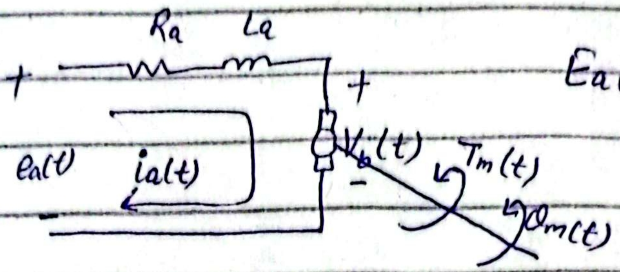
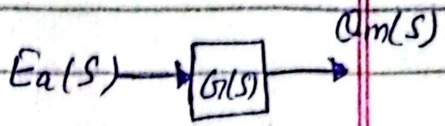
M E & S Finals

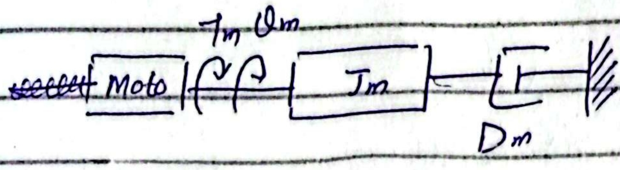
ElectroMechanical





$$E_a(S) = \frac{(R_a + L_a S)\,T_m(S)}{K_t} + K_b S \theta_m(S) \quad — \text{Eq } (1)$$

Now Load



$$(J_m S^2 + D_m S)\,\theta_m(S) = T_m(S)$$

put this in Eq (1)

$$E_a(S) = \frac{(R_a + L_a S)(J_m S^2 + D_m S)\,\theta_m(S)}{K_t} + K_b S \theta_m(S)$$

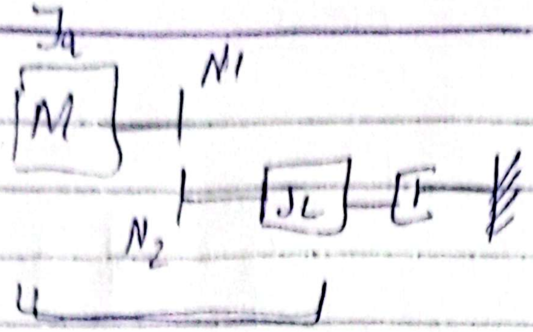
$L_a = 0$ because it is small as compared to armature Resistance R_a

$$E_a(S) = \frac{R_a (J_m S^2 + D_m S)\,\theta_m(S)}{K_t} + K_b S \theta_m(S)$$

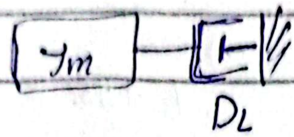
Transfer function.

$$\frac{\theta_m(S)}{E_a(S)} = \frac{K_t / R_a J_m}{S\left[S + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(S)}{E_a(S)} = \frac{K}{S(S+\alpha)}$$



Equivalent



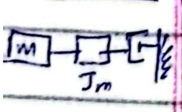
$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

$J_m, D_m \rightarrow$ motor constants.

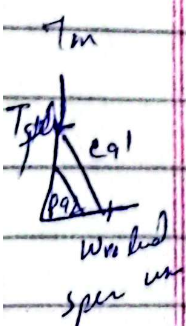
$K_t = Nm/A$ | $\boxed{Nm/A}$ Units
$K_b = V\text{-}s/rad.$ | Vs/rad

Motor



$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/R_aJ_m}{s\left[s+\frac{1}{J_m}\left(D_m+\frac{K_tK_b}{R_a}\right)\right]}}$$

$$\boxed{T_m = -\frac{K_bK_t}{R_a}\omega_m + \frac{K_t}{R_a}e_a}$$

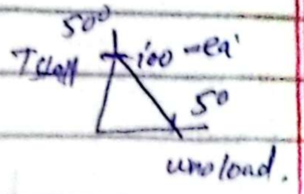


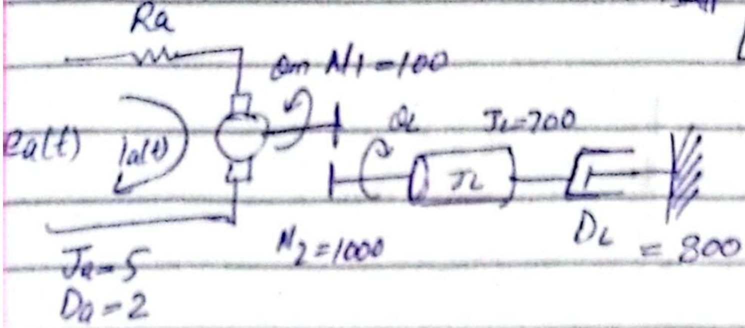
$$\boxed{T_{stall} = \frac{K_t}{R_a}e_a}$$

$$\boxed{\omega_{no\text{-}load} = \frac{e_a}{K_b}}$$

$$\boxed{\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}} \qquad \boxed{K_b = \frac{e_a}{\omega_{no\text{-}load}}}$$

