

displacements

$$\textcircled{3} \quad (-K_1)\theta_1 + (J_2 s^2 + D_3 s + K + M r^2 s^2 + f_v r^2 s + k_2 r^2)\theta_2 - (D_3 s)\theta_3 = T(s)$$

$$\begin{bmatrix} J_1 s^2 + K_1 & -K_1 & 0 \\ -K_1 & J_2 s^2 + D_3 s + K + M r^2 s^2 + f_v r^2 s + k_2 r^2 & -D_3 s \\ 0 & -D_3 s & J_3 s^2 + D_3 s \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \\ 0 \end{bmatrix}$$

Relation is b/w θ_2 & $T(s)$

using crammer rule.

$$\begin{vmatrix} J_1 s^2 + K_1 & T(s) & 0 \\ -K_1 & 0 & -D_3 s \\ 0 & 0 & J_3 s^2 + D_3 s \end{vmatrix} = \frac{\theta_2(s)}{T(s)}$$

$$\begin{vmatrix} J_1 s^2 + K_1 & -K_1 & 0 \\ -K_1 & J_2 s^2 + D_3 s + K + M r^2 s^2 + f_v r^2 s + k_2 r^2 & -D_3 s \\ 0 & -D_3 s & J_3 s^2 + D_3 s \end{vmatrix} = \Delta$$

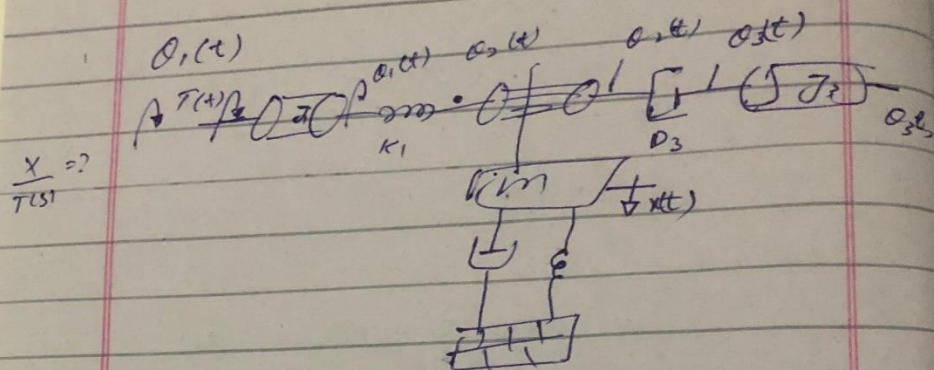
$$\frac{\theta_2(s)}{T(s)} = \frac{-(-K_1)(T(s))(D_3 s + J_3 s^2)}{\Delta}$$

$$X(s) = r\theta_2(s)$$

$$\boxed{\frac{X(s)}{T(s)} = \frac{K_1 (D_3 s + J_3 s^2)}{\Delta}}$$

$\theta_3(s)$

step 1: Linearly independent displacements



$\frac{x}{T(s)}$

$$x(t) = r q_2(t)$$

For
or

$$① \quad J_1 s^2 q_1(s) + k_1 q_1(s) - k_1 q_2(s) = T(s)$$

$$J_2 s^2 q_2(s) + k_1 q_2 - k_1 q_1 + D_3 s q_2 - D_3 s q_3 = r f(s)$$

$$② \quad J_3 s^2 q_3(s) + D_3 s q_3(s) - D_3 s q_2(s) = 0$$

$$x(t) = r q_2(t)$$

$$M s^2 x(s) + f_v s x(s) + k_2 x(s) = F(s)$$

$$J_2 s^2 q_2(s) + k_1 q_2^{(s)} - k_1 q_1^{(s)} + D_3 s q_2^{(s)} - \cancel{D_3 s q_3^{(s)}} - \cancel{r(M s^2 x(s) + f_v s x(s) + k_2 x(s))}$$

