Planck Constant from LOG

1. Starting Point — LOG Tilted Path Measure

LOG defines tilted weights on path space:

$$P^*(\omega) \propto P(\omega)\, e^{\epsilon\, \Omega[\omega]}$$

where:

- $P(\omega)$: base stochastic path measure (diffusion process).
- $\Omega[\omega]$: semantic functional (predictive information / recordability).
- ϵ: small tilt parameter.

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2. Stochastic Mechanics Link

In Nelson's stochastic mechanics, a particle's motion is modeled as Brownian diffusion with variance:

$$\langle (\Delta x)^2 \rangle = 2D\, \Delta t,$$

where D is diffusion coefficient.

Schrödinger's equation emerges if we set:

$$D = \frac{\hbar}{2m}.$$

So LOG must explain **why diffusion constant scales with $\hbar/m$**.

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3. LOG Variational Functional

We introduce LOG's cost functional:

$$\mathcal{J}[P] = \mathbb{E} \int \Big[\tfrac{1}{2} \|u\|^2 + \alpha\sigma - \epsilon\psi \Big] dt,$$

with terms:

- $\|u\|^2$: kinetic energy cost.
- σ: entropy production.
- ψ: semantic reward.

Adding Fisher information $\mathcal{I}[\rho]$ as a regularizer:

$$\mathcal{I}[\rho] = \int \frac{|\nabla \rho|^2}{\rho}\, dx,$$

weighted by a constant κ.

So total functional:

$$\mathcal{J}_{LOG} = \mathcal{J} + \kappa \mathcal{I}.$$

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4. Euler–Lagrange Equations → Schrödinger Bridge

Minimizing $\mathcal{J}_{LOG}$ yields coupled forward/backward diffusions (Schrödinger bridge). The "quantum potential" term appears:

$$Q(x) = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

From LOG side, this emerges with coefficient:

$$Q_{LOG}(x) = -\frac{2\kappa}{m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

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5. Identification of $\hbar$

Matching the LOG coefficient to the standard quantum potential:

$$\frac{\hbar^2}{2m} \quad\longleftrightarrow\quad \frac{2\kappa}{m}.$$

So:

$$\hbar^2 = 4\kappa.$$

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6. Interpretation

- κ is the **semantic curvature penalty** — the cost of having "wiggly" probability amplitudes.
- LOG says: the universe penalizes information-geometric curvature, and the conversion constant between this semantic cost and physical phase is $\hbar$.
- Thus Planck's constant is no longer fundamental "given," but emerges as the **scaling between semantic curvature and physical action**.

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7. Dimensional Check

- Fisher information density: $[\nabla\rho]^2/\rho \sim L^{-2}$.
- Multiply by κ: dimension of energy × time (action).
- Thus $\kappa \sim [\hbar^2]$.
- Identification $\hbar^2 = 4\kappa$ is dimensionally consistent.

Synthesis

- LOG tilt + Fisher regularization → Schrödinger bridge equations.
- The emergent coefficient of the quantum potential defines $\hbar$.
- $\hbar$ is the **conversion factor between semantic curvature and dynamical phase accumulation**.
- This explains *why time is phase* and *why phase cycles are quantized in Planck units*.