Example 2.23





$$J_M = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{100}{1000}\right)^2 = \boxed{12 = J_m}$$

$$D_M = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{100}{1000}\right)^2 = \boxed{10 = D_m}$$

Now $T_{stall} = 500$ find $\boxed{K_t/R_a\ ,\ K_b}$

$\omega_{no\text{-}load} = 50$ ↓ electric constants

$e_a = 100$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{500}{100} = 5$$

$$K_b = \frac{e_a}{\omega_{no\text{-}load}} = \frac{100}{50} = 2$$

Now

$$TF = \frac{\theta_m(s)}{E_a(s)} = \frac{K_t/R_a J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{(5/12)}{s\left[s + \frac{1}{12}(10 + 5(2))\right]} = \frac{0.417}{s\left[s + 0.083(10+5(2))\right.}$$

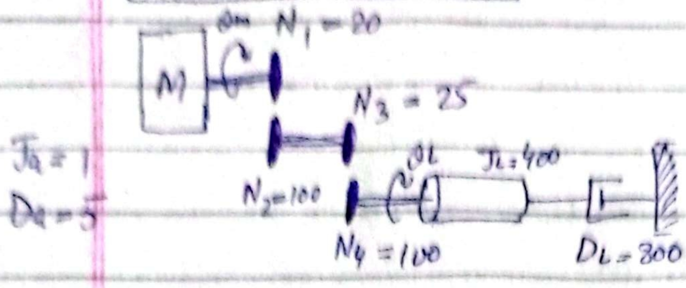
$$\frac{\theta_m(s)}{E_a(s)} = \frac{0.417}{s[s+1.667]}$$

Now θ_L terms × with gear ratio (1/10) N_1/N_2 with θ_m

$$\frac{\theta_L}{E_a(s)} = \frac{0.417 \times \frac{1}{10}}{s[s+1.667]} = \boxed{\frac{0.0417}{s[s+1.667]} = \frac{\theta_L(s)}{E_a(s)}}$$

Answer

2.11 Example



Step 1

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\cdot\frac{N_3}{N_4}\right)^2 = 1 + 400\left(\frac{20}{100}\cdot\frac{25}{100}\right)^2$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\cdot\frac{N_3}{N_4}\right)^2 = 5 + 800\left(\frac{20}{100} + \frac{25}{100}\right)^2$$

$\boxed{J_m = 2}$ $\boxed{D_m = 7}$

Step 2

$e_a = 100V$

$T_m = -8\omega_m + 200$

$$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a} e_a$$

$$T_m = -8\omega m + 200$$

$$T_m = -8\,\omega_m + 2(100)$$

$$T_m = -K_b \cdot \frac{K_t}{R_a}\omega_m + \frac{K_t}{R_a} \cdot e_a$$

$$T_m = -K_b \cdot 2\omega_m + 2 \cdot 100$$

will be 4

$\boxed{\frac{K_t}{R_a} = 2}$

So, $\boxed{K_b = 4}$

Step 3

Now find

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{R_a}/J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{2/2}{s\left[s + \frac{1}{2}(7 + 2(4))\right]}$$

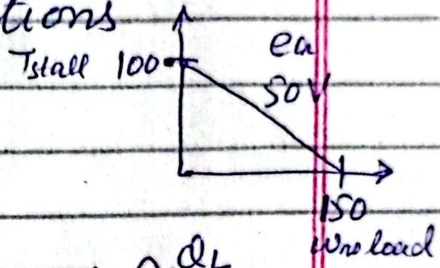
$$\frac{\theta_m(s)}{E_a(s)} = \frac{1}{s[s + 7.5]}$$

In terms of θ_L,
multiply gear ratio with θ_m $\left(\frac{N1}{N2}\frac{N3}{N4}\right)$

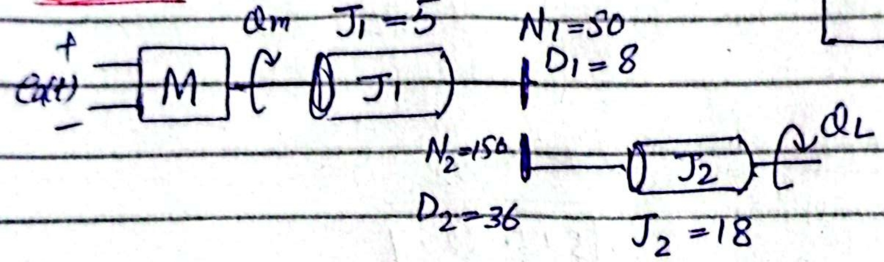
$$\frac{\theta_L(S)}{E_a(S)} = \frac{1 \times \left(\frac{20}{100} \times \frac{25}{100}\right)}{S[S+7.5]}$$

$$\boxed{\frac{\theta_L(S)}{E_a(S)} = \frac{1/20}{S[S+7.5]}}$$ Answer.

