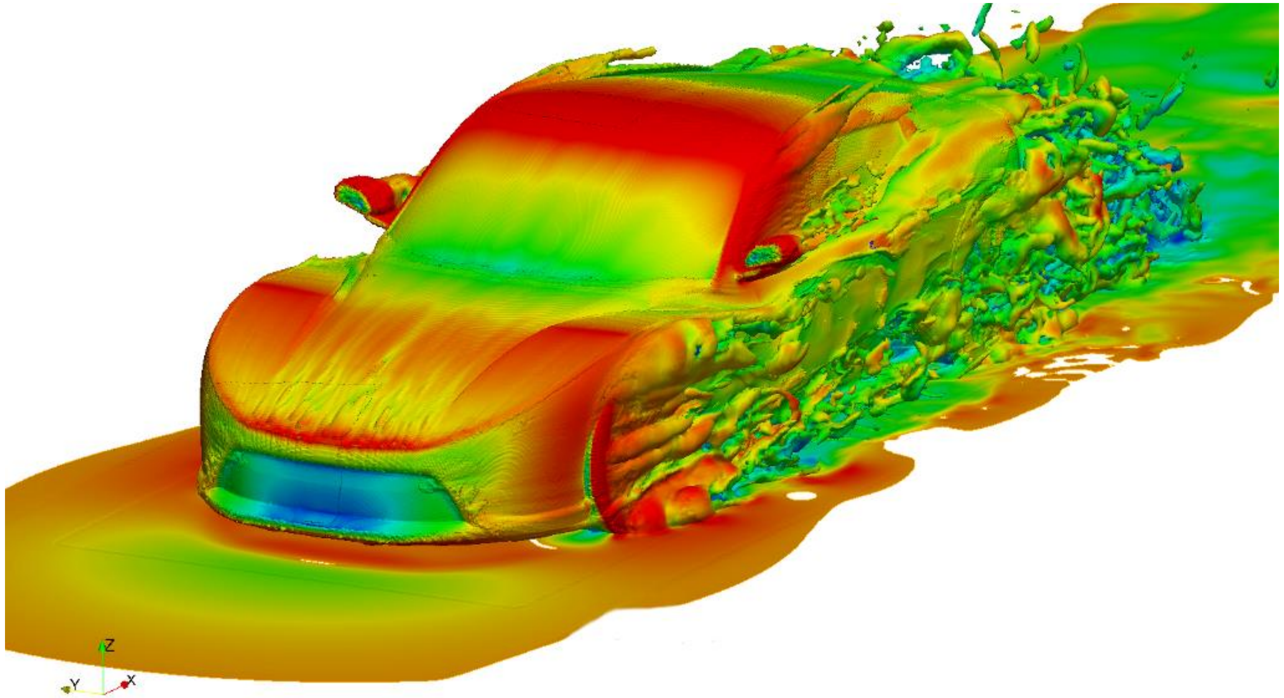
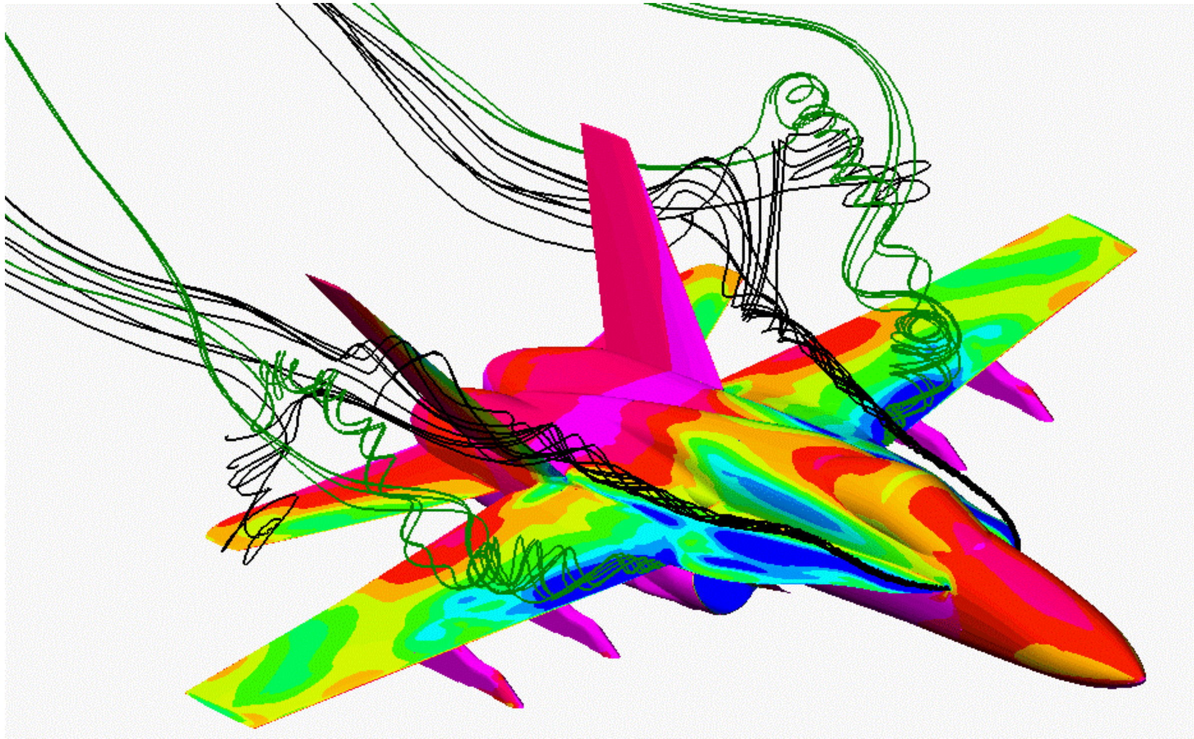
Modeling and Simulation

Week# 01& 02
Introductory Lectures





Summarized Contents of the Course

- **Introduction to Modeling and Simulation**
  - ✓ Importance of the Course
  - ✓ Fundamental Terminologies (Would also be helpful in Control System-1)
  - ✓ Insights for Modeling and Simulation.
- **Models for the Dynamic Systems**
  - ✓ Formulation of the models (Certain Physical Laws).
  - ✓ Transfer Function & State Space Representation **(Most Important Content).**
  - ✓ Different Test Signals for any ***Physical Systems*** (Beneficial For Control System-1).
  - ✓ Engineering Systems Similarities.
- **Modeling of Mechanical Systems**
  - ✓ Translational Systems
  - ✓ Rotational Systems
  - ✓ Combined System(Trans.+Rot.).

What about Analysis of the Afore-mentioned Systems? (Helpful Material for CS-1)

Cont..

- **Modeling of Electrical Systems**
  - ✓ Basic Electrical Elements **(Already Studied in Electric Circuits)**
  - ✓ Passive Electrical Systems
  - ✓ Active Electrical Systems
- **Modeling of Fluidic Systems**
  - ✓ Properties of Fluids **(Fundamentals from Hydraulics & Pneumatics)**
  - ✓ What about modeling of Water Level Tanks?
- **Modeling of Thermal Systems**
  - ✓ Basic Effects such Convection, Conduction, and Radiation
  - ✓ Utilizing above-mentioned effects to model the thermal system.
- **Mixed Discipline Systems.**
  - ✓ Electromechanical Systems

Cont..

