



«

»

• •

, • •

1

2013

: 519.8(075)

:

.

—

,

,

,

• .;

,

• •

• ”

• •

:

. . 1. —

: — , 2013. — 117 .

,

.

—

.

(

—

).

.

(

),

,

,

«

».

,

,

.

.

.

,

«

»

«

».

,

,

,

,

,

.

. 15.

.: 8

.

© . . , 2013

© . . , 2013

© , 2013

1.

,

—

(

)

—

.

—

,

(

optimus —

).

,

,

.

.

.

,

,

,

,

.

,

,

,

.

,

,

,

,

,

.

1.1.

“

”

,

,

,

:

?

—

—

?

—

,

?

—

?

,

, . .

,

.

:

?

,

. .

.

,

.

,

,

.

,

,

,

,

, —

.

.

.

,

,

.

()

,

.

.

,

,

—

()

.

.

,

.

... “ ”.

.

1.2.

,
,
()
.
— ,

. — ,

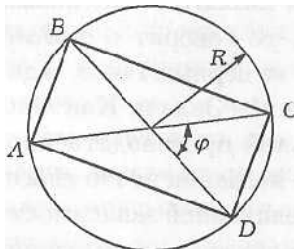
— ,

,

.

1.1

, R
 S (. 1.1).



. 1.1

,

.

. S

, ... $S = 2R^2 \sin$,
 $\sin = 1$, $= /2$.

$$2R^2.$$

$$\sqrt{a^2 + b^2} = 2R$$

$$\begin{cases} S = ab \rightarrow \max; \\ a^2 + b^2 = 4R^2, a \geq 0, b \geq 0. \end{cases}$$

$$S = ab$$

$$4R^2 - 2S = a^2 + b^2 - 2ab = (a - b)^2, \quad S = 2R^2 - \frac{(a - b)^2}{2}.$$

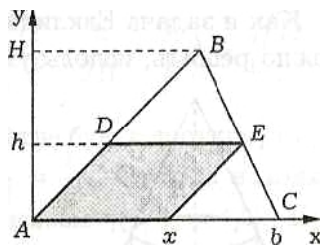
$$S^* = 2R^2 = b^2, \quad b = R\sqrt{2}.$$

1.2 (

ABC

b

(1.2).



1.2

.

,
(. 1.2),

—

.

h .

S

$S = hx,$,

, — (H

$- h)/H = x/b,$ DBE $ABC.$,

$$\begin{cases} S = hx \rightarrow \max; \\ \frac{H-h}{H} = \frac{x}{b}, h \geq 0, x \geq 0. \end{cases}$$

(.

1.1).

$x^* = b/2$

$h^* = (1 - x/b)H = H/2.$

,

,

,

,

$ABC.$

1.3.

.

,

.

.

.

1.3

,

.

$V,$

R

S

$$S = 2 \ R(H + R)$$

$$R^2H = V, \quad \dots$$

$$S(R) = \frac{V}{R} + \frac{V}{R} + 2\pi R^2,$$

$R,$

$$S(R) = \frac{V}{R} + \frac{V}{R} + 2\pi R^2 \geq 3\sqrt[3]{\frac{V}{R} \frac{V}{R} 2\pi R^2} = 3\sqrt[3]{2\pi V^2} = S_*.$$

$$\frac{V}{R} = 2\pi R^2, \quad R_* = \sqrt[3]{\frac{V}{2\pi}}.$$

$$H_* = \frac{V}{\pi R_*^2} = \sqrt[3]{\frac{4V}{\pi}} = 2R_*,$$

R^2

$4R^2,$

$$\tilde{S} = 2\pi R H + 8R^2$$

$$R^2H = V.$$

$$\tilde{R}_* = \sqrt[3]{V}, \quad \tilde{S}_* = 6\sqrt[3]{V^2}, \quad \tilde{H}_* = \frac{8}{\pi} \tilde{R}_*.$$

$$\begin{array}{c} 2R. \\ 2\sqrt{3}R^2 \\ R. \end{array}$$

$$\tilde{S} = 2\pi RH + 4\sqrt{3}R^2$$

$$\hat{R}_* = \sqrt[3]{\frac{V}{4\sqrt{3}}}, \; \hat{S}_* = 3\sqrt[3]{4\sqrt{3}V^2}, \; \hat{H}_* = \frac{4\sqrt{3}}{\pi}\tilde{R}_*.$$

,

,

,

.

.

1.4.

,

,

.

.

“ — ”.

,

:

,

.

,

.

.

