<u>Module 4: Modelling of Fluidic Systems</u>

- ✓ Likewise, the mechanical, electrical, and electromechanical systems, the fluidic systems have been primarily divided into two main categories:
- ✓ In **<u>Hydraulic Systems</u>**, the working fluid is relatively incompressible and therefore liquid like water is usually considered to transmit the **<u>power</u>**.
- ✓ In **<u>Hydraulic Systems</u>**, the tanks will always be needed to store the working fluidic; therefore, what about **generic modeling of Tanks (Water level Tank)?**
- ✓ In **<u>Pneumatic Systems</u>**, the working fluid is relatively compressible and therefore gas (air) is usually considered to transmit the **<u>power</u>**.
- ✓ In **<u>Pneumatic Systems</u>**, the model can withdraw the air from atmosphere and simply a filter would be utilized for purification (If desired).

https://www.theengineerspost.com/hydraulics-and-pneumatics/

What is the difference between mass flow rate and Volumetric flow rate?

Mass Flow Rate Vs Volumetric Flow Rate

Flow Meters

Mass Flow Meters

The measure of amount of mass moving through a device(Flow meter) per unit time.

$\dot{m}$=ρAv

Volumetric Flow meters

The measure of amount of volume of the fluid flowing through a device (Flow meter) per unit time.

$\dot{Q}$=Av

Building Blocks of Hydraulic Systems

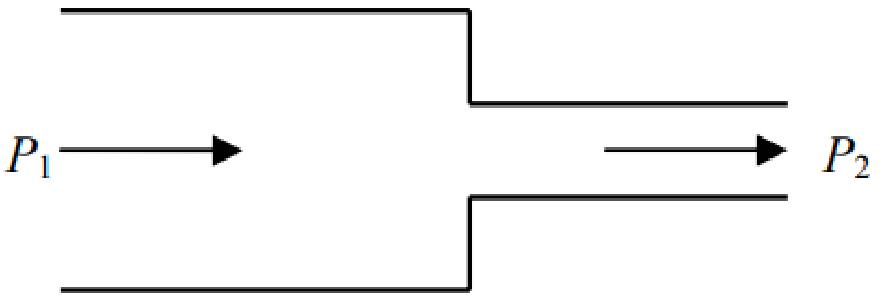
Volumetric Flow Rate (q) → **Fluidic System (Hydraulic; Pneumatic)** → **Pressure Difference (P1-P2)**

- ✓ During the flow of the working fluid in the hydraulic systems, an opposition force will always be noticed (Primarily due to change in diameter of the pipe or valve) and is known as **hydraulic resistance**.

$$Pressure\ Difference \propto Volumetric\ Flow\ Rate:\ P_1 - P_2 \propto \text{q} : P_1 - P_2 = \text{Rq}$$

$$R = \frac{\text{Pressure difference}}{\text{Change in flow rate, m}^3/\text{s}} = \frac{p_1 - p_2}{q}$$

- ✓ Where R is called the hydraulic resistance. The higher the resistance the higher the pressure difference for a given rate of flow.



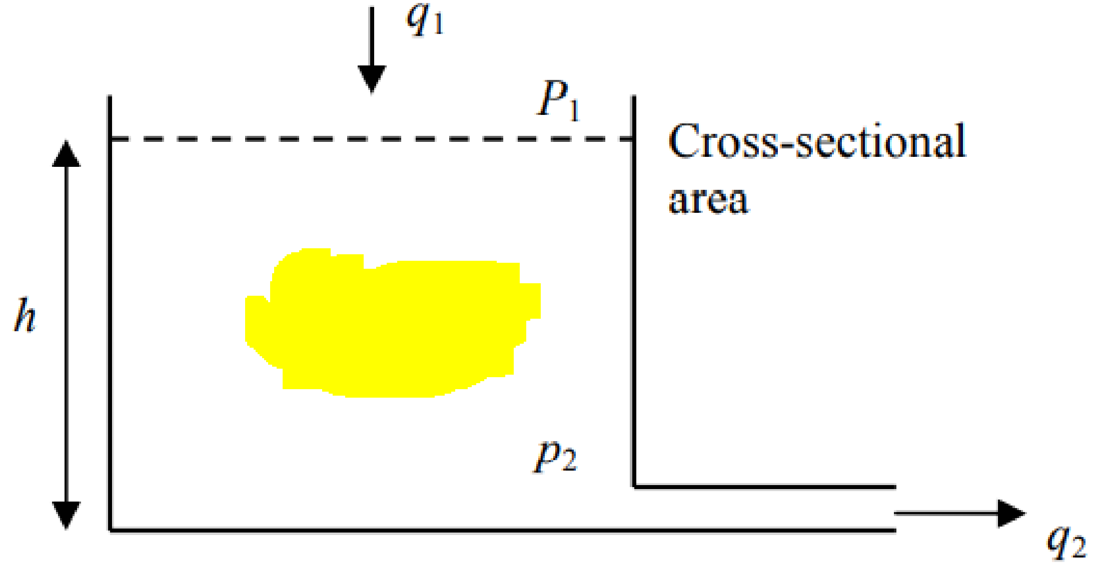
Hydraulic Capacitance

- ✓ There will always be an imperative need to store the working fluid in some unit (Such as tank). **Hydraulic Capacitance** describes that energy storage (Usually in the form of Potential energy) as in the figure, and height of the working fluid in a container (Pressure Head) is one form of the storage.

$$\text{Rate of Change of Volume in the tank} = \text{Difference between the inward and outward flow rate}$$

$$\frac{dV}{dt} = q_1 - q_2$$

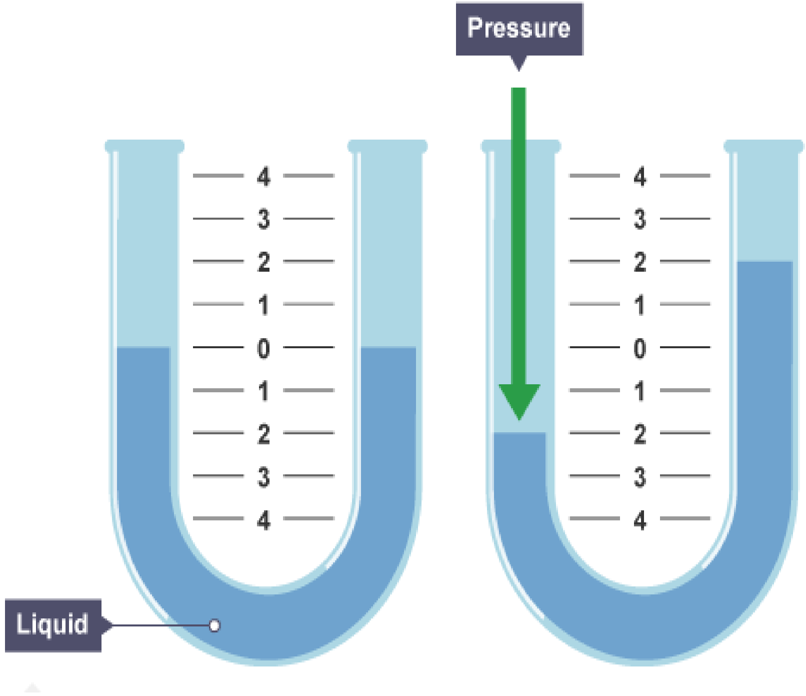
- ✓ Volume of the tank can be written as the product of cross-sectional area and height of the working liquid.
- ✓ Recall the measurement of Pressure difference in manometers



Hydraulic Capacitance

- ✓ Pressure difference can be obtained by considering the manometer's interpretation as given below;

$$P=\rho gh$$



- ✓ By putting above well-justified terms into the main equation of flow rate;

$$q_1 - q_2 = \frac{dV}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$

$$q_1 - q_2 = A\frac{d(\frac{P}{\rho g})}{dt} = \frac{A}{\rho g}\frac{dP}{dt}$$

$$q_1 - q_2 = C\frac{dP}{dt}$$

Where $C=\frac{A}{\rho g}$ is the hydraulic Capacitance but what about Capacitance of the tank?

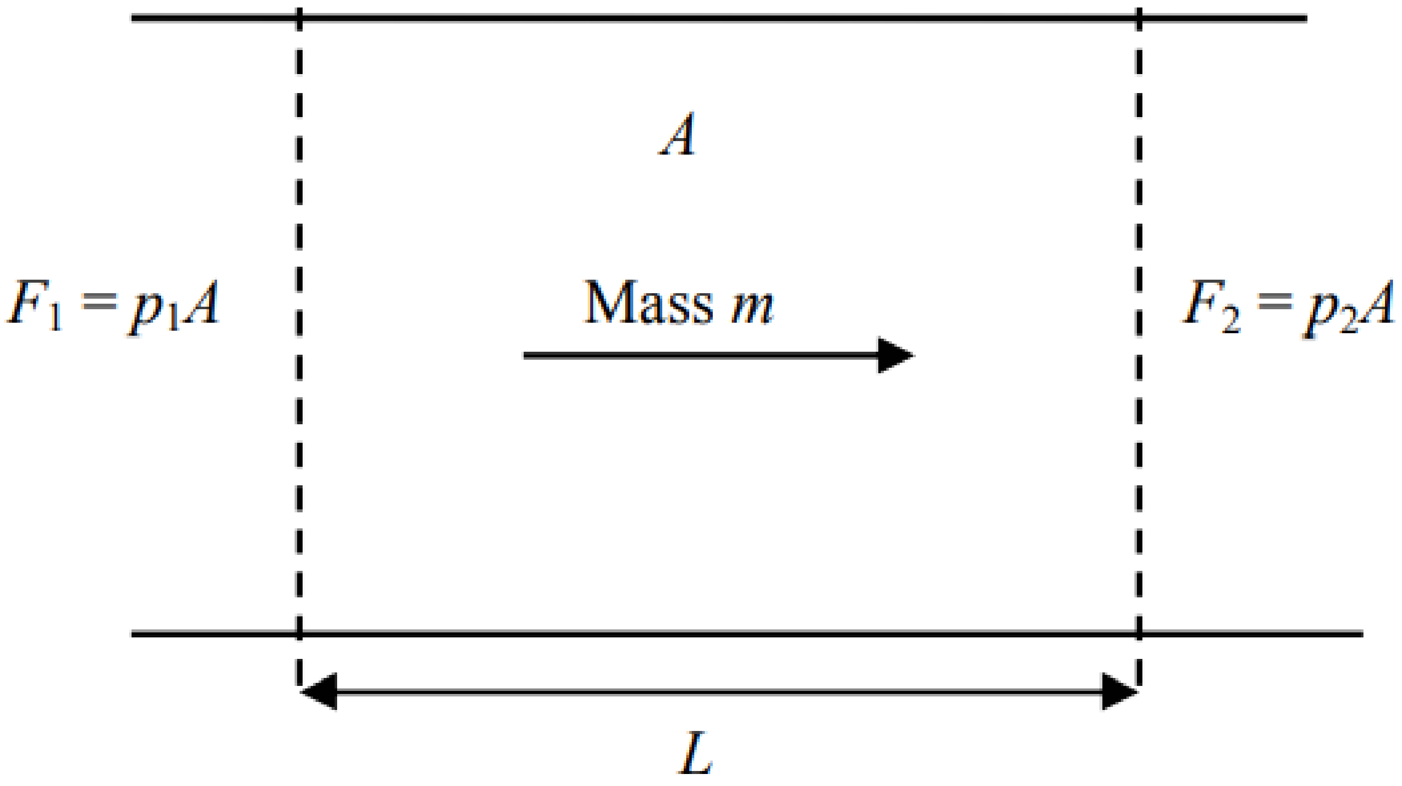
Can you now develop an analogy of hydraulic Capacitance with the electric capacitance?

