P and **NP**

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Complexity v.s. Algorithm

Hardness in Computation (NP-completeness)

Hardness

Question:

How to capture the hardness of a problem?

Reduction

We reduced the largest matching problem to the max-flow problem.

Reduction

Decision Problem

- The output of a decision problem Yes or No
- In many cases, search problems can be naturally transformed into decision problems.

Notations

- For a decision problem P and an instance x,
  If the answer of x is yes, we denote $P(x) = 1$;
  If the answer of x is no, we denote $P(x) = 0$;
- We denote $L_P := \{x : P(x) = 1\}$

Solve Decision Problems by Algorithms

An algorithm A solves a problem P, if for every x

- If $x \in L_P$, $A(x)$ terminates and output 1;
- If $x \in L_P$, $A(x)$ terminates and output 0.

Running Time

Running time

Let $T : \mathbb{N} \to \mathbb{N}$ be a function. The running time of an algorithm A is $T(n)$ if for every input x, $A(x)$ terminates in $T(|x|)$ time.

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What is the running time for calculating Fibonacci numbers?

Polynomial Time

An algorithm A runs in polynomial time if there exists constants $a, c > 1$ such that the running time of A is $T(n) = a \cdot n^c$.

Non-deterministic Algorithms

Sudoku Problem

Proof v.s. Verification

Non-deterministic Polynomial Time

Definition

Let P be a computational problem and denote $L := \{x : P(x) = 1\}$. We say that P is in **NP**, if there is an algorithm $A(\cdot, \cdot)$ such that

- If $x \in L$, there is a string y, such that $A(x, y) = 1$;
- If $x \notin L$, then for every y, $A(x, y) = 0$;
- For every x and y, the running time of $A(x, y)$ is polynomial in $|x|$

Boolean satisfiability problem

SAT problem

For a given Boolean formula f, is there an assignment that satisfies f?

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SAT problem

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Theorem

$SAT \in \mathbf{NP}$.

$\mathbf{P}$ v.s. $\mathbf{NP}$

Theorem

$\mathbf{P} \subseteq \mathbf{NP}$.

Thanks!