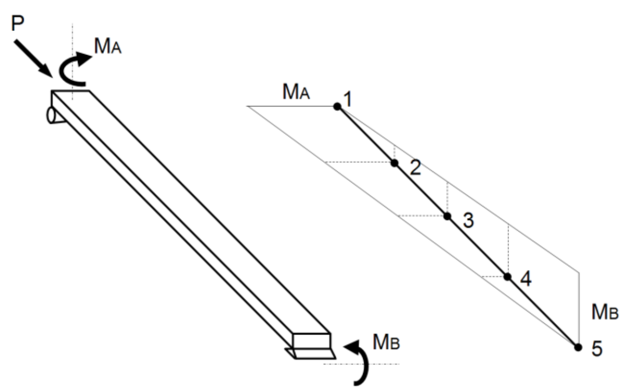
The
C_m Factor

Effect of Moment Gradient on In-Plane Instability of Beam-Columns



15 March 2011

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Effect of Moment Gradient on In-Plane Instability of Beam-Columns – the C_m Factor

Introduction

The purpose of this document is to show the relevance and description of the *concept of equivalent moment* and the necessity of using maximum bending moment on the definition of equivalent moment in opposition to the local bending moment.

Beam–columns are members subjected to axial forces and bending moments simultaneously.

Our software allows two possibilities: local or maximum bending moments. The document also concludes that the best engineering practice is to use the maximum bending moment in each direction. From the picture below, and taking the numbers as code check positions, we can realize that using combination of the local bending moments leads to a usage factor always smaller or equal than using the maximum bending moments.

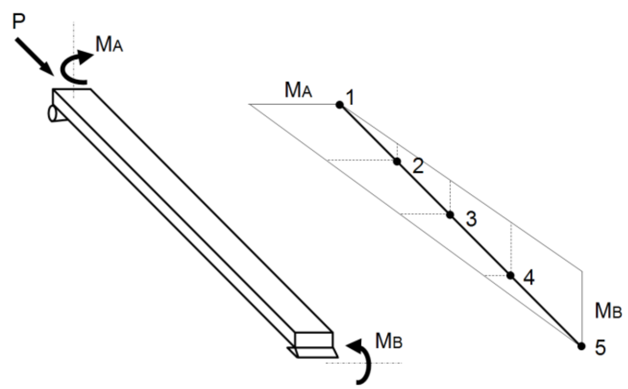


Figure 1 - Schematic view of a beam subjected to end bending moments and axial force and bending moment distribution

For design purpose, it is the value of the maximum bending moment that governs the design. The location of the maximum bending moment is of secondary importance since member cross section properties and length are normally selected based on the magnitude rather than location of the maximum bending moment. As consequence, having the possibility of use of an equivalent bending moment would be useful, and this is fulfilled by introducing the *concept of equivalent moment*.

Beam-columns

Beam–columns are members subjected to axial forces and bending moments simultaneously. Practical applications of the beam-column are numerous. They occur as chord members in trusses, as elements of rigidly connected frameworks, and as members of pin-connected structures with eccentric loads.

The manner in which the combined loads are transferred to a particular beam-column significantly impacts the ability of the member to resist those loads. Starting with the axially loaded column, bending moments can occur from various sources. Lateral load can be applied directly to the member, as is the case for a truss top chord or a column supporting the lateral load from a wall. Alternatively, the axial force can be applied at some eccentricity from the centroid of the column as a result of the specific connections. In addition, the member can receive end moments from its connection to other members of the structure such as in a rigid frame. In all cases, the relation of the beam-column to the other elements of structure is important in determining both the applied forces and the resistance of the member.

Second-order effects

The single most complicating factor in the analysis and design of a beam-column is what is known as *second-order effects*. Second-order effects are the changes in member forces and moments as the direct result of structural deformations. Because the common elastic methods of structural analysis assume that all deformations are small, these methods are not able to account for the additional second order effects. The results of that type of analysis are called *first-order effects*, that is, first-order forces, first –order moments, and first order displacements. To account for the influence of the deformations, an additional analysis must be performed. The results of additional analysis are referred to as the second-order effects.

Several approaches are available for including second-order effects in the analysis. A complete second-order inelastic analysis would take into account the actual deformation of the structure and the resulting forces, as well as the sequence of loading and the behavior of the structure after any of its components are stressed beyond the elastic limit. This approach to analysis is generally more complex than is necessary for normal design.

Nowadays, modern code check standards incorporate parameters in order to model second-order effects. This is evident for compression and bending moment acting together. Two different deflection components that occur in a beam-column influence the moments in that beam-column. The first is the deflection along the length of the member that results from the moment gradient along the member. In this case, the member ends must remain in their original position relative to each other, thus no sway is considered. The moment created by the load P, acting at an eccentricity δ, from the deformed member is superimposed on the moment gradient resulting from the applied end moments. The magnitude of this additional, second-order moment depends on the properties of the column itself. This effect is also known in literature as the P- δ effect.