- **System Response and Analysis.**
  - ✓ Time Response (Laplace Transform).
  - ✓ Frequency Response(Modern Control).
- **Methods of Simulation**

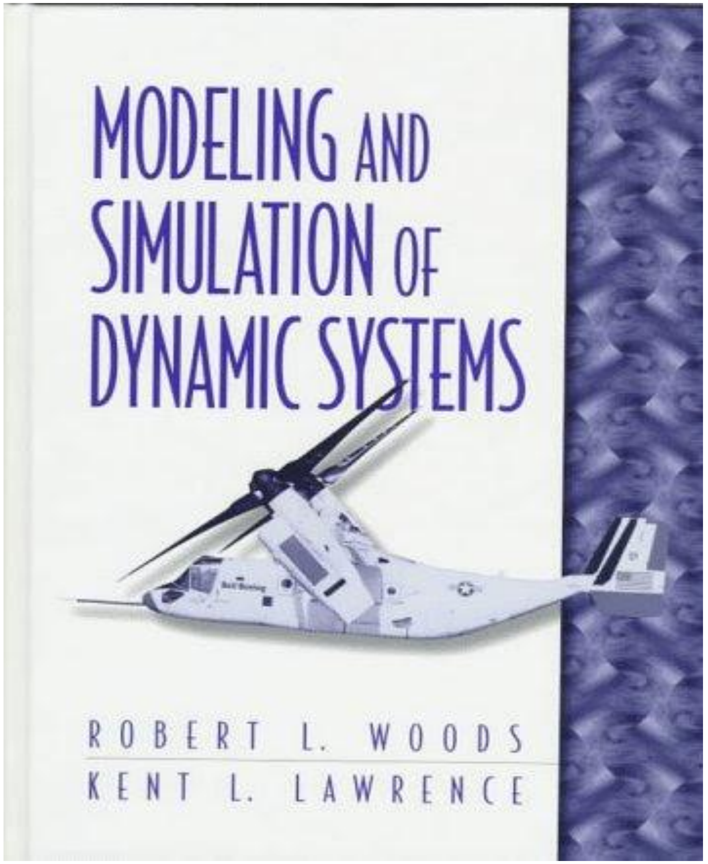
Additional Requirements of the Course

Apart from the advantageous nature of the Washington accord accreditation, CLOS can also help you to crack the exams.

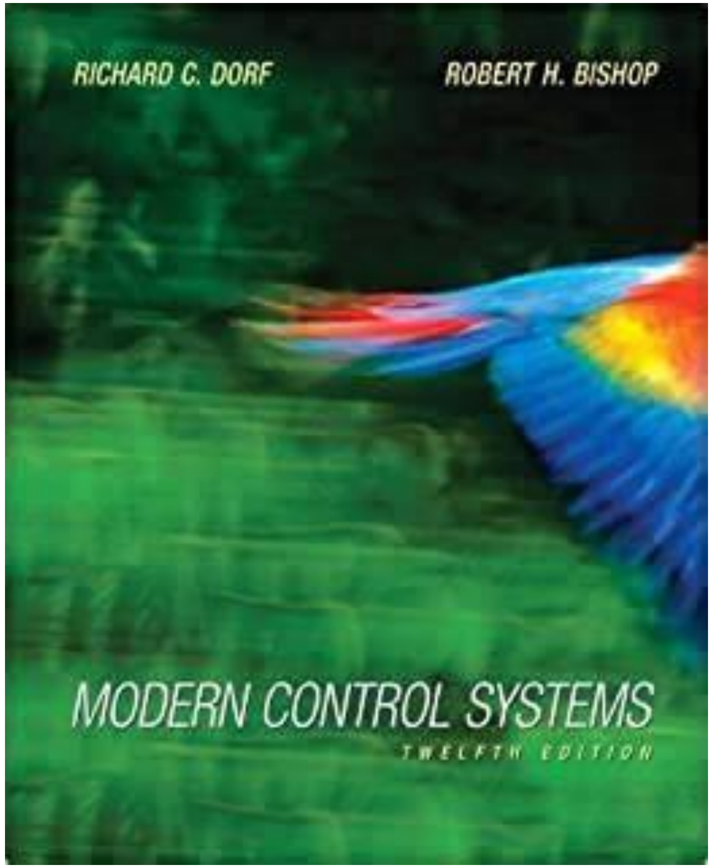
After successful completion of this course the students will be able to		PLOs
CLO 1	**Explain** the fundamentals of system modeling including system states, inputs, outputs and representations.	PLO1
CLO 2	**Apply** basic principles to model various types of mechanical components and systems	PLO3
CLO 3	**Apply** basic modeling principles to represent and simulate different electrical components and systems.	PLO3
CLO 4	**Use** mixed disciplinary systems modelling techniques to analyze the response of fluidic, electromechanical and thermal systems.	PLO2
CLO 5	**Assess** the response of dynamical systems using differential equations.	PLO2
CLO 6	**Use** standard simulation tools (e.g. MATLAB/Simulink) to simulate and analyze systems response for various types of inputs.	PLO5

Recommended/Reference Books

- Modeling and Simulation of Dynamic Systems by Robert L Woods & Kent L. Lawrence



- Modern Control Systems by Richard C. Dorf & Robert H. Bishop



- Contents would also be outside of Ref Books

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Lecture Hands Out

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You have to attend the lectures.

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IF Attendance< 75%
{
Cout("Not Allowed to sit in the exam");}

Why Modeling & Simulation?

A Cantilever Beam for Structural Applications

Capable of Sustaining at least 20 N(Axially)

Design Verification: Material for the beam, Whether there is need of coating at the fixed end?

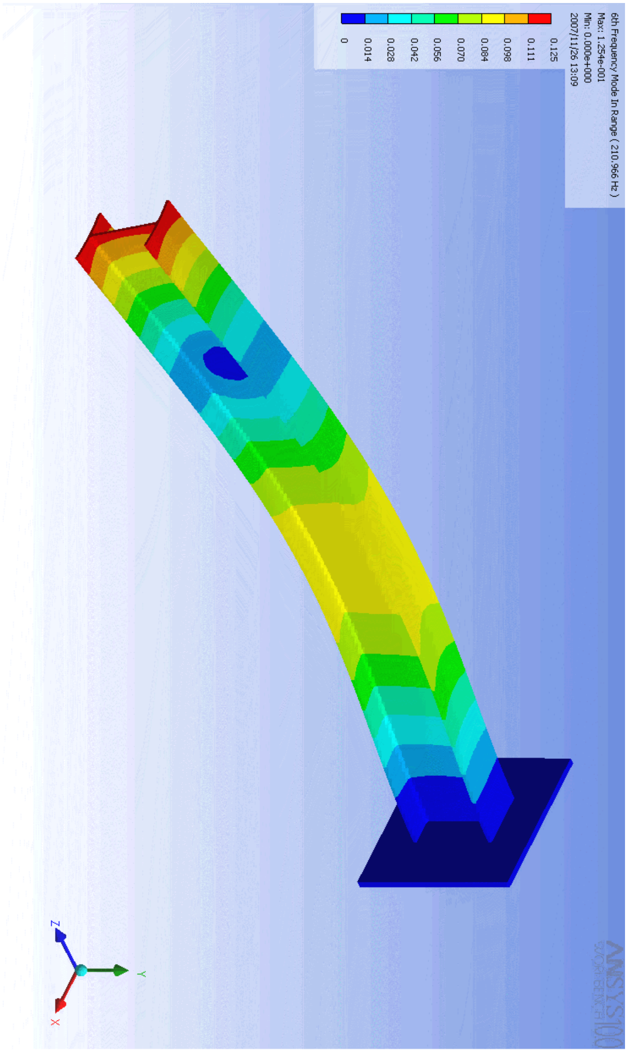
How Identifying the parameters of designed beam under the given conditions?: Modeling Step

Rendering of the defined parameters?: Simulation Step

Design Verified: Start Building the Beam

Design Refinements

Failed



Why Modeling & Simulation?