Exercise Questions



Q42



Step 1

$$J_m = J_1 + J_2\left(\frac{N_1}{N_2}\right)^2 = 5 + 18\left(\frac{50}{150}\right)^2 = \boxed{7 = J_m}$$

$$D_m = D_1 + D_2\left(\frac{N_1}{N_2}\right)^2 = 8 + 36\left(\frac{50}{150}\right)^2 = \boxed{12 = D_m}$$

Step 2

$T_{stall} = 100$

$\omega_{no\text{-}load} = 150$

$e_a = 50$

Step 3

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a}, \quad K_b = \frac{e_a}{\omega_{no\text{-}load}}$$

$$\frac{K_t}{R_a} = \frac{100}{50}, \quad K_b = \frac{50}{150}$$

$$\boxed{\frac{K_t}{R_a} = 2}, \quad \boxed{K_b = \frac{1}{3}}$$

Step 4

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/R_a \cdot J_m}{s\left[s+\frac{1}{J_m}\left[D_m+\frac{K_t K_b}{R_a}\right]\right]}$$

$$= \frac{2/7}{s\left[s+\frac{1}{7}\left[12+2\left(\frac{1}{3}\right)\right]\right]}$$

$$= \frac{2/7}{s[s+1.809]}$$

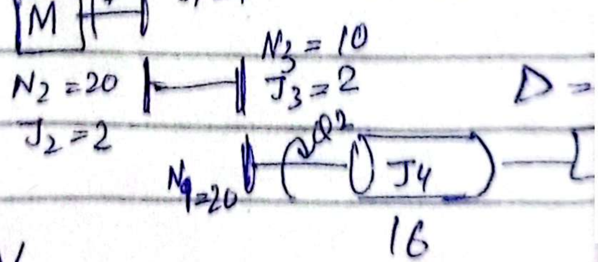
Now in terms of θ_L

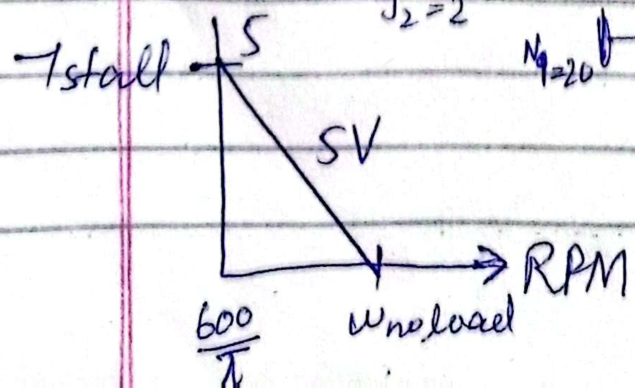
$$\frac{\theta_L(s)}{E_a(s)} = \frac{2/7 \times \frac{N_1}{N_2}}{s[s+1.809]}$$

$$= \frac{2/7 \times \left[\frac{50}{150}\right]}{s[s+1.809]}$$

$$\boxed{\frac{\theta_L(s)}{E_a(s)} = \frac{0.095}{s[s+1.809]}}$$ Answer

Q43





$$J_m = J_1 + (J_2 + J_3)\left(\frac{N1}{N_2}\right)^2 + J_4\left(\frac{N1}{N_2}\cdot\frac{N3}{N_4}\right)^2$$

$$J_m = 1 + (2+2)\left(\frac{10}{20}\right)^2 + 16\left(\frac{10}{20}\cdot\frac{10}{20}\right)^2$$

$$J_m = 1 + (4)\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{4}\right)^2$$

$$J_m = 1 + 4\left(\frac{1}{2}\right)^2 + 16\left(\frac{1}{4}\right)^2$$

$$\boxed{J_m = 3}$$

$$D_m = D\left(\frac{N1}{N_2}\cdot\frac{N3}{N4}\right)^2$$

$$D_m = 32\left(\frac{10}{20}\cdot\frac{10}{20}\right)^2 = 32\left(\frac{1}{4}\right)^2$$