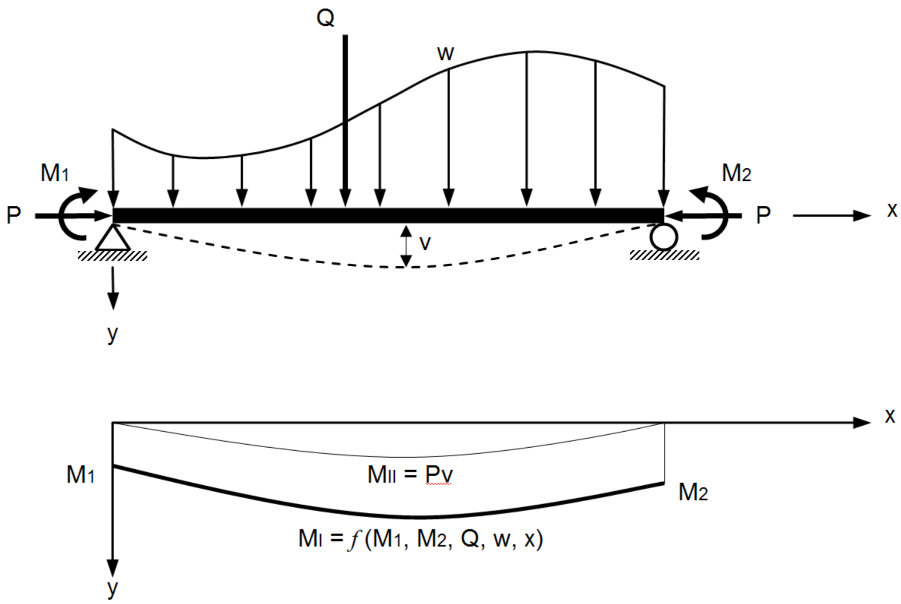


Figure 2 – P-δ effect

When the beam-column is part of a structure that is permitted to sway, the displacements of the overall structure also influence the moments in the member. For a beam-column that is permitted to sway an amount Δ, the additional moment is given by *P*-Δ. Because of the lateral displacement of a given member is a function of the properties of all of the members in a given story, this effect is commonly called *P*- Δ effect.

Both of these second-order effects must be accounted for in the design of beam columns, although, on this document we will just write about *P*- *δ* because is the only that is possible to incorporate directly on code check rules as is observed today.

P-δ effect

Let us considerer the beam-column shown above. The forces and bending moments M_1, M_2, Q and w lead to the primary moment M_I and primary deflection v_I. The axial force P will act on the primary deflection to produce additional moment M_{II} and additional deflections v_{II}. These additional or secondary moments and deflections are the result of the so called *P-δ* effect. Since its effect will increase the instability of the member.

The total bending moment along the beam-column is equal to the sum of the primary and second moments

$$M = M_I + M_{II} \tag{1}$$

and the total deflection is given by

$$v = v_I + v_{II} \tag{2}$$

The most important design variable is the maximum bending (M_{max}) moment along the member. In order to compute the maximum bending moment we may found it by solving the ordinary differential equation of the beam-column with the proper boundary conditions, although, for design purpose, it is more suitable to use a simplified procedure.

For the sake of simplicity let us assume the secondary moment M_{II} has the shape of a half sine wave and the maximum deflection v_{max} ($=\delta = \delta_I + \delta_{II}$) occurs at the midspan, thus M_{II} can be written on the following form

$$M_{II} = P\delta \sin\frac{\pi x}{L} \tag{3}$$

The elastic behavior, the moment in the member is related to the second derivative of the lateral deflection by

$$M_{II} = -EIv''_{II} \tag{4}$$

In the above equation, the double prime denotes differentiation with respect to x axis twice. The minus sign is due to the fact the moment increases with a decreasing slope from $x = 0$ to $x = L/2$.

Eliminating M_{II} from equations (3) and (4), we obtain

$$v''_{II} = -\frac{P\delta}{EI}\sin\frac{\pi x}{L} \tag{5}$$

Integration (5) twice and enforcing the boundary conditions $v_{x=0} = 0$ and $v_{x=L} = 0$ the secondary deflection is given by

$$v_{II} = \frac{P\delta}{EI}\left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L} \tag{6}$$

The secondary deflection at midspan is

$$\delta_{II} = v_{II}\big|_{x=L/2} = \delta\frac{P}{P_e} \tag{7}$$

where $P_e = \pi^2 EI/L^2$ is the *Euler buckling load* for a *pinned-pinned* column.