1.4

,

,

(, , . .).

a_{ij} $i-$ $j-$ o ,

, $i = \overline{1,m}$,
 $j = \overline{1,n}$, b_i x_j —

.

a_j A_j

,

$$\sum_{j=1}^n a_{ij}x_j \leq b_i, \; i = \overline{1,m}, \; a_j \leq x_j \leq A_j, \; j = \overline{1,n}.$$

,

= (1,

2, ..., n)

$$S=\sum_{j=1}^nd_jx_j\,,$$

$$d_j-\hspace{1.5cm}j-\hspace{1.5cm}.$$

$$x_j, \hspace{0.5cm} j \hspace{0.5cm} = \hspace{0.5cm} \overline{1,n}.$$

$$1.5\,(\hspace{1.5cm})$$

m

$$\hspace{1.5cm},\hspace{1.5cm}i-\hspace{1.5cm},\hspace{1.5cm}i=\overline{1,m},\hspace{1.5cm}a_i$$

$$,\hspace{0.5cm}a\hspace{0.5cm}j-\hspace{1.5cm},\hspace{0.5cm}j=\overline{1,n},\hspace{1.5cm}b_j,\hspace{1.5cm}$$

$$\hspace{1.5cm}i-\hspace{1.5cm}j-\hspace{1.5cm}ij.$$

$$x_{ij}\hspace{1.5cm},\hspace{1.5cm}i-\hspace{1.5cm}j-\hspace{1.5cm}$$

$$\hspace{1.5cm},\hspace{1.5cm}ij\hspace{0.5cm}x_{ij}.$$

$$-\hspace{1.5cm}.$$

$$S=\sum_{i=1}^m\sum_{j=1}^nc_{ij}x_{ij}\rightarrow\min\,,$$

$$\min\hspace{1.5cm}x_{ij}\hspace{0.5cm}0,$$

$$\hspace{1.5cm}.$$

$$,\hspace{1.5cm}\cdot\cdot\cdot$$

$$\sum_{i=1}^mx_{ij}=b_j,\hspace{0.5cm}j=\overline{1,n}.$$

,

$$\hspace{1.5cm},\hspace{1.5cm}$$

$$\sum_{j=1}^nx_{ij}\leq a_i,\hspace{0.5cm}i=\overline{1,m}.$$

,

,

,

,

,

..

$$\sum_{j=1}^n b_j \leq \sum_{i=1}^m a_i.$$

1.6 ()

.

$$\begin{aligned} &: - & ; - \\ & ; a_{ij} - i- \\ j- & ; b_j - i- \\ ; c_j - j- . , \end{aligned}$$

() , ,

,

$$= (x_1, x_2, \dots, x_n)^T, \quad x_j =$$

$$j=0 \quad . \quad ,$$

.

:

$$S = \sum_j^n c_j x_j$$

$$\sum_j^n a_{ij} x_j \geq b_i, \quad i = \overline{1, m};$$

$$x_j \geq 0, \quad j = \overline{1, n}.$$

1.7

,

,

$$x \quad i- \quad S(x_i) = a_i x_i^{k_i},$$

$$a_i = \quad i- \quad ($$

$$k_i = 1, \quad - k_i <$$

1).

,

:

$$b_i$$

$$i- \quad , \quad \dots \quad x_i \geq b_i, \quad i = \overline{1, n}.$$

,

$$\sum_{i=1}^n d_i x_i \geq b,$$

$$d_i, \; i = \overline{1,n} \; - \qquad i- \qquad , \; b -$$

$$S=\sum_{i=1}^na_ix_i^{k_i},$$

$$\mathbf{1.8}$$

$$m \qquad i,$$

$$i=\overline{1,m}, \qquad B_j, j=\overline{1,n},$$

$$p_0 \qquad 0$$

$$g_i \; i-$$

$$, \; i \; = \; \overline{1,m},$$

$$G_i, \; \ldots$$

$$0 \leq g_i \leq G_i.$$

$$i-$$

$$, \; i \; = \; \overline{1,m},$$

$$\varphi_i(g_i)$$

$$g_i.$$

$$K$$

$$k-$$

$$,$$

$$k=\overline{1,K},$$

$$f_k(g_k) \qquad g_k$$

$$.$$

$$\sum_{k\in B_j^+}g_k=p_j+\sum_{k\in B_j^-}g_k\,,j=\overline{1,n}\,,$$

$$B_j^+ \qquad B_j^- \; -$$

$$j-$$

$$.$$

$$g_i=\sum_{k\in A_i^-}g_k\,,i=\overline{1,m}.$$

$$,$$

$$S=\sum_{i=1}^m\varphi_i(g_i)+\sum_{k=1}^Kf_k(g_k)$$

$$.$$

$$\varphi_i(g_i) \quad f_k(g_k)$$

$$,$$

$$-$$

$$.$$

1.5.