COBOT comprises of 7 DOF(Multiple Components)

Payload should not be less than 2Kg
Design should be flexible for multipurpose applications

Design Verification: Motors, Materials of Links, & End Effector ?

How each DOF will be obtained by defining Parameters?: Modeling Step

How Integration/togetherness of those parts perform the desired functions?: Simulation Step

Failed

Design Refinements

Design Verified: Start Building the COBOT

Why Modeling & Simulation?

Space Shuttle should be landed successfully on MARS!

A common Space Shuttle contains 2.5 million parts

↓

Acceleration from 0 Km/h to 28 e3 Km/h in ~ 9mins
Survival Temp= -150 C & 1600 C

↓

Why would NASA go without Design Verification?

↓

How each of 2.5 million parts would work efficiently?: Modeling Step

↓

How Integration/togetherness of those parts perform the desired functions?: Simulation Step

↓

Design Verified: Start Building the Shuttle

Failed

Design Refinements

System along with it’s Categories

- ✓ An interconnection of the components, elements, or objects to perform a specific task (Function) is known as **System** (Concisely, we can say anything that has input & Output is a system).
- ✓ A Series of actions or steps to achieve the above-defined function is known as **Process**.

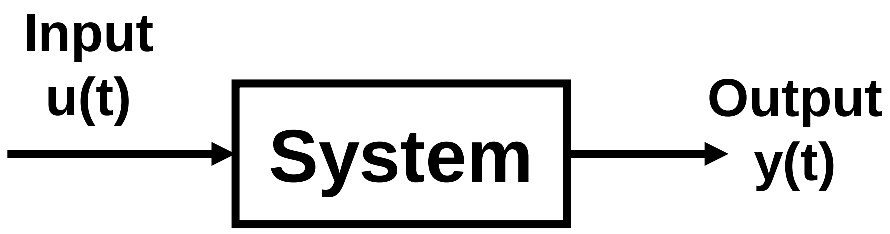
Categories of the System

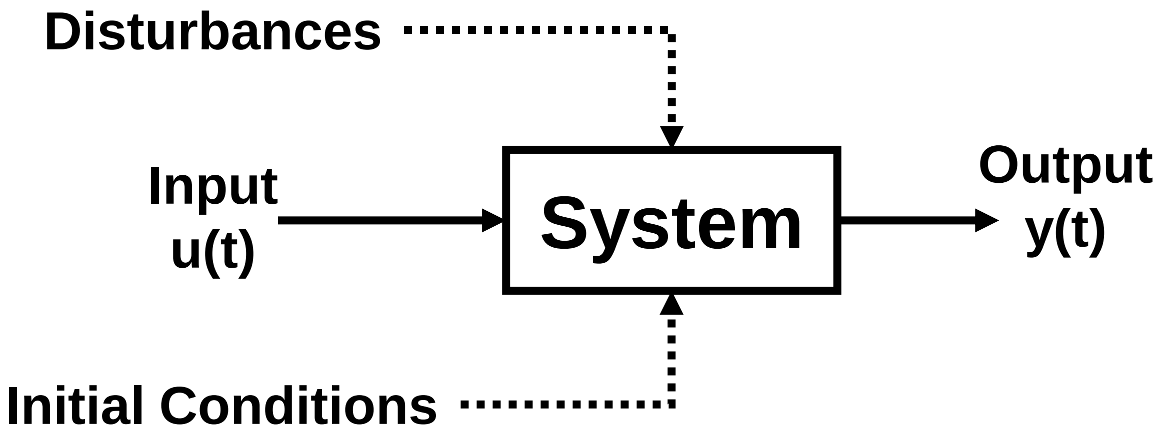
- **Chemical System**
  - Further comprises system and Surroundings. The solution in a beaker.
- **Physical System**
  - That can be modeled with the laws of Physics. For example, A Thermometer.
- **Biological System**
  - A Complex network of the biologically relevant entities. For Example, our digestive system

Human-made **physical systems** are called tools, appliances, devices, instruments, or event the gadgets.

Types of Physical System

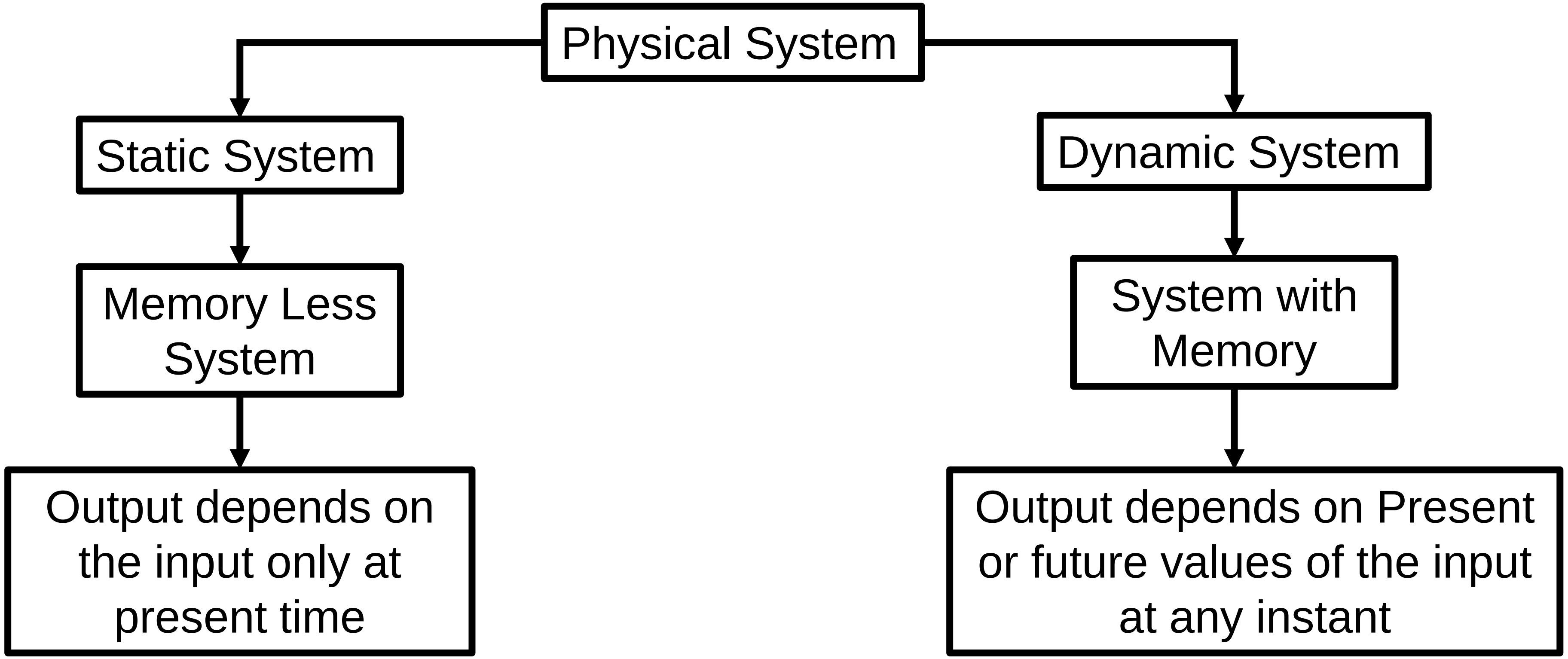
- ✓ A system in which the output response does not change with the time is known as **static system**. More concisely, in this type, system's current output does not depend on the past output.
- ✓ A system in which the output response changes with the time is known as **dynamic system**. A dynamic system can respond input, disturbance signal as well as even the initial conditions.