Since

$$\delta = \delta_I + \delta_{II} \tag{8}$$

Substituting equations (7) into (8) and solving for δ gives

$$\delta = \left(\frac{1}{1-\frac{P}{P_e}}\right)\delta_1 \tag{9}$$

Now, if let us assume that the maximum primary moment occurs at or near midspan, we can write

$$M_{\max} = M_{I\max} + P\delta \tag{10}$$

Substituting (9) into (10) and rearranging we can write

$$M_{\max} = \left(\frac{1 + \psi\, P/P_e}{1 - P/P_e} \right) M_{I\max} \tag{11}$$

where

$$\psi = \frac{\delta_1 P_e}{M_{I\max}} - 1 \tag{12}$$

Let us define

$$C_m = 1 + \psi\, P/P_e \tag{13}$$

This way we can re-write equation (11) as

$$M_{\max} = \left(\frac{C_m}{1 - P/P_e} \right) M_{I\max} = B_1 M_{I\max} \tag{14}$$

where

$$B_1 = \frac{C_m}{1 - P/P_e} \tag{15}$$

and is usually referred to as the *P*- δ amplification factor. By multiplying the maximum first-order moment amplification factor, we can obtain the moment in the member due to the action of the increase in bending moment and action on axial force. If P = 0, the *P*- δ effect does not exist and B1 will equal unit.

It should have in mind that the definition used for ψ in equation (12) is applicable in case of the maximum primary bending moment occurs at or near the midspan. If that is not the case ψ must be redefined.

Beam-columns in a structural frame can be subjected to a variety of loadings. Very often, the maximum moment does not occur at or near midspan of the member. For such cases, the determination of the exact location and magnitude of the maximum moment will require the use of *structural stability theory*. Let us take a look on the determination of the *P*- δ moment magnification factor in such cases.

Basic differential equation of an elastic beam-column

Now, let us explore the location of the maximum bending moment along the beam and its effect combined with the axial compression.

Let us recall Figure 1 and set the basic differential equation governing the elastic in-plane behavior of a beam-column.

The differential equation can be found in any elementary book about structural stability, as for instance, Timoshenko and Gere. The differential equation takes the following form:

$$EI\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + P\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = w \tag{16}$$

The general solution to this differential equation has the form:

$$v = A\sin kx + B\cos kx + Cx + D + f(x) \tag{17}$$

where $k = \sqrt{P/EI}$ and $f(x)$ is a particular solution satisfying the differential equation. The constants *A*, *B*, *C* and *D* must be determined from the boundary conditions of the beam-column under investigation.

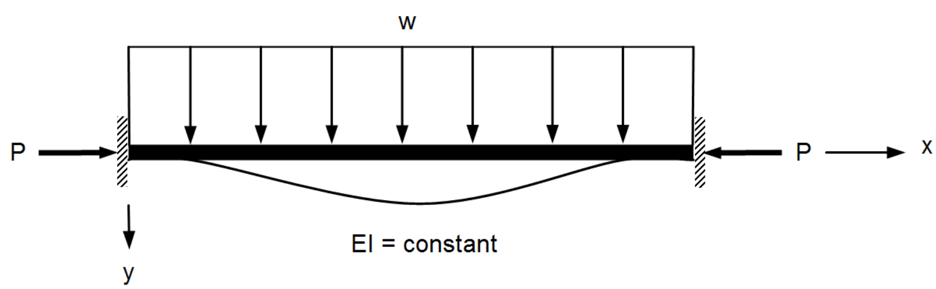


Figure 3 – Fixed-ended beam-column

The general solution to the differential equation is

$$v = A\sin kx + B\cos kx + Cx + D + \frac{w}{2EIk^2}x^2 \tag{18}$$

Using the boundary conditions

$$v_{x=0} = 0,\quad v'_{x=0} = 0,\quad v_{x=L} = 0,\quad v'_{x=L} = 0 \tag{19}$$

In which a prime denotes differentiation with respect to *x*, it can be shown

$$A = \frac{wL}{2EIk^3},\quad B = \frac{wL}{2EIk^3\tan(kL/2)},\quad C = -\frac{wL}{2EIk^2},\quad D = -\frac{wL}{2EIk^3\tan(kL/2)} \tag{20}$$

We can re-write the deflection in the following way

$$v = \frac{wL}{2EIk^3}\left[\sin kx + \frac{\cos kx}{\tan(kL/2)} - kx - \frac{1}{\tan(kL/2)} + \frac{kx^2}{L}\right] \tag{21}$$

The maximum moment for this beam-column occurs at the fixed ends and is equal to

$$M_{\max} = -EIv''\big|_{x=0} = -EIv''\big|_{x=L} = -\frac{wL^2}{12}\left[\frac{3(\tan u - u)}{u^2 \tan u}\right] \tag{22}$$

where $u = kL/2$.

The maximum first-order bending moment occurs at the fixed ends and has the magnitude of $wL^2/12$ and this is present on equation (22). The terms in brackets describe the theoretical moment amplification factor due to P-δ effect.

