A Faster Approach: **Segment Trees**

A Divide-and-Conquer Approach

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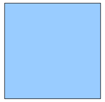

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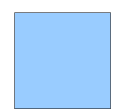

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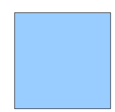

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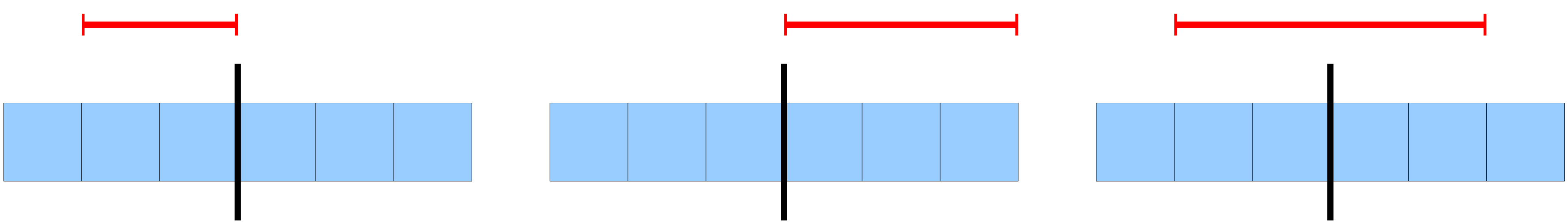

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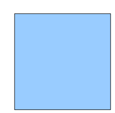

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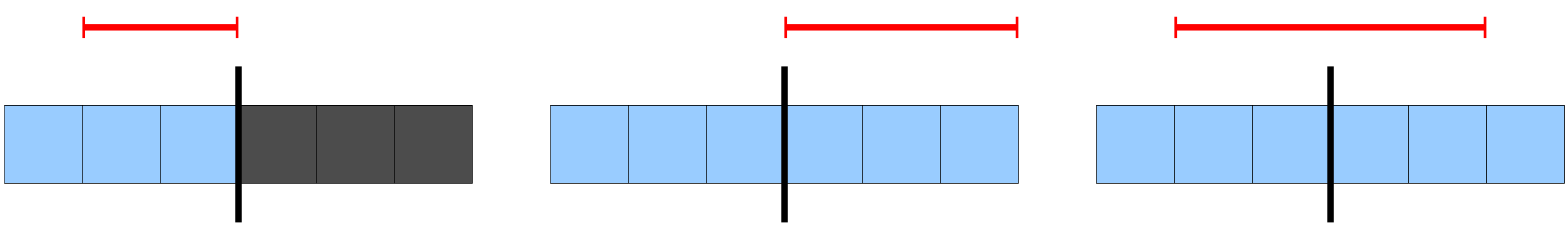

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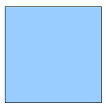

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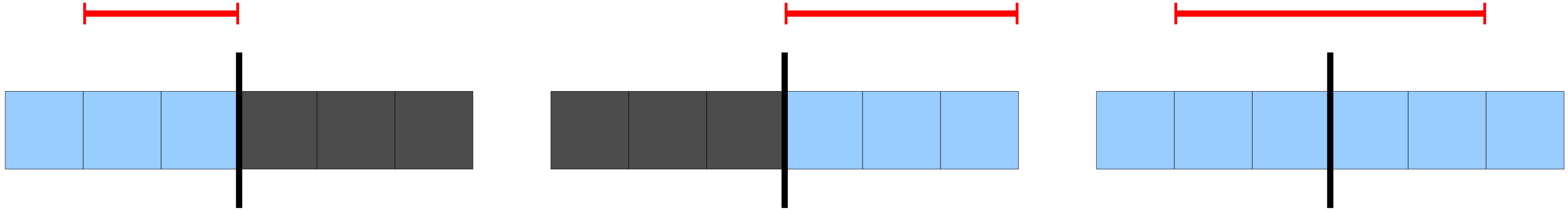

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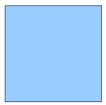

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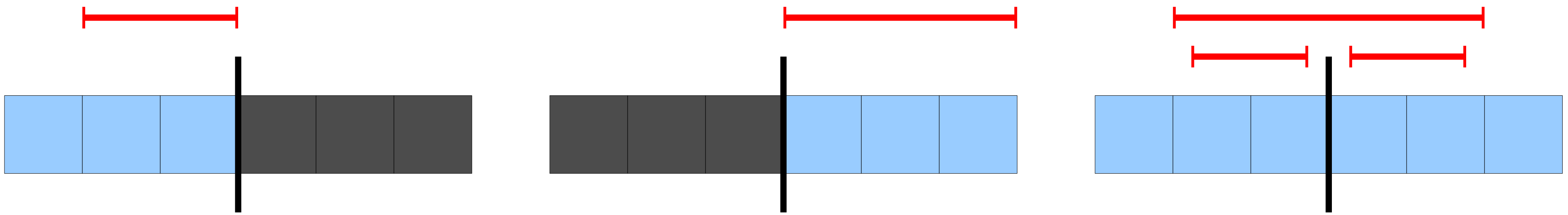

A Divide-and-Conquer Approach

31	41	59	26	53	58	97	93	23	84	62	64	33	83	27	95

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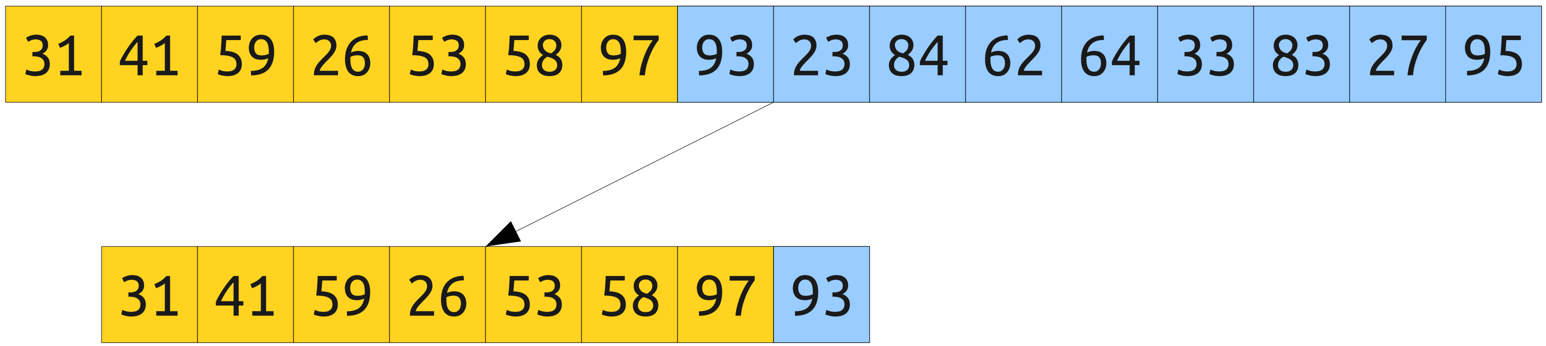

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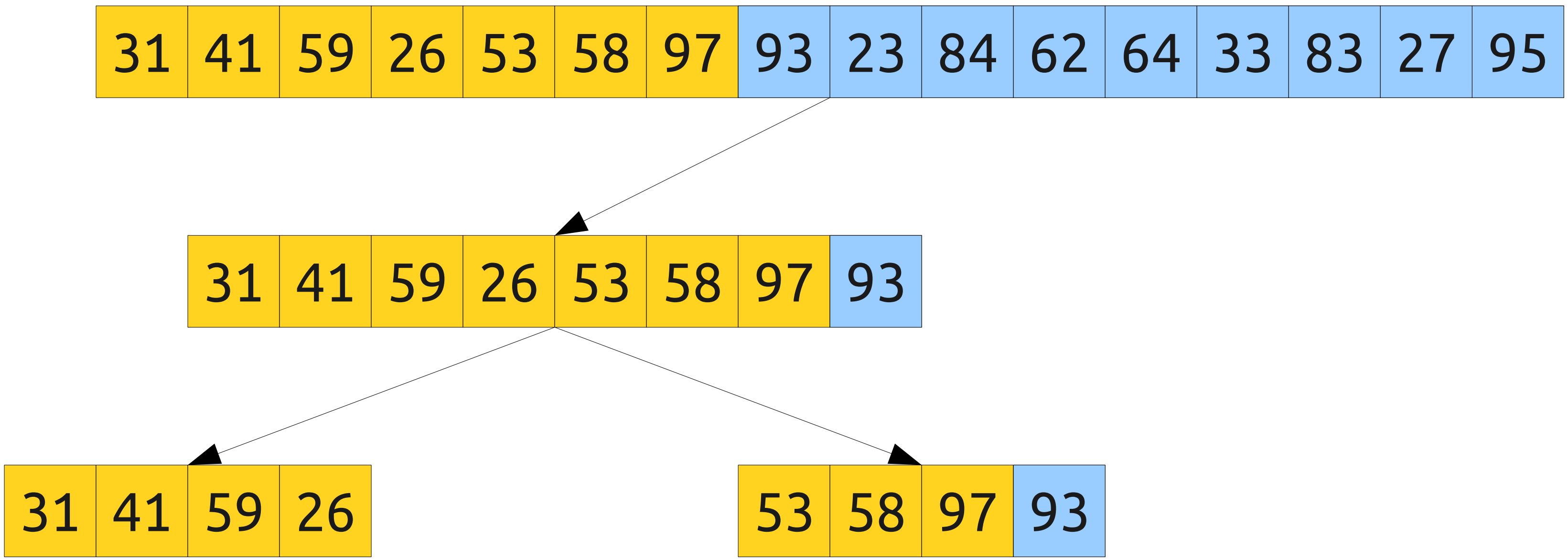

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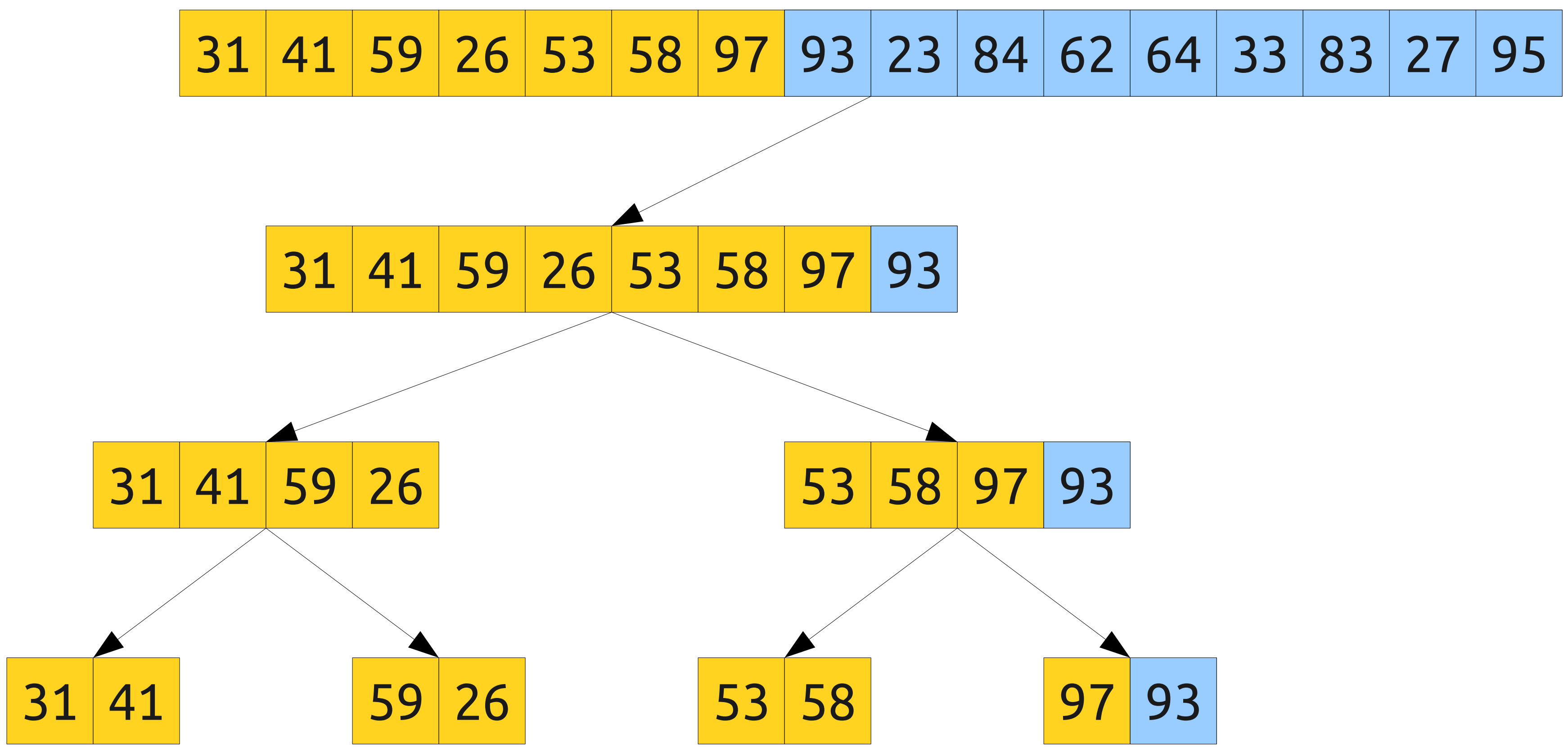

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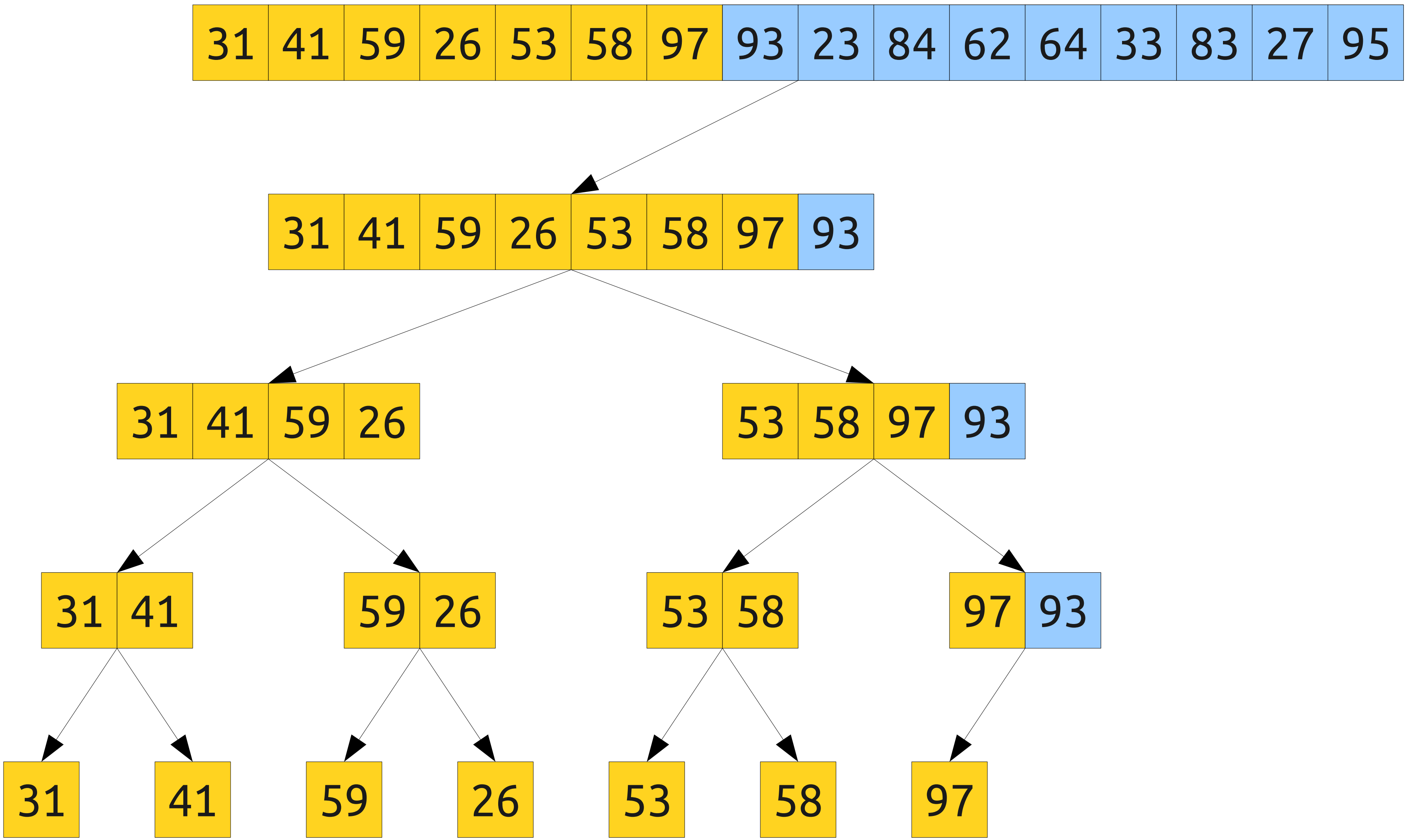

