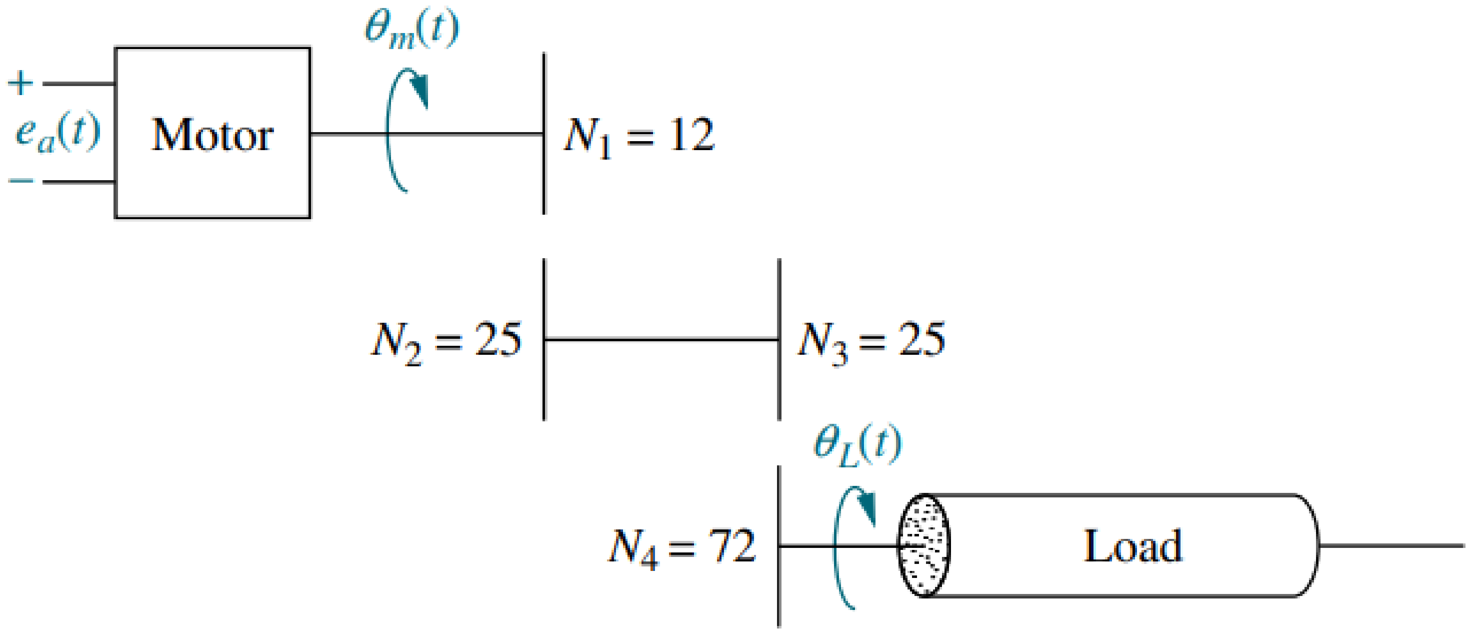
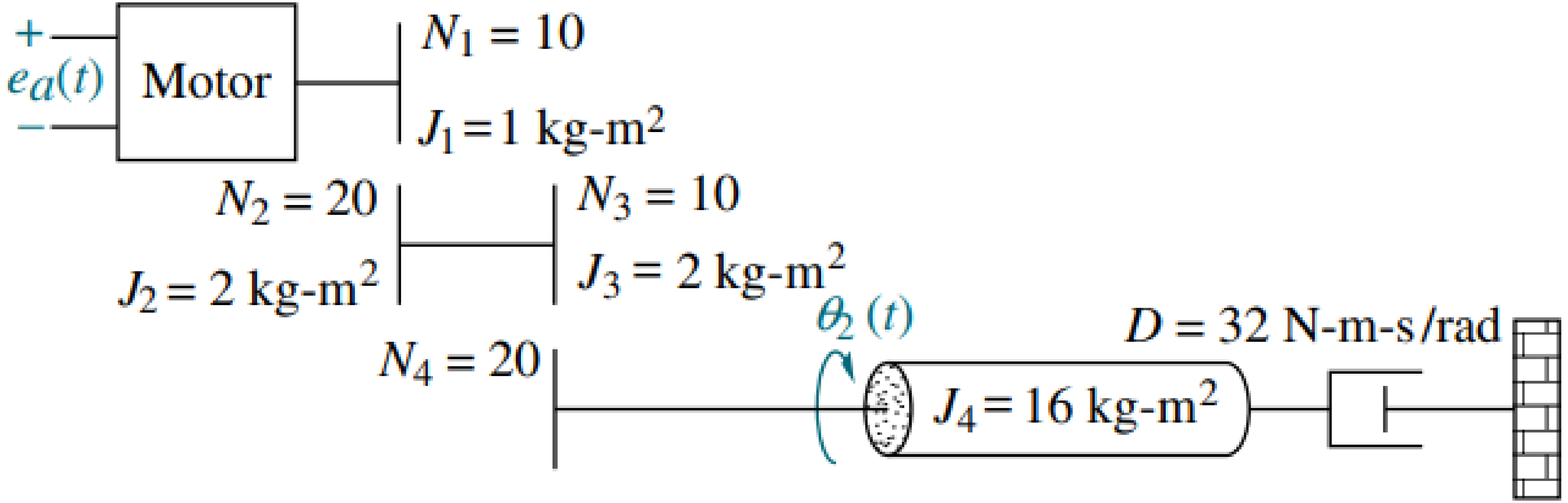