Hydraulic Inertance

- ✓ Hydraulic Inertance is analogous to the inductance in the electrical circuit.
- ✓ A measure of the pressure difference in a fluid required to cause a unit change in the rate of change of **volumetric flow-rate** with time.
- ✓ Consider a **fluidic system** in the form of rectangular block with **mass "m"** as shown in Figure. There will always be a need of force to move this system.

$$\sum Forces = ma$$

$$(P_1 - P_2)A = m\frac{d\text{v}}{dt}$$



Hydraulic Inertance

- Due to incompressible nature of the working fluid, the before-described equation would now become;

$$(P_1 - P_2)A = \rho V \frac{dv}{dt} = \rho A L \frac{dv}{dt}$$

- Volumetric flow rate always plays a pivotal role in the flow-regimes and therefore, the velocity of the fluid can also be rewritten in the form of volumetric flow rate as given below;

$$(P_1 - P_2)A = \rho A L \frac{d\left(\frac{q}{A}\right)}{dt} = \rho L \frac{dq}{dt}$$

$$(P_1 - P_2) = \frac{\rho L}{A} \frac{dq}{dt}$$

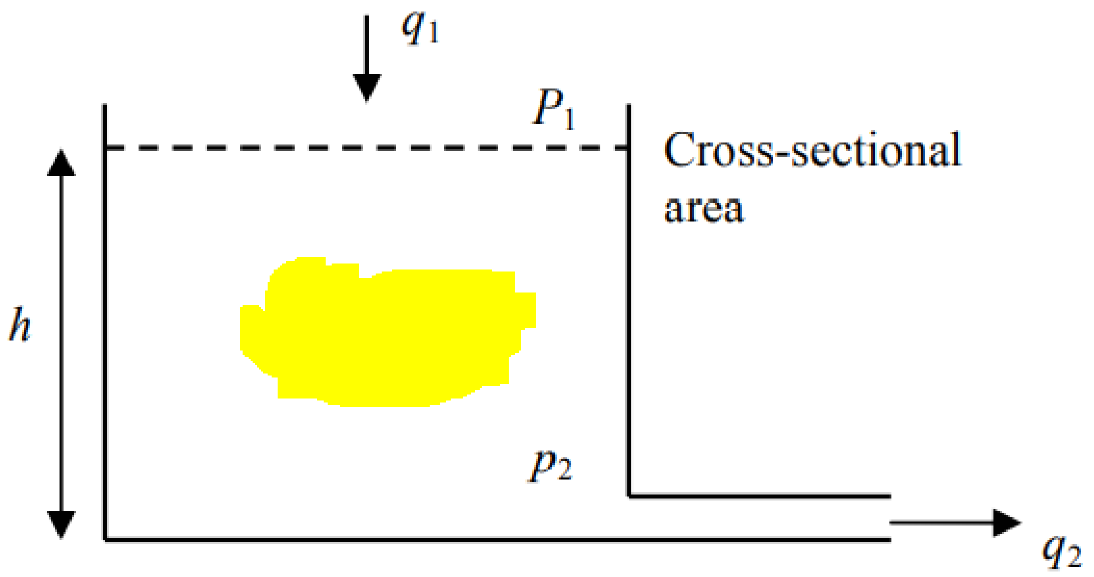
$$(P_1 - P_2) = I \frac{dq}{dt}$$

Where **I=**$\frac{\rho L}{A}$ is the hydraulic inertance but can you relate the derived notation with the definition?

Can you relate the final derived expression with the electrical system?

Simplest Model of Water Level Tank

✓ Consider a simple hydraulic system in which the liquid enters and leaves a container as shown in Figure. It is assumed that the liquid is not accelerating and you are required to



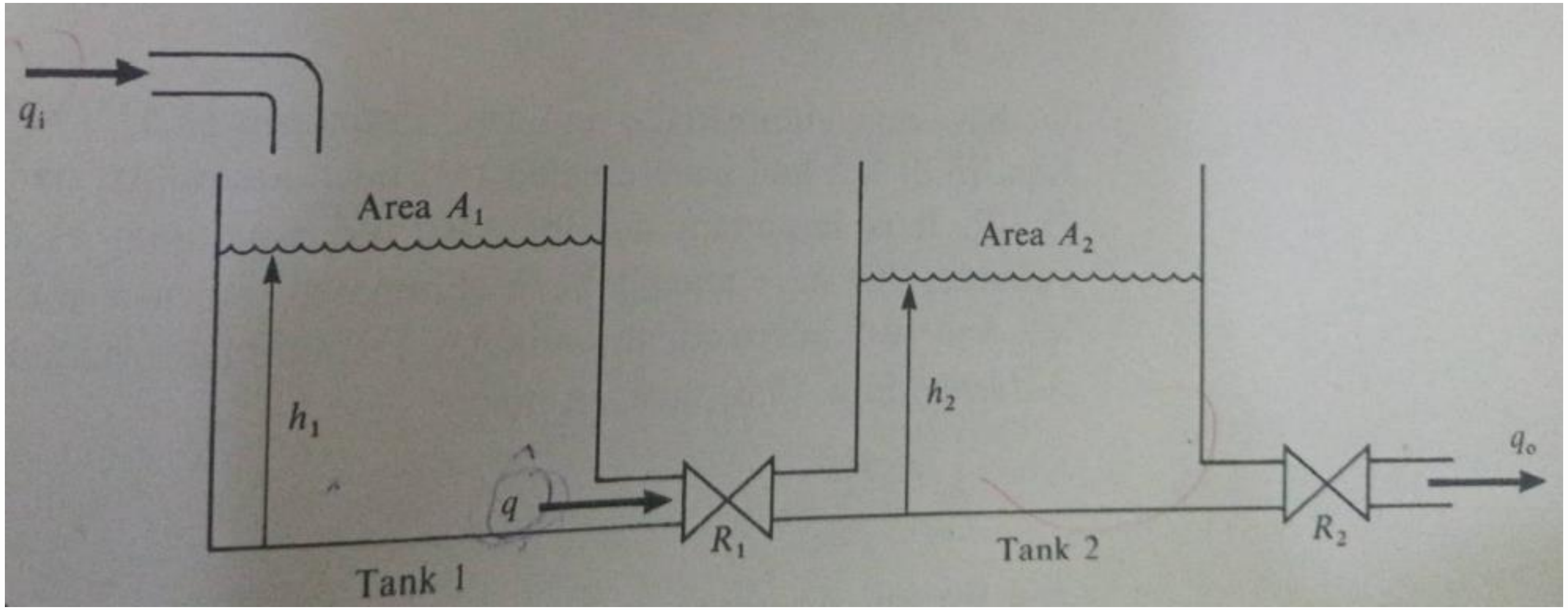
(a) Render the complete dynamics of system into a **single differential equation** relating the input flow rate with the height of the working fluid.

(b) Find the transfer function between the height to the inward flow rate.

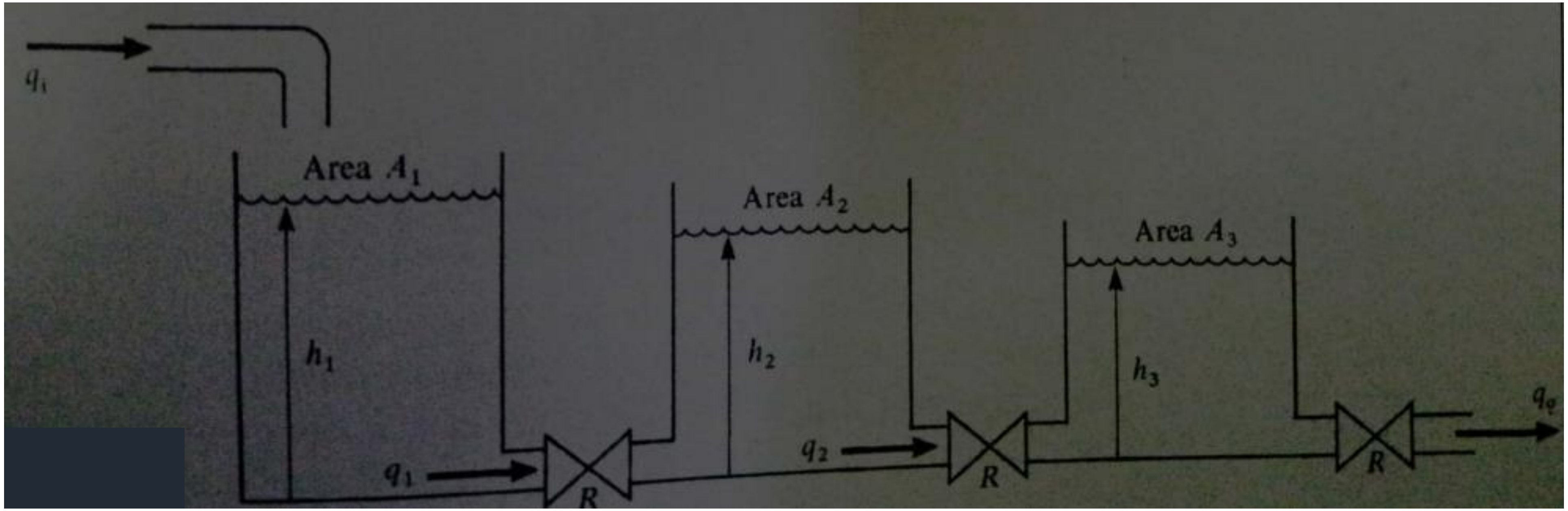
(c) Find the state space from part (b).

