算法:
  - 在小波变换域中，对变换系数做非线性增强变换，使得小的系数更小，大的系数更大



增强流程图

# 应用: 基于小波分解的图像增强

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图像增强实例图



(a) Original Image

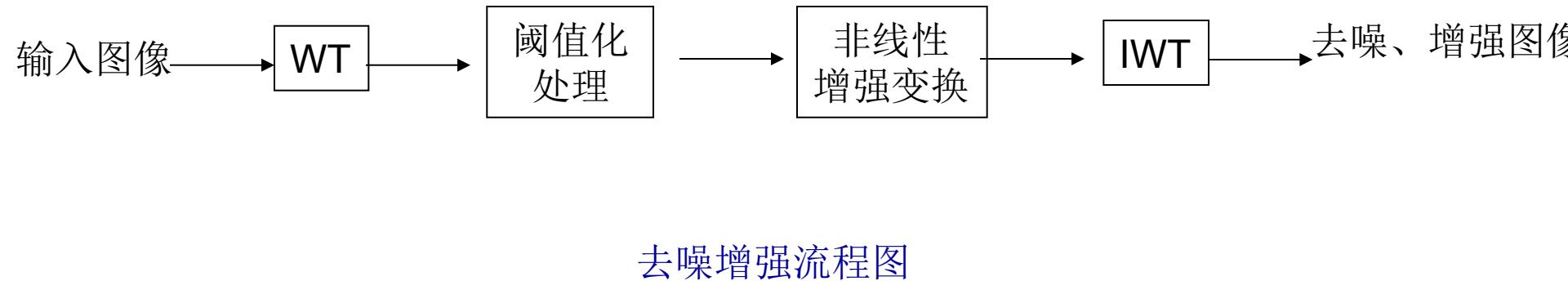


(c) Proposed Method

# 应用: 基于小波分解的图像去噪、增强

- 小波域内同时去噪和增强:

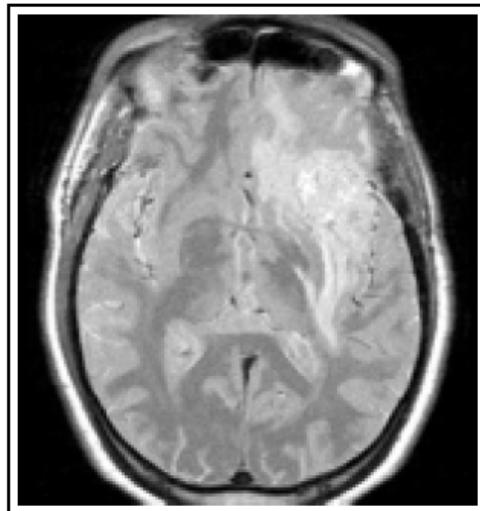
- 在小波变换域中，对变换系数先做**阈值化**处理，然后做非线性增强变换，使得小的系数更小，大的系数更大。