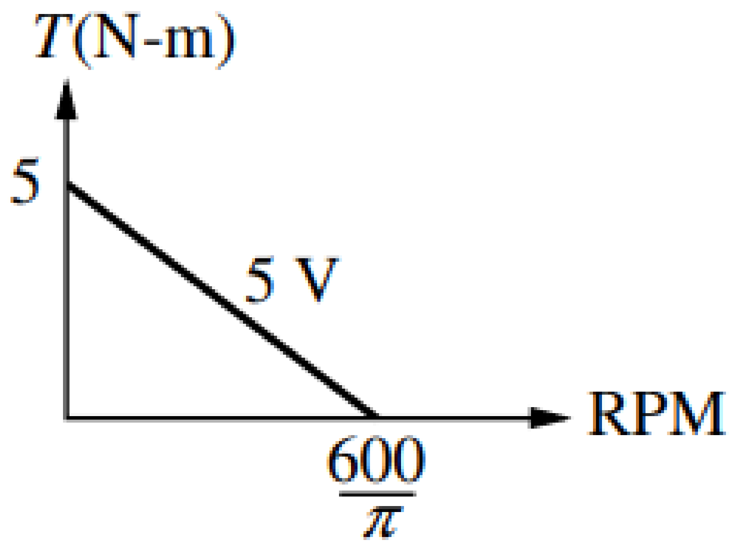
Practice Problems