Throughout the course, **u(t)** will be used as notation of input and **y(t)** would be the output. This u(t) can be any signal as opposed to the assumptions of calculus (step function).

Insights for the System



Examples:

- $y(t)=u(3(t))$

Put t=1: $y(1)=u(3(1))=u(3)$

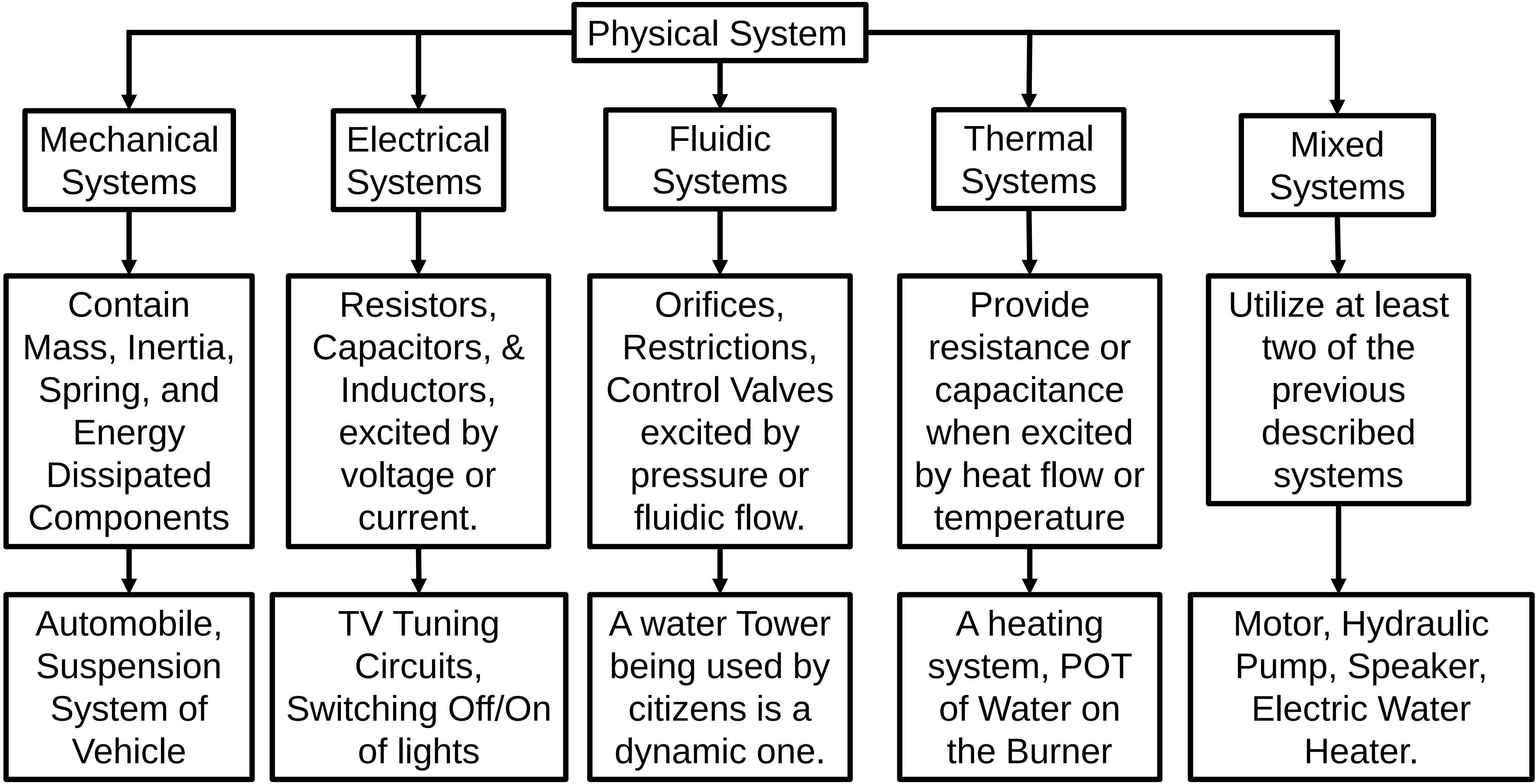
- $y(t)=u(t)$

Put t=1: $y(1)=u(1)=u(1)$

Which one is dynamic system?