The theoretical moment amplification factors are different according different loading conditions and boundary conditions. For design purposes, it is more convenient to approximate the theoretical moment amplification factor by a design moment amplification factor similar to the presented in equations (15). However, adaptations to reflect boundary conditions must be included and it is more appropriate to replace the Euler load P_e by the critical load P_{cr} where K represents the effective length factor of the member.

$$P_{cr} = P_{ek} = \frac{\pi^2 EI}{(KL)^2} = \frac{P_e}{K^2} \tag{23}$$

Thus, equation (15) will have the following form

$$B_1 = \frac{1 + \psi P/P_{cr}}{1 - P/P_{cr}} \tag{24}$$

Now, we will determine the value of ψ arising from the fact of fixed-ended boundary conditions. This is done by expanding part of equation (22) by using Taylor series expansion

$$\frac{3(\tan u - u)}{u^2 \tan u} \approx 1 + \frac{u^2}{15} + \frac{2u^4}{315} + \frac{u^6}{1575} + \frac{2u^8}{31185} + \dots \tag{25}$$

Since $u = kL/2 = (\pi/2)\sqrt{(P/P_e)}$,

$$\frac{3(\tan u - u)}{u^2 \tan u} \approx 1 + 0.1645\left(\frac{P}{P_e}\right) + 0.03865\left(\frac{P}{P_e}\right)^2 + 0.09538\left(\frac{P}{P_e}\right)^3 + 0.002377\left(\frac{P}{P_e}\right)^4 + \dots \tag{26}$$

Substituting equation (23) into (26) with K = 0.5 representing fixed-end boundary conditions, we get

$$\begin{aligned}
\frac{3(\tan u - u)}{u^2 \tan u} &\approx 1 + 0.658\left(\frac{P}{P_{cr}}\right) + 0.618\left(\frac{P}{P_{cr}}\right)^2 + 0.610\left(\frac{P}{P_{cr}}\right)^3 + 0.608\left(\frac{P}{P_{e}}\right)^4 + \ldots \\
&= 1 + 0.6\left(\frac{P}{P_{cr}}\right)\left[1.097 + 1.030\left(\frac{P}{P_{cr}}\right) + 1.017\left(\frac{P}{P_{cr}}\right)^2 + 1.013\left(\frac{P}{P_{cr}}\right)^3 + \ldots\right] \qquad (27) \\
&= 1 + 0.6\left(\frac{P}{P_{cr}}\right)\left[\frac{1}{1 - P/P_{cr}}\right] = \frac{1 - 0.4P/P_{cr}}{1 - P/P_{cr}}
\end{aligned}$$

Thus, after carried out the calculations we verify that ψ =-0.4, for this particular case. For beams under other loads and/or boundary conditions a similar approach can be followed to determine ψ. Table 1 and Table 2 will give a synopsis of the most typical cases.

Table 1 - Values of ψ and C_m

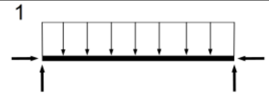
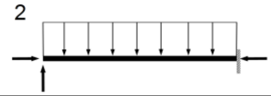
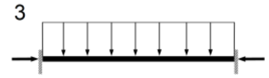
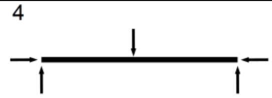
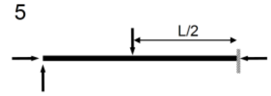
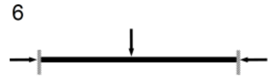
Case	ψ	C_m
1 	0	1.0
2 	-0.4	$1-0.4\frac{P}{P_{cr}}$
3 	-0.4	$1-0.4\frac{P}{P_{cr}}$
4 	-0.2	$1-0.2\frac{P}{P_{cr}}$
5 	-0.3	$1-0.3\frac{P}{P_{cr}}$
6 	-0.2	$1-0.2\frac{P}{P_{cr}}$

Table 2 - Theoretical and design moment amplification factors $u = kL/2 = (\pi/2)\sqrt{(P/P_e)}$

Case	End Conditions	P_{cr}	Location of M_{max}	Moment amplification factor	
				Theoretical	Design
1	Hinged-hinged	$\frac{\pi^2 EI}{L^2}$	Center-line	$\frac{2(\sec u - 1)}{u^2}$	$\frac{1}{1-(P/P_{cr})}$
2	Hinged-fixed	$\frac{\pi^2 EI}{(0.7L)^2}$	End	$\frac{2(\tan u - u)}{u^2(1/2u - 1/\tan 2u)}$	$\frac{1-0.4(P/P_{cr})}{1-(P/P_{cr})}$
3	Fixed-fixed	$\frac{\pi^2 EI}{(0.5L)^2}$	End	$\frac{3(\tan u - u)}{u^2 \tan u}$	$\frac{1-0.4(P/P_{cr})}{1-(P/P_{cr})}$
4	Hinged-hinged	$\frac{\pi^2 EI}{L^2}$	Center-line	$\frac{\tan u}{u}$	$\frac{1-0.2(P/P_{cr})}{1-(P/P_{cr})}$
5	Hinged-fixed	$\frac{\pi^2 EI}{(0.7L)^2}$	End	$\frac{4u(1-\cos u)}{3u^2 \cos u(1/2u - 1/\tan 2u)}$	$\frac{1-0.3(P/P_{cr})}{1-(P/P_{cr})}$
6	Fixed-fixed	$\frac{\pi^2 EI}{(0.5L)^2}$	Center-line and end	$\frac{2(1-\cos u)}{u \sin u}$	$\frac{1-0.2(P/P_{cr})}{1-(P/P_{cr})}$

Analysis of a beam subjected to end moments

Let us working on simple but important example, beam-column subjected to end moments. This example is important in order to understand some code check approaches and meaning of variables involved on the compression and bending situations.