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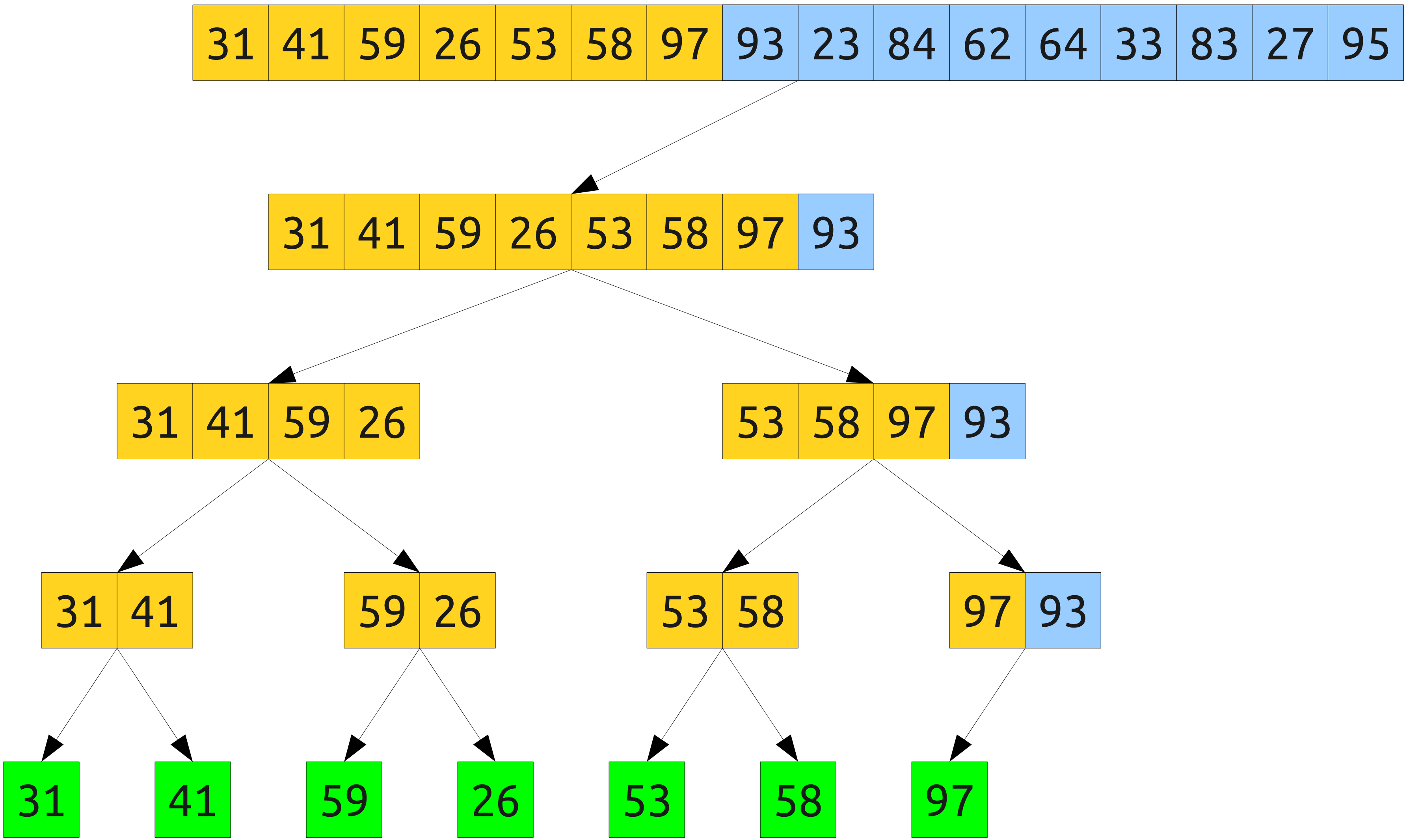

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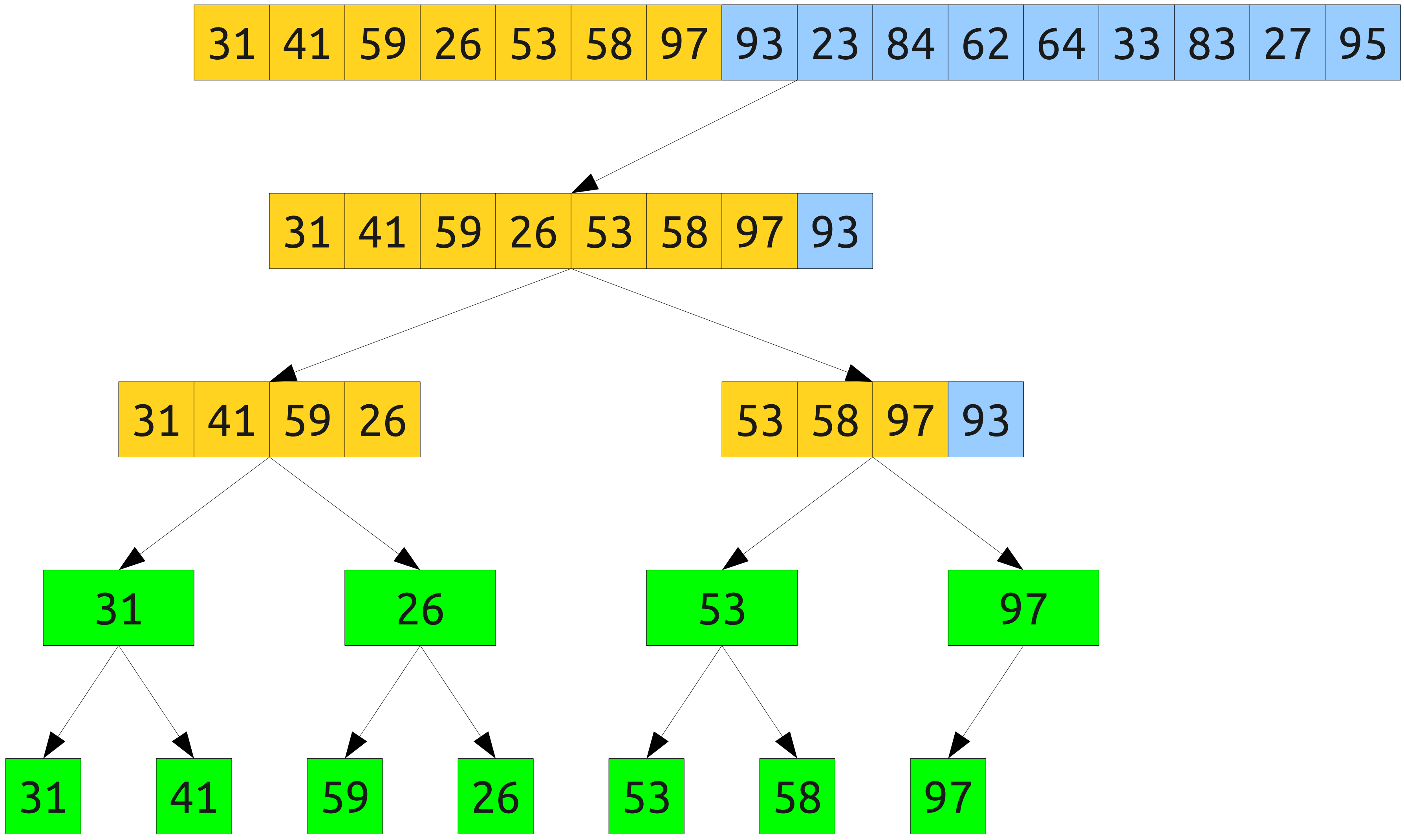

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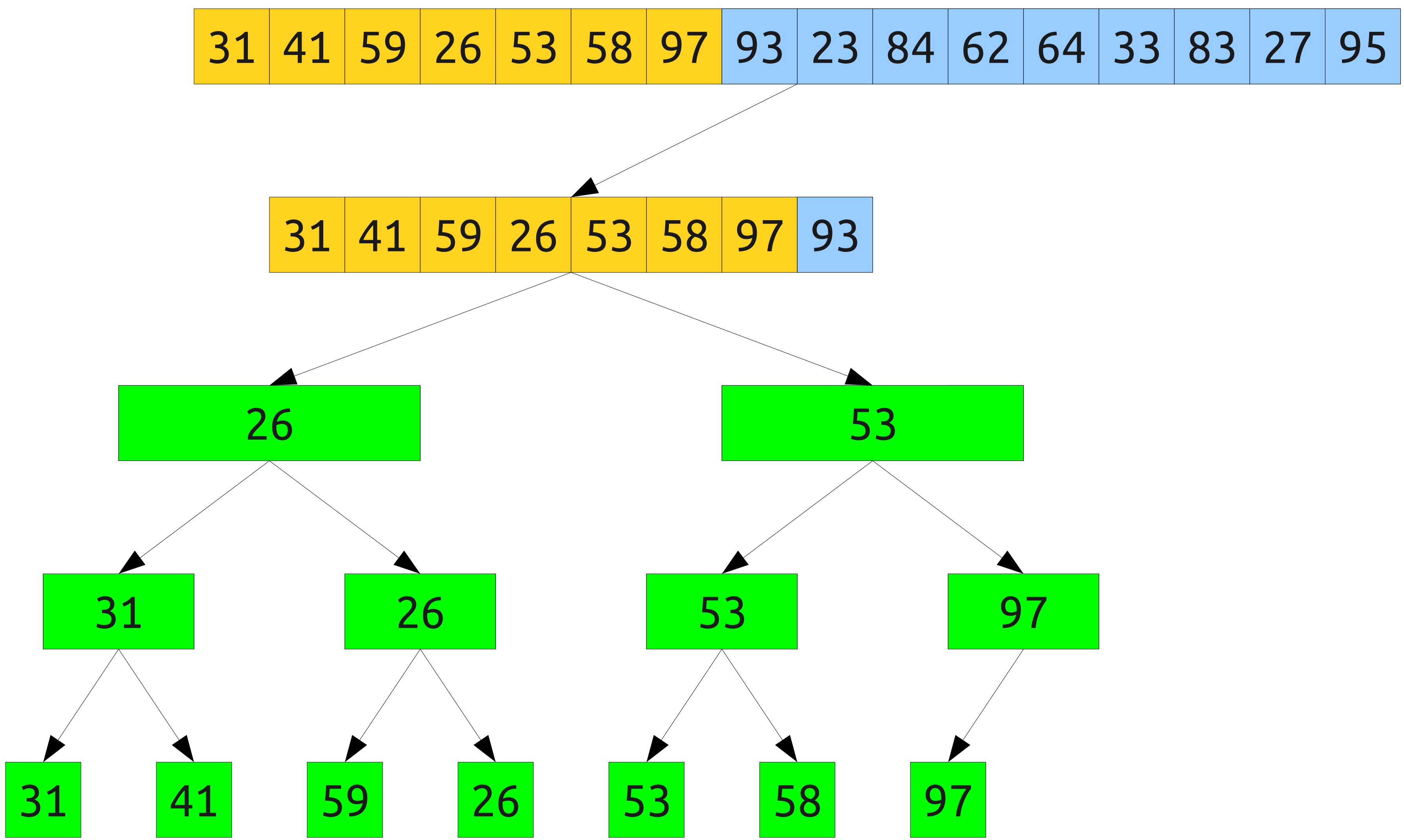

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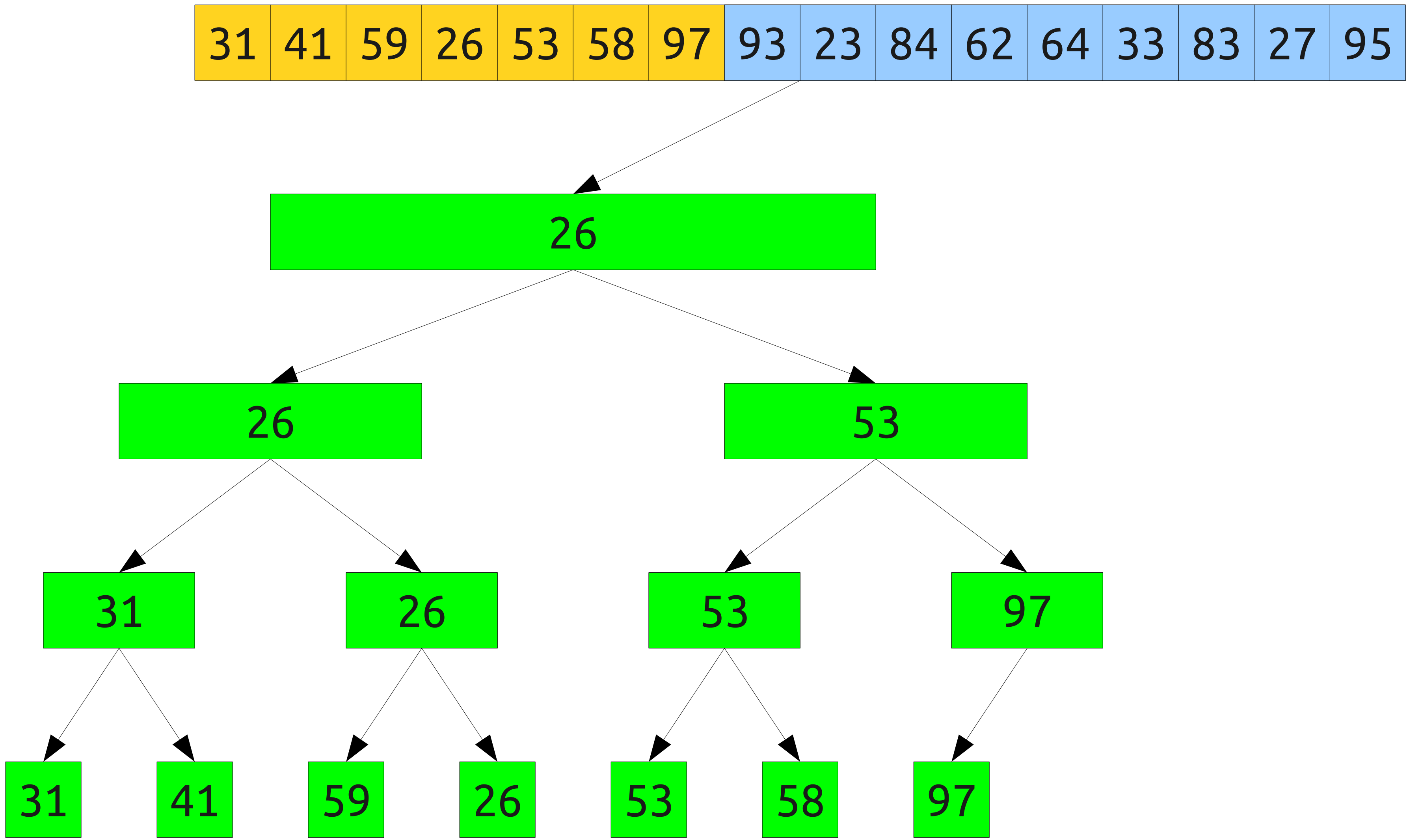

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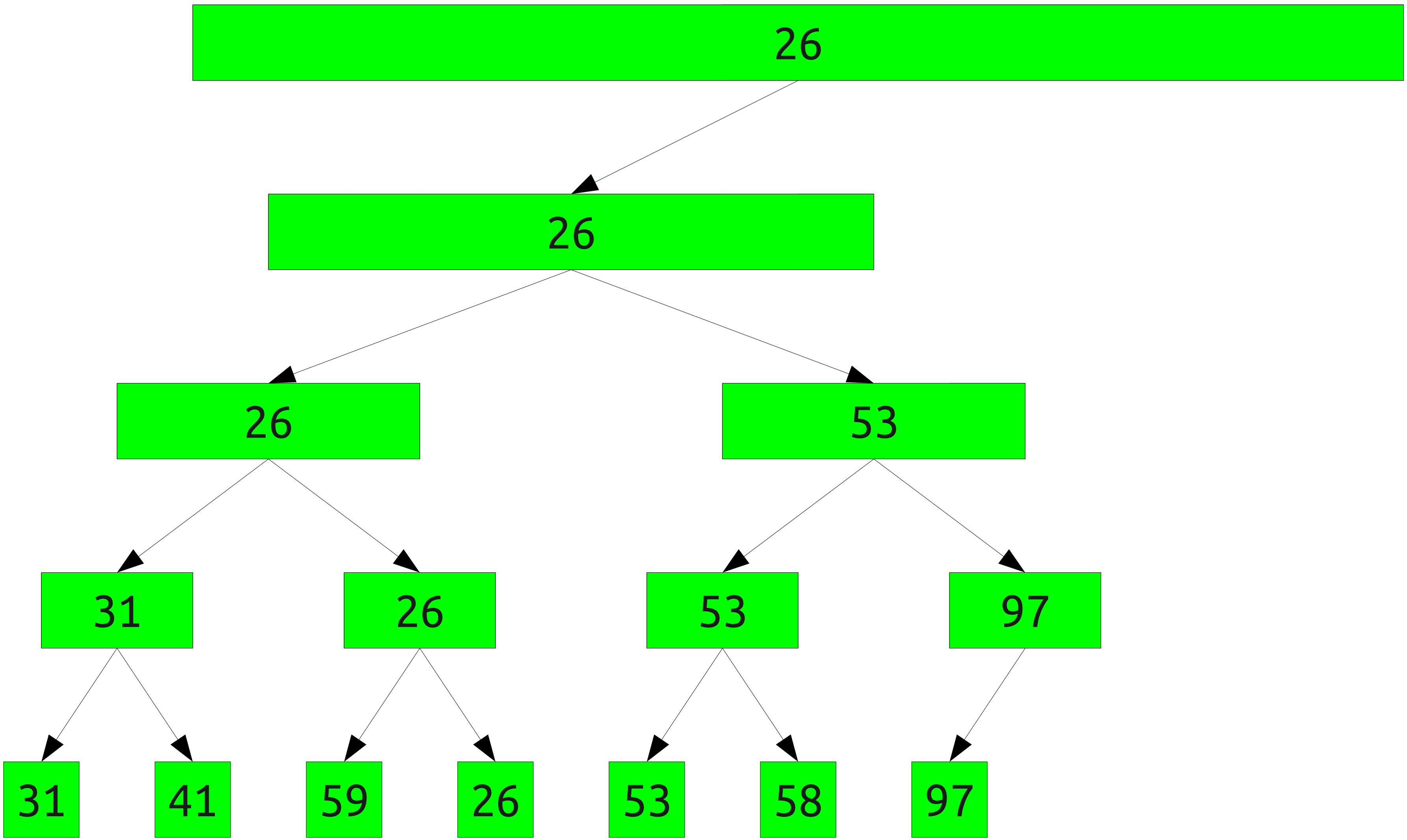

Analyzing Efficiency

- Each recursive call fires off at most two recursive calls and does O(1) work to combine them.
- Recurrence relation:

$$T(n) = 2T(n / 2) + O(1)$$

- Using the Master Theorem, this solves to **O(*n*)**.
- This is no better than our initial solution!

