Network Flows and Graph Cuts

August 23, 2025

Residual Capacity

- Forward edge $e = (u, v)$: with residual capacity $c'(e) = c(e) - f(e)$
- Backward edge $e = (v, u)$: with residual capacity $c'(e) = f(e)$

Connectivity in the Residual Graph

Residual Graph and Ford-Fulkerson Algorithm

- Start with empty flow $f = 0$;
- While the residual graph G_f has a s-t path
  Find a s-t path P (using BFS/DFS)
  Augment f along with P ($f \leftarrow f + P$)
- EndWhile
- Output f

Augment with Paths in Residual Graph

$f + P$.

Correctness of Ford-Fulkerson

Lemma

Let f be a flow on G. For any s-t path in G_f, the flow $f + P$ is also a flow on G.

Ford-Fulkerson is Optimal

Theorem

- *Ford-Fulkerson flow* $\leq$ *Maximum flow*
- *Maximum flow* $\leq$ *Minimum cut*
- *Minimum cut* $\leq$ *Ford-Fulkerson flow*

Ford-Fulkerson is Optimal

Running Time of Ford-Fulkerson

Two Improved Algorithms

Thanks!