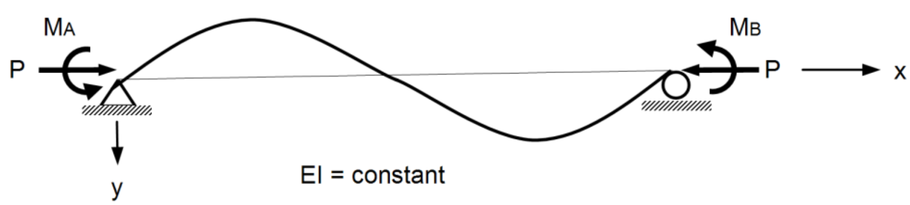


Figure 4 - Beam-column subjected to end moments

Let us start with differential equation governing the buckling problem as we so previously on equation (16)

$$EI\frac{d^4 v}{dx^4} + P\frac{d^2 v}{dx^2} = w$$

The general solution is given by

$$v = A\sin kx + B\cos kx + Cx + D \qquad (28)$$

The constants A, B, C, D are determined by imposing four boundary conditions

$$v\big|_{x=0} = 0, \quad v''\big|_{x=0} = \frac{M_A}{EI}, \quad v\big|_{x=L} = 0, \quad v''\big|_{x=L} = \frac{-M_B}{EI} \tag{29}$$

The solution for the above mentioned constants are:

$$A = \frac{M_A \cos kL + M_B}{EIk^2 \sin kL}, \quad B = -\frac{M_A}{EIk^2}, \quad C = -\left(\frac{M_A + M_A}{EIk^2 L}\right), \quad D = -\frac{M_A}{EIk^2} \tag{30}$$

Substituting these constants into (28) gives

$$v = \frac{1}{EIk^2}\left[\frac{\cos kL}{\sin kL}\sin kx - \cos kx - \frac{x}{L} + 1\right]M_A + \frac{1}{EIk^2}\left[\frac{1}{\sin kL}\sin kx - \frac{x}{L}\right]M_B \tag{31}$$

In order to compute the maximum bending moment, first we proceed to compute the location of the maximum bending moment by setting $\mathrm{d}M/\mathrm{d}x = 0$ and substitute the result into $M = -EIv''$.

$$M_{\max} = \frac{\sqrt{M_A{}^2 + 2M_A M_B \cos kL + M_B{}^2}}{\sin kL} \tag{32}$$

Let us assume M_B as the larger of the two end bending moments, equation (32) can be expressed as

$$M_{\max} = M_B \frac{\sqrt{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}}{\sin kL} \tag{33}$$

As mentioned above, M_B represent the maximum first-order bending moment, the expression in brackets is the theoretical moment amplification factor. The ratio (M_A/M_B) is positive if the member is bent in double curvature and negative if bent in single curvature. In case of equal and opposite (M_B=-M_A =M_0) we will get

$$M_{\max} = M_0\left[\frac{\sqrt{2(1-\cos kL)}}{\sin kL}\right] \tag{34}$$

For this special case the maximum bending moment will occurs at midspan.

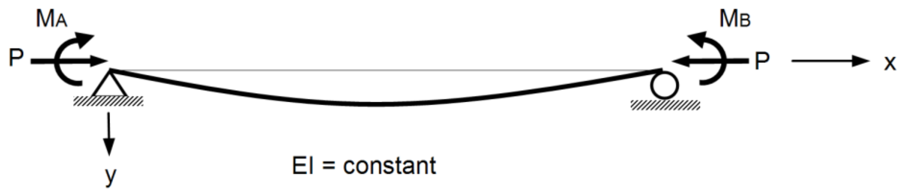


Figure 5 - Beam-column subjected to equal and opposite end moments

The concept of equivalent moment

The case presented in Figure 4 it's a particular case once by symmetry the maximum bending moment occurs at the midspan. For other cases, in order to compute the location of the maximum bending moment we need to set $\mathrm{d}M/\mathrm{d}x = 0$. Knowing $M = -EIv''$, and by consequence $v''' = 0$. Taking the derivatives thrice of equation (31) and set it equal to zero we get the following expression

$$\tan kx = -\left[\frac{(M_A/M_B)\cos kL + 1}{(M_A/M_B)\sin kL}\right] \tag{35}$$

Equation (35) is function of M_A, M_B and P. Let us take as domain solution $0 < x < L$, then the maximum moment occurs within the beam-column length and we can use equation (33) to compute it. Otherwise, the maximum moment occurs at the end and is equal to the large of the two end moments.