An Observation

31	41	59	26	53	58	97	93

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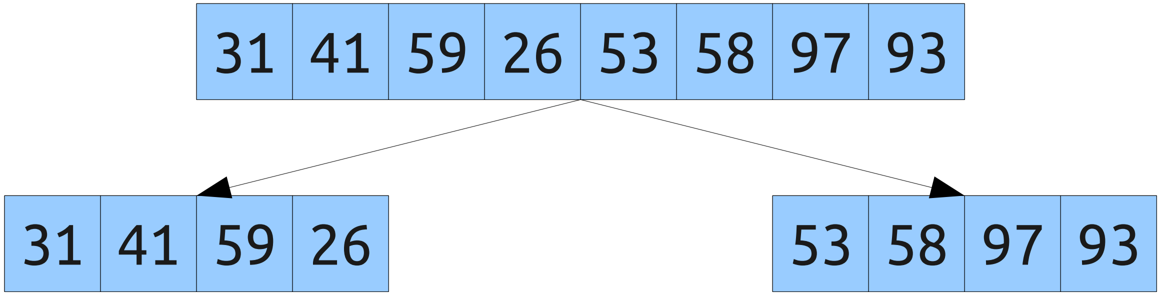

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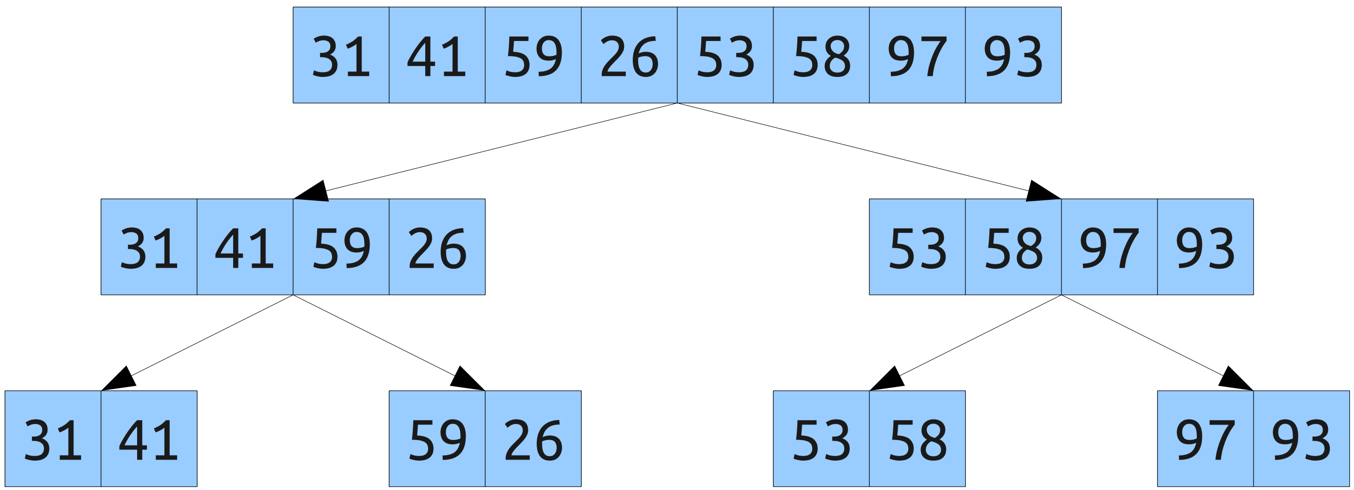

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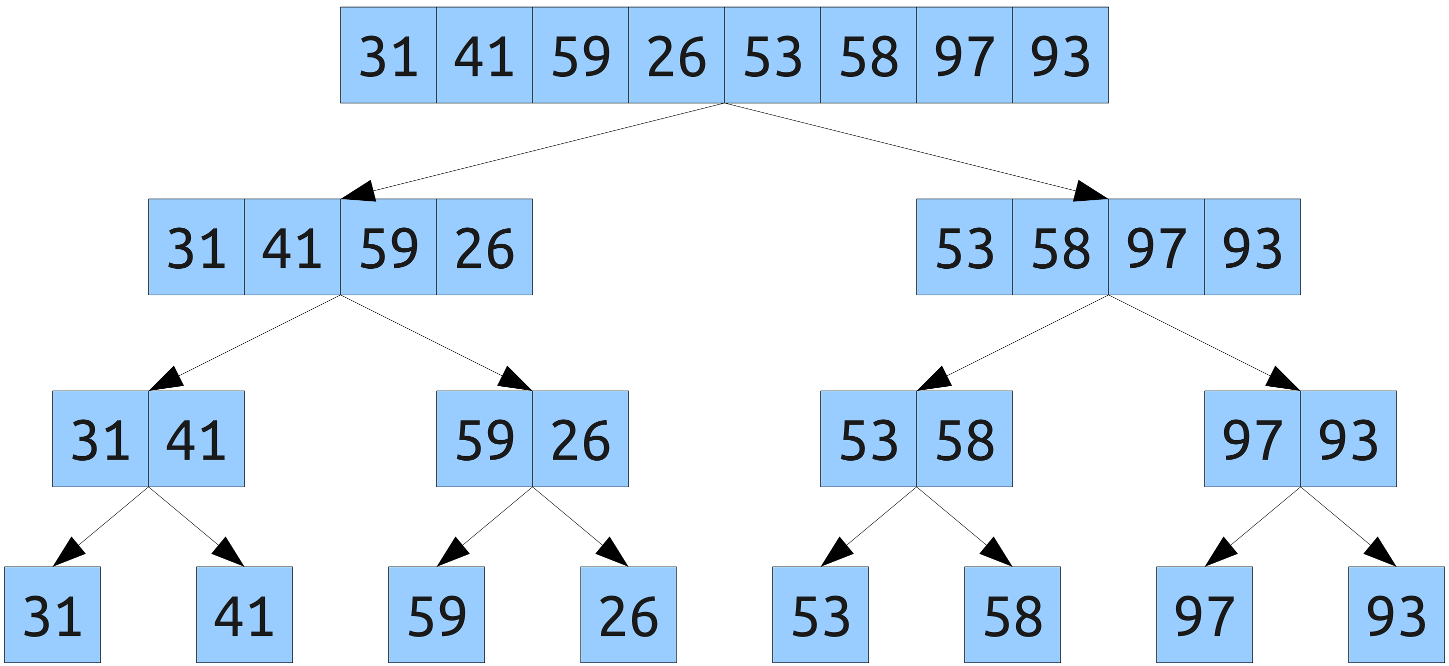

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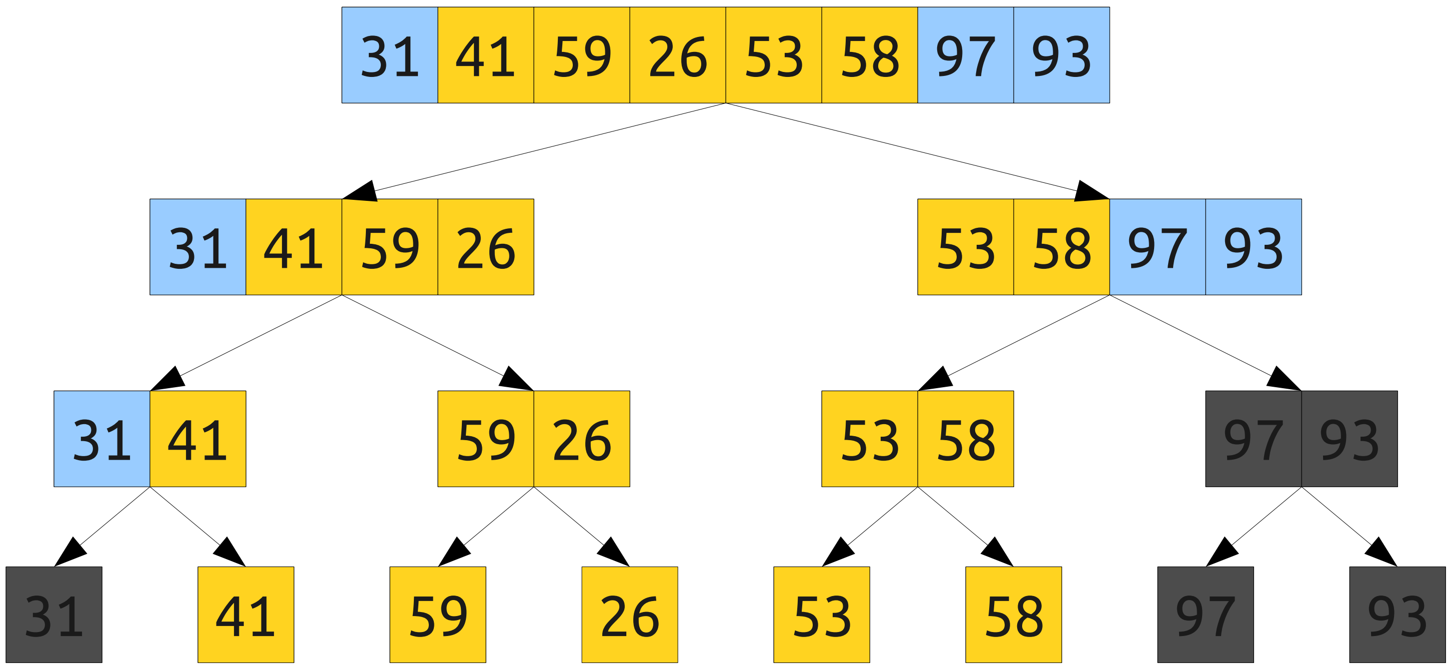

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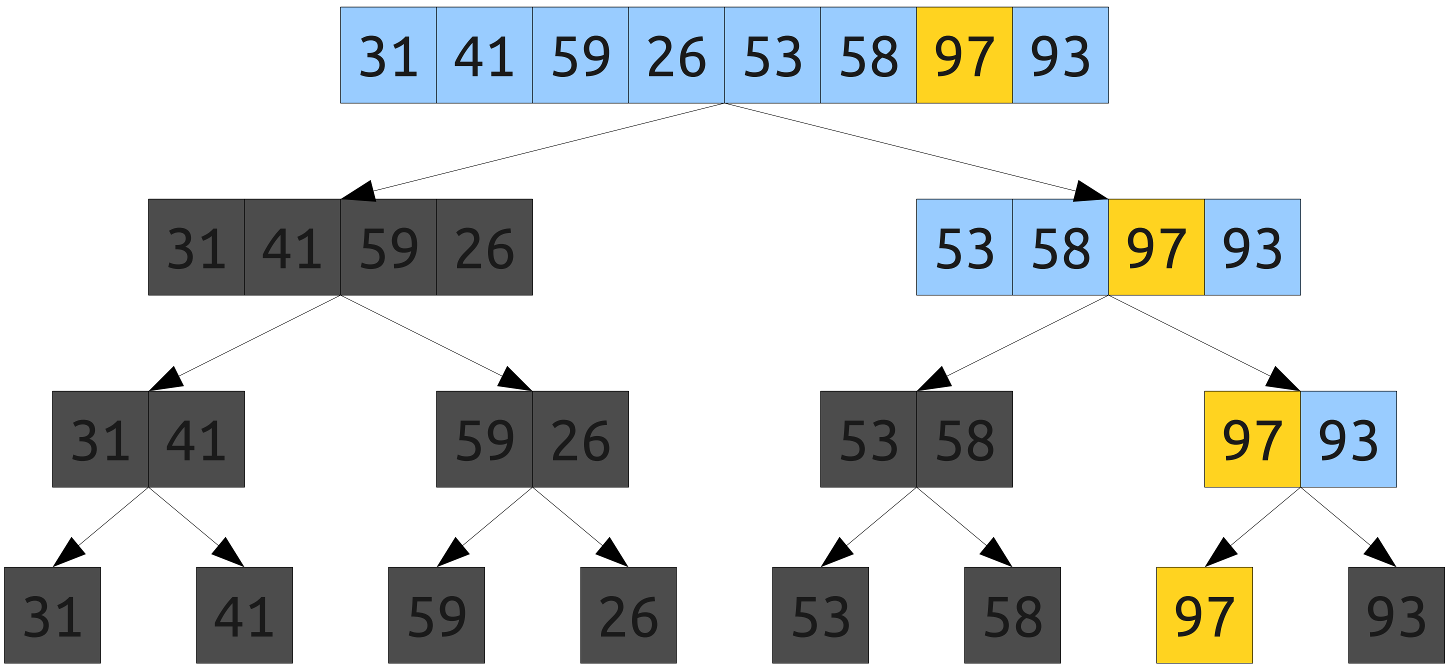

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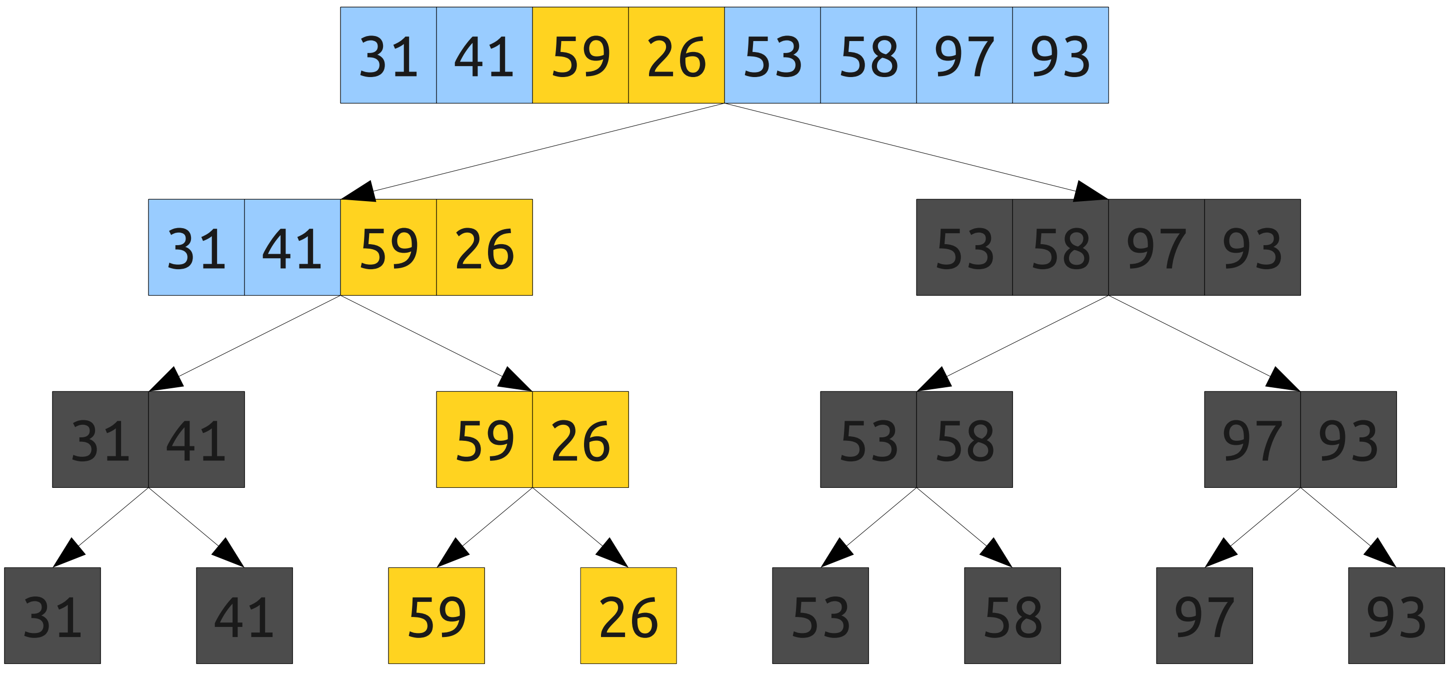

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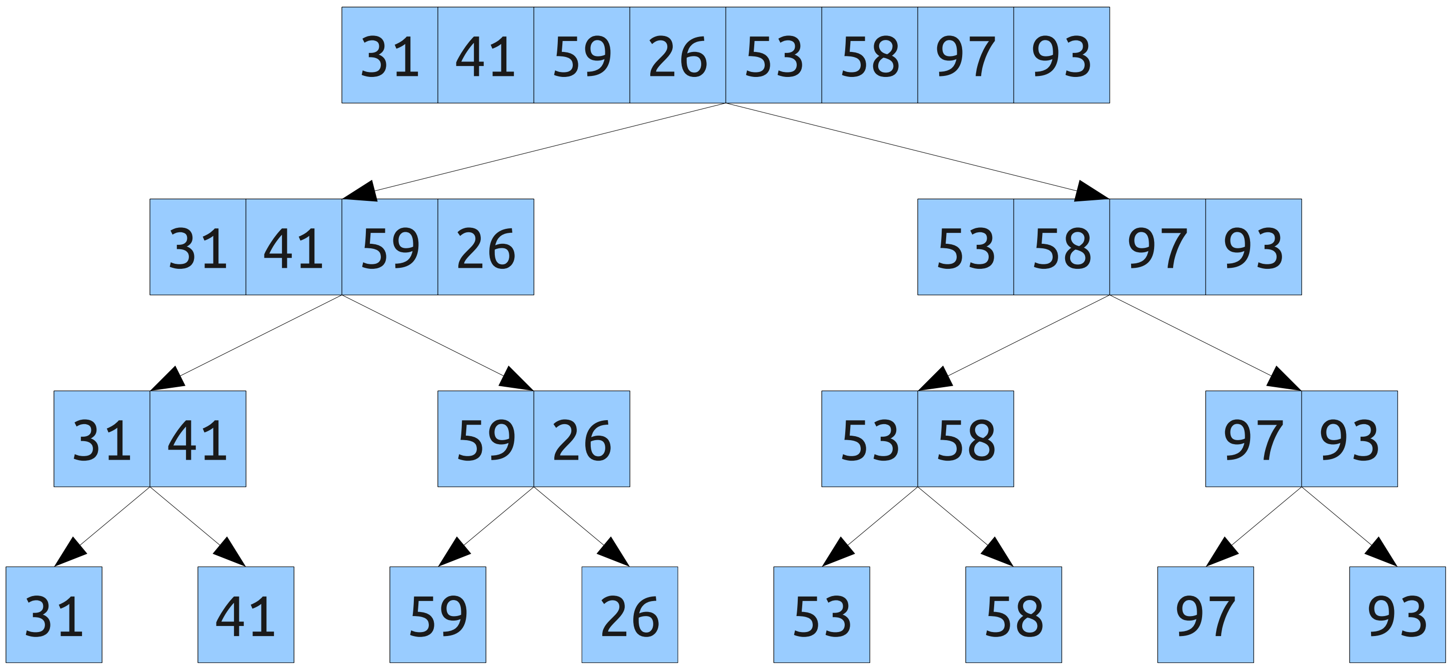

