Ошибка метода:

$$\varepsilon_м = \left| A \frac{f^{(3)}(\xi)}{6!} \left(\frac{h}{2}\right)^3 + C \frac{f^{(3)}(\xi)}{6!} h^3 \right|$$

$$\exists\ |f^{(3)}(\xi)| \le M_3:$$

$$\varepsilon_м = \left| \frac{M_3}{3} \left(\frac{h^2}{6 \cdot 4} - \frac{4h^2}{64} \right) \right| = \frac{M_3 h^2}{6 \cdot 9}$$

Ошибка вычисл.: $\varepsilon_в = \dfrac{\varepsilon_{изм} \cdot 3 \cdot f(x_i)}{h}$

$$\exists\ \varepsilon_в = \varepsilon_{изм} f(x_i),\ \varepsilon_в = \frac{3\varepsilon}{h}$$

$$\varepsilon_п = \frac{3\varepsilon}{h} + \frac{M_3 h^2}{6 \cdot 9};\ \varepsilon_п' = 0;\ \frac{3\varepsilon}{h^2} = \frac{M_3 h}{3}$$

$$h_{опт} = \sqrt[3]{\frac{9\varepsilon}{M_3}}$$

№1 $f(x) = e^x$; $[-3; 4]$.

$x_i = \frac{1}{2}\left\{1 + 7\cos\left(\frac{\frac{\pi}{2} + \pi k}{N}\right)\right\}$, $k = \overline{0, N-1}$

$N = 5$;

№2 $\{x_i - \frac{h}{2};\ x_i;\ x_i + h\}$

$f' = A f(x_{i-0,5}) + B f_i + C f_{i+1}$;

$f(x_i - \frac{h}{2}) = f(x_i) - f'(x_i)\frac{h}{2} + \frac{f''(x_i)}{2}\left(\frac{h}{2}\right)^2 + \frac{f^{(3)}(\xi)}{6}\left(\frac{h}{2}\right)^3$

$f(x_i + h) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2}h^2 + \frac{f^{(3)}(\xi)}{6}h^3$

$f' = A f_i - A f_i' \frac{h}{2} + A\frac{f''}{2}\left(\frac{h}{2}\right)^2 + A\frac{f^{(3)}(\xi)}{6}\left(\frac{h}{2}\right)^3 + B f_i'$

$+ C f(x_i) + C f'(x_i)h + C\frac{f''(x_i)}{2}h^2 + C\frac{f^{(3)}}{6}(\xi)h^3$

$f_i(A + B + C)$; $f_i'(-A\frac{h}{2} + Ch)$

$f_i''\left(\frac{1}{2}A\left(\frac{h}{2}\right)^2 + \frac{C \cdot h^2}{2}\right)$;

$A = -\frac{4}{3h}$; $B = \frac{1}{h}$; $C = \frac{1}{3h}$

$$\begin{cases} A + B + C = 0 \\ 2Ch - A\frac{h}{2} = 1 \\ \frac{Ah^2}{8} + \frac{Ch^2}{2} = 0 \end{cases}$$

$$f' = \frac{-\frac{4}{3}f(x_i - \frac{h}{2}) + f(x_i) + \frac{1}{3}f(x_i + h)}{h}$$

Порядок аппроксим. $h = 2$.