Lecture 21

August 23, 2025

Segment Problem

You are given a social network, and want to segment it into two groups. (Stephen Curry's fans and Lebron James' fans). Two conflicting pieces of evidence:

- for each one, a score of being Curry's fan and
- for each one, a score of being James' fan.
- for each pair, a score of them to belong to the same group.

Segment Problems

Input:

- A undirected graph $G = (V, E)$;
- For each $v \in V$, a foreground score $a(v) \geq 0$
- For each $v \in V$, a background score $b(v) \geq 0$
- For each edge $e \in E$, a separation penalty $p(u, v) \geq$

Goal:

Output a partition $(S, V \setminus S)$ maximizing

$$Q(S, \overline{S}) = \sum_{u \in S} a(u) + \sum_{v \in \overline{S}} b(v) - \sum_{e=(u,v): u \in S, v \in \overline{S}} p(u, v) \geq 0$$

Example

Rewrite $Q(S, \overline{S})$

Construct a Flow-Problem

Complexity v.s. Algorithm

Hardness in Computation (NP-completeness)

Hardness

Question:

How to capture the hardness of a problem?

Reduction

We reduced the largest matching problem to the max-flow problem.

Reduction

Thanks!