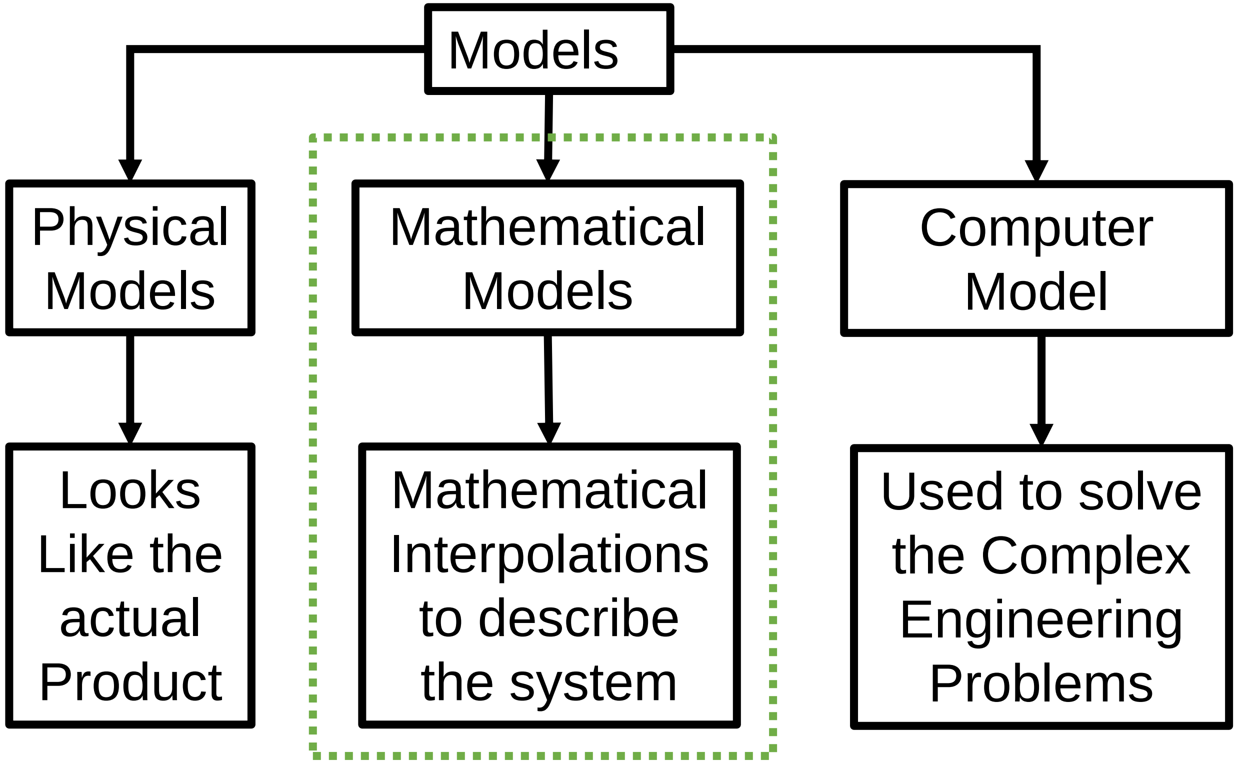
The association between the input and output defines the characteristics of the system.

Categories of the Physical System



Modeling

- ✓ Modeling is a process of creating a ‘model’ which represents an object or system with its all or subset of properties. A model may be exactly the same as the original system or sometimes approximations make it deviates from the real system.

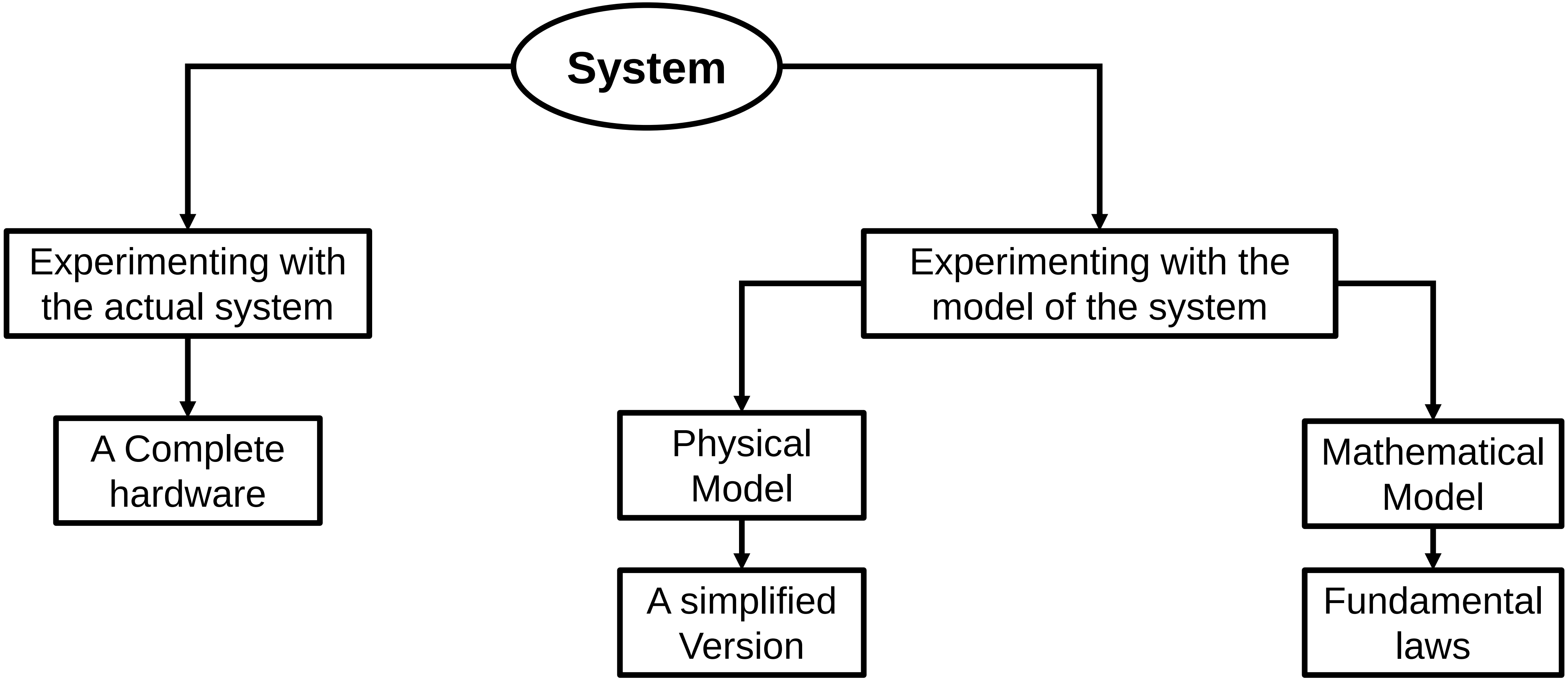


Should I assume that now you are able to define the process model?

https://www.differencebetween.com/difference-between-modelling-and-vs-simulation/