For design purpose, it is the value of the maximum bending moment that governs the design. The location of the maximum bending moment is of secondary importance since member cross section properties and length are normally selected based on the magnitude rather than location of the maximum bending moment. As consequence, having the possibility of use of an equivalent bending moment would be useful, and this is fulfilled by present the *concept of equivalent moment*. This concept allow users to base the design of beam-column subjected to any combinations of end moments on an equivalent beam-column subjected to a pair of equal and opposite end moments. The magnified moment of the equivalent beam-column is numerically equal to that of the original beam-column. Figure 5, represent graphically this concept.

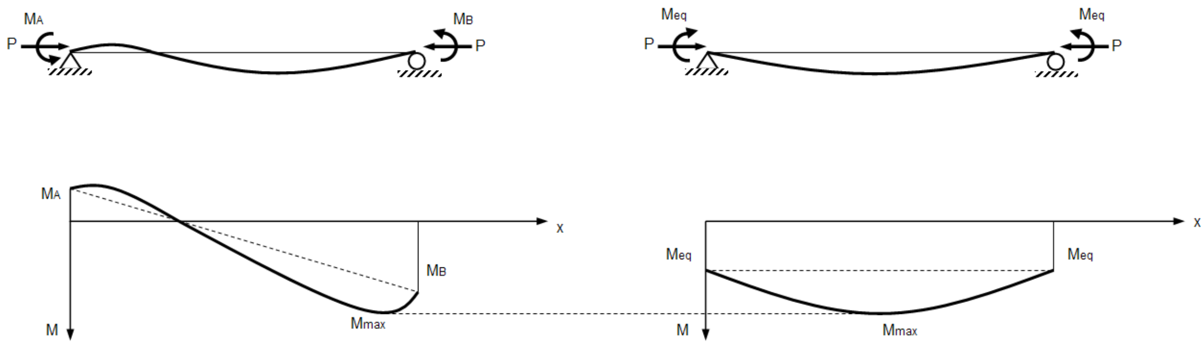


Figure 6 - Equivalent moment concept

Now, let us to determine the magnitude of the equivalent end moment M_A, we set equation (33) equal to equation (34) with M_0 replaced by M_{eq}

$$M_{\max} = M_B \frac{\sqrt{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}}{\sin kL} = M_0 \frac{\sqrt{2(1-\cos kL)}}{\sin kL} \tag{36}$$

Solving for M_{eq} we get

$$M_{eq} = \sqrt{\frac{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}{\sqrt{2(1-\cos kL)}}} M_B = C_m M_B \tag{37}$$

finding the expression for the equivalent moment factor C_m

$$C_m = \sqrt{\frac{(M_A/M_B)^2 + 2(M_A/M_B)\cos kL + 1}{\sqrt{2(1-\cos kL)}}} \tag{38}$$

Once the equivalent moment M_{eq} is obtained, the maximum moment M_{max} in the member can be evaluated using equation (34) with M_0 replaced by M_{eq}.

$$M_{\max} = M_{eq}\left[\frac{\sqrt{2(1-\cos kL)}}{\sin kL}\right] = C_m M_B \left[\frac{\sqrt{2(1-\cos kL)}}{\sin kL}\right] \tag{39}$$

Since M_B is recognized as the maximum primary moment acting on the member, equation (40) can be seen as the moment amplification factor for a beam-column subjected to end moments.

$$\text{moment amplification factor} \xrightarrow{d} C_m \left[\frac{\sqrt{2(1-\cos kL)}}{\sin kL}\right] \tag{40}$$

Let us use equation (14) in order to express equation (39)

$$\frac{\sqrt{2(1-\cos kL)}}{\sin kL} = \frac{\sqrt{2\left(1-\cos \pi\sqrt{P/P_e}\right)}}{\sin \pi\sqrt{P/P_e}} = \sec\left[(\pi/2)\sqrt{P/P_e}\right] \approx \frac{1}{1-P/P_e} \tag{41}$$

Then

$$M_{\max} \approx \left(\frac{C_m}{1-P/P_e}\right) M_B = B_1 M_B \tag{42}$$

where we recall M_B being the maximum bending moment along the beam.