$$\boxed{q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}}$$

Coupled Tank Systems



Two tanks System: Find the transfer function between the outward flowrate to the inward flow rate



Coupled tanks System

Building blocks of the Pneumatic Systems

- ✓ Unlike the hydraulic systems, the working fluid is relatively compressible in the pneumatic systems and therefore, density will be the prominent in the modeling of pneumatic systems (i.e., Due to change in pressure, volume changes and therefore, density will also be changed).
- ✓ Likewise, the other dynamic systems, there are three building blocks of the pneumatic systems: (1) Pneumatic Resistance (2) Pneumatic Capacitance (3) Pneumatic Inertance.
- ✓ **Pneumatic Resistance:** The force that restricts the mass flow rate of the working fluid (i.e., gas) through a device.

$$Pressure\ Difference \propto Mass\ Flow\ Rate$$

$$P_1 - P_2 \propto \frac{dm}{dt} : P_1 - P_2 = \mathrm{R}\dot{m}$$

Analogy with the hydraulic resistance and Electrical Resistance?

Pneumatic Capacitance.

- ✓ Pneumatic Capacitance is related to the **change in volume** as well as **compressibility of the gas** with reference to rate of change in pressure (i.e., It could be due to the compressibility of the gases).
- ✓ Comparable to the compression of the spring which stores the energy.
- ✓ Consider a container in which inward mass flow rate is $\dot{m}_1$and flow rate leaving the container is $\dot{m}_2$. Due to the rate of change in the pressure, volume of the container certainly changes, and we are required to develop the relation for the pneumatic capacitance!

V=Volume of the container; $\dot{m}_1$=Mass flow rate entering the container, $\dot{m}_2$=Mass flow rate leaving the container; ρ=Density of the working fluid; P=Pressure Difference

Pneumatic Capacitance.

Difference in the mass flow rate through a container=Rate of change of mass within a container

$$\dot{m}_1 - \dot{m}_2 = \frac{dm}{dt}; \dot{m}_1 - \dot{m}_2 = \frac{d(\rho V)}{dt}$$

- ✓ Since the working fluid is compressible; therefore, rate of change in the density will also be observed in the above equation.

$$\dot{m}_1 - \dot{m}_2 = V\frac{d(\rho)}{dt} + \rho\frac{d(V)}{dt}$$

- ✓ As per the definition and theoretical interpretations of the pneumatic system, controllable parameter could be the pressure; therefore,

$$\dot{m}_1 - \dot{m}_2 = V\frac{d(\rho)}{dt} + \rho\frac{d(V)}{dP} \times \frac{d(P)}{dt}$$

Pneumatic Capacitance.

- ✓ According to the ideal gas law; $PV = mRT; P = \rho RT$; $\rho = \frac{P}{RT}$; $\frac{d\rho}{dt} = \frac{1}{RT}\frac{dP}{dt}$

$$\dot{m}_1 - \dot{m}_2 = V\left(\frac{1}{RT}\frac{dP}{dt}\right) + \rho\frac{d(V)}{dP} \times \frac{d(P)}{dt}$$

- ✓ What about writing the final equation in terms of the rate of change of pressure and difference in the mass flow rate.

$$\dot{\mathrm{m}}_1 - \dot{\mathrm{m}}_2 = \left(\frac{\mathrm{V}}{\mathrm{RT}} + \rho\frac{\mathrm{d(V)}}{\mathrm{dP}}\right)\frac{\mathrm{dP}}{\mathrm{dt}}$$

$$\dot{\mathrm{m}}_1 - \dot{\mathrm{m}}_2 = (C_C + C_V)\frac{\mathrm{dP}}{\mathrm{dt}}\ ;\ \dot{\mathrm{m}}_1 - \dot{\mathrm{m}}_2 = C\frac{\mathrm{dP}}{\mathrm{dt}}$$

- ✓ Where C_V =Pneumatic Capacitance due to the change in volume of the container, and $C_C =$ Pneumatic Capacitance due to the compressibility of the gas via the rate of change of pressure difference

Can you now develop an analogy of Pneumatic Capacitance with the electric capacitance?

Pneumatic Inertance.

- The amount of pressure difference required to accelerate the block of gas is known as pneumatic inertance.
- A measure of the pressure difference in a fluid required to cause a unit change in the rate of change of mass flow-rate with time.

$$\sum Forces = ma$$

$$(\mathrm{P}_1 - \mathrm{P}_2)\mathrm{A} = m\frac{d\mathrm{v}}{dt}$$

- But in the pneumatic systems, mass is also changing so what about writing the equation in terms of momentum?

$$(\mathrm{P}_1 - \mathrm{P}_2)\mathrm{A} = \frac{d(m\mathrm{v})}{dt} = \mathrm{L}\frac{d(\rho \mathrm{q})}{dt}; \quad (\mathrm{P}_1 - \mathrm{P}_2) = I\frac{d\dot{m}}{dt}$$

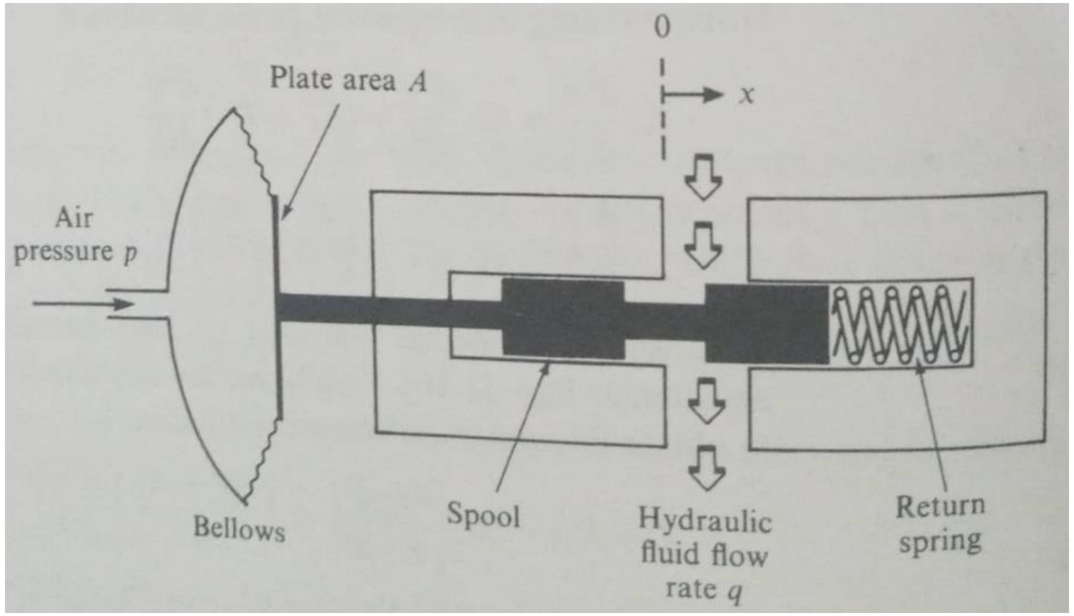
Modelling of a Pneumatic Bellow.

Consider the pneumatic bellow in which gas flows into the body through the applied pressure P_1 which results in the extraction or contraction of the bellow depending upon the generated pressure inside the bellow (P_2). However, before the volume compressibility certain constriction is observed which restricts the inward mass flow rate $(\dot{m})$ and therefore, the modeler wants to analyze the generated pressure (Inside the bellow) as well as extraction or contraction via the applied pressure.

- ✓ Find the differential equation which should relate the generated pressure(Inside the bellow) with the applied pressure.
- ✓ Differential equation comprises of extraction of the bellow with the applied pressure.
- ✓ Transfer function between extraction (x) and applied pressure.