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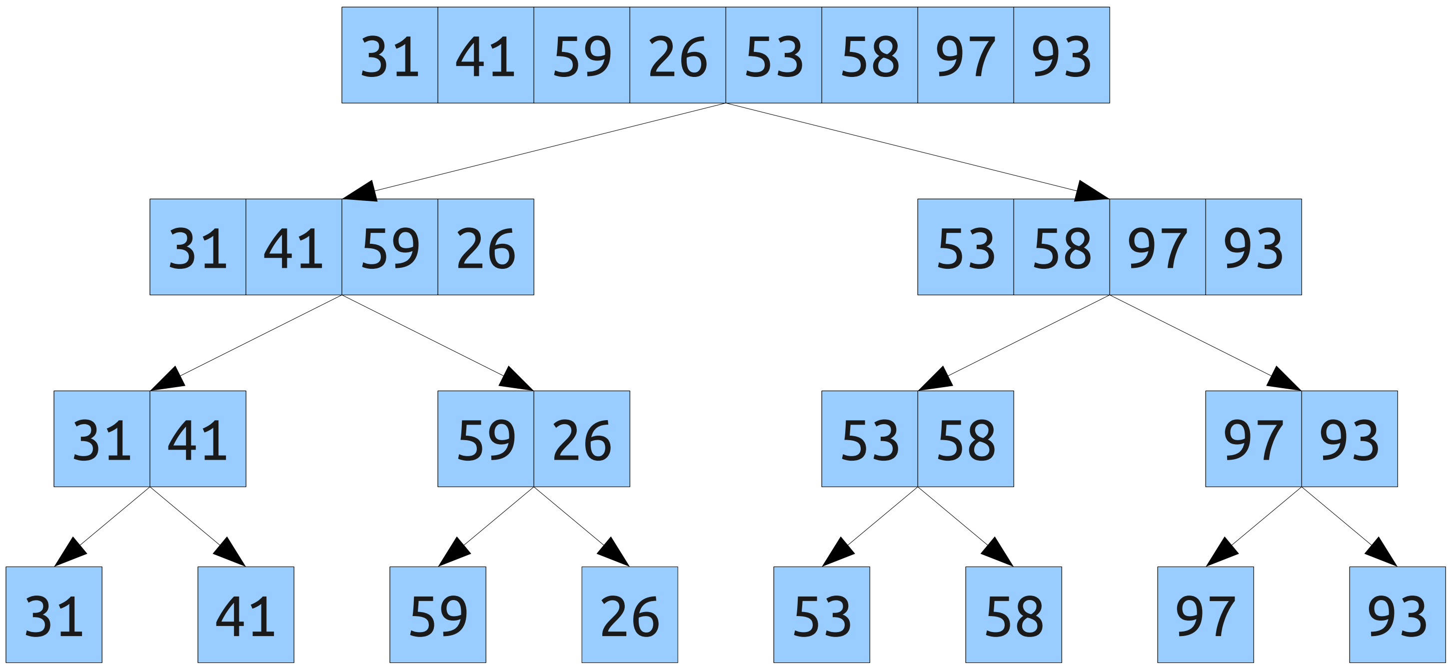

Observation 1: Every recursive call that will ever be made doing RMQ this way must use one of the subarrays given here.

A Second Observation

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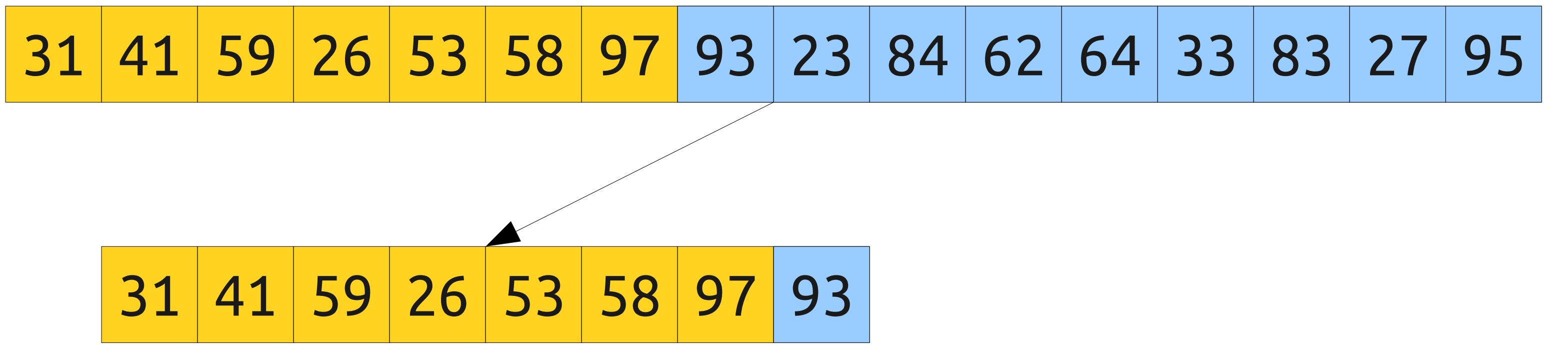

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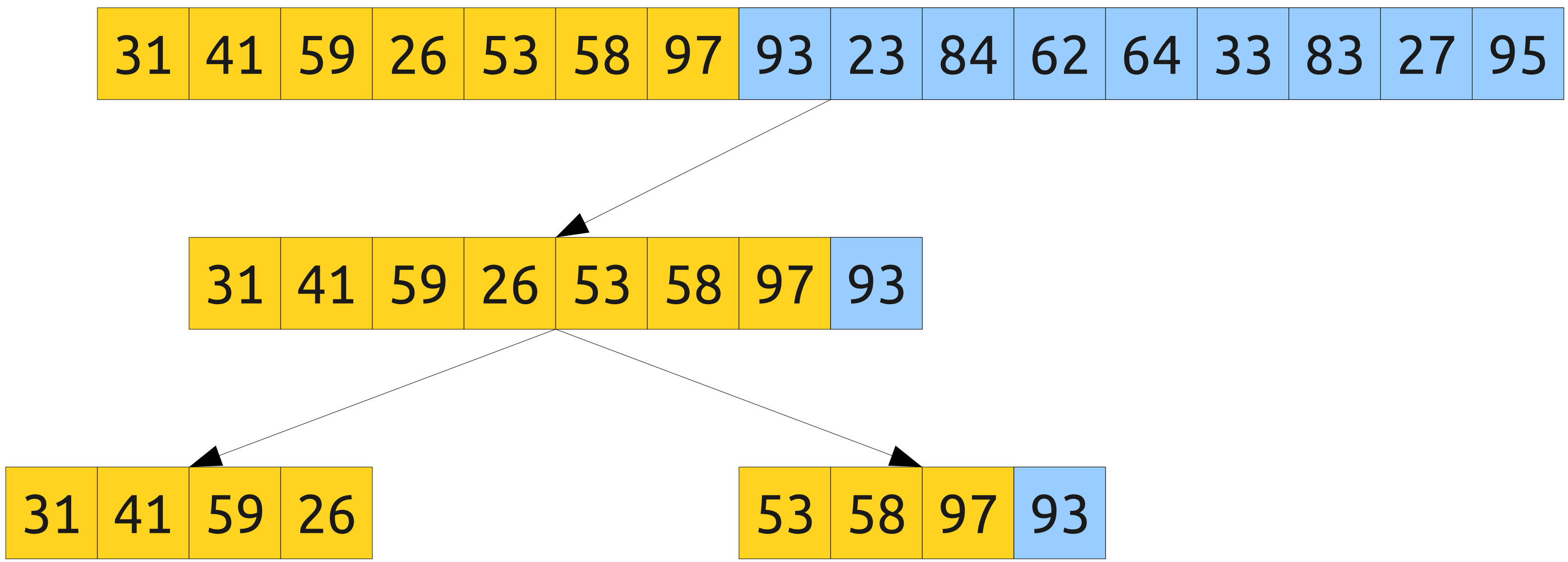

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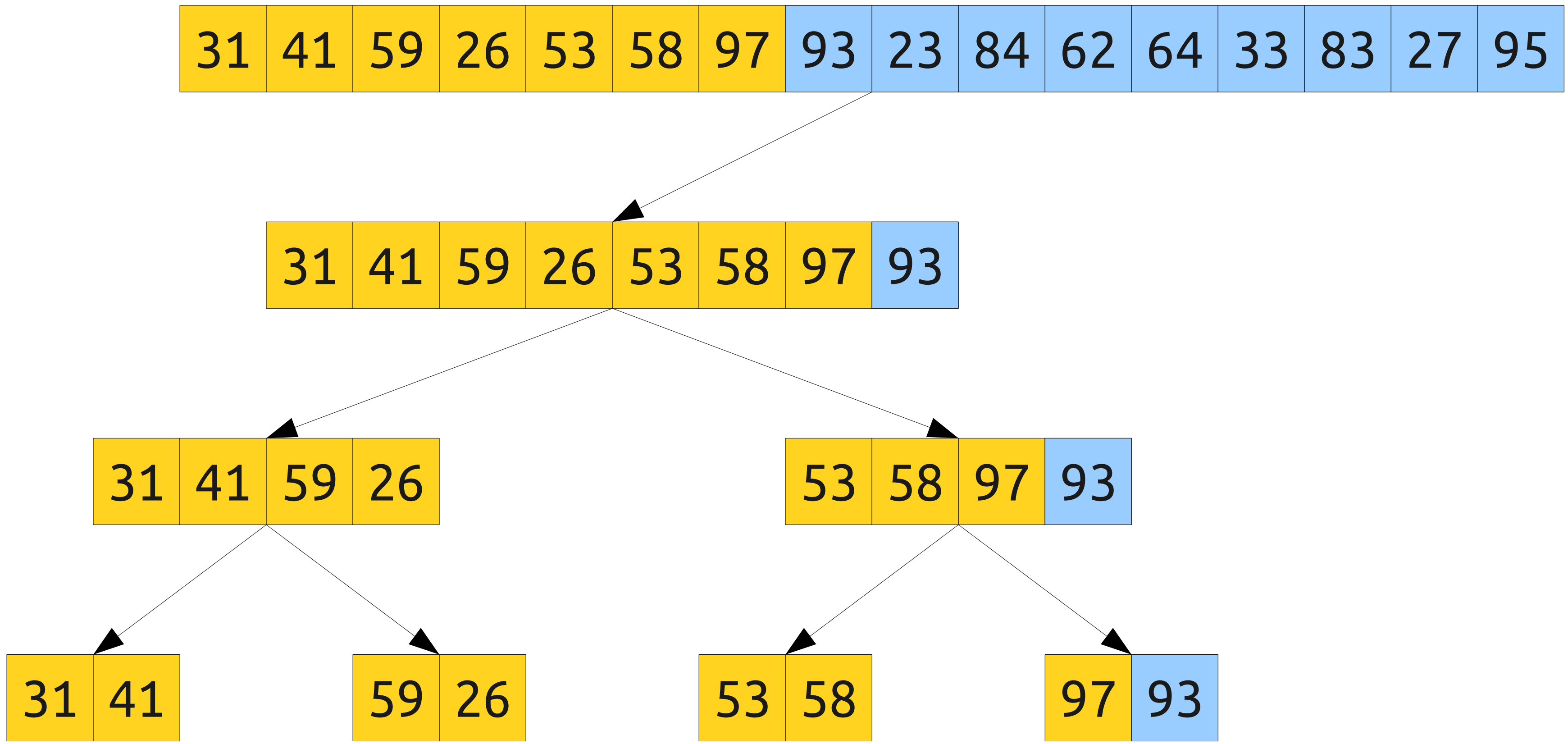

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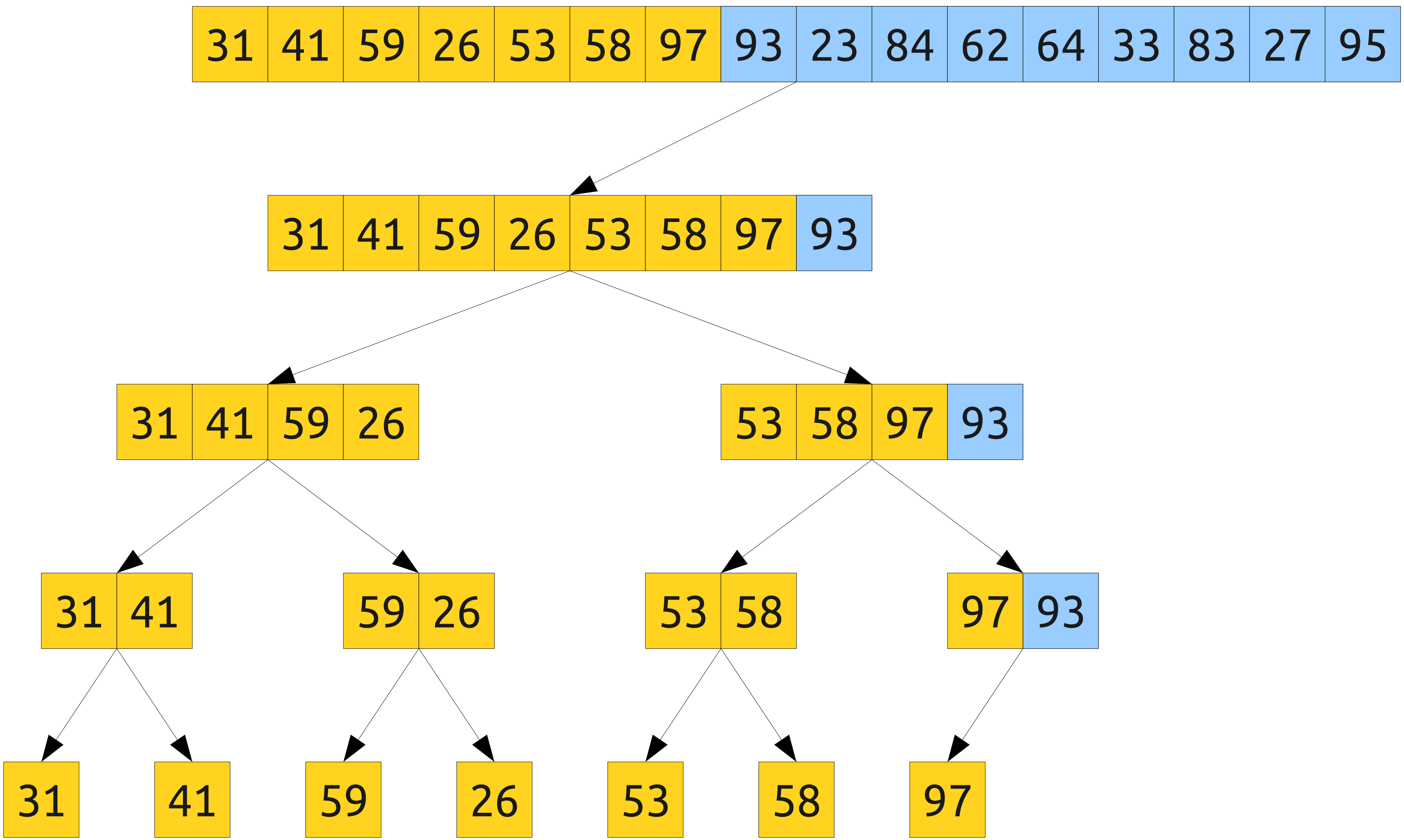

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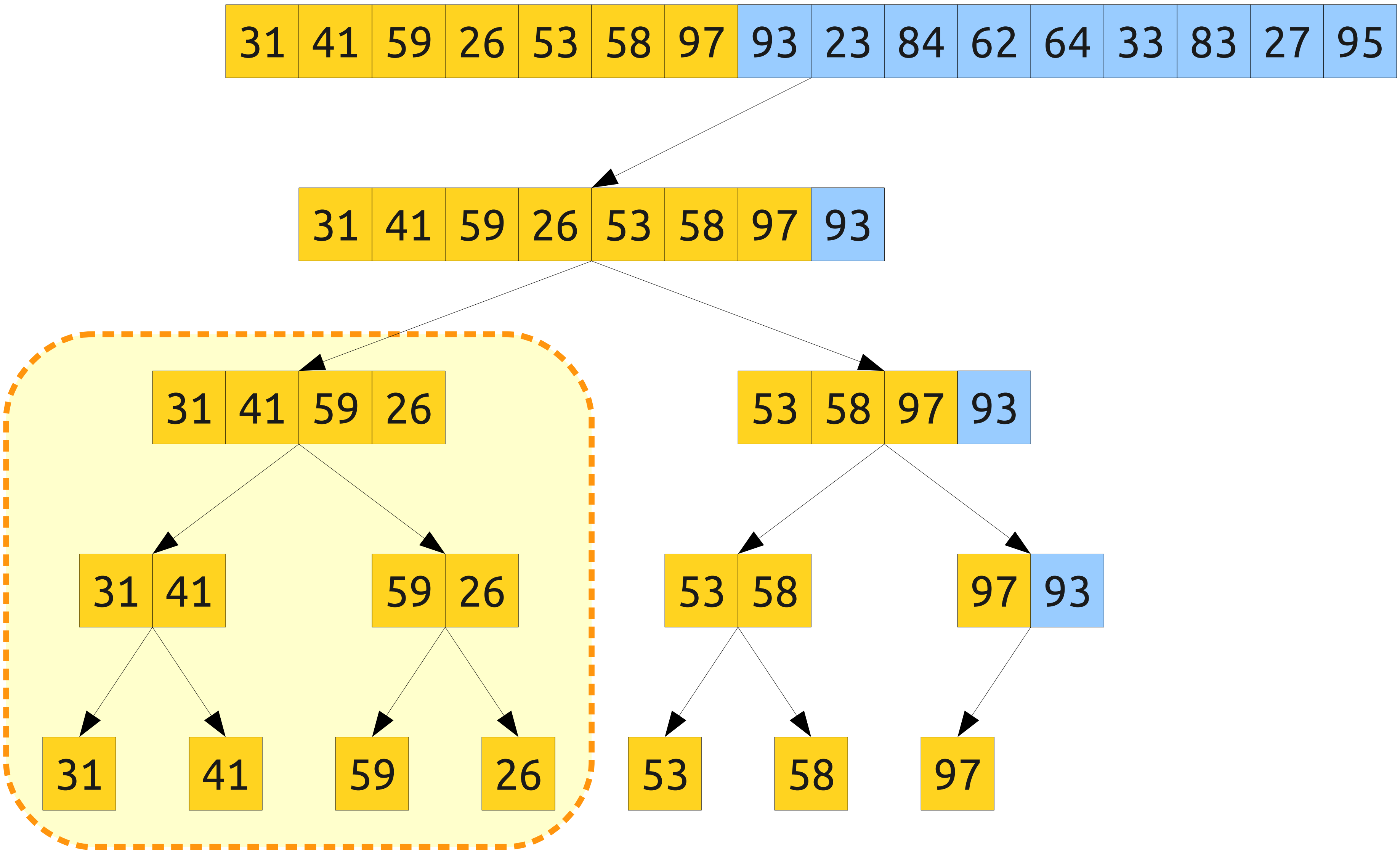