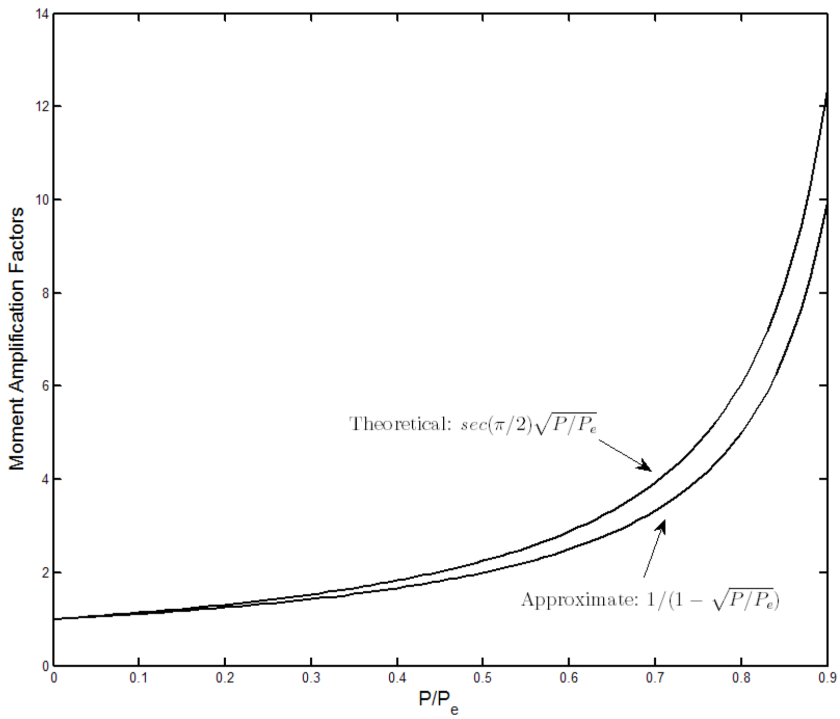


Figure 7 - Theoretical and approximate moment amplification factors.

Example using GeniE

Let us use *GeniE* in order to compute code check usage factors were moment amplification takes part. The results will show the difference between using local bending moments and maximum bending moments. The maximum bending moment should be used by default and is considered to be the best engineering practice.

The example is based on a compressed circular hollow beam subjected to end moments. The beam dimensions are: length (*L*) 10 [m], diameter (*D*) 0.25 [m] and thickness (*t*) 0.01 [m]. Material properties are: Yield stress (σ_y) 300 [MPa], Young Modulus (*E*) 210 [GPa], Poisson coefficient (ν) 0.3 and density (ρ) 7850 [kg/m^3]. The end moments are: M_y = 10 [kN.m] and M_z = 8 [kN.m] and the axial force is 10 [kN] The data above is displayed on Figure 8.

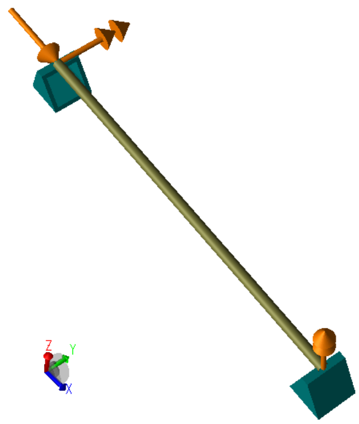


Figure 8 - Circular hollow beam under bi-axial bending and compression.

The need of represent an example with bi-axial bending is due to show the difference on the results by choosing local bending moments or maximum bending moments.

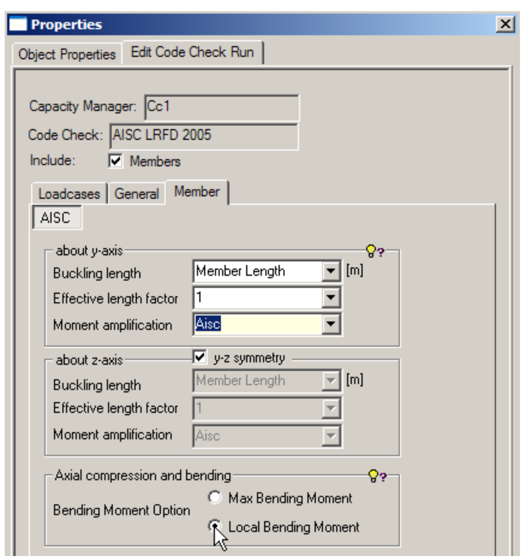


Figure 9 - Axial compression and bending: set computations for local bending moment.

After performing the finite element analysis, the distribution of forces and bending moments along the five code check spots is given in Figure 10. The code check chosen for this example is AISC LRFD 2005, and the usage factor under consideration is ufH1.

Position	NXX [N]	NXY [N]	NXZ [N]	MXX [N*m]	MXY [N*m]	MXZ [N*m]	Wpress [Pa]
0.00	-10000	800	1000	0	10000	0	0
0.25	-10000	800	1000	0	7505.01	1995.99	0
0.50	-10000	800	1000	0	5000	4000	0
0.75	-10000	800	1000	0	2494.99	6004.01	0
1.00	-10000	800	1000	0	0	8000	0

Figure 10 - Forces and bending moments along code check positions.

The results obtained at the relevant positions are given on Table 3. The usage factor ufH1 uses the data displayed on Figure 10 and at each code check position the input data inserted into AISC LRFD 2005, section H1 correspond to each of the lines of Figure 10.

Table 3 - GeniE code check results for AISC LRFD 2005 - Local Bending Moment, ufH1.