$$\boxed{D_m = 2}$$

Step 2

$T_{stall} = 5$

$\omega_{no\,load} = \frac{600}{\pi}$

Conversion: $\frac{600}{\pi} \times \frac{2\pi}{60}$

$= \boxed{20 \Rightarrow \omega_{no\text{-}load}}$

$e_a = 5V$

Step 3

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{5}{5} = 1$$

$$K_b = \frac{e_a}{\omega_{no\text{-}load}} = \frac{5}{20} = \frac{1}{4}$$

Step 4

$$\frac{\theta_m(s)}{E_a(s)} = \frac{1/3}{s\left[s + \frac{1}{3}\left[2 + (1)\left(\frac{1}{4}\right)\right]\right]} = \frac{1/3}{s(s+0.75)}$$

$\theta_2 = \frac{1}{4}\theta_m$

$$\boxed{\frac{\theta_2(s)}{E_a(s)} = \frac{1/12}{s(s+0.75)}}$$ Answer

Q44

$T_m = 55\ Nm$

speed $\omega = 600$ rad/s

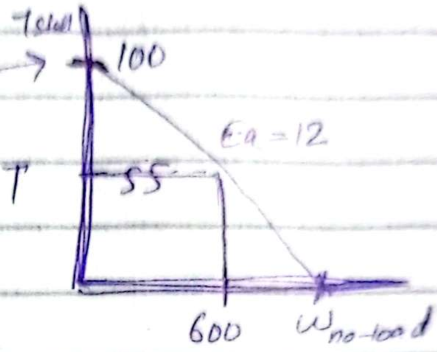
$e_a = 12$

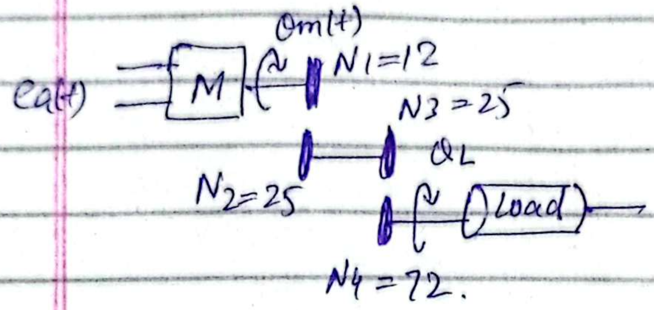
$T_{stall} = 100\ Nm \rightarrow$

$J_a = 7$

$D_a = 3$

Load $= 105$





$$T_m = -\frac{K_b K_t}{R_a}\omega_m + \frac{K_t}{R_a} e_a \quad \text{-- Eq (1)}$$

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a} = \frac{100}{12} = \boxed{8.33 = \frac{K_t}{R_a}}$$

Put in Eq (1)

$$\cancel{55 = -K_b(8.33)(600) + (8.33)(12)}$$

$$\cancel{\frac{55}{(8.33)(600)} - (8.33)(12) = -K_b}$$

$$e_a(t) = \frac{R_a T_m}{K_t} + K_b \omega_m$$

$$12 = \frac{12}{100} \times 55 + K_b\, 600$$

$$12 = 6.6 + 600 K_b$$

$$K_b = \frac{12 - 6.6}{600}$$

$$\boxed{K_b = 9 \times 10^{-3}}$$

$$J_m = J_a + \text{Load}$$

$$= 7 + 105\left(\frac{12}{25} \times \frac{25}{72}\right)^2$$

$$= 7 + 105\left(\frac{1}{6}\right)^2$$

$$\boxed{J_m = 9.92}$$

$$D_m = 3$$

$$\frac{\theta_m(S)}{E_a(S)} = \frac{\left(\frac{100}{12}\right)\left(\frac{1}{9.92}\right)}{s\left(s + \frac{1}{9.92}(3.075)\right)} = \frac{0.84}{s(s+0.31)}$$

Now

$$\boxed{\frac{\theta_L(S)}{E_a(S)} = \frac{0.14}{s(s+0.309)}}$$ Ans-

$$E_a = \frac{R_a}{K_t} T_m + K_b \omega_m$$

$$T_m = -\frac{K_t K_b}{R_a}\omega_m + \frac{K_t}{R_a} e_a$$

q_1 = higher
q_2 = lower

T_1 = higher Temp
T_2 = lower Temp

Thermal Systems

Thermal Resistance:

$$T_1 - T_2 \propto q \Rightarrow T_1 - T_2 = Rq \qquad \boxed{q = \frac{T_1 - T_2}{R}}$$

Conduction

$$R = \frac{h}{KA}, \quad T_1 - T_2 = \frac{h}{KA} q, \quad \boxed{q = \frac{KA}{h} T_1 - T_2}$$