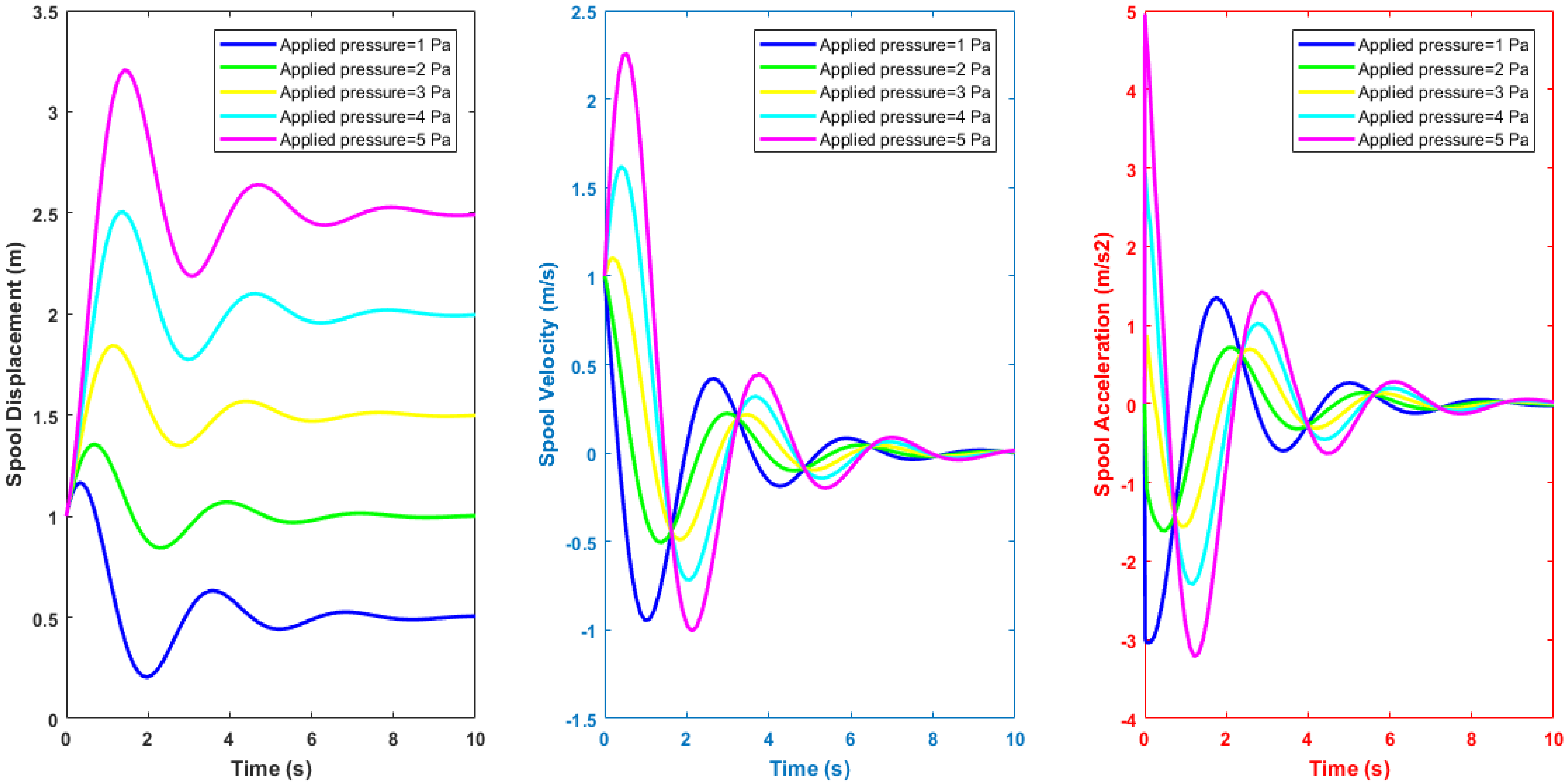
Modelling of a Pneumatically Controlled Spool Valve

Consider the **hybrid system** in which air pressure is indirectly utilized to displace the spool as shown in Figure. Due to change in the displacement, position of the valve relatively changes, and hydraulic fluid flows outward as represented by the outward arrows. You should consider the viscous effects between the spool and valve. You are required to;

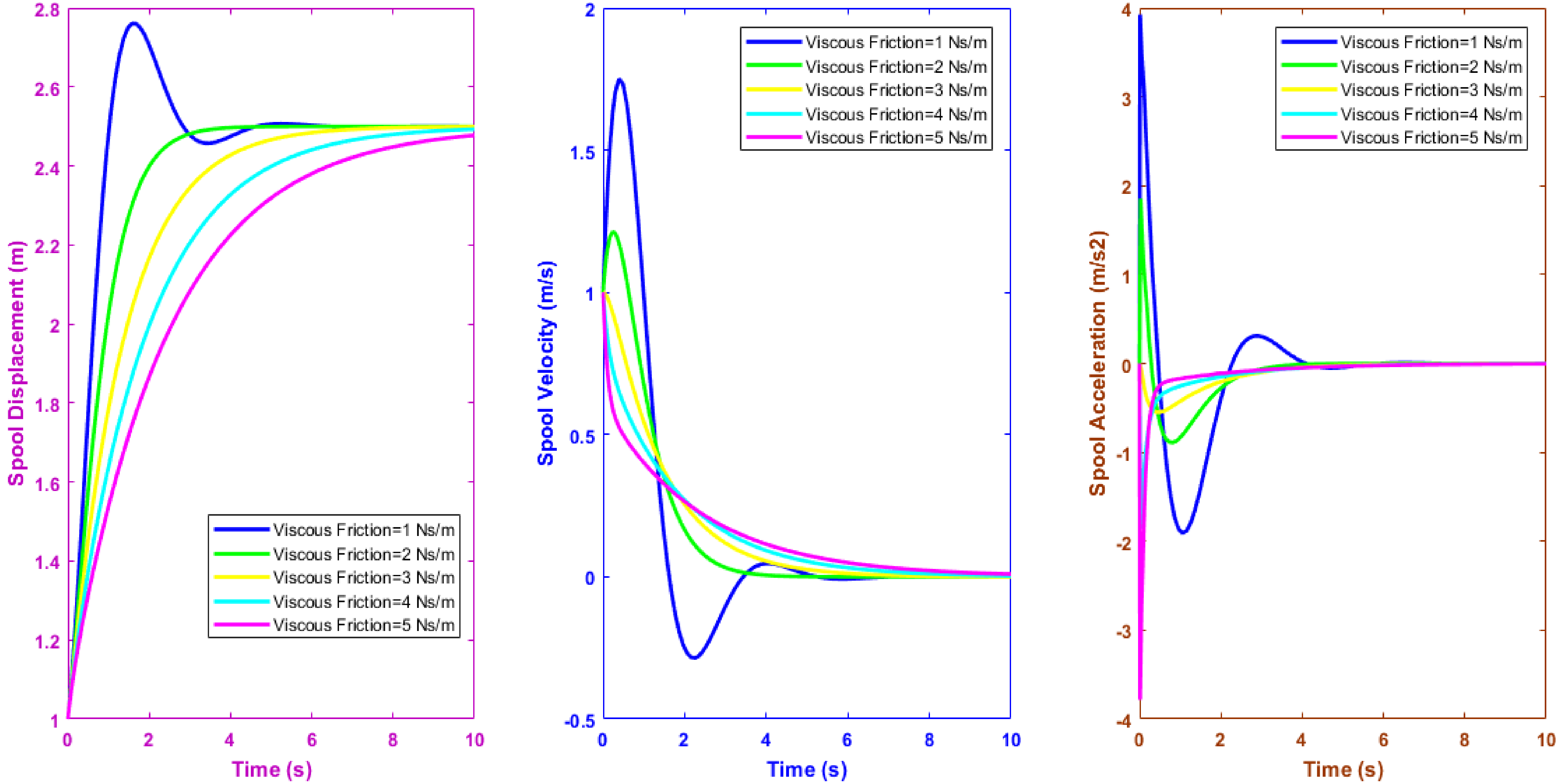


- ✓ Find the differential equation which should relate the spool displacement (x) with the air pressure. Afterward, illustrate the transfer function and state space of the system.
- ✓ Find the differential equation which should relate the hydraulic flow rate with the air pressure. Afterward, illustrate the transfer function and state space of the system.

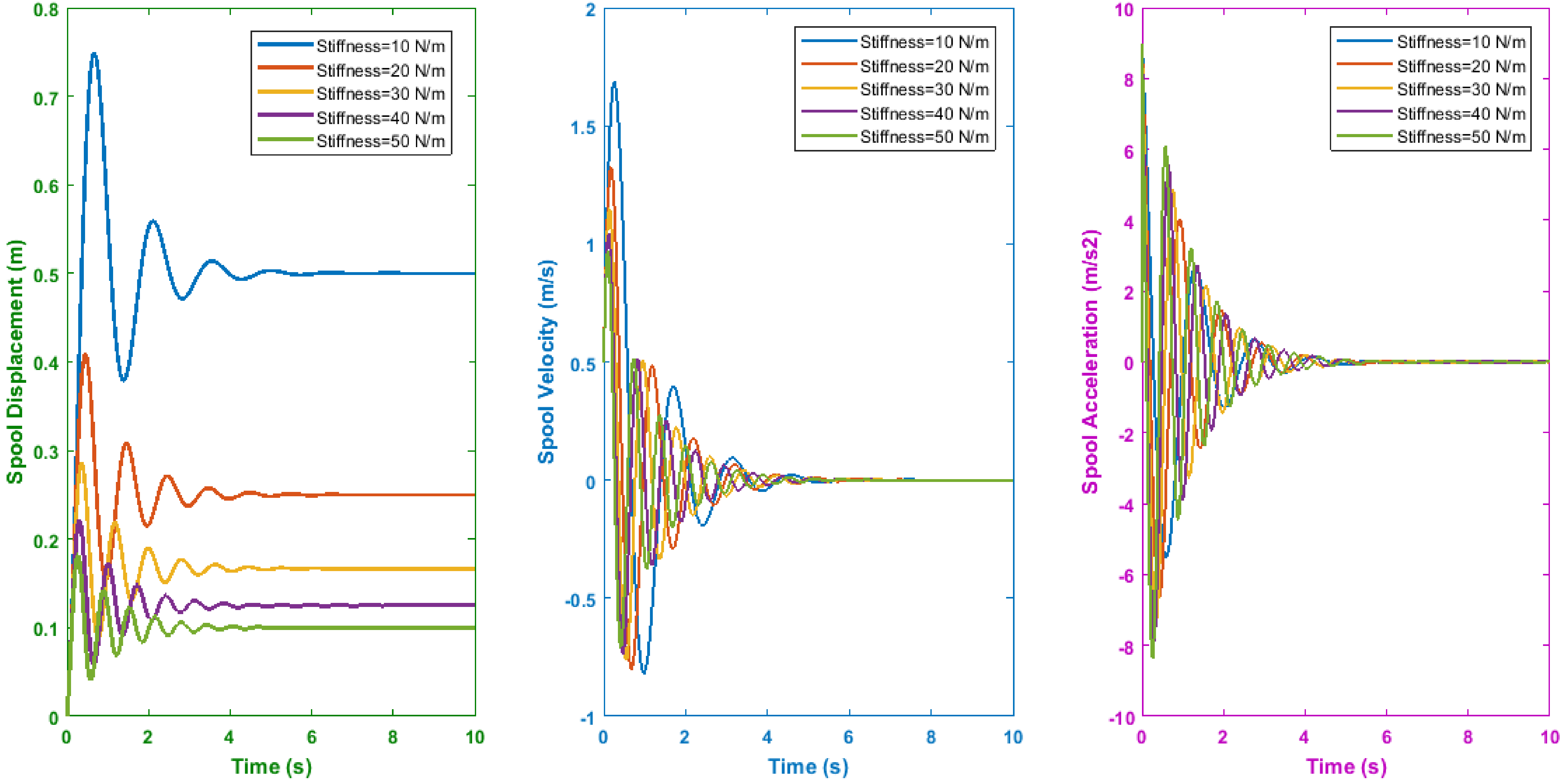
Dynamic Response of Spool Valve via the Applied Pressure