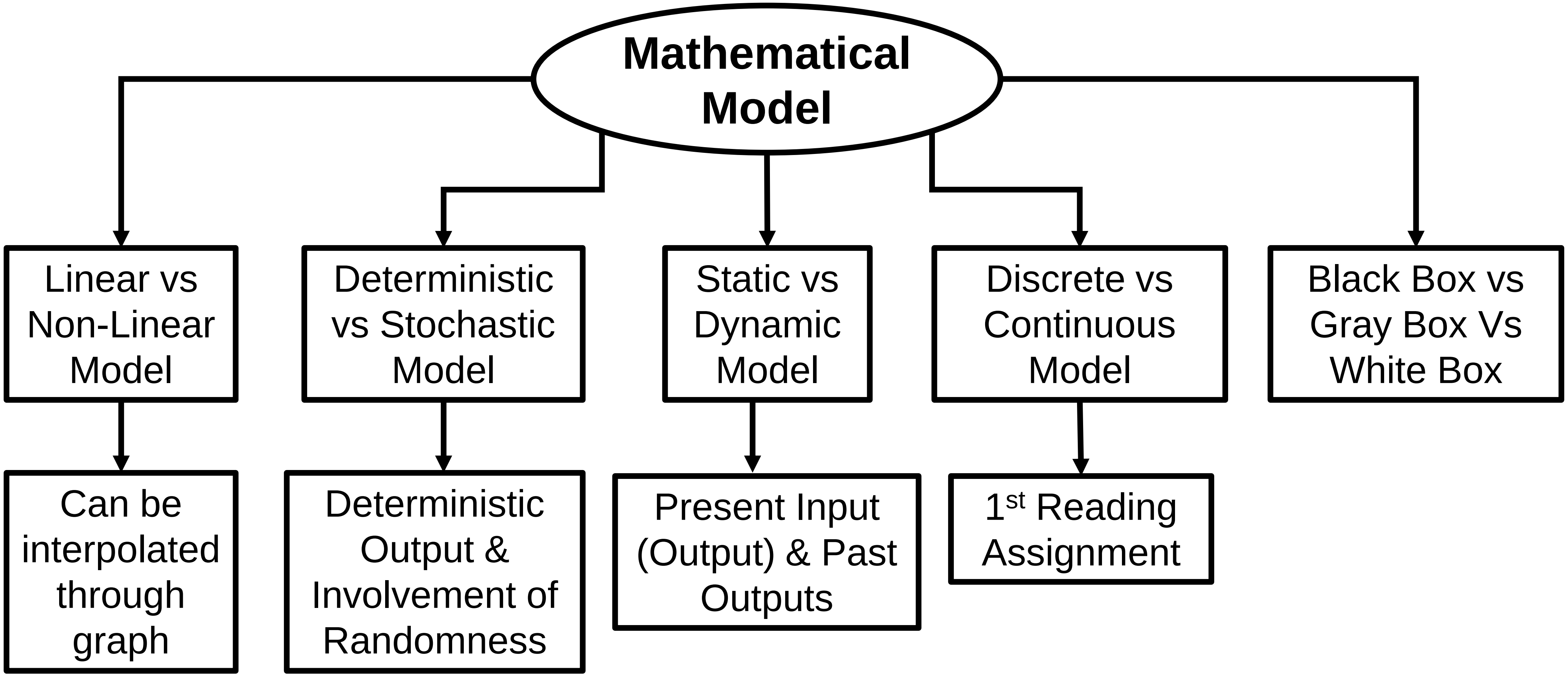
How can we study a system?

- ✓ We can primarily study the system in two different ways.
- ✓ What does study mean?



Mathematical Model

- ✓ A set of mathematical equations that interpolates the characteristics of the system.
- ✓ What does characteristics of the system mean?



Black Box Vs Grey Box Vs White Box

- ✓ A model in which only the input and output are given is known as black box model.
- ✓ In **black box**, we don't care about the internal dynamics and therefore, it's much easier to model.
- ✓ A **grey box** is a black box plus the information about the internal dynamics(To Certain Extent).
- ✓ Should have complete Knowledge of the system to derive a **white box model**.

Approaches of the Mathematical Modeling

- ✓ **Transfer-Function Approach:** A transfer function (also known as system function or network function) of a system, sub-system, or component is:
  - A mathematical function which theoretically models the system's output for each possible input.
  - Interlinked to Classical Control Approach.
  - Highly Applicable for LTI Systems and **Why not for Non-LTI's (2nd Reading Assignment) ?**
- ✓ **State-Space Approach: The** basic idea with a state space approach is to define the **inputs**, define the **outputs**, and a set of **equations** to represent the behavior of the system.

https://en.wikipedia.org/wiki/Transfer_function https://www.sciencedirect.com/topics/engineering/state-space-approach

Mathematical Modeling of a Dynamic System

Mathematical model can be established by considering the :

- ✓ Input-Output of the system
- ✓ Fundamental Parameters and Assumptions.
- ✓ Applicable Physical Laws.
- ✓ Differential Equations describing the Model.
- ✓ Any one approach for the desired output of the system.
- ✓ Analyze the solution by tuning the variables along with the assumptions.

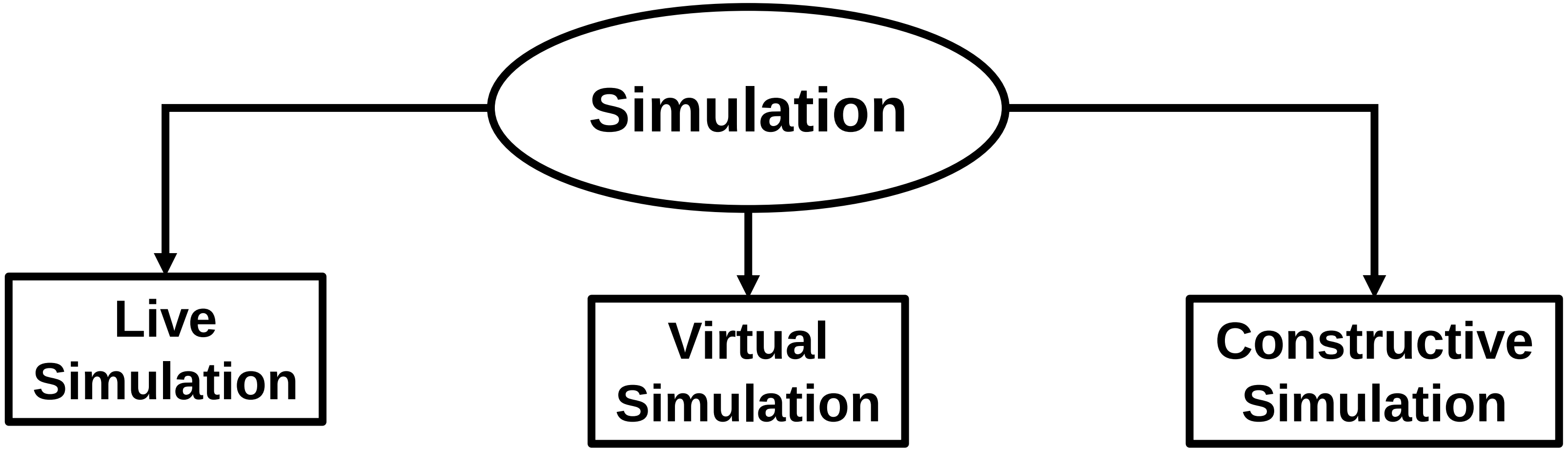
A race car driver sits at the starting line. When the gun fires, he fully accelerates at “a” m/s2 for “t” seconds. How fast are the car and driver going when the driver stops pushing the gas pedal to the floor?

What about the complex system? Is it really possible to analyze the system by considering only the mathematical model?

Absolutely Not!

Simulation

- ✓ Computer-based model that mimics the functionality of the actual system.
- ✓ A numerical simulation is a calculation that is run on a computer following a program that implements a mathematical model for a physical system.
- ✓ Numerical simulations are required to study the behavior of systems whose mathematical models are too complex to provide analytical solutions, as in most nonlinear systems (**Modeling of Brain or Heart?**).