A dc motor develops 55 N-m of torque at a speed of 600 rad/s when 12 volts are applied. It stalls out at this voltage with 100 N-m of torque. If the inertia and damping of the armature are 7 kg-m2 and 3 N-m-s/rad, respectively, find the transfer function between load displacement and applied voltage of this motor if it drives an inertia load of 105 kg-m2 through a gear train, as shown in Figure.



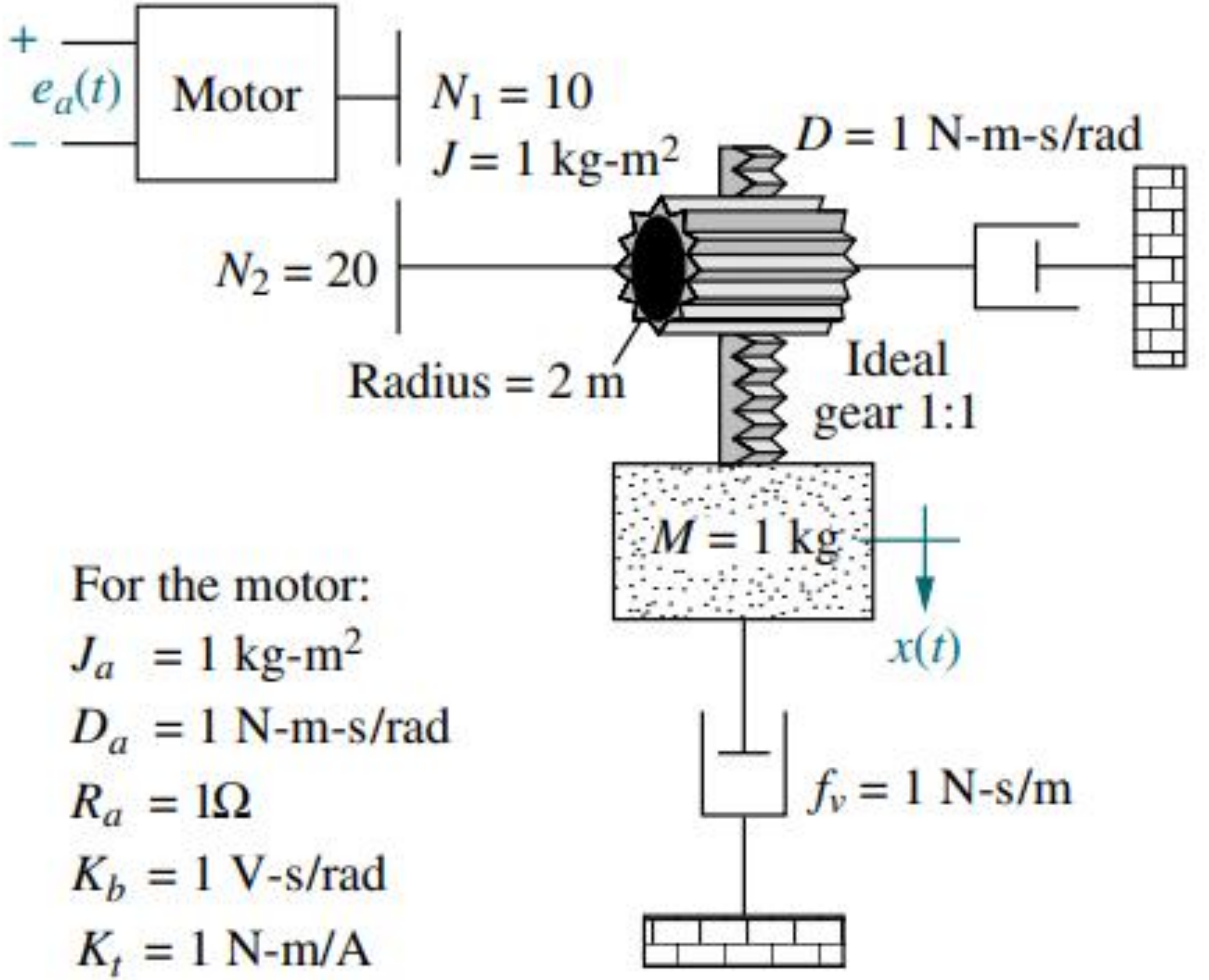


+
$e_a(t)$
−
Motor
$N_1 = 10$
$J_1 = 1$ kg-m^2
$N_2 = 20$
$J_2 = 2$ kg-m^2
$N_3 = 10$
$J_3 = 2$ kg-m^2
$\theta_2(t)$
$N_4 = 20$
$J_4 = 16$ kg-m^2
$D = 32$ N-m-s/rad