Position	0.00	0.25	0.50	0.75	1.00
ufH1	0.070	0.067	0.064	0.060	0.057

So, using the local bending moment the governing usage factor will be ufH1 and the computed value of 0.070 occurring at position 0.00 where is located the absolute value of maximum bending moment. Taking a brief look on GeniE's we get a ratio $P_r/P_c = 0.011 < 0.2$ meaning that ufH1 will be given by formula (H1-1b) from AISC LRFD 2005 Standard.

In this example C_m is set to be computed internally using AISC definitions, as it can be observed in Figure 9.

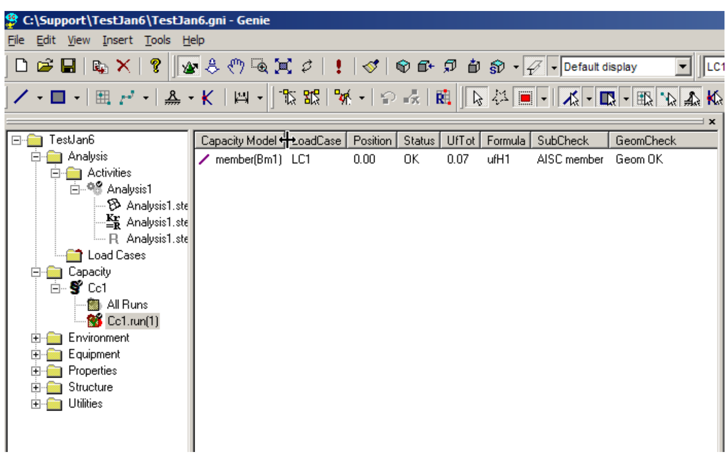


Figure 11 - Member result using AISC LRFD 2005 code check: local bending moment.

Now, lets us run the code check using the maximum bending moment from each direction. The maximum bending moment M_y (10 [kN.m]) and M_z (8 [kN.m]) will be chosen and the result will be the same for all the code check points along the beam. Thus, the result we get for ufH1 = 0.12 as we can see on Figure 13.

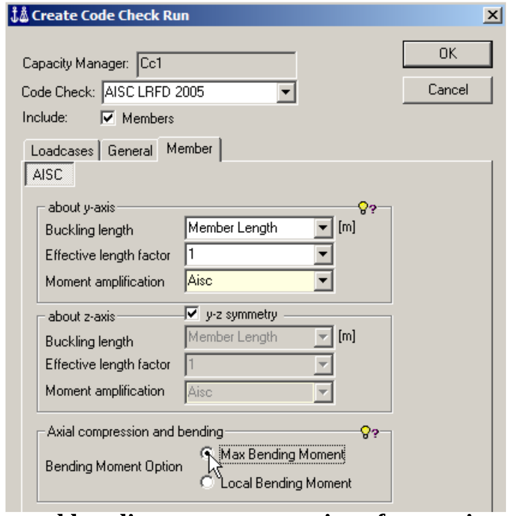


Figure 12 - Axial compression and bending: set computations for maximum bending moments

Once more we emphasize the fact of maximum bending moments produce higher usage factors when compared with the local bending moments. Previously, we demonstrated and concluded on the necessity of using the maximum bending moments when bending and compression are acting. The proof was carried out for one-dimensional case but it can be applied for two-dimensional cases without loss of generality.

Cases where the distribution of bending moment on both directions is constant there are no differences between the use of local bending moments or the maximum bending moments.

Historically, some in-house software's (code check based softwares like FRAMEWORK) only used the local bending moments approach once it was not specified the necessity of the use of maximum bending moments. The case was triggered when Eurocode 1993-1-1 presents a formulation where the maximum bending moment was required. Some research on this topic was carried out leading to the reported conclusions.

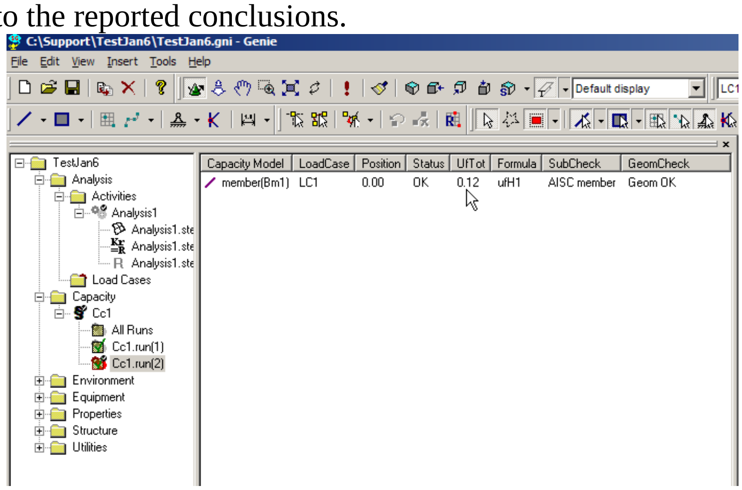


Figure 13 - Member result using AISC LRFD 2005 code check: maximum bending moment.