# 应用: 基于小波分解的图像去噪、增强

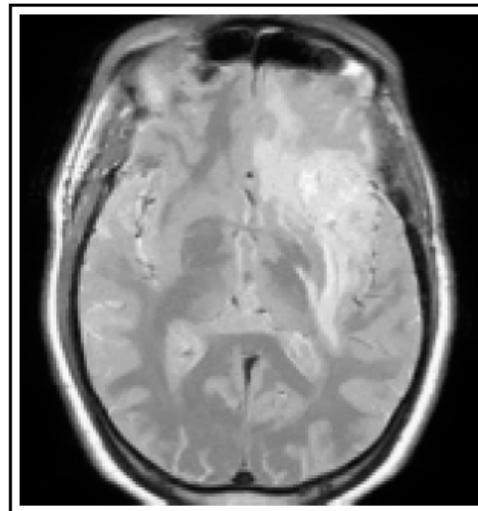
---

原始图像



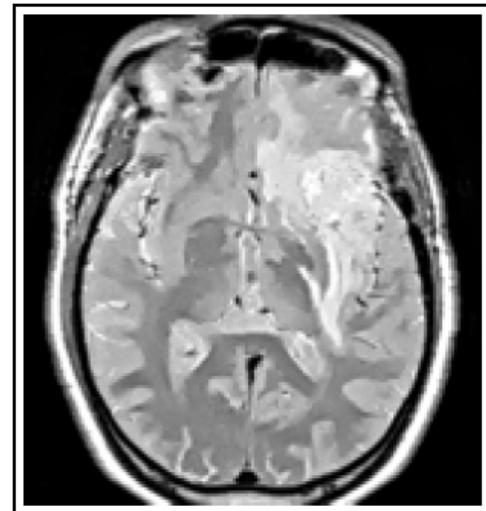
(a)

去噪图像



(b)

同时去噪和增强图像



(c)

# 应用: 基于小波分解的图像融合

- 小波域内图像融合:

- 分别将要融合的图像进行小波变换。
- 在小波变换域中，不同图像的相同小波子带系数进行融合。
- 对融合的图像进行逆小波变换得到融合后图像。



(a) Visible image



(b) Infrared image

“Multisensor Image Fusion using the Wavelet Transform,” by H. Li, B.S. Manjunath, and S.K. Mitra; *Graphical Models and Image Processing*, May 1999.