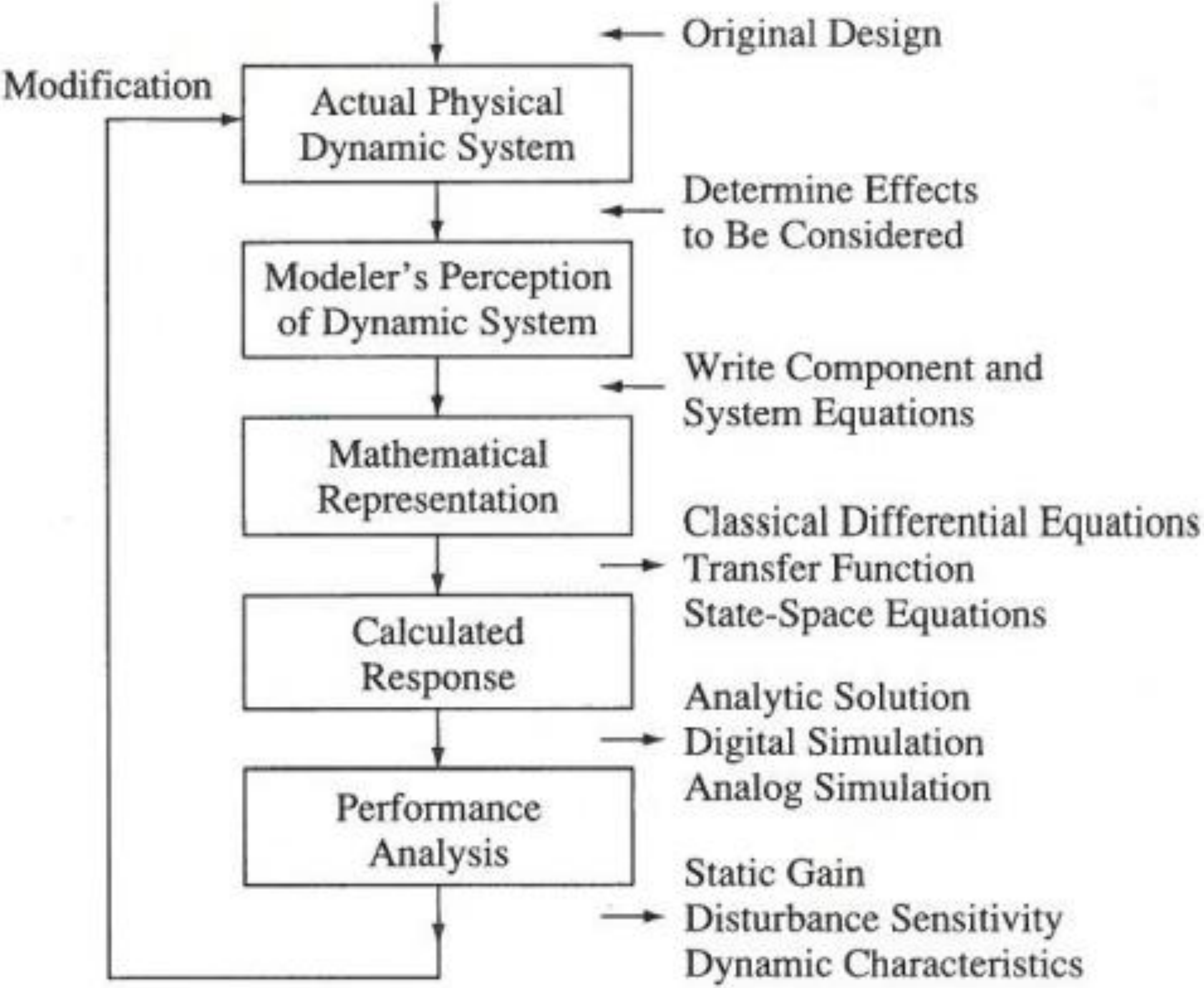


Advantages & Disadvantages of Simulation

- Help the Engineers in iterating and testing designs without developing the actual system.
- Behavior (Characteristics) of the system before delving into the actual product.
- Solve Complex Engineering problems quickly.
- A flexible environment where we can check the endurance limits of the system.

- No Compromise on the skills of the modeler.
- The quality is highly influenced by wrong interpretations of the model.
- The outcomes are sometimes hard to interpret.

" Simulation develops the Theoretical Hypothesis"


Original Design
Modification
Actual Physical Dynamic System
Determine Effects to Be Considered
Modeler's Perception of Dynamic System
Write Component and System Equations
Mathematical Representation
Classical Differential Equations
Transfer Function
State-Space Equations
Calculated Response
Analytic Solution
Digital Simulation
Analog Simulation
Performance Analysis
Static Gain
Disturbance Sensitivity
Dynamic Characteristics

Linear Dynamic Systems (Models for Dynamic System with Similarity)

✓ How can we define the Linearity of the system?

Linear System

- **Principle of Homogeneity** → **Scalar rule or the scaling property**
- **Principle of Additivity** → **X1=Y1 & X2=Y2: Xc=X1+X2 so Yc=Y1+Y2**

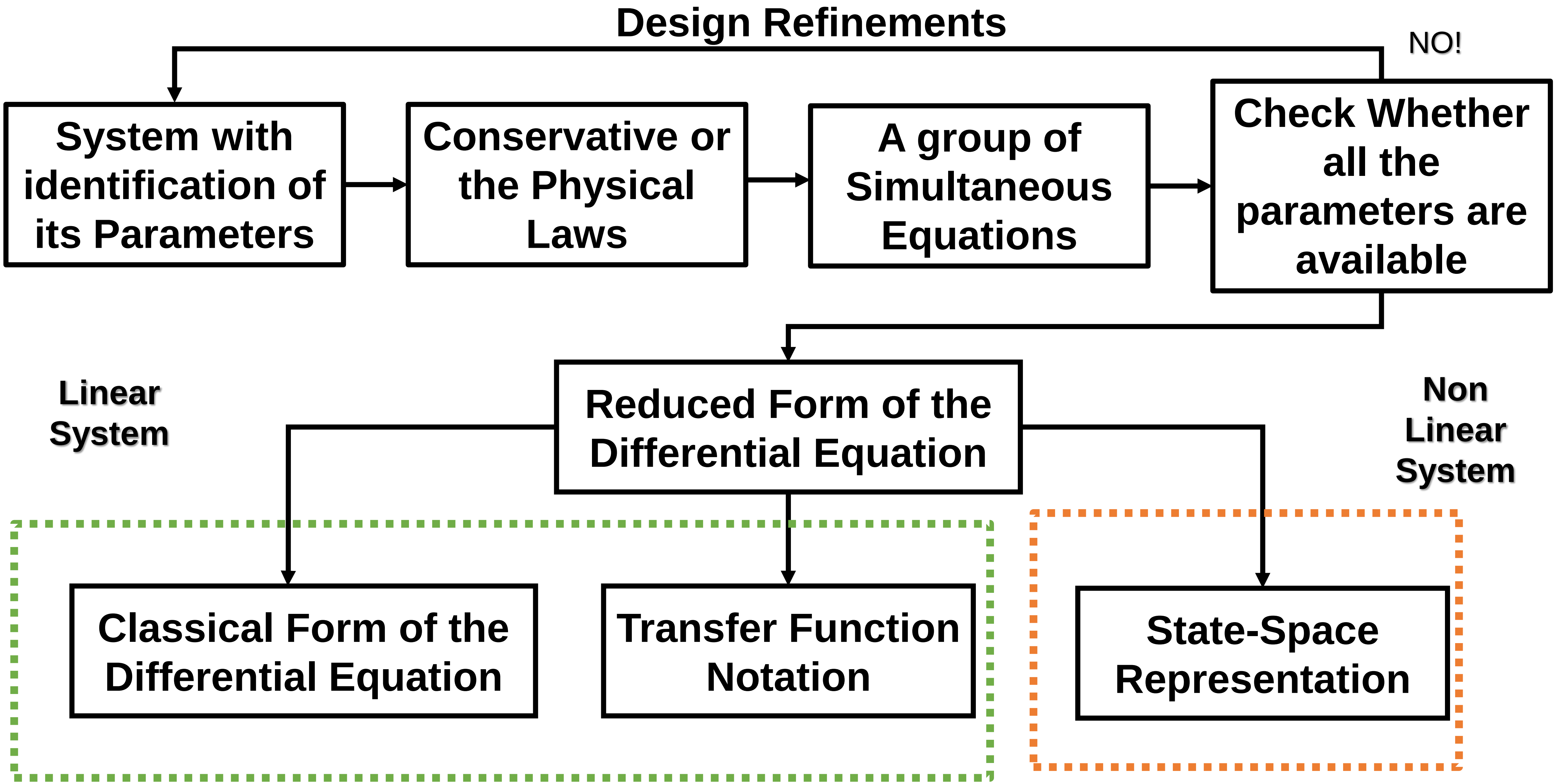
In circuit design, we often strive for linearity because it leads to desirable output characteristics.