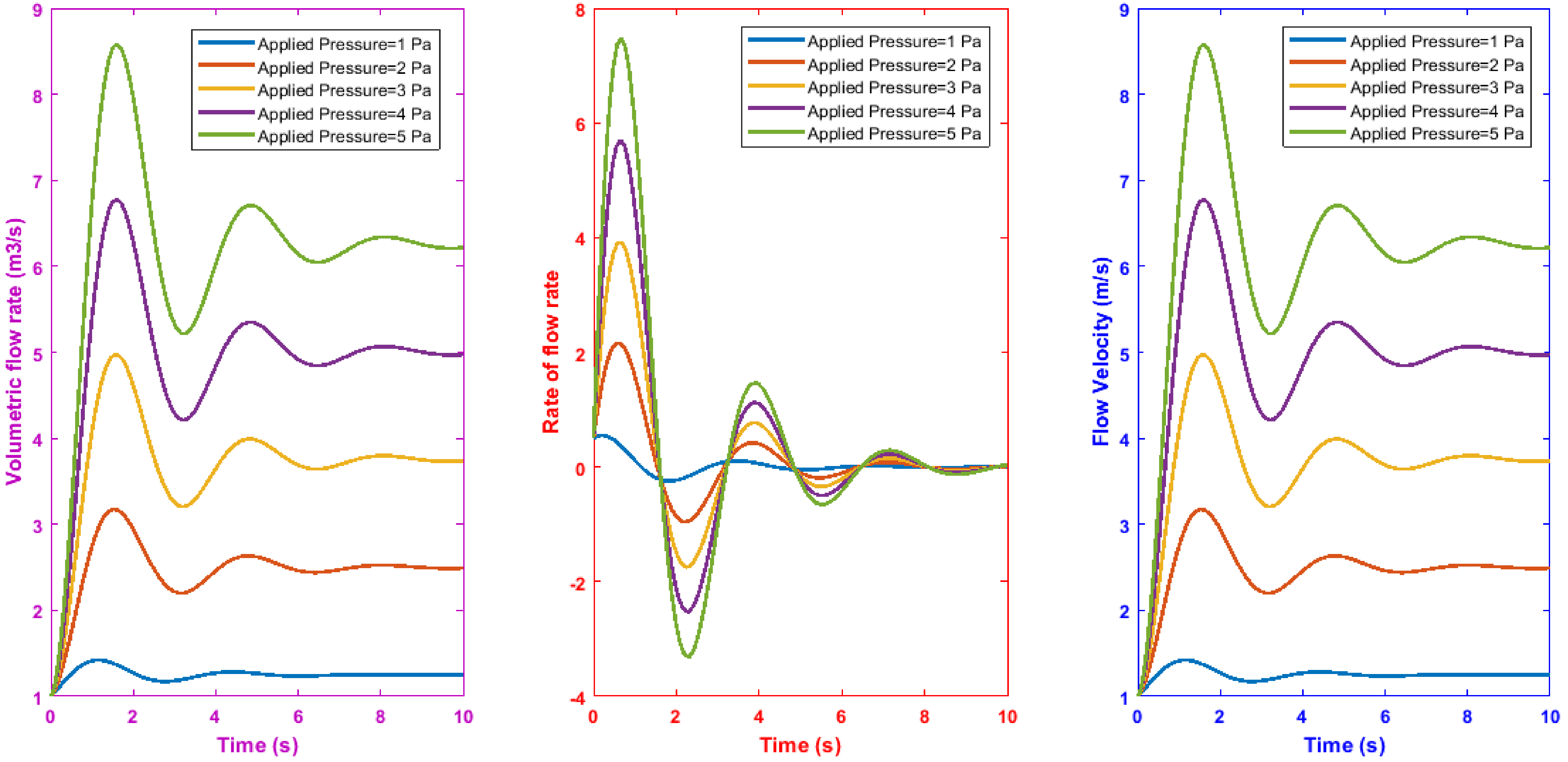
Dynamic Response of Spool Valve via the Piston viscous friction



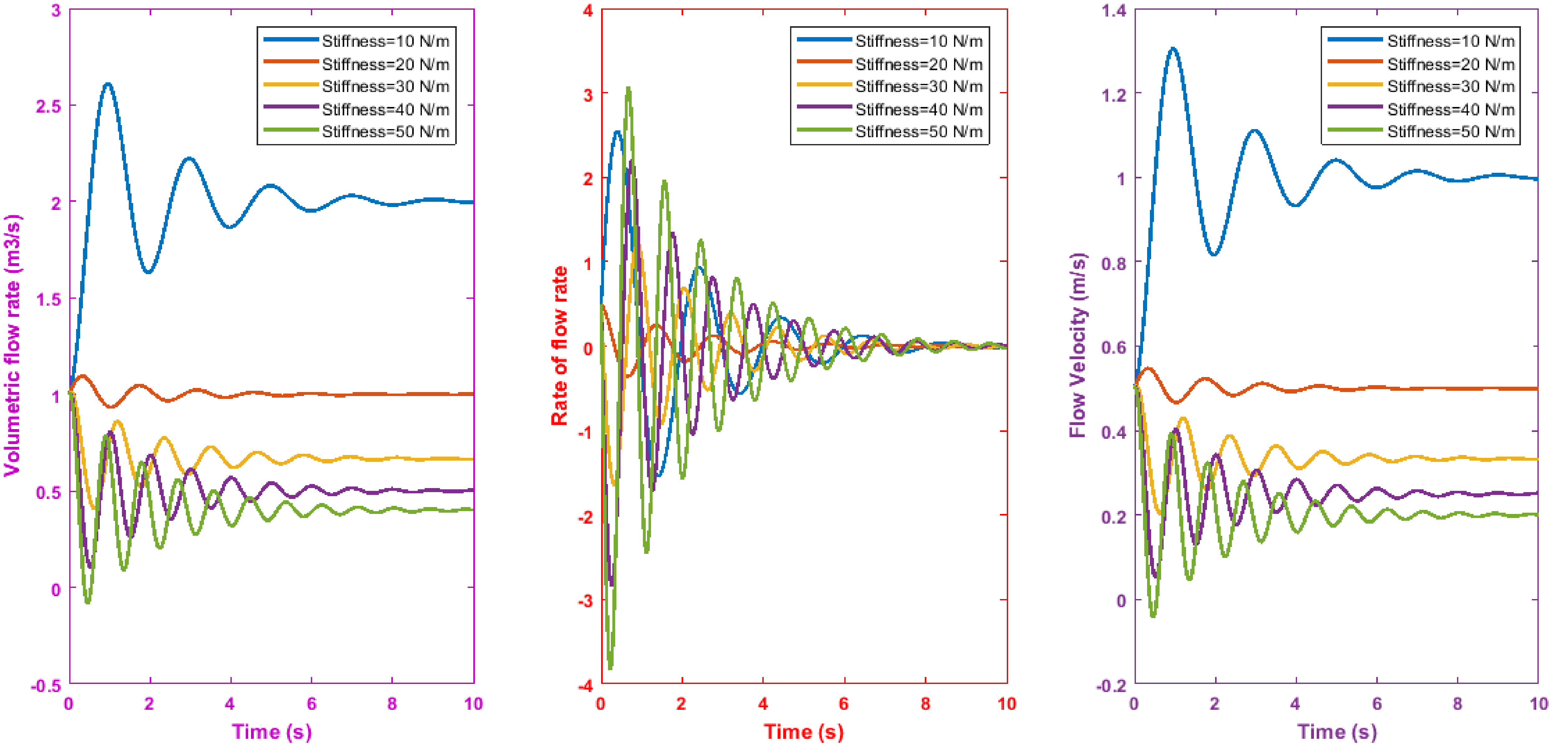
Dynamic Response of Spool Valve via the spring's stiffness



Dynamic Response of flow-rate via the Applied Pressure

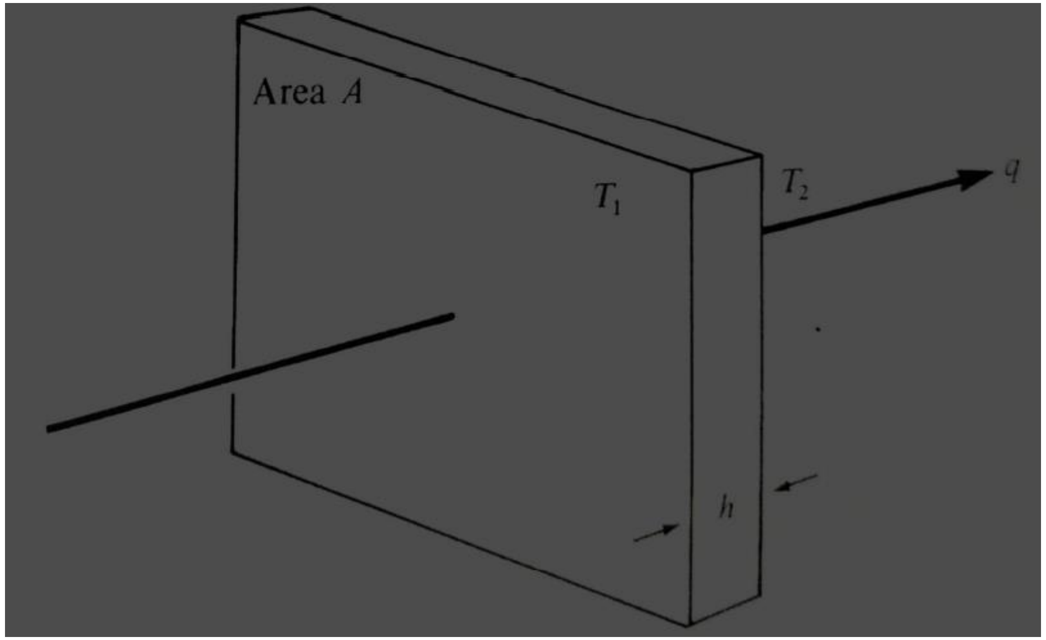


Dynamic Response of flow-rate via the Spring’s Stiffness



Building Blocks of Thermal Systems

- In the modeling of thermal systems, the main quantity of interest will be **heat.**
- The model of thermal systems are obtained by using **thermal resistance** and **thermal capacitance** which are the basic elements of the thermal system.