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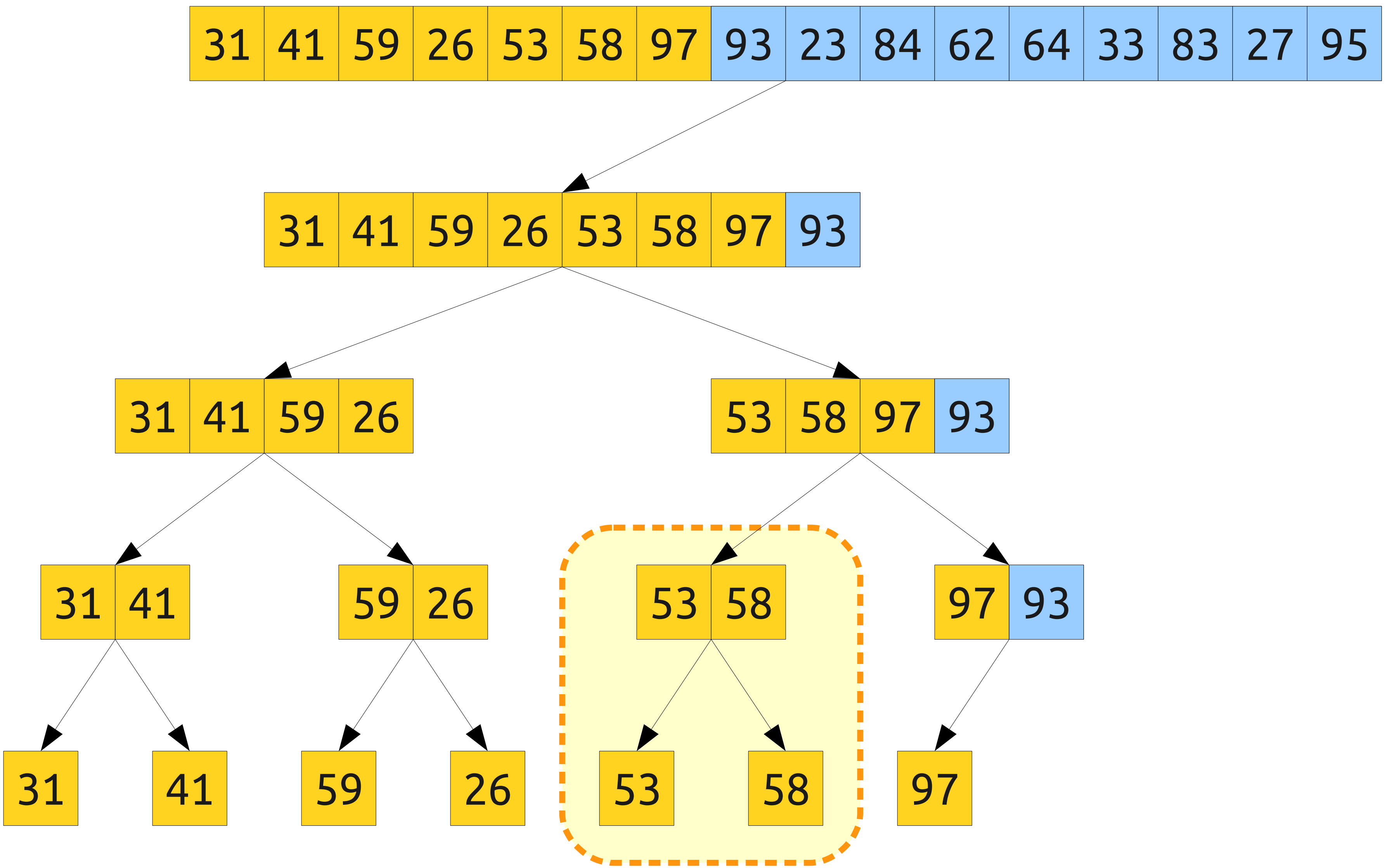

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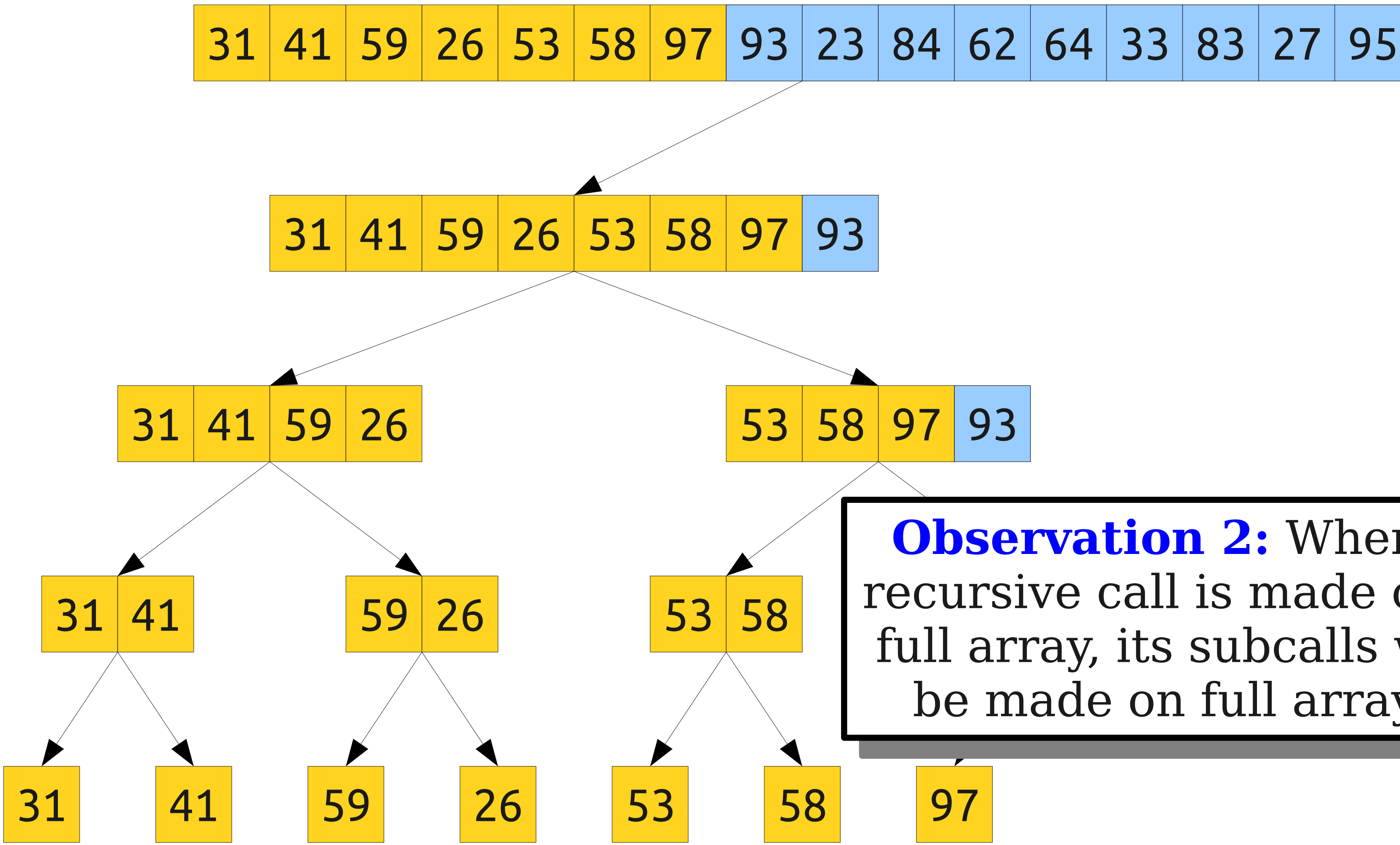
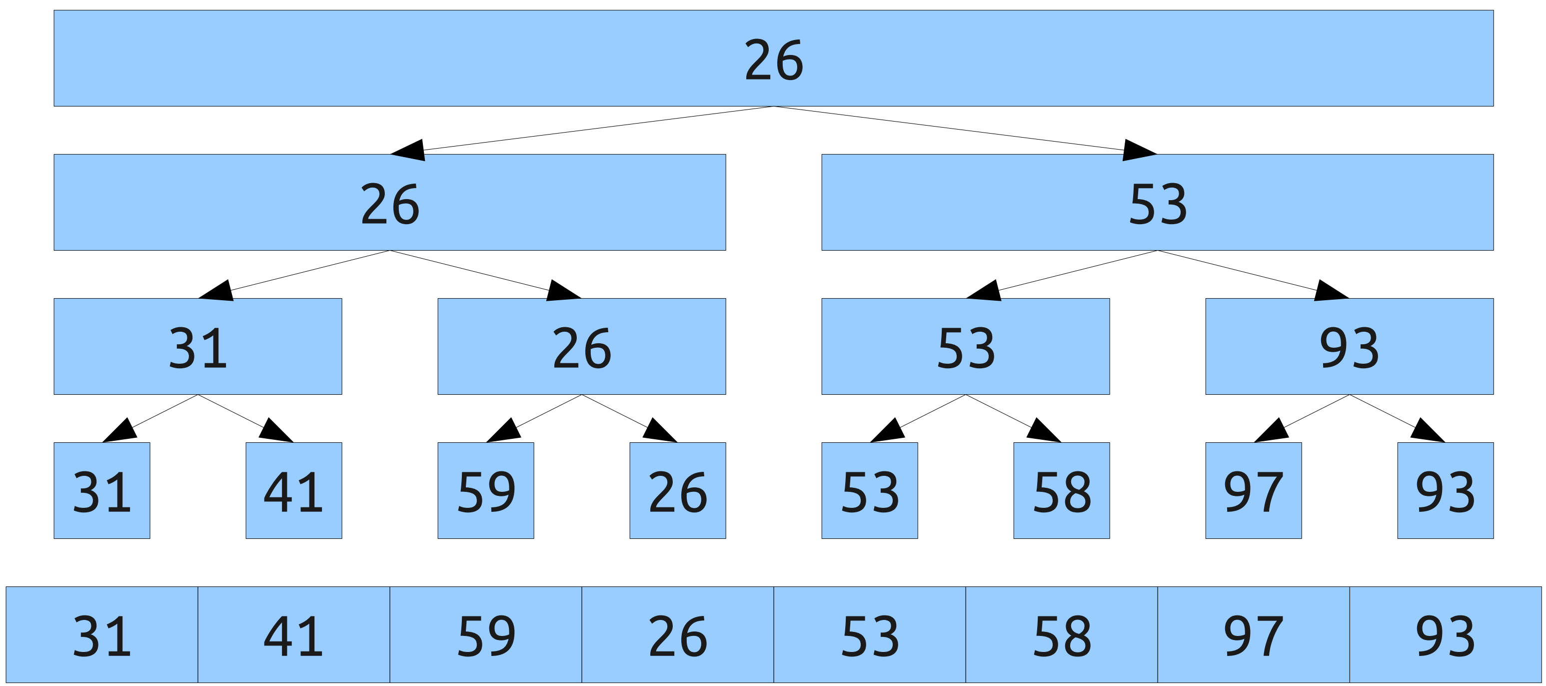


Observation 2: When a recursive call is made on a full array, its subcalls will be made on full arrays.

A Revised Idea

- For each subarray that could *ever* be visited by a recursive call, compute the minimum of that subarray and store it.
- Store result as a **segment tree**:



- Can be built with O(n) preprocessing. *(why)?*

A Revised Idea

- Modify the recursive algorithm to use the segment tree.
- If range to search equals the range at the current node, return the minimum value in that range.
  - We precomputed this; takes time O(1).
- Otherwise:
  - If range is purely in the first or second half, recurse on that subrange.
  - Otherwise, split the range in half, then recursively search the left and right halves and take the minimum.

Segment Trees

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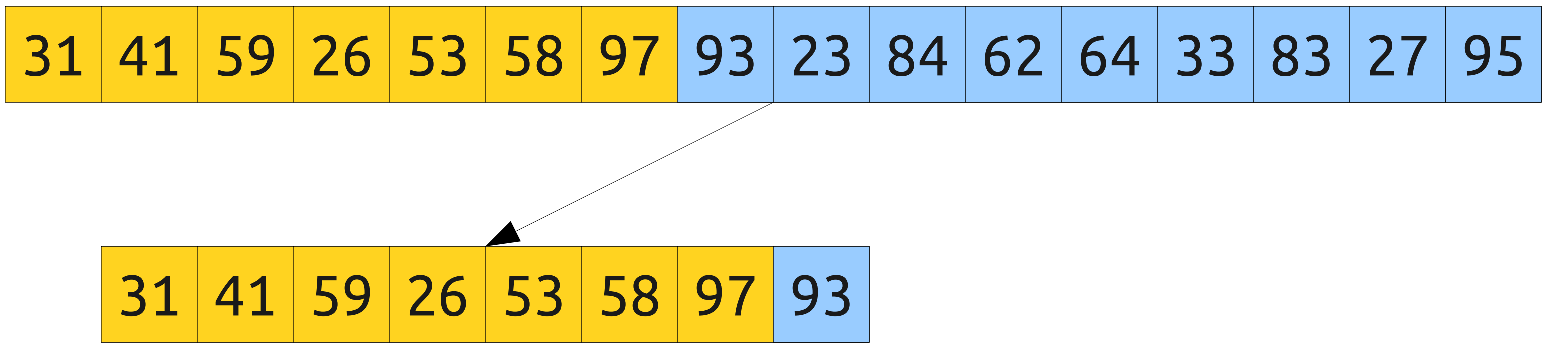

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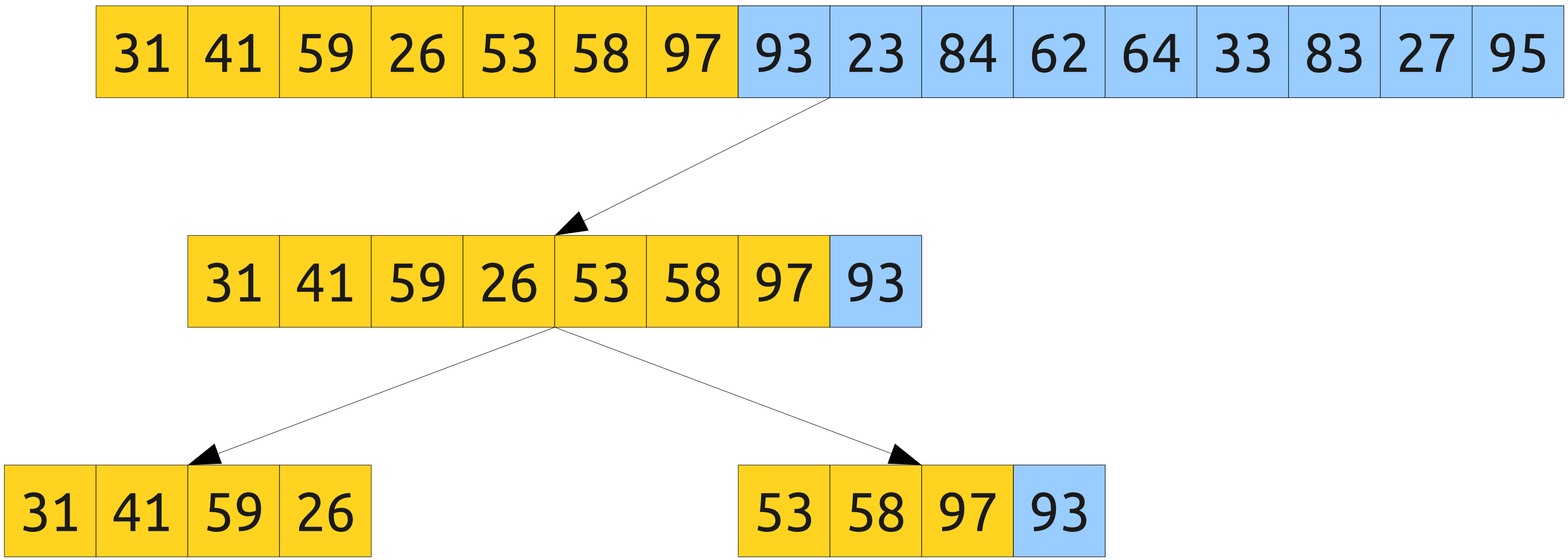