Conduction

Convection

$$R = \frac{1}{hA}, \quad q = \frac{T_1 - T_2}{R} \Rightarrow \boxed{q = hA(T_1 - T_2)}$$

convection

Thermal capacitance:

$$q_1 - q_2 = mc \frac{dT}{dt} \Rightarrow \boxed{q_1 - q_2 = C \frac{dT}{dt}}$$

capacitance

$$q_1 - q_2 = \rho V c \frac{dT}{dt}$$

$m\rho$

$\rho = \frac{m}{V}$

$\rho V = m$

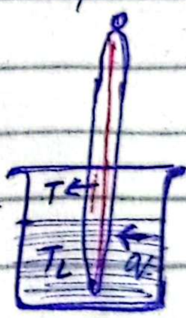
$\boxed{\text{Example 1:}}$ Thermometer in the Liquid.

liquid Temp T_L

Thermometer Temp T

q flows from Liquid to Thermometer

Convection



Equation:

$$T_L - T = Rq \quad \text{Thermal Resistance} \quad Eq\ 1$$

$$\cancel{q = \frac{T_L - T}{R}}$$

because ~~...~~ No heat is transferring outside from thermometer only absorbtion ~~...~~ is happend no dissipation.

$$q - 0 = C\frac{dT}{dt}$$

$$q = C\frac{dT}{dt} \quad \text{Thermal capacitance} \quad Eq\ 2$$

Put q in Eq 1

$$T_L - T = Rq$$

$$T_L - T = R \cdot C \frac{dT}{dt}$$

$$T_L = T + RC\frac{dT}{dt}$$

$$Convection = \frac{1}{hA}$$

$$T_L(t) = T(t) + RC\frac{dT}{dt} \quad (1)$$

Laplace

$$T_L(S) = T(S) + RC\,S \cdot T(S)$$

$$T_L(S) = T(S)\left[1 + RCS\right]$$

Transfer function

$$\boxed{\frac{T(S)}{T_L(S)} = \frac{1}{1+RCS}}$$

$$\frac{T(S)}{T_L(S)} = \frac{1}{1+\frac{1}{hA}\,mc\cdot S}$$

$$\boxed{\frac{T(S)}{T_L(S)} = \frac{1}{1+\frac{mc}{hA}S}}$$

Example 2

Mercury in Glass Thermometer

heat flow ⇒ Surrounding liquid → Glass → Mercury.

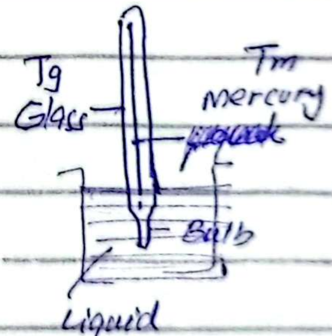
Liquid Temp → T

Glass Temp → T_G

Mercury Temp → T_m

Glass Res → R_g

Mercury Res → R_m



☆ Liquid Surrounding to Glass

$$\boxed{T - T_g = R_g q} \quad \text{Eq } (1)$$

$$\boxed{q - q_g = C_g\frac{dT_g}{dt}} \quad \text{Eq } (2)$$

★ Glass to Mercury

$$\boxed{T_g - T_m = R_m q_g} \quad \text{— Eq (3)}$$

$$\boxed{q_g - q_m = C_m \frac{dT_m}{dt}} \quad \text{— Eq}$$

no ~~heat~~ is ~~following~~ outside from mercury, only absorbtion is happened no dissipation happend.

so,

$$\boxed{q_g - 0 = C_m \frac{dT_m}{dt}} \quad \text{— Eq (4)}$$

We need to Find TF between T and T_m $\frac{T_m(s)}{T(s)}$

Make all terms in form of this

from Eq (1)

$$q = \frac{T - T_g}{R_g}$$

put in Eq (2)

$$q - q_g = C_g \frac{dT_g}{dt}$$

$$\frac{T - T_g}{R_g} - q_g = C_g \frac{dT_g}{dt}$$

$$T - T_g - q_g R_g = C_g R_g \frac{dT_g}{dt} \quad \text{— Eq (5)}$$

Now put q_g using Eq (4) in Eq (5)

$$q_g = \frac{C_m dT_m}{dt}$$

$$\boxed{T - T_g - C_m \frac{dT_m}{dt} R_g = C_g R_g \frac{dT_g}{dt}} \quad \text{— Eq (6)}$$

Now separate q_g from eq (4)