- **Thermal Resistance**:

$$Change\ in\ temperature \propto Heat\ flow\ rate$$

$$T_1 - T_2 \propto q: T_1 - T_2 = \mathrm{R}q$$

$$\text{Thermal Resistance} = \frac{\text{Change in temperature}}{\text{Heat flow rate}}$$

Can you write an analogy between the resistances of electrical, pneumatic, hydraulic, and thermal systems?

- The value of thermal resistance will always be dependent on the mode of the heat transfer (i.e., Conduction, Convection, and Radiations)

Building Blocks of Thermal Systems

- For a uni-directional conduction: $R = \frac{h}{KA}$, therefore, q= $\frac{T_1 - T_2}{R}$; q=$\frac{\text{(KA)}}{h} T_1 - T_2$
- Where A=Cross-sectional Area(Normal to the rate of heat transfer), K=Thermal Conductivity, and h=Thickness of the material.
- **For convection**, we must consider the **Newton's law of cooling,** and final form of the thermal resistance will be dependent on the convection heat transfer coefficient and area through which heat is being transferred.

$$q = hA(T_1 - T_2); \; q = \frac{T_1 - T_2}{R}; \; R = \frac{1}{hA}$$

- **Thermal Capacitance:** The ability of a material to store heat energy.

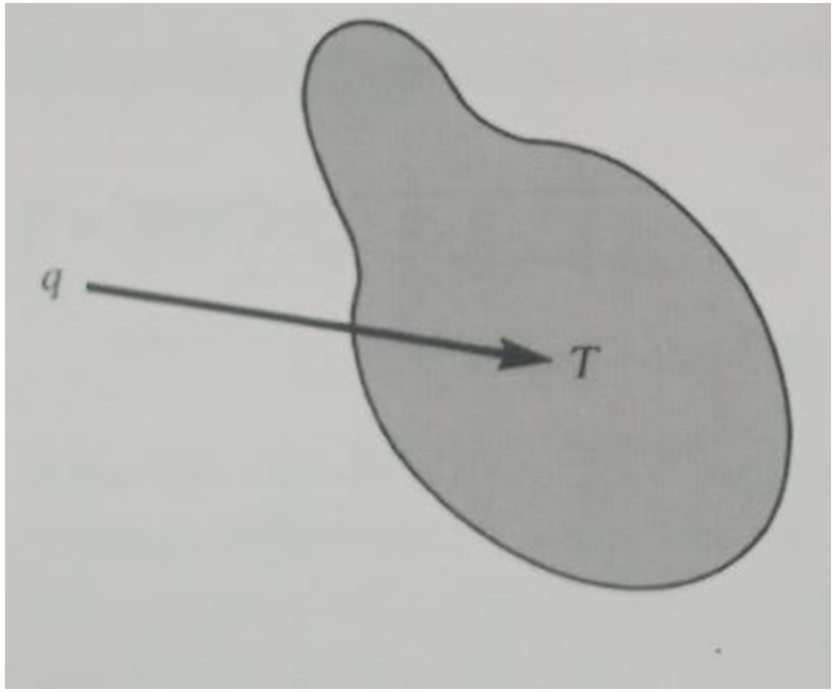
$$\text{Rate of change of internal energy} = q_1 - q_2 = mc\frac{dT}{dt}$$

Building Blocks of Thermal Systems

- ✓ This change in internal energy is likely to be dependent on the mass, volume, and specific heat of the material.

$$q_1 - q_2 = q = \rho V c\,\frac{dT}{dt};\; q = mc\,\frac{dT}{dt}$$

$$q_1 - q_2 = \boldsymbol{C}\,\frac{dT}{dt}$$

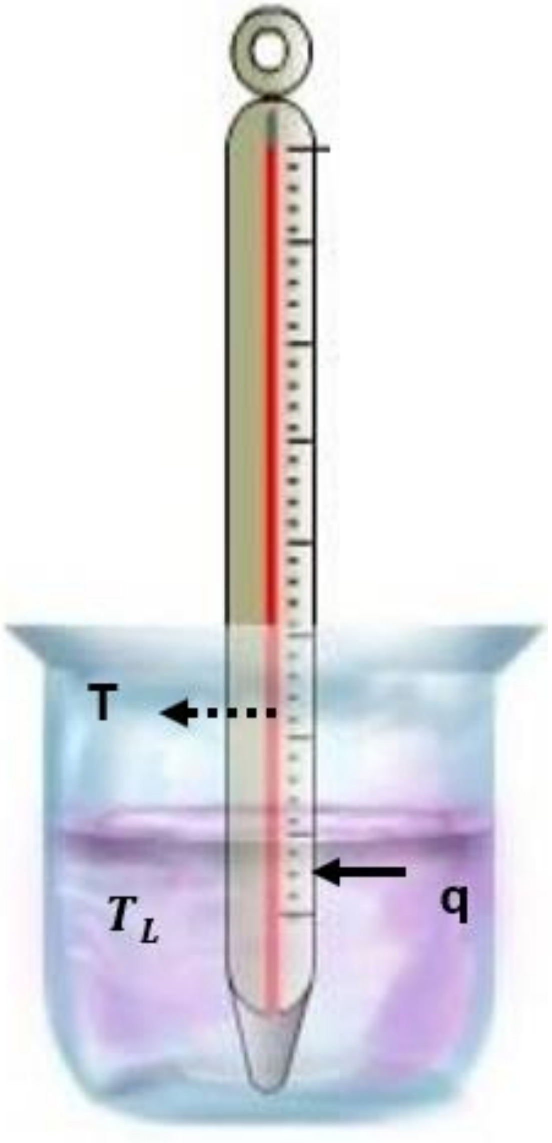


- ✓ Where C defines the thermal capacitance, whose value will be primarily dependent on mass and specific heat of the material.

Can we write an analogous equations for electrical, pneumatic, hydraulic, and thermal Capacitances?