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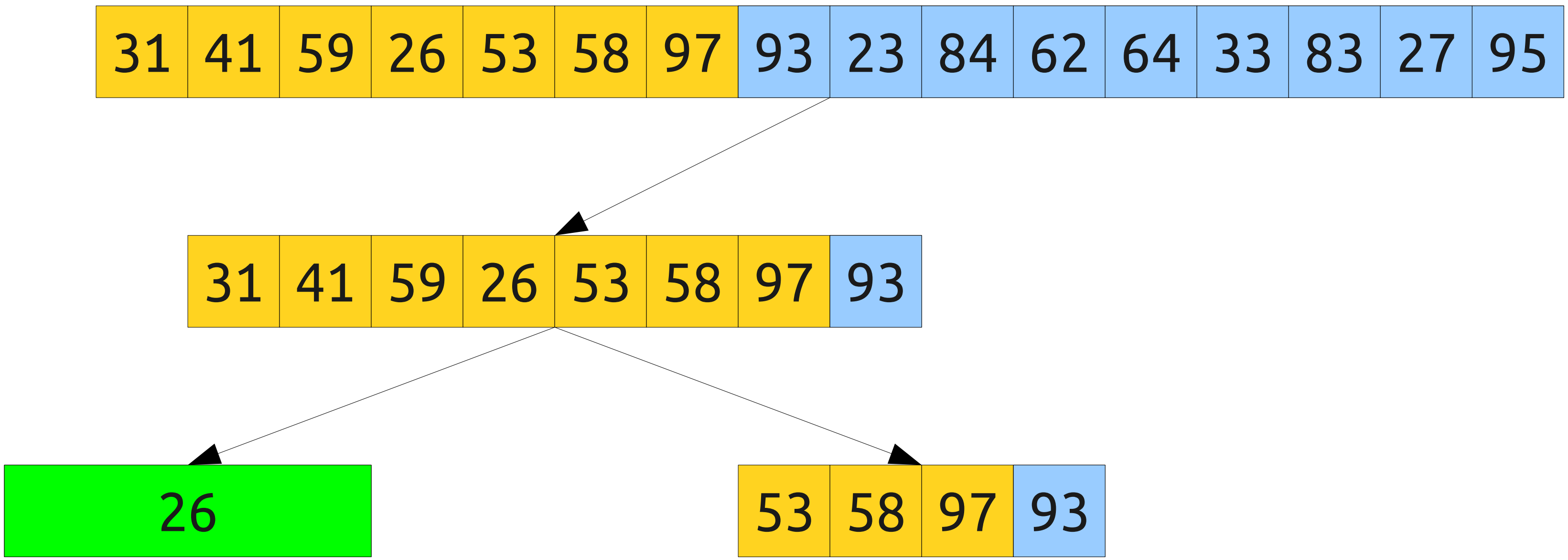

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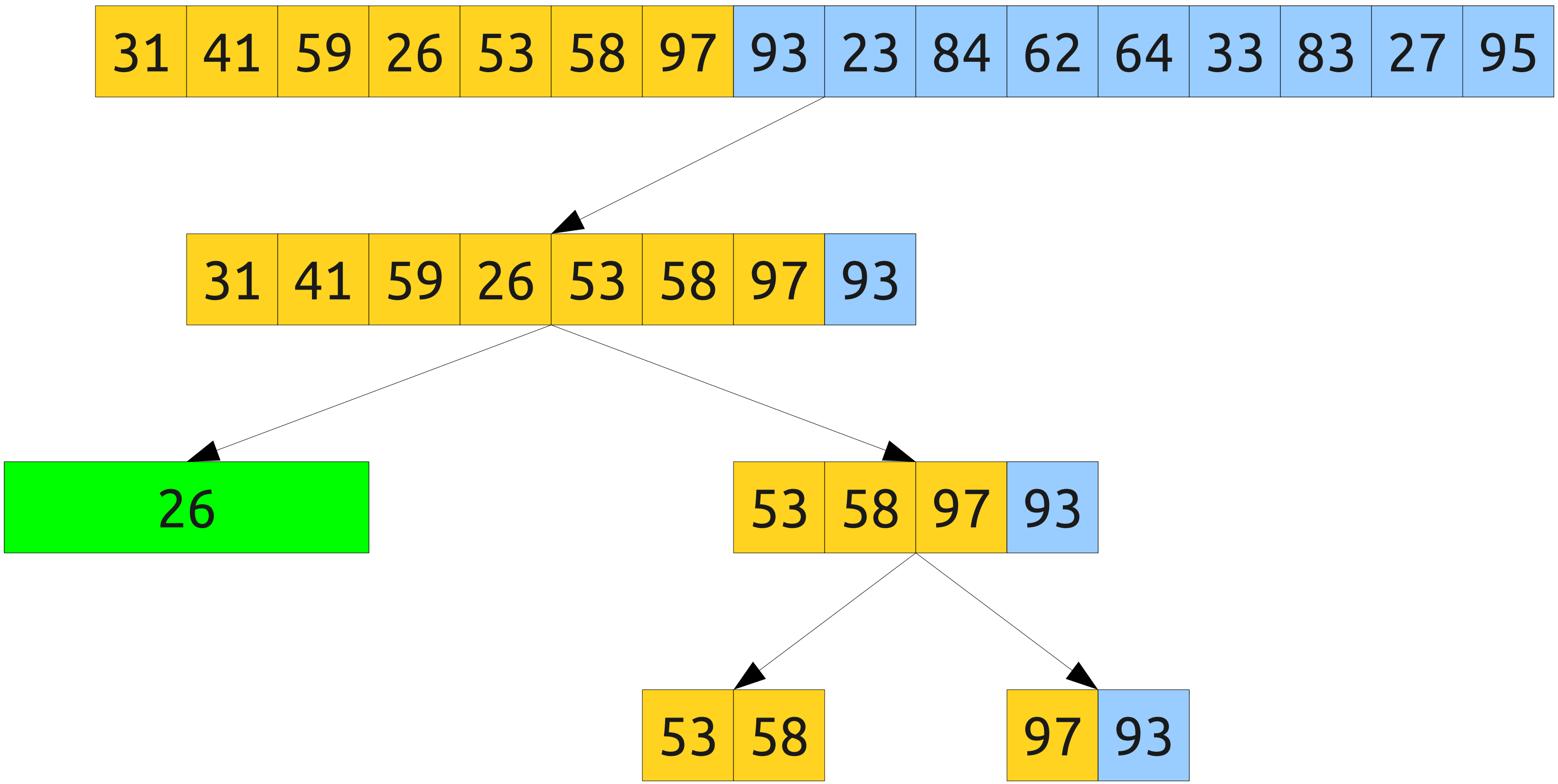

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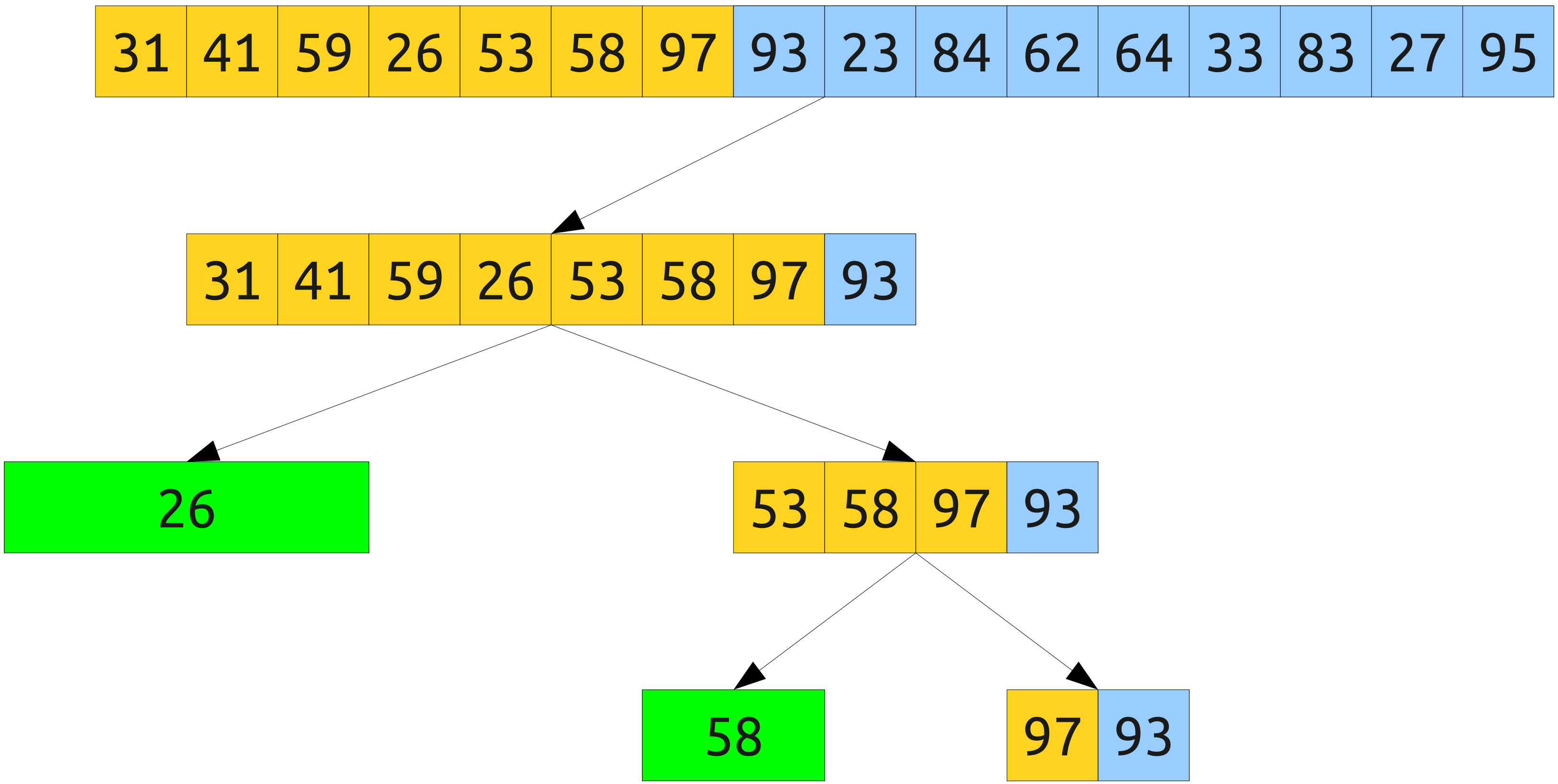

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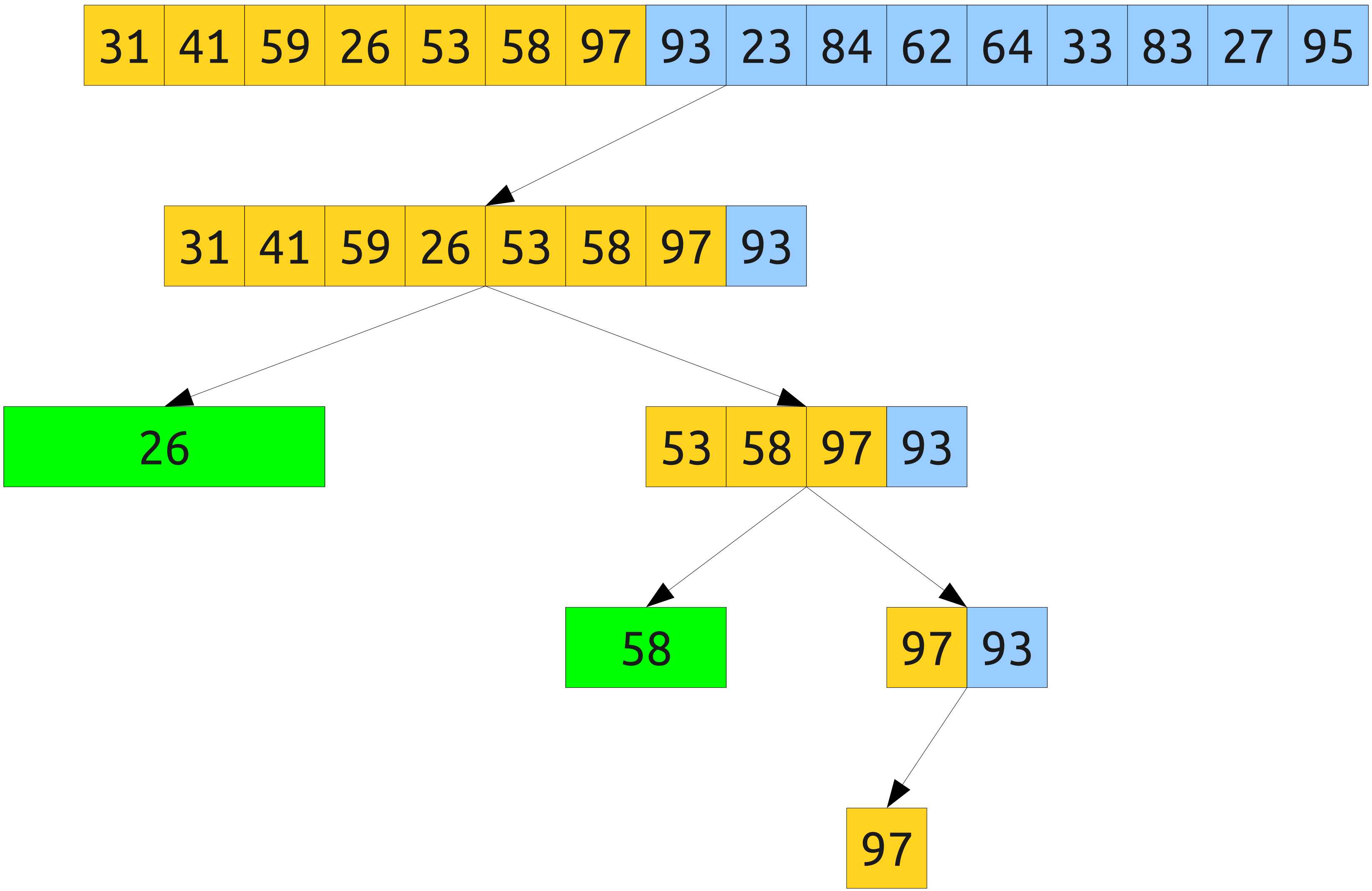

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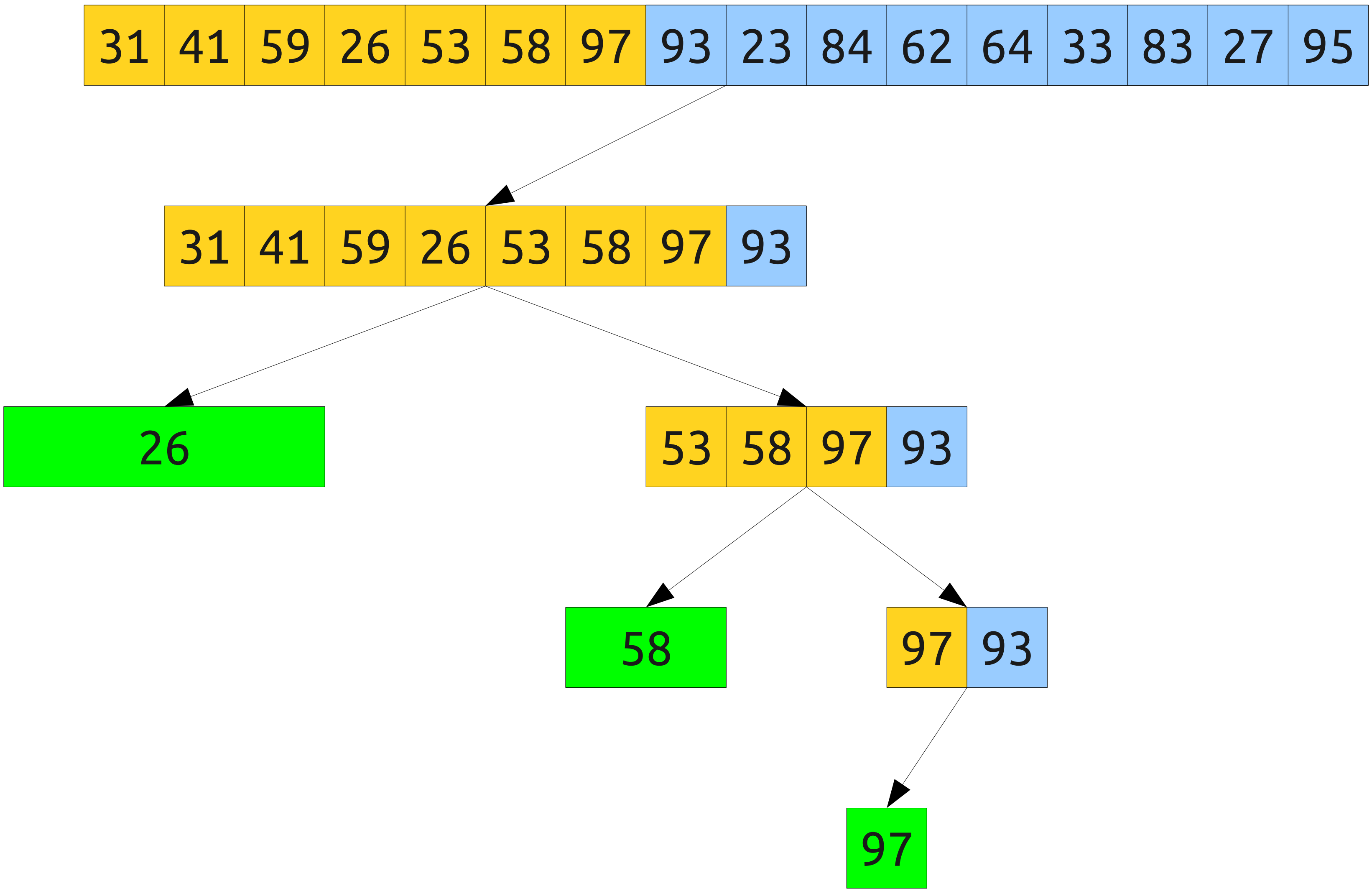