# 应用: 基于小波分解的图像融合

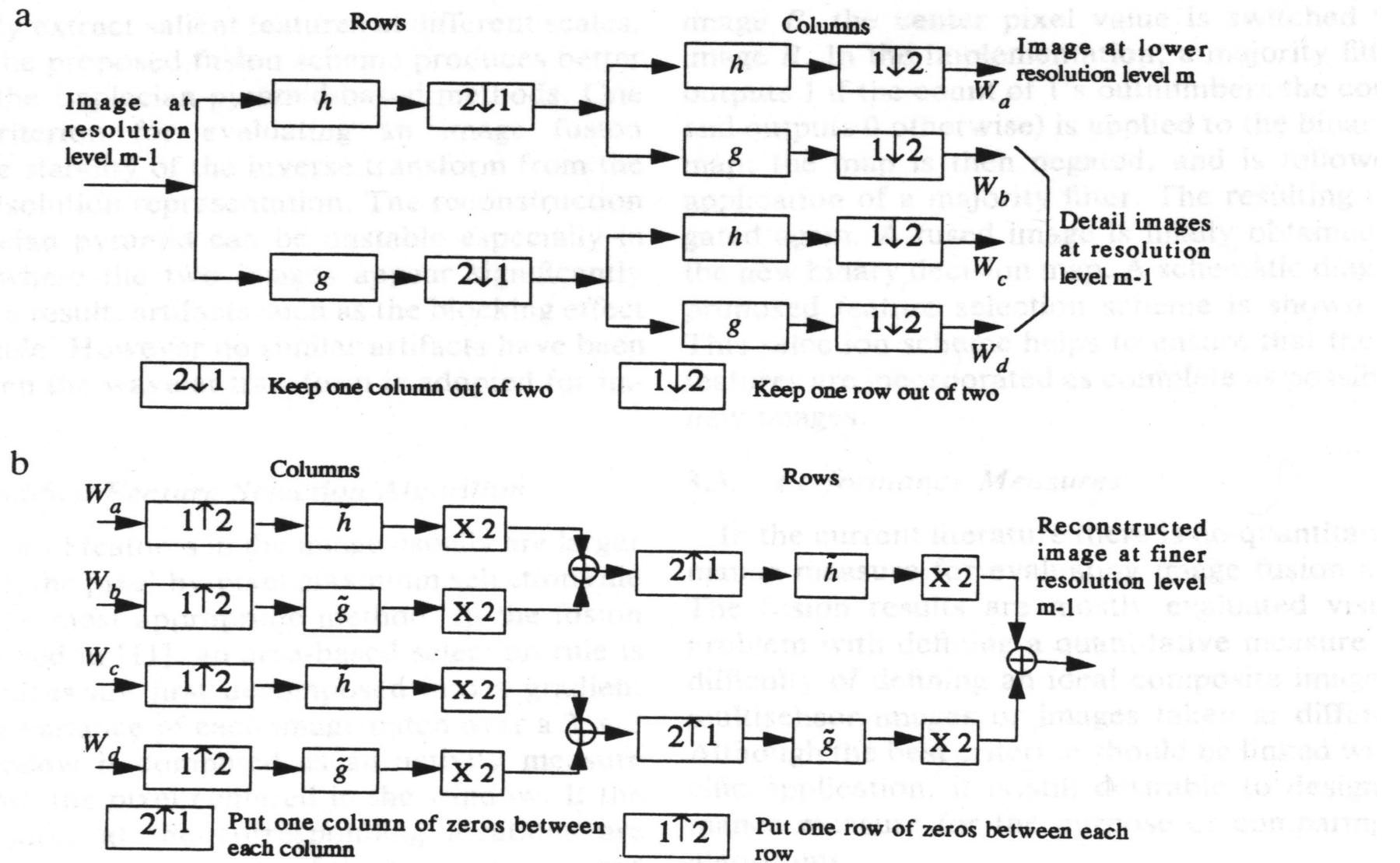


FIG. 2. (a) One stage in a multiresolution image decomposition. (b) One stage in a multistage image reconstruction.

# 应用: 基于小波分解的图像融合

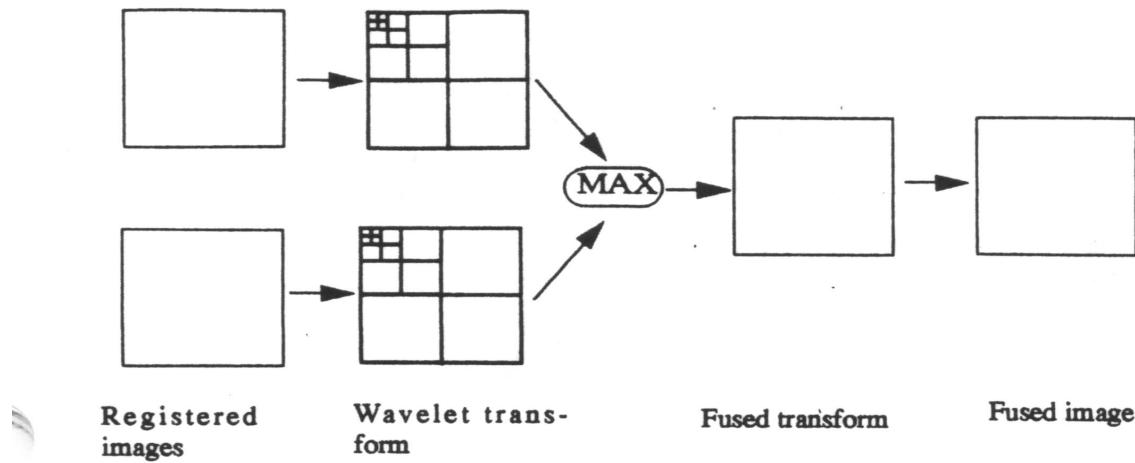


FIG. 3. Schematic diagram for the basic image fusion scheme.

融合策略（极大准则）：

$$c_{ij}^{fusion} = \max\{c_{i,j}(I_1); c_{i,j}(I_2)\}$$

融合策略（代权平均准则）：

$$c_{ij}^{fusion} = w c_{i,j}(I_1) + (1-w) c_{i,j}(I_2)$$

# 应用: 基于小波分解的图像融合



输入图像1



输入图像2



融合图像

注: 采用3层小波分解, 使用DB2 小波

# 应用: 基于小波分解的图像融合

I/P 1



I/P 2



Orig-  
inal



O/P





(a) Visible image



(b) Infrared image



(f) DWT fusion



(g) Gradient pyramid fusion



(h) CWT fusion



(i) NSCT fusion

# 作业

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- 用matlab尝试生成高斯金字塔和拉普拉斯金字塔
- 用matlab小波工具箱，尝试图像的离散小波变换，并进行图像去噪