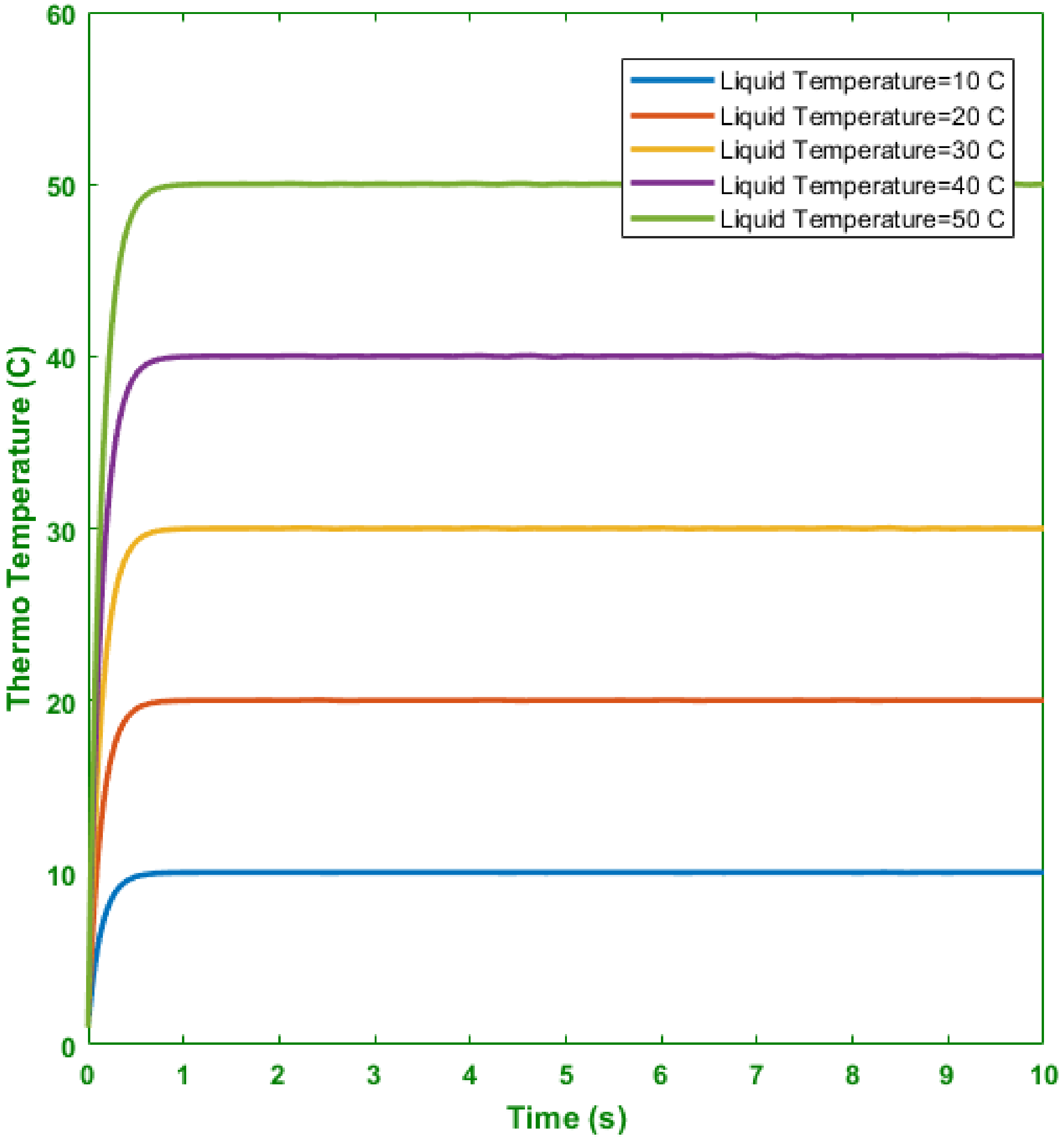
Thermometer in the Liquid

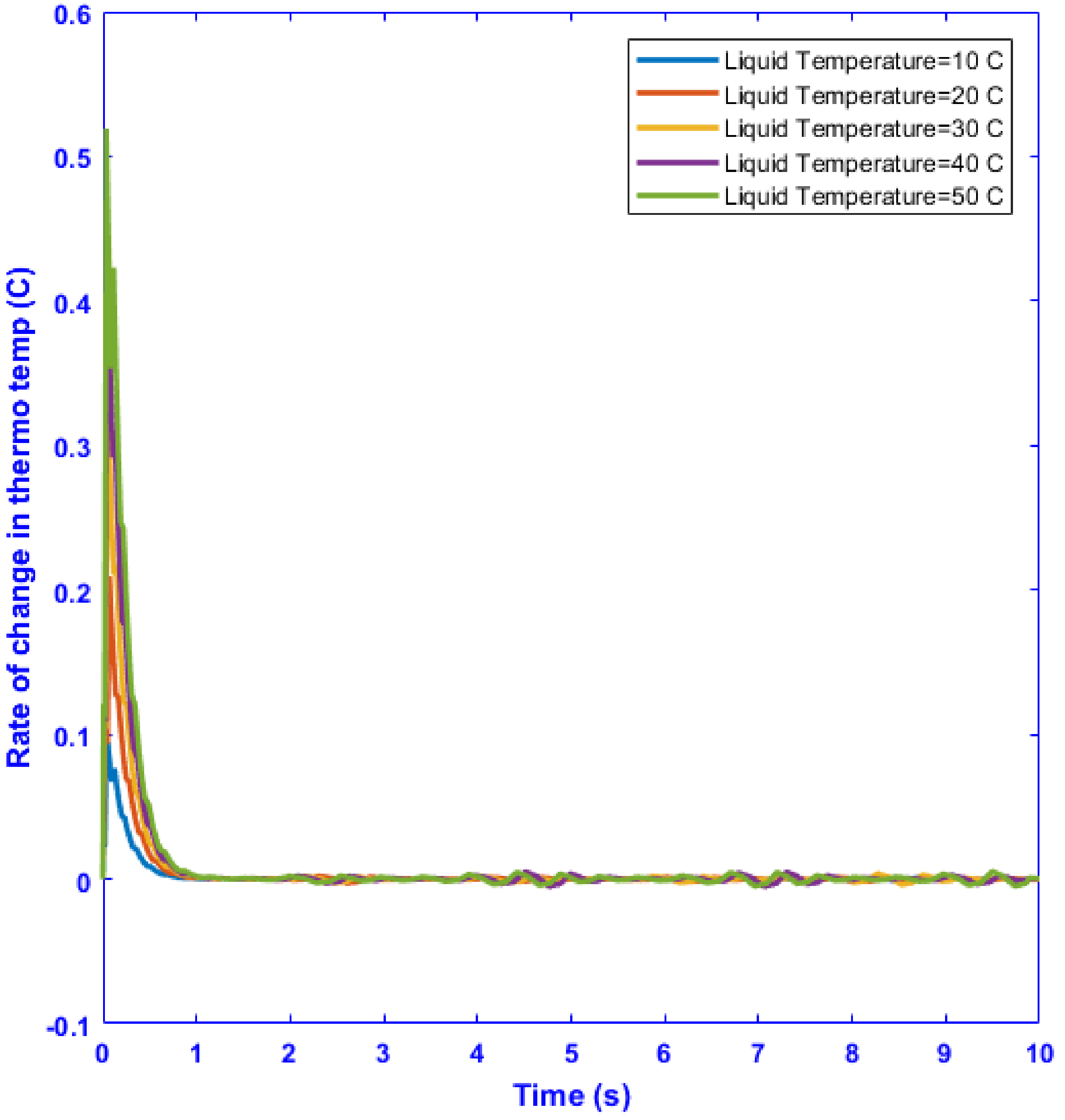
Consider a container of the liquid (With temperature T_L) and a thermometer is inserted in it to measure the temperature of the liquid. Our hypothesis enlightened that the temperature of the liquid is greater than the initial temperature of the thermometer (Tolerance value before measuring the temperature); therefore, heat q will flow from the liquid to the thermometer and consequently, a rise in the temperature of the thermometer will be noticed (T) as shown in **Figure**.



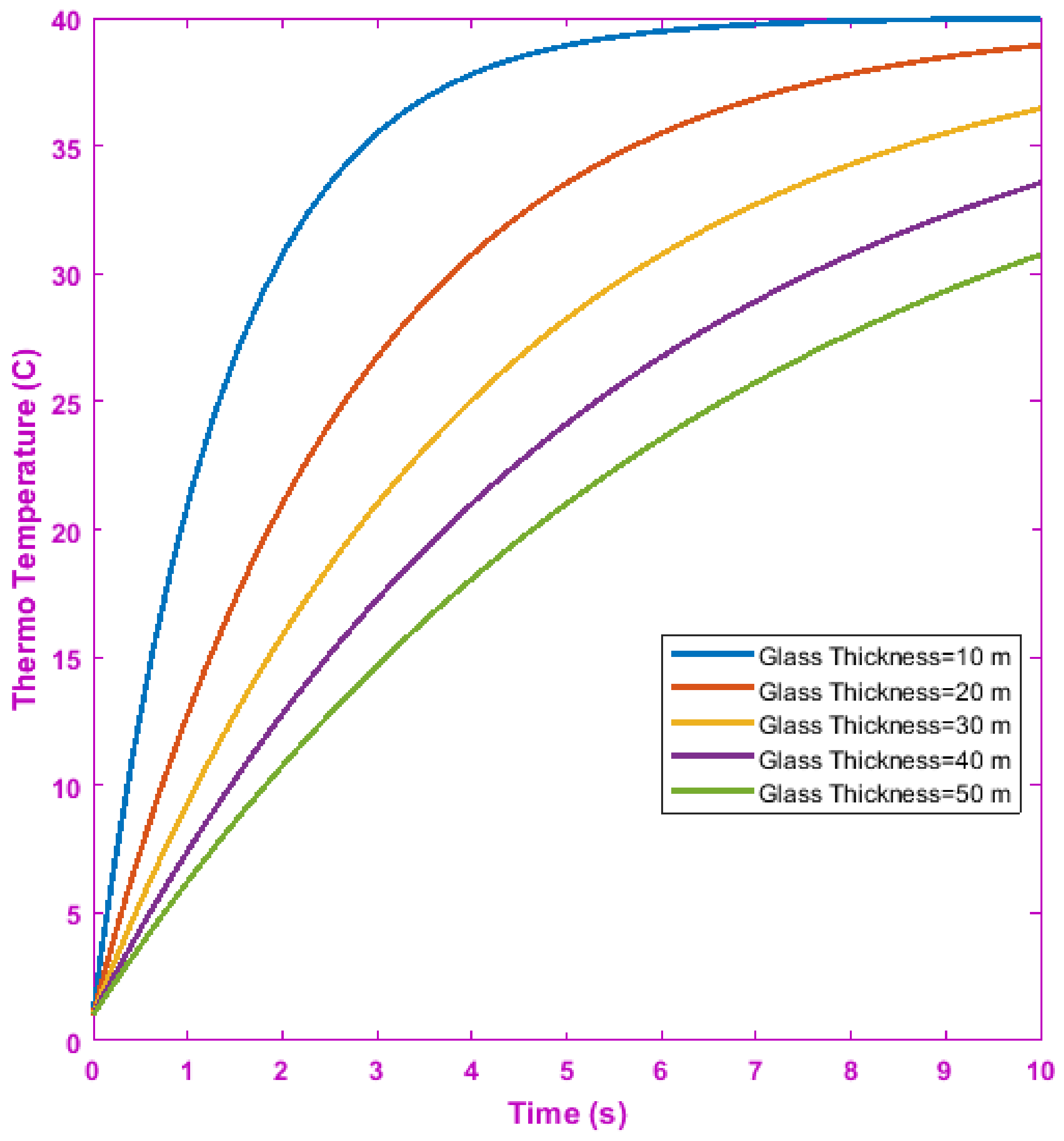
- ✓ Find the differential equation which should relate the thermo-temperature (T) with the temperature of the liquid (T_L).
- ✓ Illustrate the transfer function and state space of the dynamic system.