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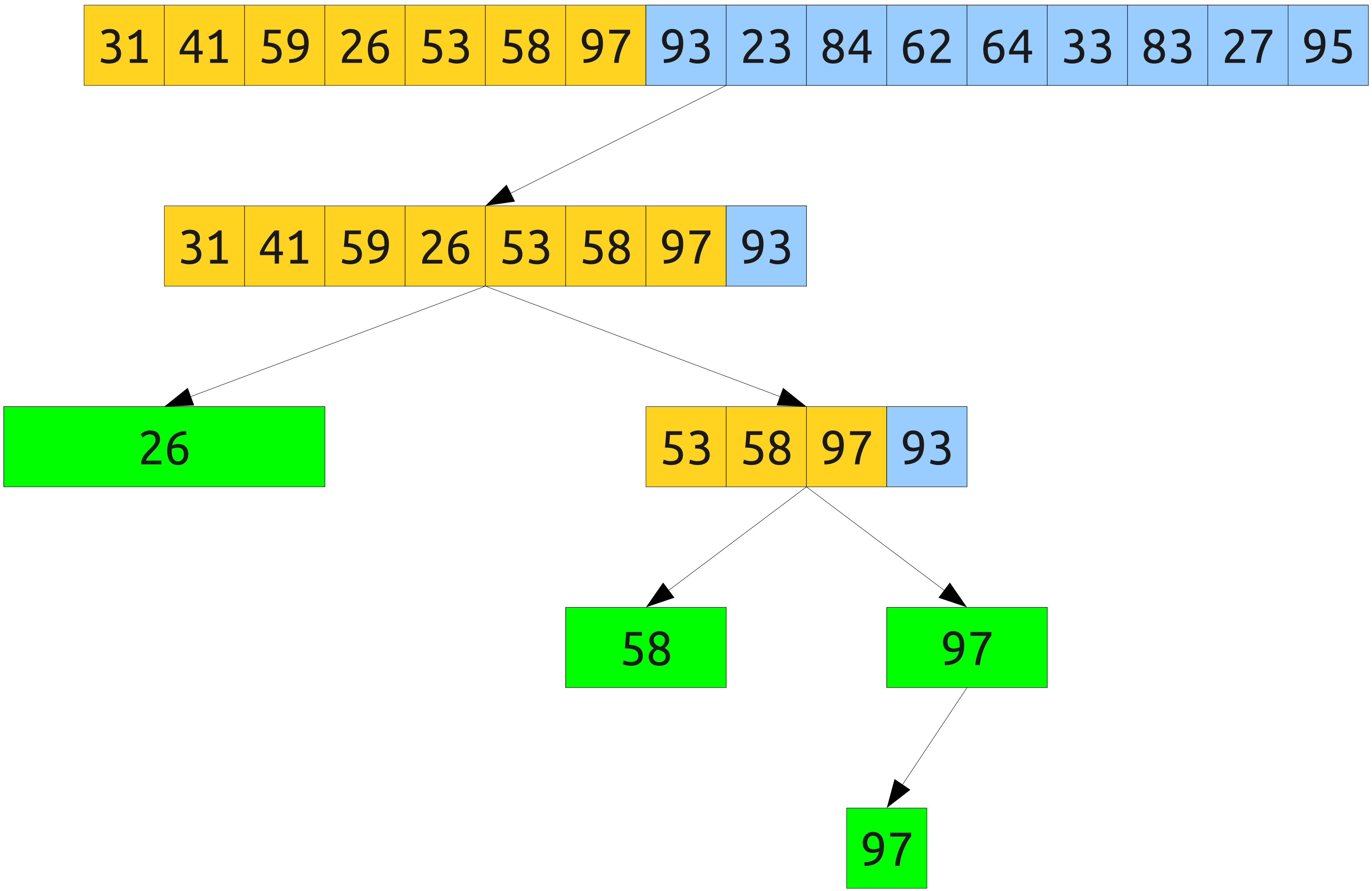

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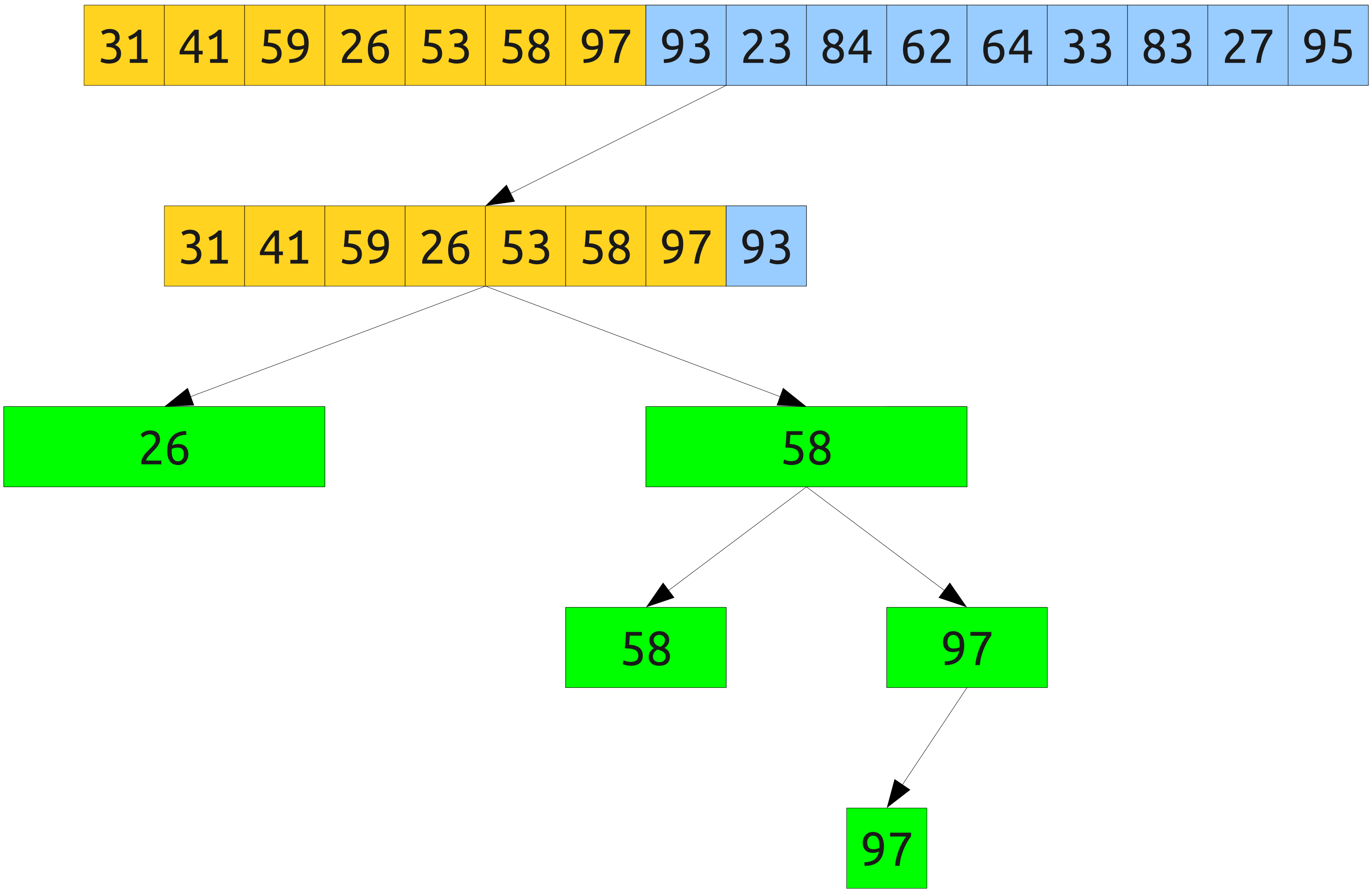

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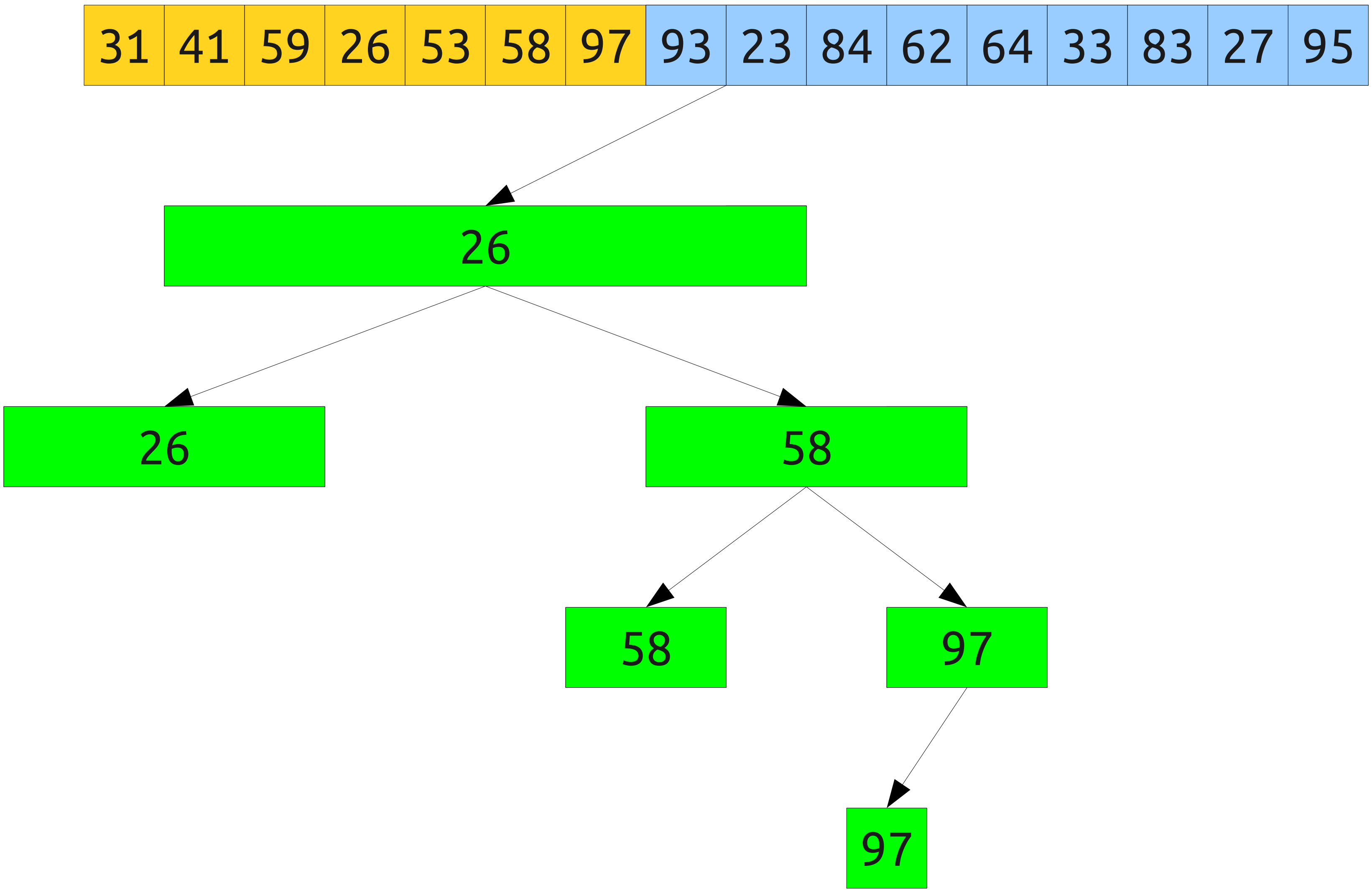

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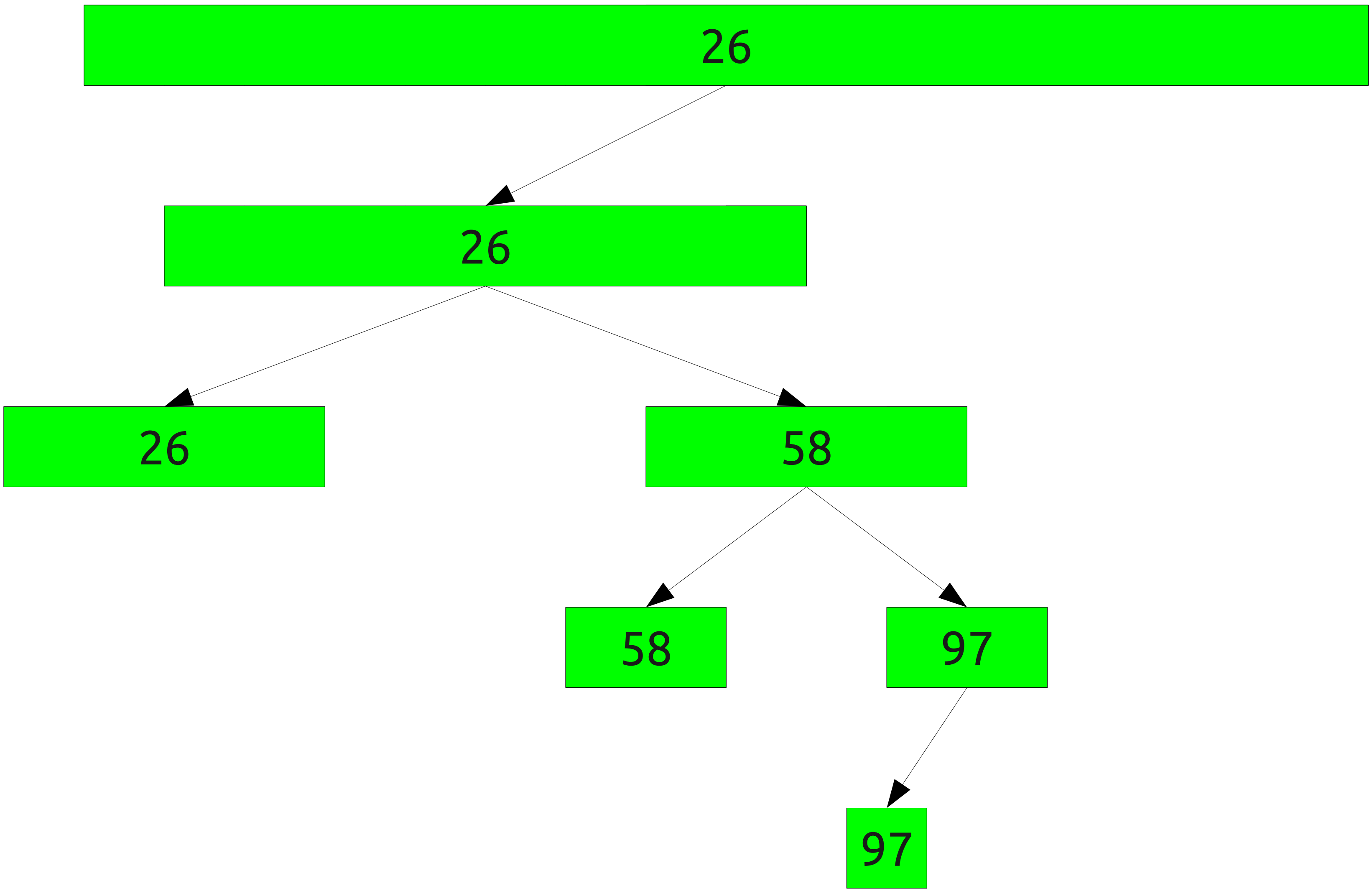
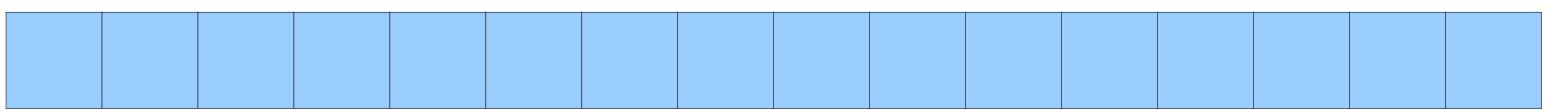


Is This Faster?

- The root cause of the inefficiency in the initial approach was the branching recursion, which is still present in this new solution.
- Is this new approach any faster than what we had before?
- **Claim:** Yes! In fact, queries only take time O(log n).

