Solution 1:

Let V_t^j be the probability of the most probable path of the symbol sequence

$x_1, x_2, \ldots, x_t$ ending in state j, then:

$$V_{t+1}^j = b_j(x_{t+1}) \underset{i}{max}(V_t^i a_{ij})$$

For the matrix V_t^j , where $j \in S$ and $1 \leq t \leq n$.

Iteration:$V_t^j = b_j(x_t) \underset{i}{max}(V_{i-1}^i * a_{ij})$ for all states $i, j \in S, t \geq 2$.

So we have the initial table as following:

Initialization:

$V_1^j = b_j(x_1)p(q_1 = j)$, $V_1^j = \frac{b_j(A)}{state}$

$V_2^j = b_j(C) \underset{i}{max}(V_1^i a_{ij})$

$V_3^j = b_j(B) \underset{i}{max}(V_3^i a_{ij})$

From the title, we can get the information of status transition matrix and states generated matrix:

$A = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0 & 0.2 & 0.8 \end{pmatrix}$,$B = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0 & 0.1 & 0.9 \end{pmatrix}$.

	A	C	B	A	C
good	$\frac{1}{3} * 0.7$				
neutral	$\frac{1}{3} * 0.3$				
bad	$\frac{1}{3} * 0$				

and then, further we have

	A	C	B	A	C
good	0.23	$0.23 * 0.2 * 0.1$			
neutral	0.1	$0.3 * 0.23 * 0.3$			
bad	0	$0.9 * 0.23 * 0.5$			

and then, further we have

	A	C	B	A	C
good	0.23	0.0046	$0.2 * 0.0207 * 0.2$		
neutral	0.1	0.0207	$0.4 * 0.1035 * 0.2$		
bad	0	0.1035	$0.1 * 0.1035 * 0.8$		

and then, further we have

	A	C	B	A	C
good	0.23	0.0046	0.000828	$0.7 * 0.00828 * 0.2$	
neutral	0.1	0.0207	0.00828	$0.3 * 0.00828 * 0.2$	
bad	0	0.1035	0.00828	0	

and then, further we have

	A	C	B	A	C
good	0.23	0.0046	0.000828	0.0011592	$0.1 * 0.0011592 * 0.2$
neutral	0.1	0.0207	0.00828	0.0004968	$0.3 * 0.0011592 * 0.3$
bad	0	0.1035	0.00828	0	$0.9 * 0.0011592 * 0.5$

so the result table is following:

	A	C	B	A	C
good	**0.23**	0.0046	0.000828	**0.0011592**	0.000023184
neutral	0.1	0.0207	**0.00828**	0.0004968	0.000104328
bad	0	**0.1035**	0.00828	0	**0.00052164**

So most probable mood curve is following:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Assignment	A	C	B	A	C
Mood	good	bad	neutral	good	bad

Solution 2:

For each data point x, we can introduce a latent variable $Y_i \in 1, 2, \ldots, m$ denoting the component that point belongs to. For the E-step, we compute the posterior over the classes and have to normalize:

$$V_j(x_i) = \frac{\pi_j f_L(x_i; \mu_j, \beta_j)}{\sum_{l=1}^{m} \pi_l f_L(x_l; \mu_l, \beta_l)}.$$

In the M-step, we optimize:

$$
\begin{aligned}
\sum_{i=1}^{n}\sum_{j=1}^{m} V_j(x_i)\log P(x_i, y_i=j) &= \sum_{i=1}^{n}\sum_{j=1}^{m} V_j(x_i)\log \pi_j f_L(x_i;\mu_j,\beta_j) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{m} V_j(x_i)(\log \pi_j + \log \frac{1}{2\beta_j} e^{-\frac{1}{\beta_j}|x-\mu|}) \\
&= \sum_{i=1}^{n}\sum_{j=1}^{m} V_j(x_i)(\log \pi_j - \frac{1}{\beta_j}|x_i-\mu_j|) + C
\end{aligned}
\tag{1}
$$

And then, we add a Lagrange multiplier λ to make that $\sum_{j=1}^{m}\pi_j = 1$ and get the Lagrangian:

$$
L(\pi,\mu,\lambda) = \sum_{i=1}^{n}\sum_{j=1}^{m} V_j(x_i)(\log \pi_j - \frac{1}{\beta_j}|x_i-\mu_j|) + \lambda(\sum_{j=1}\pi_j - 1).
$$

Exactly as in the previous problem, by setting the gradient with respect to π_j to zero, we get

$$
\begin{aligned}
\frac{\partial}{\partial \pi_j} L(\pi,\mu,\lambda) &= \frac{\sum_{i=1}^{n} V_j(x_i)}{\pi_j} + \lambda = 0 \\
\implies \pi_j &= \frac{\sum_{i=1}^{n} V_j(x_i)}{-\lambda}
\end{aligned}
$$

The multiplier is again equal to $\lambda = -n$. If we want yo maximize (1) with respect to the variables μ_j, we have to solve m separate optimization problems, one for each μ_j, These m problems have the following form:

$$
\underset{\mu_j}{maximize} - \sum_{i=1}^{n} \frac{V_j(x_i)}{\beta_j}|x_i-\mu_j|
$$

These are one-dimensional convex optimization problems. While one can try soling this via an iterative process lick subgradient descent, a direct approach is also possible if we observe that function is piecewise linear and the breakpoints ara $x_1, x_2, \ldots x_n$. Hence,the optimum must be attained at one of these n points and we can simply set μ_j to the point x_i with the largest objective value.