$\boxed{\cancel{q_g = \frac{T_g - T_m}{R_m}}}$ ~~put in eq 6~~

$$q_g = C_m \frac{dT_m}{dt} \quad \text{put in eq (3)}$$

$$T_g - T_m = R_m q_g$$

$$T_g - T_m = R_m C_m \frac{dT_m}{dt}$$

Take its derivative

$$\boxed{\frac{dT_g}{dt} = \frac{dT_m}{dt} + R_m C_m \frac{d^2 T_m}{dt^2}}$$

put value of $\frac{dT_g}{dt}$ in eq (6)

$$T - T_g - R_g C_m \frac{dT_m}{dt} = R_g C_g \left(\frac{dT_g}{dt}\right)$$

$$T - T_g - R_g C_m \frac{dT_m}{dt} = R_g C_g \left(\frac{dT_m}{dt} + R_m C_m \frac{d^2 T_m}{dt^2}\right)$$

$$T - T_g - R_g C_m \frac{dT_m}{dt} = R_g C_g \frac{dT_m}{dt} + R_g C_g R_m C_m \frac{d^2 T_m}{dt^2}$$

Taking laplace

$$T(s) - T_g(s) - R_g C_m \cdot s\, T_m(s) = R_g C_g \cdot s\, T_m(s) + R_g C_g R_m C_m \cdot s\, T_m$$

$$T(s) = T_g(s) + R_g C_m \cdot s\, T_m(s) + R_g C_g\, s\, T_m(s) + R_g C_g R_m C_m \cdot s^2\, T_m(s)$$

Now remove T_g term

using

$$T_g - T_m = R_m C_m \frac{dT_m}{dt}$$

$$T_g = T_m + R_m C_m \frac{dT_m}{dt}$$

laplace

$$\boxed{T_g(s) = T_m(s) + R_m C_m s \, T_m(s)}$$

put this in

$$T(s) = T_g(s) + R_g C_m s \, T_m(s) + R_g C_g s \, T_m(s) + R_g C_g R_m C_m s^2 \, T_m(s)$$

$$T(s) = T_m(s) + R_m C_m s \, T_m(s) + R_g C_m s \, T_m(s) + R_g C_g s \, T_m(s) + R_g C_g R_m C_m s^2 \, T_m(s)$$

$$T(s) = T_m(s) \left[1 + R_m C_m s + R_g C_m s + R_g C_g s + R_g C_g R_m C_m s^2\right]$$

$$\frac{T(s)}{T_m(s)} = \left[1 + R_m C_m s + R_g C_m s + R_g C_g s + R_g C_g R_m C_m s^2\right]$$

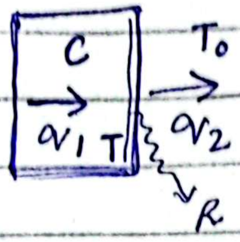
$$\frac{T_m(s)}{T(s)} = \frac{1}{\left[1 + R_m C_m s + R_g C_m s + R_g C_g s + R_g C_g R_m C_m s^2\right]}$$

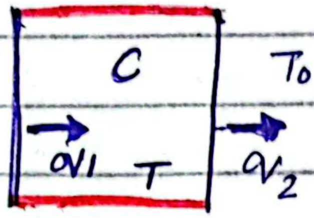
$$\frac{T_m(s)}{T(s)} = \frac{1}{(R_m C_m + R_g C_m + R_g C_g)s + 1 + R_g C_g R_m C_m s^2}$$

$$\boxed{\frac{T_m(s)}{T(s)} = \frac{1}{(R_g C_g R_m C_m)s^2 + (R_m C_m + R_g C_m + R_g C_g)s + 1}}$$

Transfer function

Example 3 :





red → isolated
no heat
transfer

$$T - T_0 = Rq_2$$

$$q_1 - q_2 = C\frac{dT}{dt}$$

put q_2 in above eq.

$$\boxed{\frac{T-T_0}{R} = q_2}$$

$$q_1 - \left(\frac{T-T_0}{R}\right) = C\frac{dT}{dt}$$

$$\left(R = \frac{1}{hA}\right)$$

convection

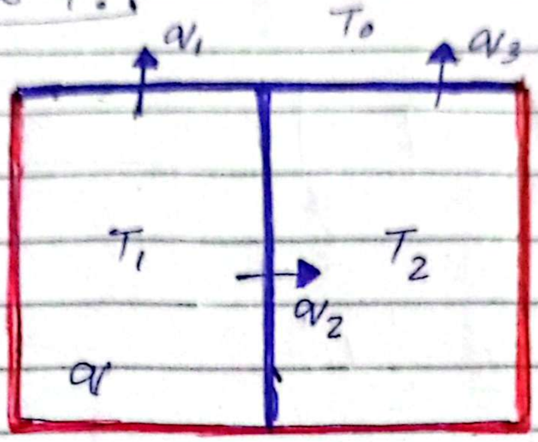
$$C\frac{dT}{dt} = mc\frac{dT}{dt}$$

$$q_1 - (T-T_0)hA = C\frac{dT}{dt}$$

$$q_1 = hA(T-T_0) + mc\frac{dT}{dt}$$

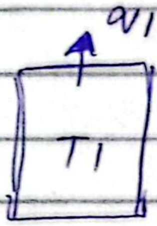
Example 4:

red → no heat transfer.
blue: allow heat transfer.