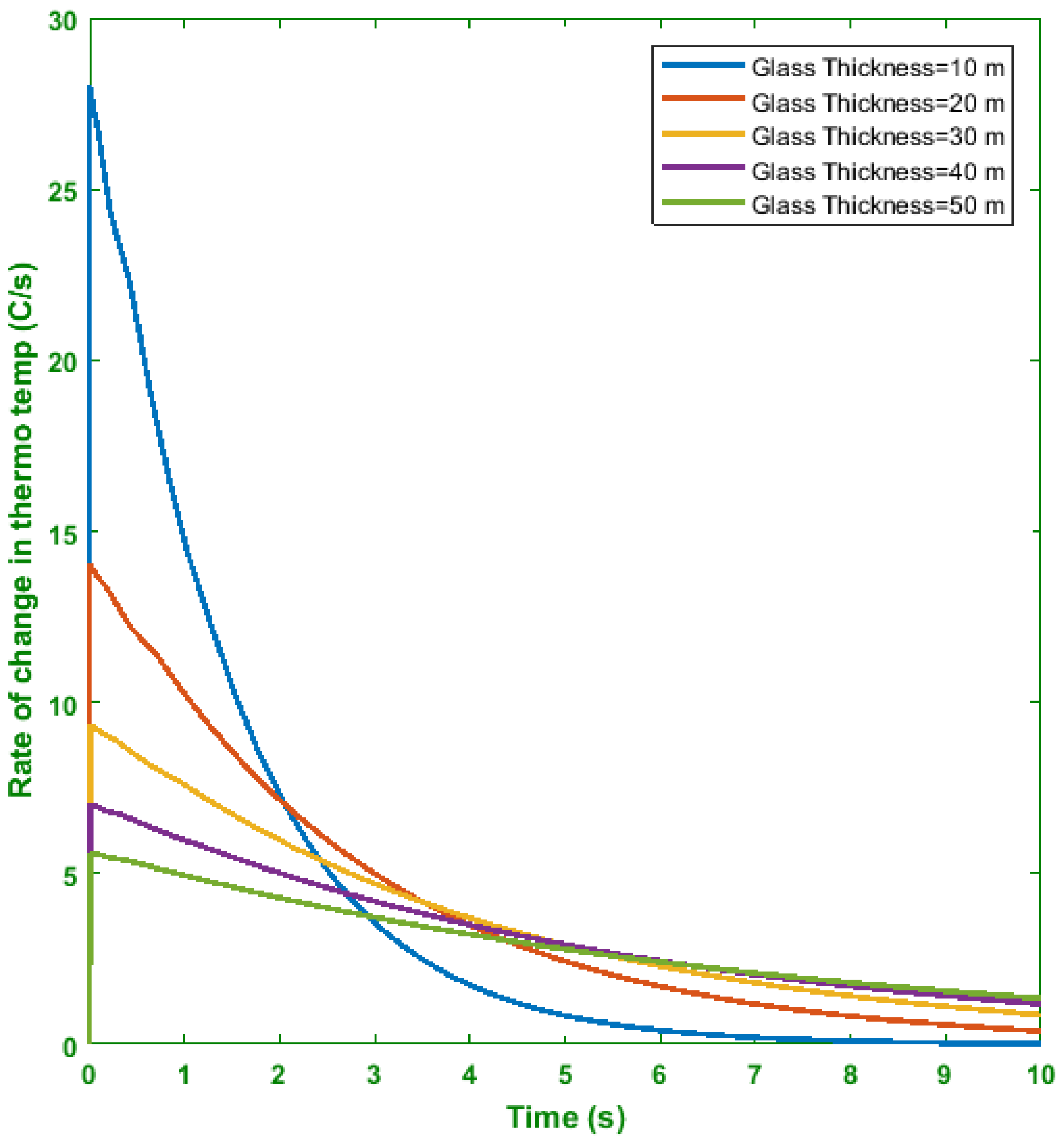
Dynamic Response of Thermo-Temp via the liquid temp





Dynamic Response of Thermo-Temp via the glass thickness

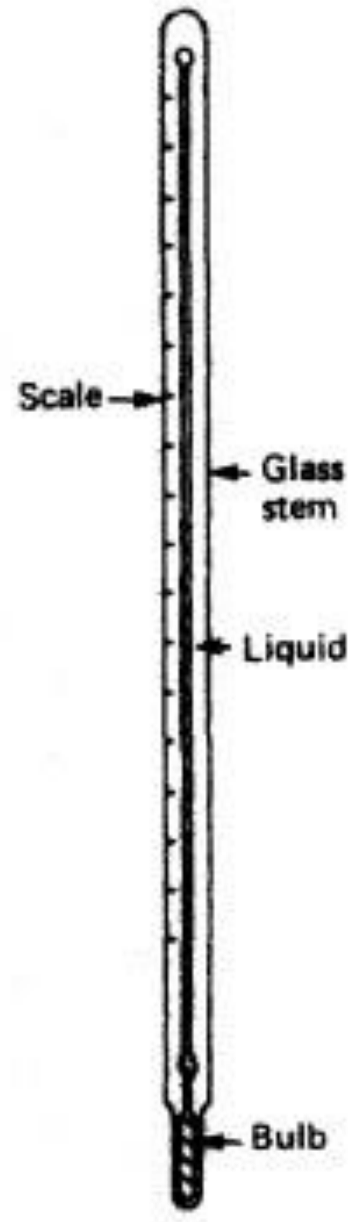




Mercury in Glass Thermometer

Consider mercury in the glass thermometer which is being used to measure the temperature of the liquid (T) and it is hypothesized that the temperatures of the glass and mercury are T_g and T_m, respectively. The interface between the glass and the outside environment is defined by thermal resistance R_g ; likewise, the interface between mercury and the glass is equivalent to thermal resistance R_m. You are required to;

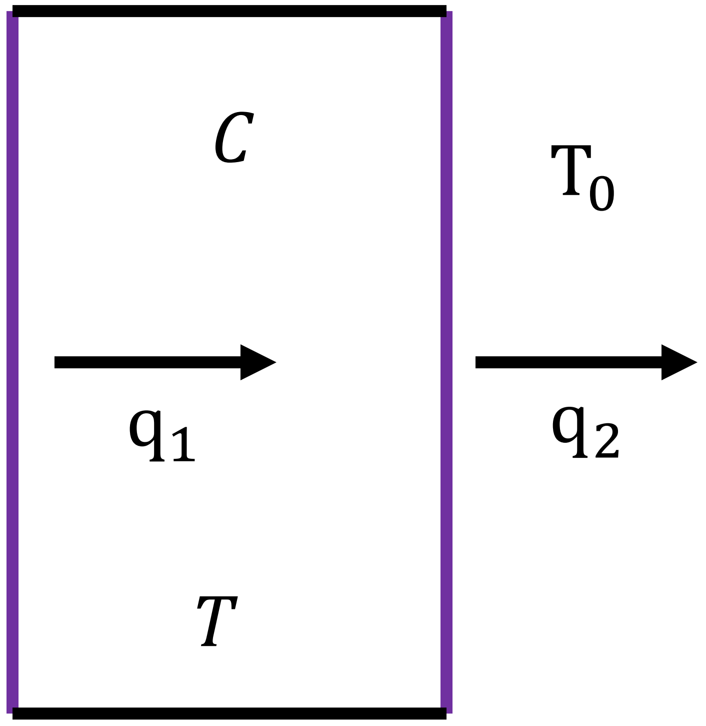
- ✓ Find the differential equation which should relate the actual temperature of the liquid (T) with the temperature of the mercury (T_m).
- ✓ Illustrate the transfer function and state space of the dynamic system.



Simplified Model of Compartment Heating

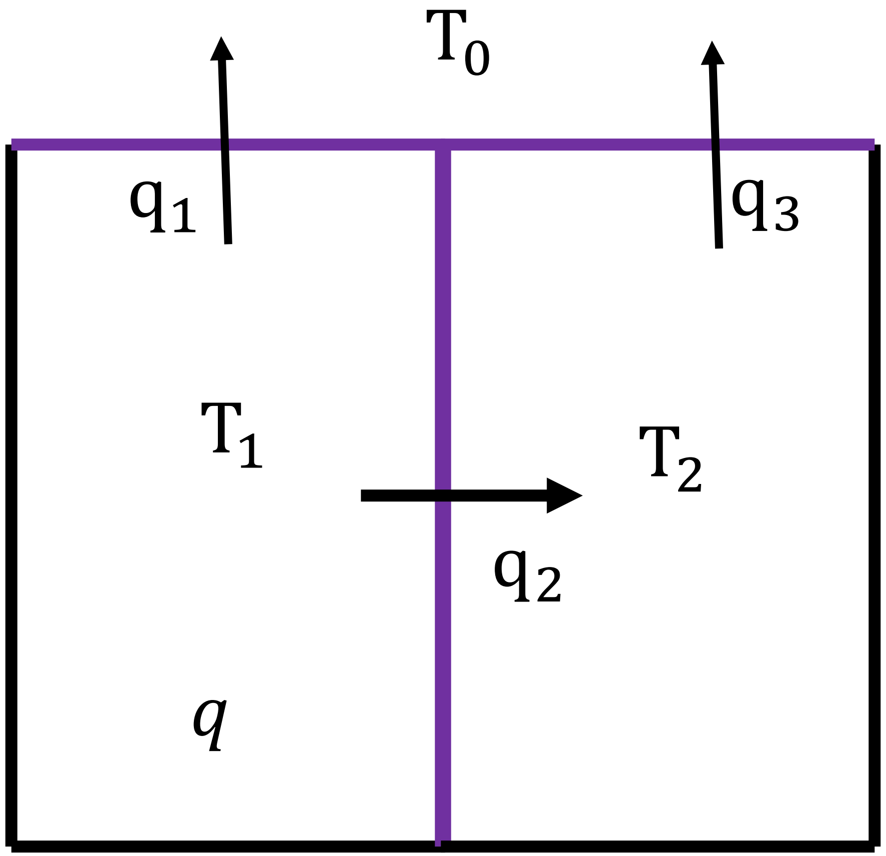
Consider a compartment in which auxiliary ignition is present due to which heat (q_1) is being transferred within the walls of the compartment. This heat transfer results in the rise in temperature T, however, due to the lowest resistance path (Walls of the compartment), heat (q_2) will cross the compartment to achieve the equilibrium state with the ambient (Surrounding) temperature (T_0).

- ✓ Find the differential equation which should include the heat flow due to the ignition (q_1), compartment temperature (T), and surrounding temperature (T_0).



Simplified Model of Compartment Heating

Consider two compartments in which auxiliary ignition is present due to which heat (q) is being transferred within the walls of the first compartment. This heat transfer results in the rise in temperature T_1, however, due to the lowest resistance path (Walls of the compartment), heat (q_1) will cross the compartment to achieve the equilibrium state with the ambient (Surrounding) temperature (T_0). Furthermore, heat (q_2) will also flow into the second compartment resulting in the rise in temperature (T_2) and heat (q_3) will also cross the compartment to achieve the equilibrium state with the ambient (Surrounding) temperature (T_0).



Analyse the change in temperature of both compartments with time?