T(N-m)
5
5 V
RPM
$\frac{600}{\pi}$

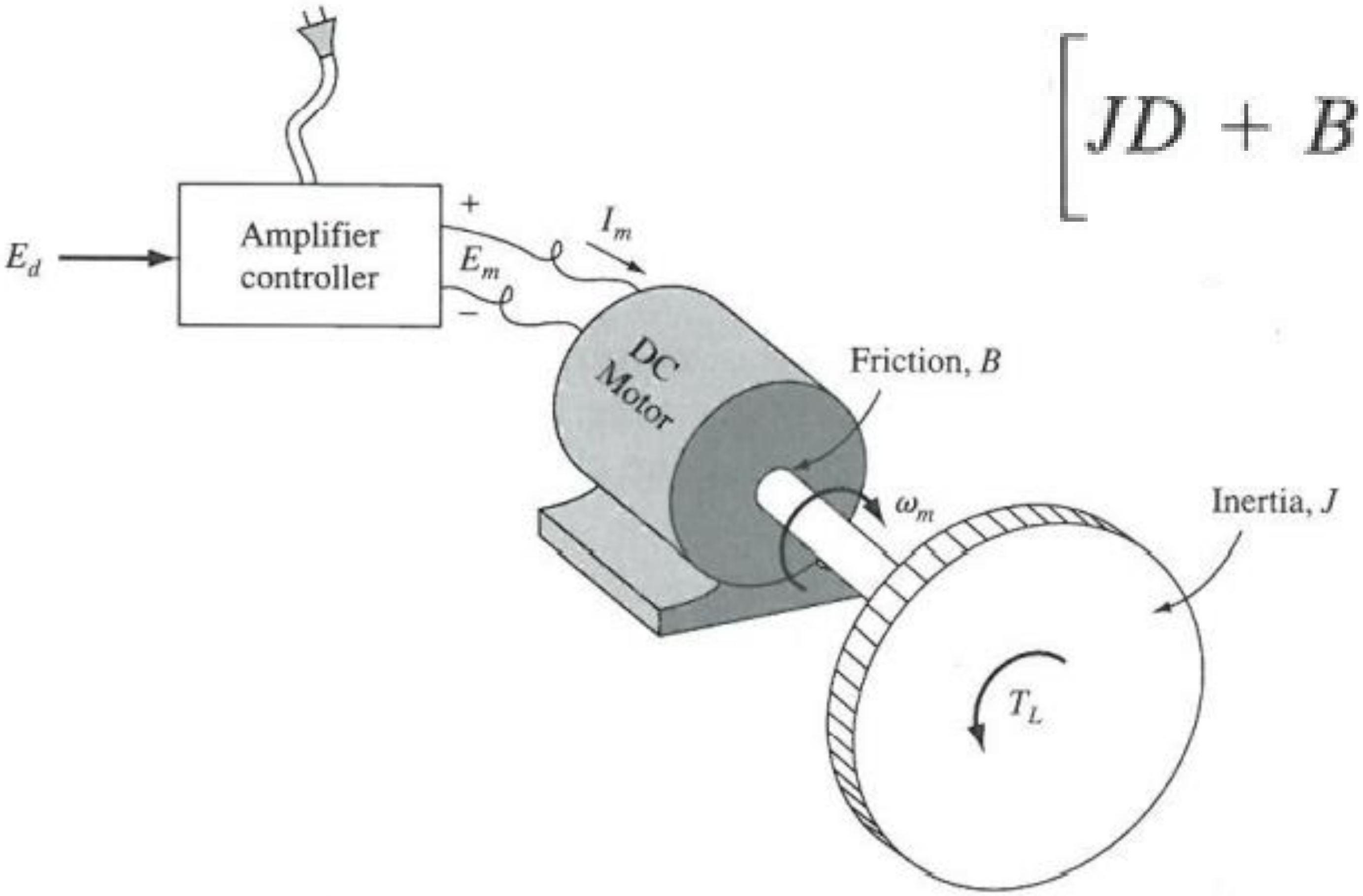
Try it on your own!



- Find the transfer function between (a) Motor displacement and applied voltage (b) Load displacement and Applied voltage (C) Translational displacement and Applied Voltage

Open-Loop Speed Control of DC Motor

- Find the static gain, disturbance sensitivity, and time constant of the DC motor(With out feedback) as shown in **Figure**.