Resistance

★ $T_1 - T_0 = R_1 q_1$

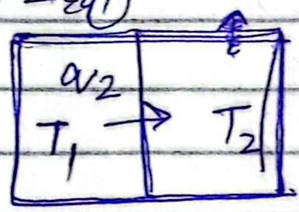


$$\boxed{q_1 = \frac{T_1 - T_0}{R}}$$

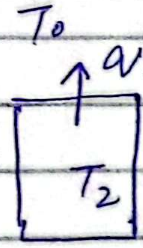
★ capacitance

$$q - (q_1 + q_2) = C\frac{dT_1}{dt} \quad \text{— Eq 1}$$

★ $T_1 - T_2 = R_2 q_2$



$$\boxed{q_2 = \frac{T_1 - T_2}{R}}$$



★ $T_2 - T_0 = R_3 q_3$

$$\boxed{q_3 = \frac{T_2 - T_0}{R}}$$

★ Capacitance

$$q_2 - q_3 = C\frac{dT_2}{dt} \quad \text{— Eq 2}$$

put q_1, q_2 in Eq 1

$$\boxed{q - \left(\frac{T_1 - T_0}{R_1} + \frac{T_1 - T_2}{R_2}\right) = C\frac{dT_1}{dt}}$$

diff Eq. for Compartment 1

put q_2, q_3 in Eq 2

$$\boxed{\left(\frac{T_1 - T_2}{R_2}\right) - \left(\frac{T_2 - T_0}{R_3}\right) = C\frac{dT_2}{dt}}$$

for Compa
2

Hydraulics Systems

Resistance

$P_1 - P_2 \propto q \Rightarrow \boxed{P_1 - P_2 = Rq}$ hydraulic Resistance

Capacitance

$$q_1 - q_2 = \frac{dV}{dt} \quad (V = Ah)$$

$$(P = \rho g h)$$

$$q_1 - q_2 = \frac{d(Ah)}{dt} \quad \left(h = \frac{P}{\rho g}\right)$$

$$q_1 - q_2 = A\frac{dh}{dt}$$

$$q_1 - q_2 = A\frac{d\left(\frac{P}{\rho g}\right)}{dt}$$

$$q_1 - q_2 = \frac{A}{\rho g}\frac{dP}{dt} \quad \left(C = \frac{A}{\rho g}\right)$$

$$\boxed{q_1 - q_2 = C\frac{dP}{dt}}$$ Hydraulic capacitance

Inertance

Forces $= ma$

$$(P_1 - P_2)A = m\frac{dv}{dt} \quad (V = AL)$$

$$(P_1 - P_2)A = \rho V\frac{dv}{dt} \Rightarrow \rho AL\frac{dv}{dt} \quad (q = Av)$$

$v \to \frac{q}{A}$

$$(P_1 - P_2)\cancel{A} = \rho \cancel{A} L\frac{d\left(\frac{q}{A}\right)}{dt}$$

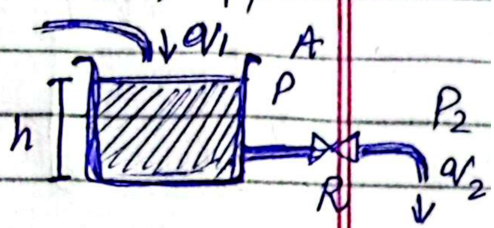
$$(P_1 - P_2) = \frac{\rho L}{A}\frac{dq}{dt} \quad \left(I = \frac{\rho L}{A}\right)$$

$$\boxed{(P_1 - P_2) = I\frac{dq}{dt}}$$ Hydraulic Inertance

Hydraulic Systems

Example: 1

ONE TANK SYSTEM



☆ if we assume that flow rate changes very slowly, we can neglect interance.