What about Resistive Heater? Whether it's linear system or not?

Linear Dynamic Systems are easy to interpret!

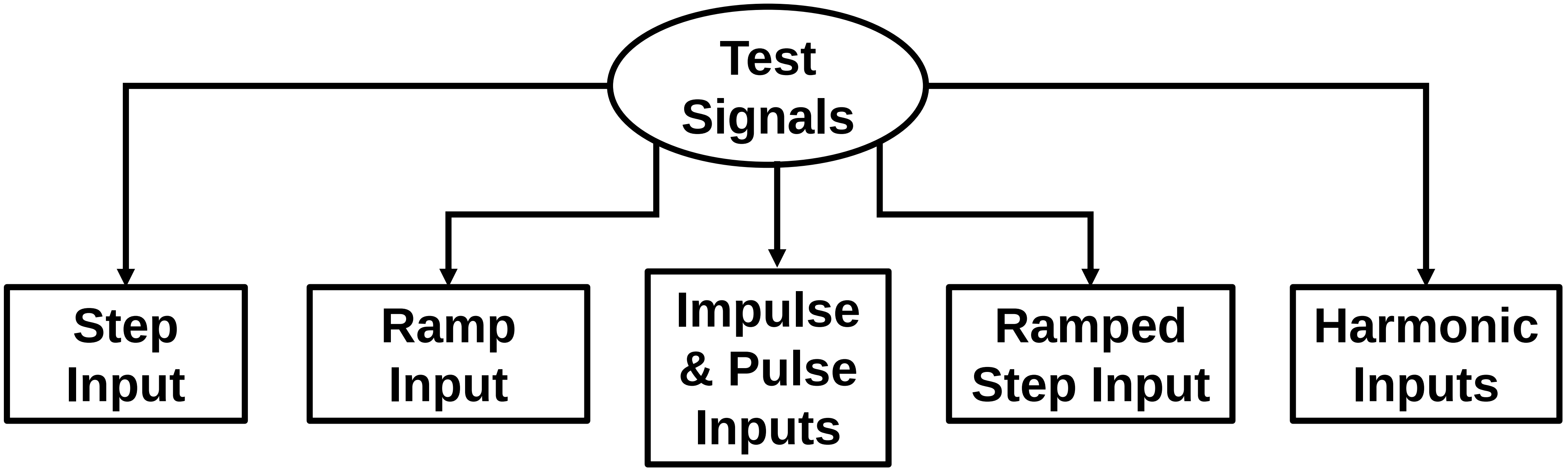
https://www.allaboutcircuits.com/technical-articles/what-is-a-linear-system/

Reduction of the Modeling Equations (Linear)



Typical Inputs to the System

- ✓ Three things are usually needed to obtain the future response of the system.
  - ➢ Initial Conditions
  - ➢ The Differential Equation
  - ➢ Test or Input Signals.



- ✓ Doublet and Quadratic inputs are not primarily utilized for the analysis of the dynamic systems but for most of the academic tests, they have significant importance.

Dynamic Model (Mathematical Modeling)

- ✓ Will share the link for modeling using classical method (You already studied in your calculus Course).
- ✓ Laplace Transform will be used to find the transfer function of any dynamic system.
- ✓ Inverse Laplace will be used to find the time-domain solution to any dynamic problem.

Item no.	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^n + 1}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Transfer Function Approach

Start with LDE that describes the system → Make use of the differential operator → Apply Laplace & factor out Y(s) and U(s) → Write in the general form of the required Transfer Function

$$\boldsymbol{a_0 y(t) + a_1 \frac{dy(t)}{dt} + a_2 \frac{dy^2}{dt^2} + \ldots\ldots = b_0 x(t) + b_1 \frac{dx(t)}{dt} + \ldots\ldots}$$

Find the transfer function of the following Dynamic Systems:

- ✓ $2\frac{\mathrm{dy(t)}^2}{\mathrm{dt}^2}-4\frac{\mathrm{dy(t)}}{\mathrm{dt}}+3\mathrm{y(t)}=5\frac{\mathrm{dx(t)}}{\mathrm{dt}}-2\mathrm{x(t)}$
- ✓ $\dddot{y}(t)+2\ddot{y}(t)-3\dot{y}(t) - y(t)=4\dot{x} - 7x$

Let's take any volunteer example either from block box, grey box, or even white box model.

What about finding the parametric relationship or the characteristics of the system?

Follow the previous procedure of TF → Relate the equation with the output & Input variables → Apply inverse Laplace transform → Formulate the equation in terms of output variable on one side & vice-versa

- ✓ Given the following differential equation, solve for y(t) if all initial conditions are zero.

$$\frac{\mathbf{dy(t)^2}}{\boldsymbol{dt^2}}+12\frac{\mathbf{dy}(\boldsymbol{t})}{\boldsymbol{dt}}+32\mathrm{y(t)}=32\boldsymbol{u}(\boldsymbol{t})$$

$$\boldsymbol{y}(\boldsymbol{t}) = (\mathbf{1} - \mathbf{2}\boldsymbol{e}^{-\mathbf{4}\boldsymbol{t}}+\boldsymbol{e}^{-\mathbf{8}\boldsymbol{t}})\boldsymbol{u}(\boldsymbol{t})$$

General form of the 1st order and second order transfer function (One of the main/fundamental contents of the control system-1)

State Space Approach

- Why we need state space approach if it's easy to interpret T.F.
- A mathematical model of a physical system as a set of input, output and **state variables** related by **first-order differential equations** or difference equations.

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\dot{\boldsymbol{X}} + \boldsymbol{B}\boldsymbol{U}$$
$$\boldsymbol{Y} = \boldsymbol{C}\boldsymbol{X} + \boldsymbol{D}\boldsymbol{U}$$

- Order of the Matrices: State Matrix (A)= n by n; B=Input Matrix=n by 1; C= Output Matrix=1 by n; D=Feedthrough Matrix; X will be representing the state vector (n by 1)

$$\boxed{\frac{Y(S)}{U(S)} = \frac{10}{S^3 + 4S^2 + 2S + 1}}$$