$$\left[JD + B + \frac{K^2}{R_o}\right]\omega_m = \frac{GK}{R_o}E_d - T_L$$

Open-Loop Speed Control of DC Motor

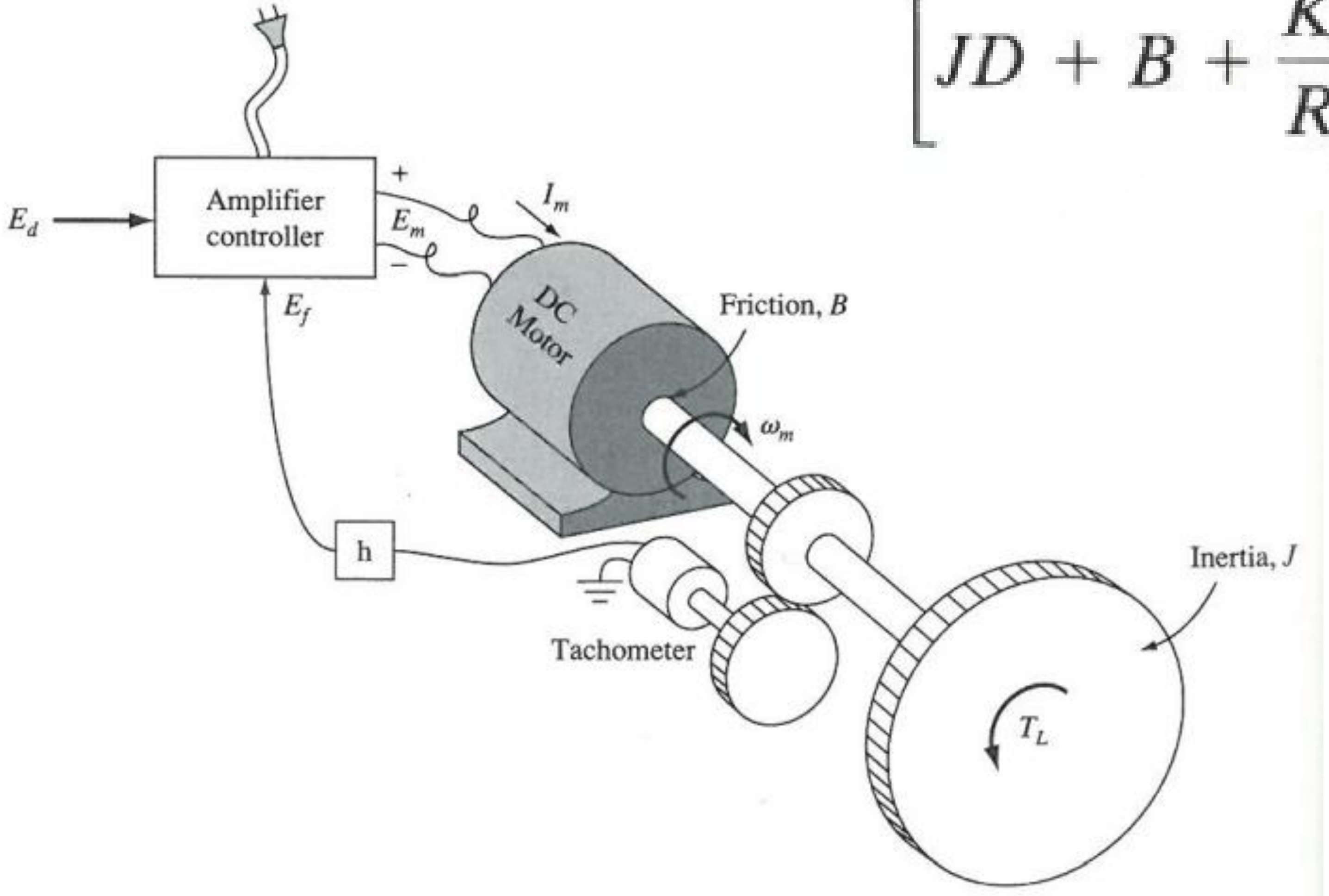
$$\omega_m = \frac{\dfrac{G/K}{\left(1+\dfrac{B}{K^2/R_o}\right)}E_d - \dfrac{R_o/K^2}{\left(1+\dfrac{B}{K^2/R_o}\right)}T_L}{\dfrac{\dfrac{J}{K^2/R_o}}{\left(1+\dfrac{B}{K^2/R_o}\right)}D+1} \tag{7.9}$$

$$\text{Static gain} = \left.\frac{\partial \omega_m}{\partial E_d}\right|_{D=0} = \frac{G/K}{\left(1+\dfrac{B}{K^2/R_o}\right)}$$

$$\text{Disturbance sensitivity} = \left.\frac{\partial \omega_m}{\partial T_L}\right|_{D=0} = -\frac{R_o/K^2}{\left(1+\dfrac{B}{K^2/R_o}\right)}$$

Closed-Loop Speed Control of DC Motor

- Find the static gain, disturbance sensitivity, and time constant of the DC motor(With feedback) as shown in **Figure**.



$$\left[JD + B + \frac{K^2}{R_o} + \frac{GKh}{R_o}\right]\omega_m = \frac{GK}{R_o}E_d - T_L$$