An Observation



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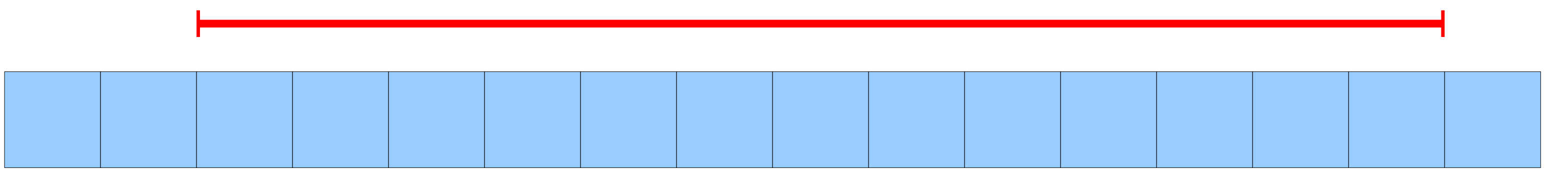

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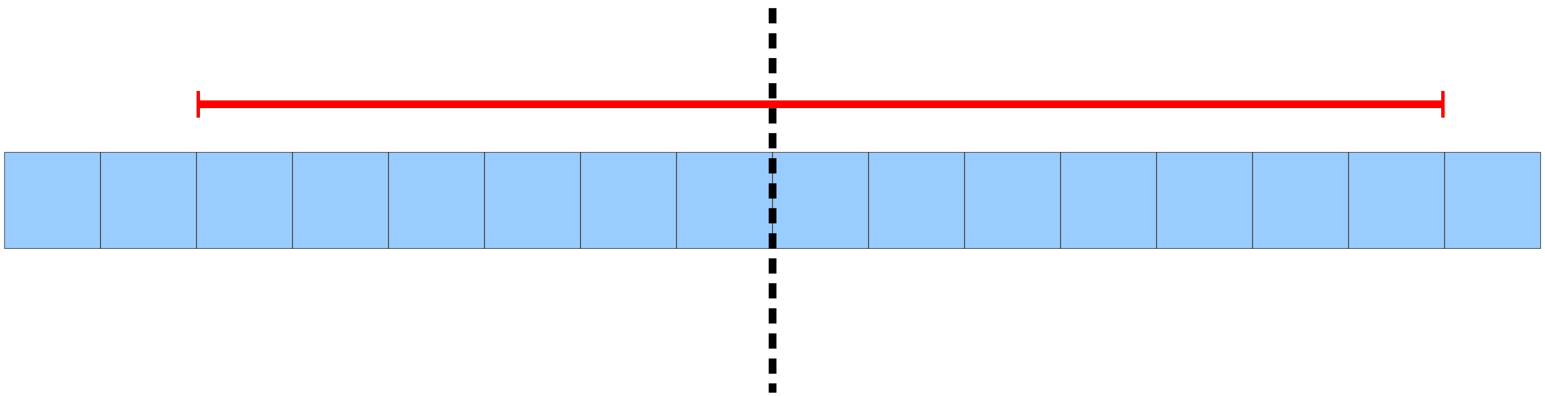

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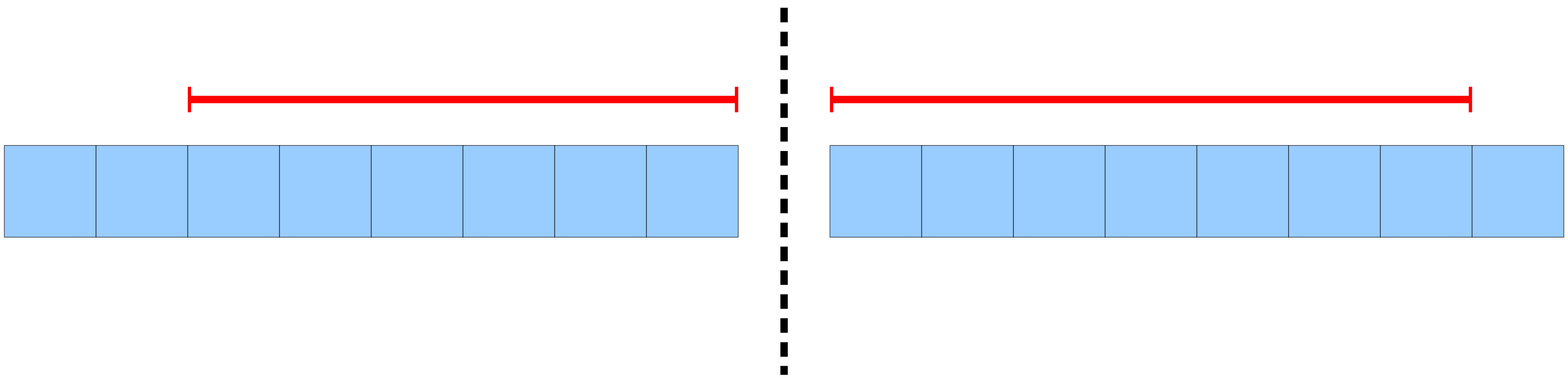

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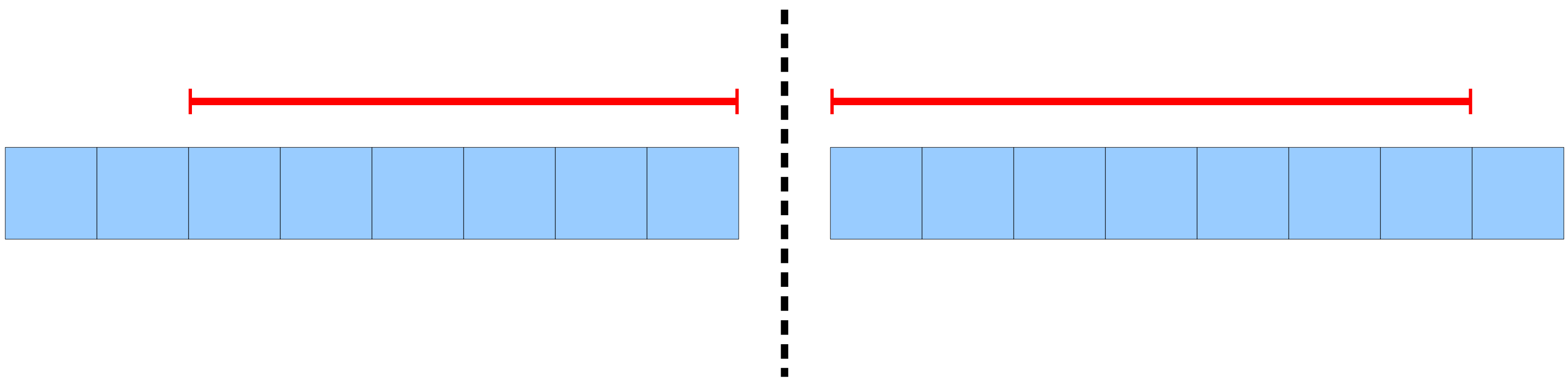

Claim 1: The first time the recursion splits, it leaves behind two ranges that are each flush against one side of the subarray.

Another Observation

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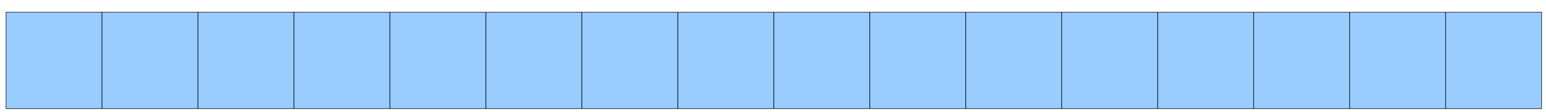

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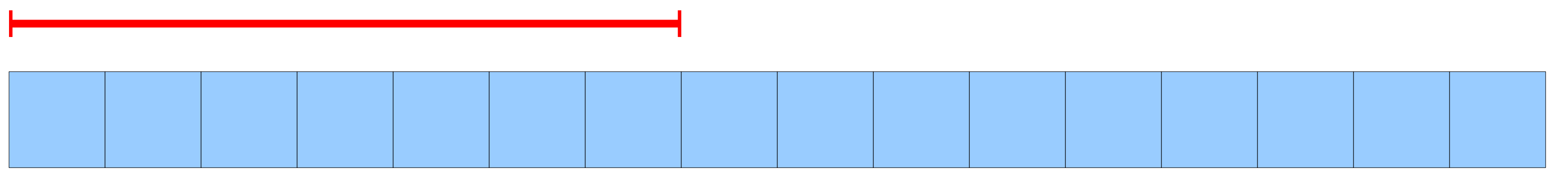

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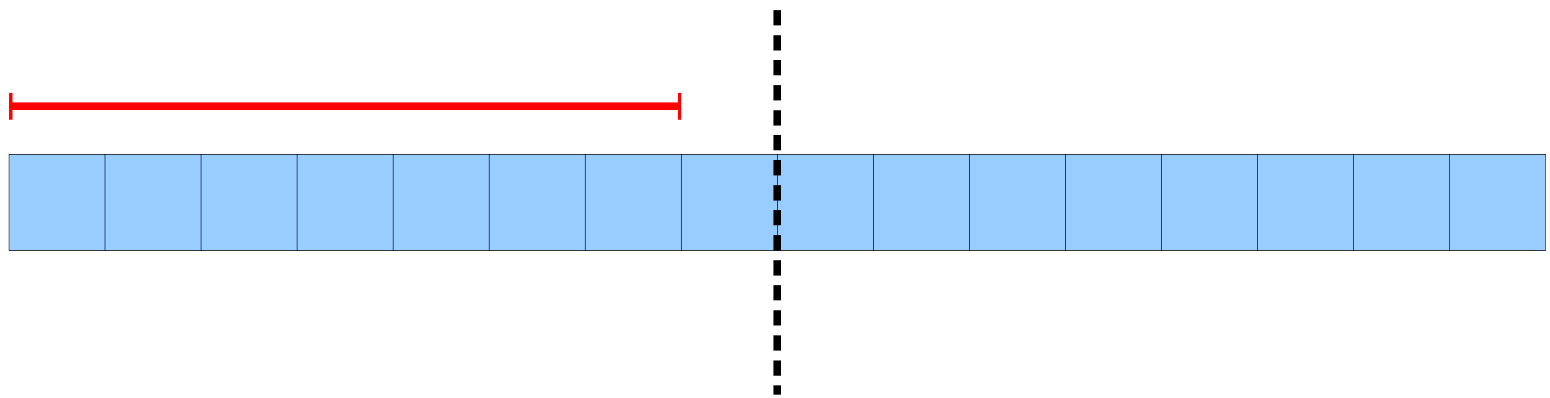

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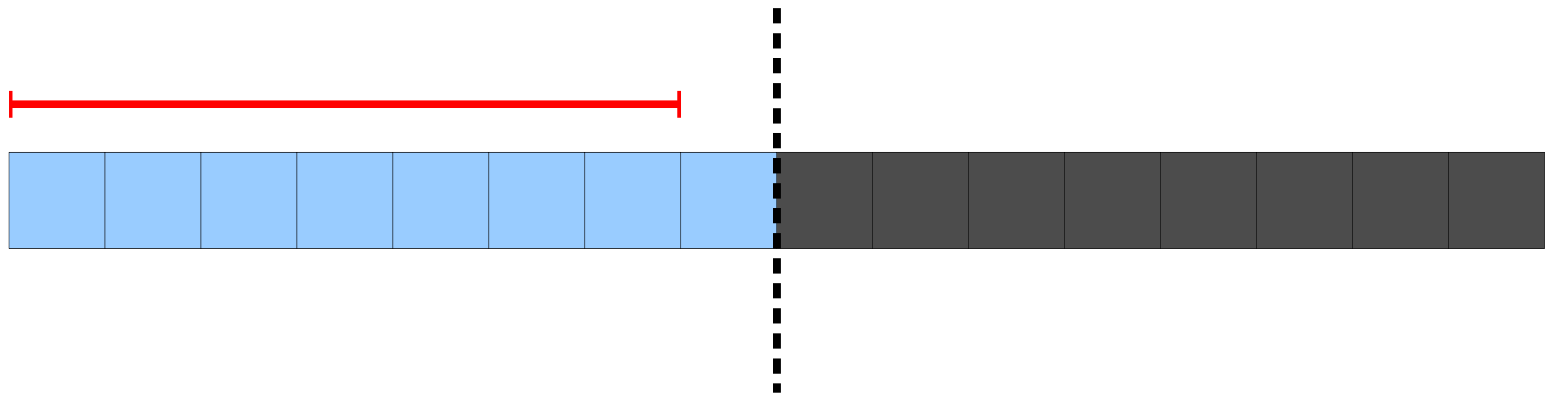

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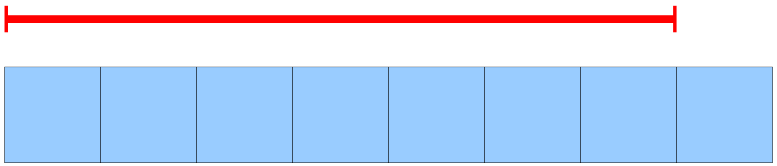

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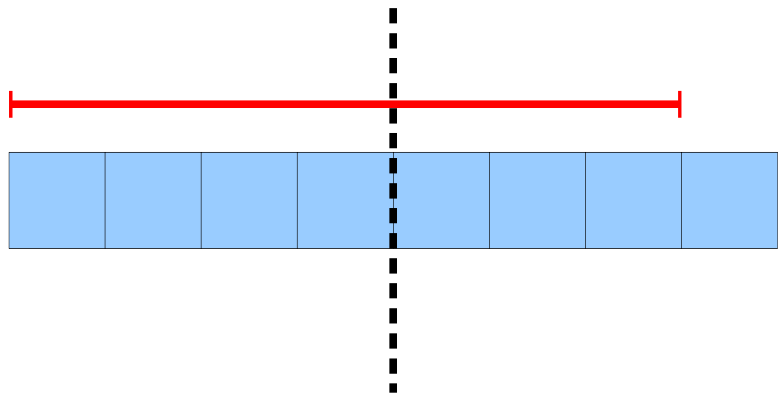

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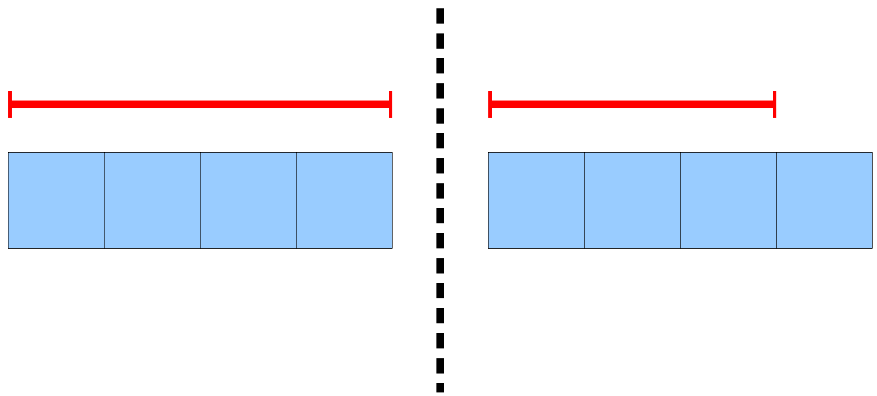

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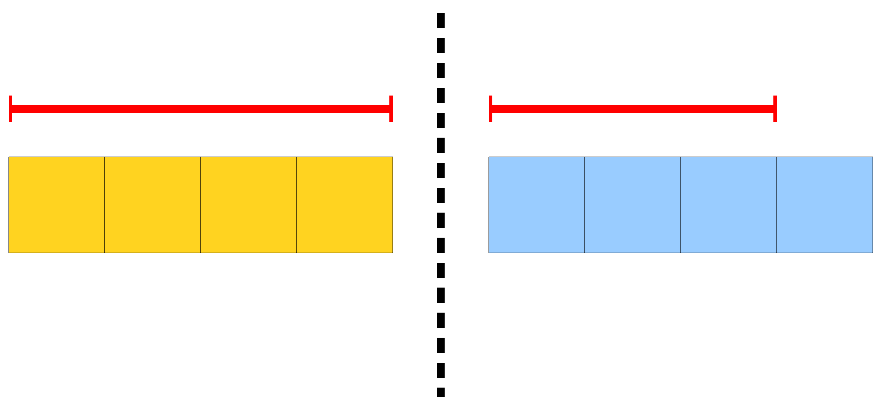

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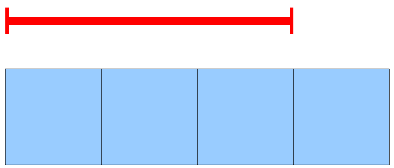

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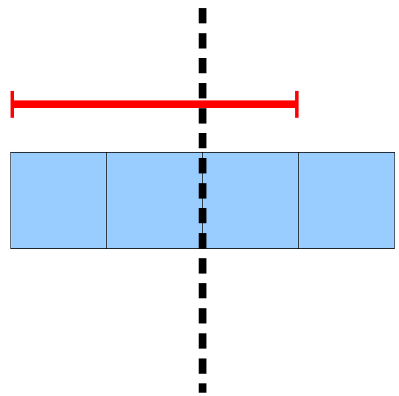

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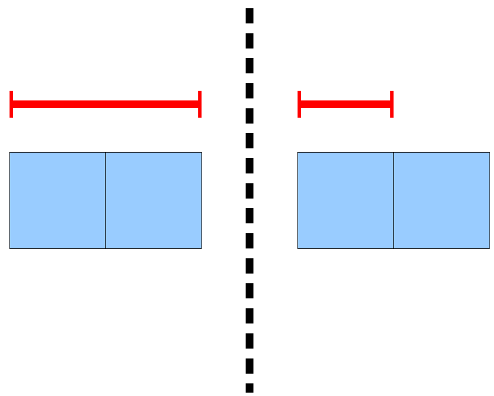

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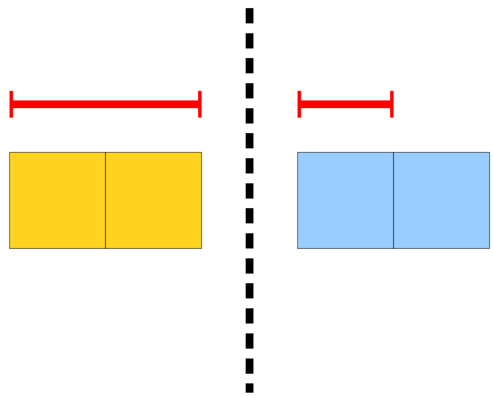

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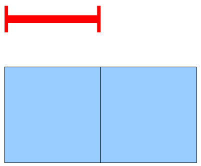
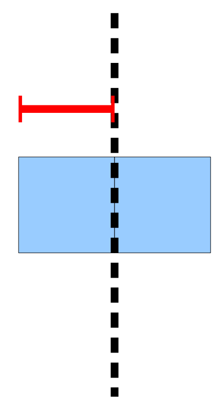
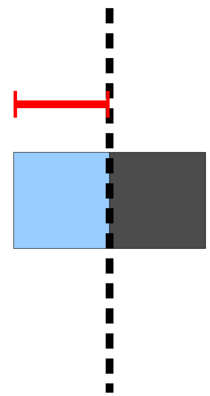


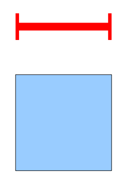
Another Observation



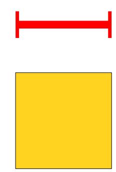
Another Observation



Another Observation



Another Observation



Another Observation

- Suppose the RMQ is over a range that is flush against one edge of the subarray.
- The recursion will then either
  - immediately terminate, or
  - recurse purely in one half of the subarray, or
  - recurse in both halves, but one of the recursive calls will immediately terminate.
- In this case, there is at most one "real" recursive call.

The Final Analysis

- If the recursion never splits into two pieces, the runtime is O(log n).
- If the recursion does split into two pieces:
  - Up until the split, we only can do O(log n) work because there is one recursive call per level.
  - After the recursion splits, each of the two pieces will have the “flush against the wall” structure and will take time only O(log n).
  - Total work done: O(log n).
- This is exponentially faster than before!

Segment Trees

- The segment tree approach requires $O(n)$ preprocessing and $O(\log n)$ time per query.
- We now have an $\langle O(n), O(\log n)\rangle$ solution to RMQ!
- To put that in perspective:
  - If we make $o(n^2 / \log n)$ queries, this is asymptotically faster than precomputing everything.
  - If we make $\omega(1)$ “large” queries, this is asymptotically faster than doing no precomputation.