Dynamic Programming

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Coin Exchange Problem

Input

- n coins with values $p_1, \ldots, p_n$;
- a number t.

Question

Could we find some coins with a total value of t?

Example

Recurrence Relation

Let $F[i,j]$ be a Boolean array such that $F[i,j] = 1$ if and only if we can get value j by first i coins, i.e., $p_1, \ldots, p_n$

- For all $i \in [n]$, $F[i,j] = 1$;
- For all $i \in [n], j \in [t]$,
  $F[i,j] = 1$ if and only if $F[i-1,j] = 1$ or $F[i-1,j-p_i] = 1$
- $F[n,t]$ is the solution.

Pseudo-code for Coin Exchanges

- Input $p_1, \ldots, p_n, t$;
- For $i = 1$ to n
  $F[i, 0] = 1$
- For $i = 1$ to n
  For $j = 1$ to t
  If $(j \geq p_i)$ Then $F[i, j] = (F[i-1, j]$ or $F[i-1, j-p_i])$
  If $(j < p_i)$ Then $F[i, j] = F[i-1, j]$
- Output $F[n, t]$

Edit Distance

Question

Given two strings x and y, how similar are they?

Edit Distance

Number of times we need to insert, remove, or rewrite characters to make x and y identical. More specifically, inserting/deleting a character costs A, and overwriting a character costs B.

Example

Recurrence Relation

Recurrence Relation

Recurrence Relation

Pseudo-code for Minimum Edit

- Input x and y;
- Let $n = |x|$ and $m = |y|$;
- For $i = 1$ to n
  $A[i, 0] = A \cdot i$
- For $j = 1$ to m
  $A[0, j] = A \cdot j$
- For $i = 1$ to n
  For $j = 1$ to m
  If $x_i = y_j$
  Then $A[i, j] = \min(A[i-1, j-1], A + a[i-1][j], A + a[i][j-1])$
  Else $A[i, j] = \min(A[i-1, j-1] + B, A + a[i-1][j], A + a[i][j-1])$
- Output $A[n, m]$

Analysis of Correctness

Thanks!