Capacitance

$$q_1 - q_2 = C\frac{dP}{dt} \qquad (P = \rho g h)$$

$$q_1 - q_2 = \frac{A}{\rho g}\frac{d(\rho g h)}{dt} \qquad \left(C = \frac{A}{\rho g}\right)$$

$$q_1 - q_2 = \frac{A}{\cancel{\rho g}}\frac{d(\cancel{\rho g}h)}{dt}$$

$$q_1 - q_2 = A\frac{dh}{dt} \quad \text{— Eq (1)}$$

Resistance

0 assume zero because it is going to atmosphere.

$$P_1 - \cancel{P_2} = Rq_2$$

$$\rho g h = Rq_2$$

$$q_2 = \frac{\rho g h}{R} \quad \text{put this in Eq (1)}$$

$$q_1 - q_2 = A\frac{dh}{dt}$$

$$q_1 - \frac{\rho g h}{R} = A\frac{dh}{dt}$$

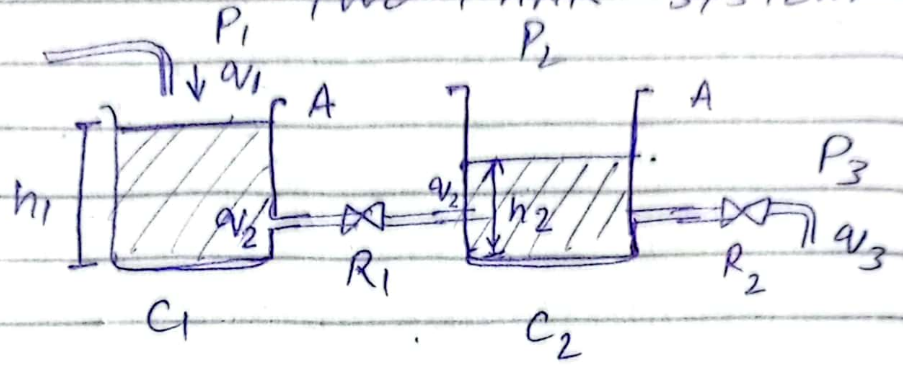
$$\boxed{q_1 = A\frac{dh}{dt} + \frac{\rho g h}{R}}$$

$C = \frac{A}{\rho g}$

$P = \rho g h$

Example 2:

TWO TANK SYSTEM



Container 1

Capacitance

$$q_1 - q_2 = C_1 \frac{dP}{dt} \Rightarrow q_1 - q_2 = \frac{A}{\rho g} \frac{dP}{dt}$$

$$q_1 - q_2 = \frac{A_1}{\rho g} \cdot \frac{d(\rho g h_1)}{dt} \Rightarrow A_1 \frac{dh_1}{dt}$$

$$\boxed{q_1 - q_2 = A_1 \frac{dh_1}{dt}} \quad \text{— Eq (1)}$$

Resistance

$$P_1 - P_2 = R_1 q_2$$

$$q_2 = \frac{P_1 - P_2}{R} \qquad (P = \rho g h)$$

$$q_2 = \frac{\rho g h_1 - \rho g h_2}{R_1}$$

$$\boxed{q_2 = \frac{\rho g (h_1 - h_2)}{R_1}} \quad \text{— ☆}$$

put q_2 in Eq (1) we have

$$\boxed{q_1 - \frac{\rho g (h_1 - h_2)}{R_1} = A_1 \frac{dh_1}{dt}} \quad \text{— Eq (A)}$$

TANK A

$$\boxed{q_1 = A_1 \frac{dh_1}{dt} + \frac{\rho g(h_1 - h_2)}{R_1}} \quad \text{— Eq (A)}$$

Container 2

Capacitance

$$q_2 - q_3 = C_2 \frac{dP}{dt}$$

$$q_2 - q_3 = \frac{A_2}{\cancel{\rho g}} \frac{d(\cancel{\rho g} h_2)}{dt}$$

$$q_2 - q_3 = \frac{A_2}{\,} \frac{d(h_2)}{dt} = \boxed{A_2 \frac{dh_2}{dt} = q_2 - q_3} \quad \text{Eq (2)}$$

Resistance

$$P_2 - \cancel{P_3}^{\,0} = R_2 q_3$$

$$\rho g h_2 - 0 = R_2 q_3$$

$$\boxed{q_3 = \frac{\rho g h_2}{R_2}} \text{ put in Eq (2)}$$

$$q_2 - q_3 = A_2 \frac{dh_2}{dt}$$

$$q_2 - \frac{\rho g h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\boxed{q_2 = A_2 \frac{dh_2}{dt} + \frac{\rho g h_2}{R_2}}$$

Put q_2 from container 1 Res Eq. ☆

$$\frac{\rho g(h_1 - h_2)}{R_1} = A_2 \frac{dh_2}{dt} + \frac{\rho g h_2}{R_2}$$

TANK (B)

$$\boxed{\frac{\rho g(h_1 - h_2)}{R_1} - \frac{\rho g h_2}{R_2} = A_2 \frac{dh_2}{dt}}$$