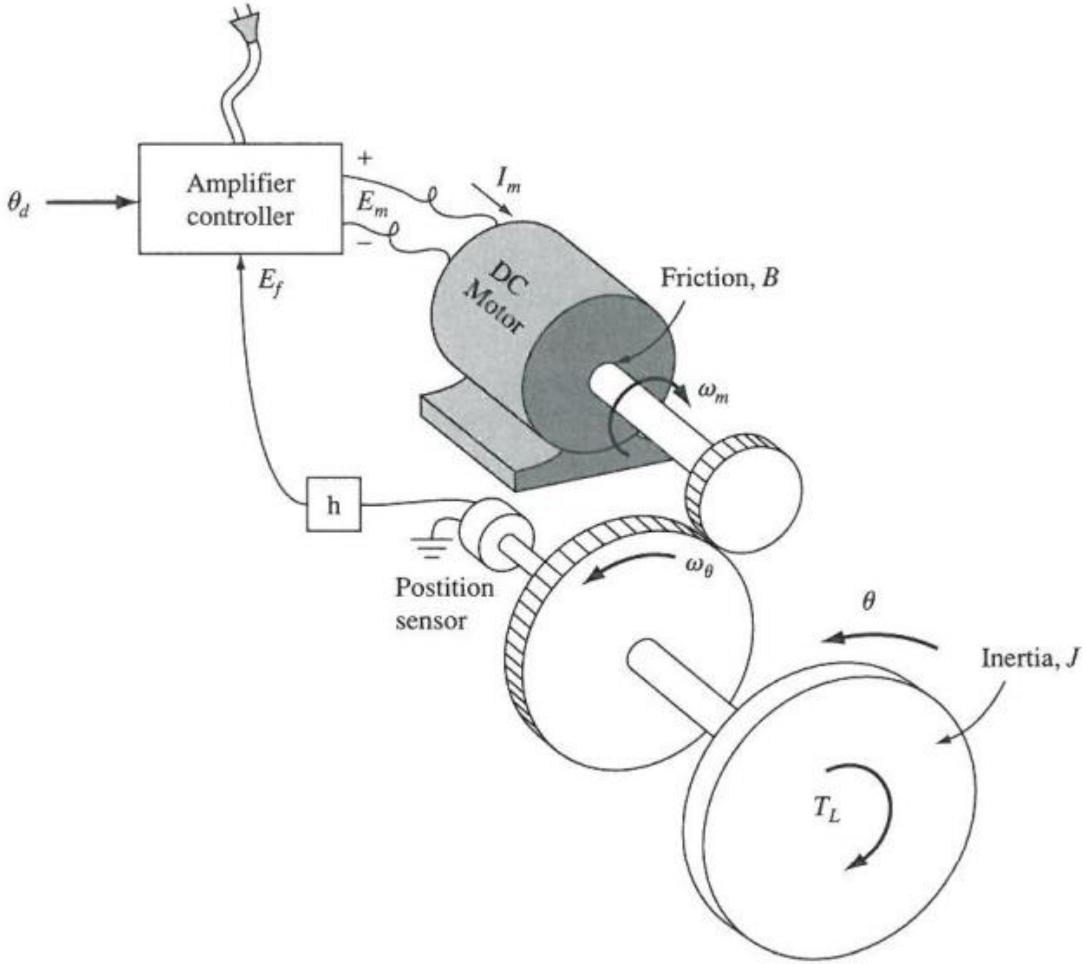
Closed-Loop Speed Control of DC Motor

$$\omega_m = \frac{\dfrac{1/h}{\left(1 + \left(\dfrac{1 + \dfrac{B}{K^2/R_o}}{\dfrac{Gh}{K}}\right)\right)} E_d - \dfrac{R_o/K^2}{\left(1 + \dfrac{B}{K^2/R_o} + \dfrac{Gh}{K}\right)} T_L}{\dfrac{\dfrac{J}{K^2/R_o}}{\left(1 + \dfrac{B}{K^2/R_o} + \dfrac{Gh}{K}\right)} D + 1}$$

$$\text{Disturbance sensitivity} = \left.\frac{\partial \omega_m}{\partial T_L}\right|_{D=0} = -\frac{R_o/K^2}{\left(1 + \dfrac{B}{K^2/R_o} + \dfrac{Gh}{K}\right)}$$

$$\text{Static gain} = \left.\frac{\partial \omega_m}{\partial E_d}\right|_{D=0} = \frac{G/K}{\left(1 + \dfrac{B}{K^2/R_o} + \dfrac{Gh}{K}\right)} = \frac{1/h}{\left(1 + \left(\dfrac{1 + \dfrac{B}{K^2/R_o}}{\dfrac{Gh}{K}}\right)\right)}$$

Closed-Loop Position Control of DC Motor



$$\left[JD^2 + \left(B + \frac{K^2}{R_oR_s^2}\right)D + \frac{GKh}{R_oR_s}\right]\theta = \frac{GKc}{R_oR_s}\theta_d - T_L$$

$$\theta = \frac{\dfrac{c}{h}\theta_d - \dfrac{1}{\left(\dfrac{GKh}{R_oR_s}\right)}T_L}{\dfrac{J}{\left(\dfrac{GKh}{R_oR_s}\right)}D^2 + \dfrac{(B + K^2/(R_oR_s^2))}{\left(\dfrac{GKh}{R_oR_s}\right)}D + 1}$$