Greedy Algorithms II

August 23, 2025

Scheduling to Minimize Late Time

Input

- n jobs
- A duration time t_i for each job i
- A deadline d_i for each job i

A feasible schedule assigns a start time s_i to each job i (with the finish time of job i being $s_i + t_i$) such that no jobs overlap.

Lateness

Lateness

Given any feasible schedule, the lateness of a job i is defined as,

$$L_i = \max\{0, f_i - d_i\}$$

The overall lateness is $L = \max_i\{L_i\}$

Lateness

Lateness

Given any feasible schedule, the lateness of a job i is defined as,

$$L_i = \max\{0, f_i - d_i\}$$

The overall lateness is $L = \max_i\{L_i\}$

Our goal is to find a schedule that minimizes L.

Example

Greedy Strategies

Greedy Strategies

Earliest Deadline First

1. Sort all jobs by deadline and let $(i_1, \ldots, i_n)$ be the sorted result;
2. Let $T \leftarrow 0$;
3. For ℓ from 1 to n
   Let $s_{i_\ell} \leftarrow T$
   Let $T \leftarrow T + t_{i_\ell}$
4. Output $(s_1, \ldots, s_n)$

Exchange Argument

An Observation

There is an optimal solution without idle time

Analysis of the Exchange Argument

Lemma

Let $(b_1, \ldots, b_n)$ be the order of jobs. If there exists an ℓ such that $d_{b_\ell} > d_{b_{\ell+1}}$, then swapping b_ℓ and $b_{\ell+1}$ does not increase max lateness.

Analysis of the Exchange Argument

Finish the Proof

Minimize Total Lateness

Jobs with Weighted Penalty

Thanks!