- **Solution of an equation**
- **Limits**
- **Differentiation**
- **Optimization**
- **Integration**

1. Solution of Equations

In some cases, the command **solve** may fail to produce all the solutions of an equation. In those cases, you can try to find solutions using **fzero** (short for "find zero") command. Just as for **solve**, you need to write equation in the form f(x)=0.

Find a solution near the *x*-value x=*a*, you can use

fzero('left side of the equation**',** a**)**

Example 1

solve the equation $e^{x^2}-2 = x+4$,

first graph the functions on the left and right side of the equation using

```
syms x
ezplot(exp(x^2)-2)
hold on
ezplot(x+4)
hold off
```

From the graph, we can see that the two functions intersect at a value near -1 and at a value near 1. To use **fzero**, we need to represent the equation in the form

$e^{x^2}-2 - (x+4) = 0$

or simplified form $e^{x^2}- x - 6 = 0$.

Then, find the positive solution by using **fzero** to find a zero near 1 and then to find the negative solution near -1 using commands

>> **fzero('exp(x^2)-2-(x+4)', 1)** **ans = 1.415**

>> **fzero('exp(x^2)-2-(x+4)', -1)** **ans = -1.248**

also that the command **solve(exp(x^2)-2-(x+4))**

It returns just the positive solution.

2. Limit of a Function

Example 2

Evaluate the limit when $x \to 2$ of the function $\frac{x^2 - 4}{x - 2}$ we have

>> **syms x**

>> **limit((x^2-4)/(x-2), x, 2) ans = 4**

You can also evaluate left and right limits. For example:

>> **limit(abs(x)/x, x, 0, 'left') ans = -1**

>> **limit(abs(x)/x, x, 0, 'right') ans = 1**

Limits at infinity:

>> **limit(exp(-x^2-5)+3, x, Inf) ans = 3**

3. Differentiation

Example 3

Differentiate x^3 -2x + 5

>> **syms x**

>> **diff(x^3-2*x+5)**

ans = 3*x^2-2

Second derivation of x^3 -2x + 5

>> **syms x**

>> **diff(x^3-2*x+5, 2)**

ans = 6*x

Similarly, the 23rd derivative of sin(x) is obtained as follows.

>> **diff(sin(x), 23)**

ans =-cos(x)

Example 4 Slope the tangent at a point

Find the slope of a tangent line to x^2+3x-2 at point 2, we need to find the derivative and to evaluate it at x=2.

>> **diff(x^2+3*x-2)** **%** (first we find the derivative)

ans = 2*x+3

>> **f = @(x) 2*x+3** **%** (then we representative the derivative as a function)

>> **f(2)** **%** (and, finally, we evaluate the derivative at 2) Obtain

ans = 7

Alternatively, using **matlabFunction** command using the following format.

f = matlabFunction(diff(x^2+3*x-2))

followed by

f(2) ans = 7

4. Optimization

In order to find minimum or maximum values of a given function (using second derivative test)

- Find first derivative
- Solve it for zeros. The x-values you obtain are called critical
- Find second derivative

- Plug critical points in second derivative. If your answer is negative, the function has a maximum value at a critical point used. If your answer is positive, the function has a minimum value at a critical point used.
- Plug critical points in your function. The y-values you obtain are your maximum or minimum values.

Example 5 Find extreme values of x^3-2x+5, start by finding first derivative.

Solution

```
>> diff(x^3-2*x+5)
    ans = 3*x^2-2
Then find critical point(s):
>> solve(3*x^2-2)
    ans = 6^(1/2)/3, -6^(1/2)/3
vpa(ans, 3)
    ans = .816, -.816
Find second derivative
>> diff(x^3-2*x+5, 2)
    ans = 6*x
Evaluate this at critical points.
>> g=@(x) 6*x g(.816)
    ans = 4.896
Positive answer means that the function has minimum at x=.816
>> g(-.816)
    ans = -4.896
Negative answer means that the function has maximum at x=.816
Finding y-values of maximum and minimum:
>>  f=@(x) x^3-2*x+5
>> f(.816)
    ans = 3.911 This is the local minimum value.
>> f(.816)
        ans = 6.088 This is the local maximum value.
```

6. Integration

We can use Matlab for computing both definite and indefinite integrals using the command **int**. For the indefinite integrals, start with **syms x** followed by the command **int(**function**)**. Definite integrals with the command:

int (function, lower bound, upper bound)

Example 6

```
>> int (x^2)              ans = 1/3*x^3
>> int (x^2,0,1)          ans = 1/3
>> int (sin (x)/x, 1, 3)  ans = sin int (3) – sin int (1)
```

By using command **vpa**, we obtain the answer in numerical form

```
>> vpa(ans, 4)            ans  = 0.9026
```

Practice problems

1. Factor $x^3+3x^2y+3xy^2+y^3$.

2. Simplify $\frac{x^3-8}{x-2}$.

3. Evaluate the following expressions. (a) $\sin(\pi/6)$ (b) $\frac{\sqrt{5}+3}{\sqrt{3}-1}$ (c) $\log_2(5)$

4. Solve the following equations and express the answers as decimal numbers.
   (a) $x^3-2x+5=0$ (b) $\log_2(x^2-9)=4$.

5. Let $f(x)=\frac{x^3+x+1}{x}$ (a) Represent *f(x)* as a function in Matlab and evaluate it at 3 and -2.
(b) Find *x*-value(s) that corresponds to *y*-value y=2. (c) Graph *f(x)* on domain [-4 4].

6. Graph $\ln(x+1)$ and $1-x^2$ on the same plot for *x* in [-2 6] and *y* in [-4 4].

7. Find the limits of the following functions at indicated values.

(a) $f(x)=\frac{x^{12}-1}{x^3-1}$, $x \to 1$ (b) $f(x)= 3+e^{-2x}$, $x \to \infty$ (c) $f(x)=\frac{6x^3-4x+5}{2x^3-1}$, $x \to \infty$

8. Let $f(x)=\frac{x^3+x+1}{x}$ Find the first derivative of *f(x)* and evaluate it at *x*=1.

9. Let $f(x)=e^{3x^2+1}$. (a) Find the first derivative of *f(x)*. (b) Find the slope of the tangent line to *f(x)* at x=1. (c) Find the critical points of f(x).

10. Find the 12th derivative of the function $(\frac{x}{2}+1)^{65}$.

11. Find the extreme values of (a) x^3-4x+8 (b) xe^{-3x}

12. Evaluate the following integrals. (a) $\int xe^{-3x}dx$ (b) $\int_0^1 xe^{-3x}dx$.