$$(J_2 s^2 + D_3 s + k) q_2(s) - (k_1) q_1(s) =$$

$$r(M s^2 x + f_v s x + k_2) r q_2 - (D_3 s) q_3(s)$$

16-october-24

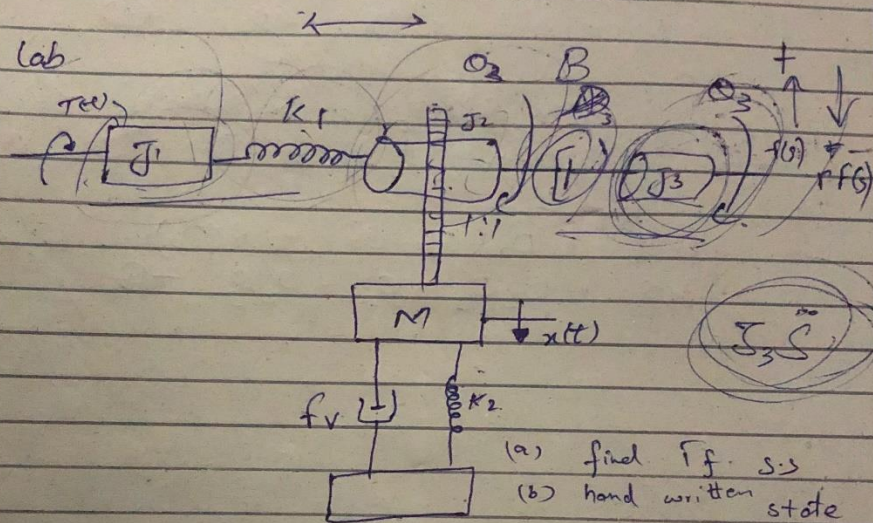
diffs → trig → exp → state space.

CEP (Example of multibody Parametric model)

Pedal → Potentiometer → Bulb → motor

$$\frac{x(s)}{s(s)} \times \frac{1(s)}{1(s)} \times \frac{D}{V_p(s)} \times \frac{V}{D} \times \frac{\omega}{V}$$

$$\frac{\omega}{F(s)}$$



- (a) find TF s.s
- (b) hand written state equation
- (c)
- (d)
- (e)

$$J_1 s^2 + K_1 - K_2 = T$$

$$(J_1 s^2 + K_1) \theta_1 - (K_2) \theta_2 = T(t)$$

$$(J_2 s^2 + K_2) \theta_2 - K_1 \theta_1 = 0$$

$$(J_2 s^2 + K_2 + D_3 s) \theta_2 - K_1 \theta_1 - D_3 \dot{\theta}_3 = -r f(s)$$

