Stable Matching and Gale-Shapley Algorithm

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Stable Matching Problem: A Simple Case

- Given n professors and n students
- $P = \{p_1, \ldots, p_n\}$, $S = \{s_1, \ldots, s_n\}$
- Each student has a preference list of P
- Similarly, each professor has a preference list of S.

Question:

Can we connect advisors and students in a manner that makes many people satisfied?

Matching and Perfect Matching

Matching and Perfect Matching

Definition

A matching M is a set of ordered pairs, each from $P \times S$, with the property that each professor and each student appears in at most one pair in M. A perfect matching is a matching with the property that each professor and each student appears in exactly one pair.

Stable Matching

Definition

For a perfect matching M, two pairs (p, s) and (p', s') are unstable if p prefers s' to s and s' prefers p to p'.

Stable Matching

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For a perfect matching M, two pairs (p, s) and (p', s') are unstable if p prefers s' to s and s' prefers p to p'. A matching is stable if there is no unstable pair.

Question:

Can we always (efficiently) find a perfect and stable matching?

Algorithm

Gale-Shapley Algorithm

Gale-Shapley Algorithm

Analysis

- Proof of accuracy.
- Running time analysis.

Running Time

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- For each student, the sequence of professors assigned to them gets better and better.
- For each professor, the sequence of students matched to them gets lower and lower rank.

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Theorem

The G-S algorithm terminates after at most n^2 iterations of the While loop.

Accuracy Analysis

Accuracy Analysis

Improved Implementation

Thanks!