$$v \cdot x = r \theta_2$$

$$M \ddot{x} + f_v \dot{x} + K_2 x = f(s)$$

$$M r \ddot{\theta}_2 + f_v r \dot{\theta}_2 + K_2 r \theta_2 = f(s) \quad \text{times}$$

```
clc ;
```

```
% Transfer function (Theta_gear(s) / T(s))
```

```
num = [1];      % Numerator
```

```
den = [J1 0 k1]; % Denominator
```

```
% Create transfer function
```

```
sys_tf = tf(num, den)
```

```
% State-space matrices
```

```
A = [0 1 0 0 0 0;
```

```
      -k1/J1 0 k1/J1 0 0 0;
```

```
      0 0 0 1 0 0;
```

```
      k1/(J2+m*r^2) 0 -(k1+k2*r^2)/(J2+m*r^2) -(fv*r^2+d3)/(J2+m*r^2) 0 d3/(J2+m*r^2);
```

```
      0 0 0 0 0 1;
```

```
      0 0 0 d3/J3 0 -d3/J3];
```

```
B = [0; 1/J1; 0; 0; 0; 0];
```

```
C = [1 0 0 0 0 0];
```

```
D = 0;
```

```
% Create the state-space system
```

```
sys_ss = ss(A, B, C, D);
```

```
%% Part A Variant Torque
```

```
clc;clear;
```

```
TR = [0 50];
```

```
X0 = [0;0;0;0;0;0];
```

```
k1=50;k2=50;fv=50;d3=30.5;T=40;m=15;J1=20;J2=10;J3=16;r=7.5;
```

```
for T=10:5:30
```

```
[t,y]= ode45(@(t,y)randFun2(t,y,T,J1,J2,J3,m,r,fv,k1,k2,d3),TR,X0);
```

```
thGear=y(:,3);
```

```
omGear=y(:,4);
```

```
x=r*thGear;
```

```
v=r*omGear;
```

```
% Compute acceleration
```

```
a1 = gradient(v, t);
```

```
a2 = gradient(omGear, t);
```

```
subplot(2,3,4)
```

```
plot(t,thGear)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('theta-Gear')
```

```
subplot(2,3,5)
```

```
plot(t,omGear)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('Angular velocity-Gear')
```

```
subplot(2,3,1)
```

```
plot(t,x)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('displacement')
```

```
subplot(2,3,2)
```

```
plot(t,v)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('velocity')
```

```
subplot(2,3,3)
```

```
plot(t, a1)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('Acceleration-1')
```

```
subplot(2,3,6)
```

```
plot(t, a2)
```

```
hold on
```

```
xlabel('time')
```

```
ylabel('Angular acceleration-Gear')
```

```
end
```

```
for i=1:6
```

```
subplot(2,3,i);legend('10N Torque', '15N Torque', '20N Torque','25N Torque', '30N Torque')  
end
```

```
%% Part B Variant Radius
```

```
clc;
```

```
clear;
```

```
TR = [0 50];
```

```
X0 = [0;0;0;0;0;0];
```

```
k1=50;k2=50;fv=50;d3=30.5;T=40;m=15;J1=20;J2=10;J3=16;r=7.5;
```

```
for r=1:1:8
```

```
    [t,y]=ode45(@(t,y)randFun2(t,y,T,J1,J2,J3,m,r,fv,k1,k2,d3),TR,X0);
```

```
    thGear=y(:,3);
```

```
    omGear=y(:,4);
```

```
    x=r*thGear;
```

```
    v=r*omGear;
```

```
    % Compute acceleration
```

```
    a1 = gradient(v, t);
```

```
    a2 = gradient(omGear, t);
```

```
    subplot(2,3,4)
```

```
    plot(t,thGear)
```

```
    hold on
```

```
    xlabel('time')
```

```
    ylabel('theta-Gear')
```

```
subplot(2,3,5)
plot(t,omGear)
hold on
xlabel('time')
ylabel('Angular velocity-Gear')
```

```
subplot(2,3,1)
plot(t,x)
hold on
xlabel('time')
ylabel('displacement')
```

```
subplot(2,3,2)
plot(t,v)
hold on
xlabel('time')
ylabel('velocity')
```

```
subplot(2,3,3)
plot(t, a1)
hold on
xlabel('time')
ylabel('Acceleration-1')
```

```
subplot(2,3,6)
plot(t, a2)
hold on
xlabel('time')
ylabel('Angular acceleration-Gear')
```



```
end
```

```
for i=1:6
```

```
subplot(2,3,i);legend('1m radius', '2m radius', '3m radius','4m radius', '5m radius', '6m radius', '7m  
radius', '8mradius')
```

```
end
```

```
function dy= randFun2(t,y,T,J1,J2,J3,m,r,fv,k1,k2,d3)
```

```
dy(1)=y(2);
```

```
dy(3)=y(4);
```

```
dy(5)=y(6);
```

```
dy(2)=1/J1*(T - k1*y(1) + k1*y(3));
```

```
dy(4)=1/(J2+m*r*r)*(k1*y(1) - (d3+fv*r^2)*y(4) -(k1+k2*r^2)*y(3) + d3*y(6));
```

```
dy(6)=1/J3*(d3*(y(4)-y(6)));
```

```
dy=dy';
```

```
end
```