$$\begin{aligned}
 & \quad , \\
 & \quad \cdot \quad , \\
 & \quad , \\
 & \quad \cdot \quad , \\
 & \quad , \\
 & \quad , \\
 & \quad , \\
 & \quad \cdot \\
 & f_0(x) - \\
 & \quad , \\
 & \quad x_j, j = \overline{1, n}, \quad x \in R^n. \\
 & \quad (\\
 & \quad). \\
 & \quad , \\
 & \quad \cdot \\
 & \quad , \quad : \\
 & f_0(x) \rightarrow \min, x \in \Omega, \quad (1.1) \\
 & \Omega \subset R^n - \\
 & \quad \cdot \\
 & x \in \Omega \\
 & \quad , \quad - \\
 & \quad , \quad , \quad \cdot \\
 & x^* \in \Omega, \quad f_0(x) \\
 & \quad \cdot \\
 & \quad \Omega \\
 & D(f_0) \subset R^n \\
 & \quad \cdot \\
 & \quad , \\
 & \quad \cdot \\
 & (1.1)
 \end{aligned}$$

$$\Omega,$$

$$f_0(x) \quad \Omega \quad x \in \Omega,$$

$$\Omega$$

$$\inf_{x \in \Omega} f_0(x)$$

$$f_0(x) \quad (1.1)$$

$$f_0(x) \rightarrow \inf, x \in \Omega. \quad (1.2)$$

$$(1.1) \quad (1.2) \quad ,$$

$$x^* \in \Omega,$$

$$\Omega,$$

$$, \quad ($$

$$x_j, j = \overline{1, n},$$

$$x^* \in \Omega$$

$$f_0(x^*) = \min_{x \in \Omega} f_0(x),$$

$$\{ \quad \} \quad x_n \in \Omega,$$

$$\{f_0(x_n)\},$$

$$\inf_{x \in \Omega} f_0(x),$$

$$(1.2)$$

$$f_0(x) —$$

$$R^n. \quad R^n$$

$$f_0(x) = (c, x), \quad c = (c_1, \dots, c_n) \in R^n —$$

$$f_0(x) = (c, x) = \sum_{j=1}^n c_j x_j$$

$$f_0(x^*) \quad \Omega$$

$$\partial \Omega$$

$$\Omega = R^n$$

$$- \infty.$$

$$(c, x) \rightarrow \min, x \in R^n \quad (1.3)$$

$$Bx = d, x \in R^n, \quad (1.4)$$

$$d \in R^k, \quad —$$

$$k \times ,$$

$$, \dots$$

$$x_j \geq 0, j = \overline{1, n}, \quad (1.5)$$

$$(1.3) - (1.5)$$

,

(

,

).

$$(1.5)$$

$$x \in \mathbb{R}^n, \quad \mathbb{R}^n -$$

 \mathbb{R}

,

.

$$(1.3) - (1.5)$$

$$\sum_{j=1}^n a_{ij} x_j \leq b_m, \quad (1.6)$$

$$a_{ij} \in \mathbb{R}, i = \overline{1, m},$$

$$(1.3) - (1.6)$$

$$(1.5)$$

.

,

.

$$(1.3), (1.5) \quad (1.6)$$

(

).

$$x_j \geq a_j, x_j \geq b_j \quad a_j \leq x_j \leq b_j, j = \overline{1, n}$$

,

,

—

.

,

$$(1.5)$$

.

.

$$(1.3)$$

, \dots

$$(c, x) + 1/2(Qx, x) \rightarrow \min, \quad (1.7)$$

$$Q -$$

,

,

$$(1.7)$$

.

,

,

,

—

.

$$(1.4) - (1.6)$$

$$\frac{(q,x)+\alpha}{(r,x)+\beta}\rightarrow\min,$$

$$q\in RR^n, r\in R^n, \alpha\in R\quad \beta\in R\quad.$$

$$g_i(x)=\left\{\begin{array}{l} f_0=\sum\limits_{j=1}^nh_j(x_j)\rightarrow\min;\\ g_i(x)=\sum g_{ij}(x_j)\leq\gamma_i,\quad i=\overline{1,m};\\ x_j\geq0,\quad j=\overline{1,n},\end{array}\right.$$

$$\gamma_i\in R\qquad\qquad\qquad,\qquad\qquad\qquad.$$

$$f_0(x)\qquad\qquad\qquad g_i(x)$$

$$,$$

$$.$$

$$f_0(x)\qquad g_i(x)$$

$$.$$

$$y(x)=\sum_{i=1}^m c_i p_i(x),$$

$$x=(x_1,\ldots,x_n)\in R_+^{\,n},\, \mathsf{c}_i\in \mathsf{R}_+,\quad R_+^{\,n}\text{ --- }n\text{ --dimensional}\\ R_+^{\,n}\qquad\qquad\qquad,$$

$$.$$

$$p_i(x)$$

$$p_i(x)=\prod_{j=1}^nx_j^{a_{ij}},$$

$$a_{ij}\in\mathsf{R}.\qquad\qquad\qquad c_i,\quad i\,=\,\overline{1,m},$$

$$,$$

$$(\qquad\qquad\qquad),$$

$$.$$

$$,$$

$$,$$

$$x_j^{a_{ij}}$$

$$ij\cdot$$

$$(\qquad\qquad)$$

$$,$$

$$.$$

$$.$$

$$.$$

()

$$f_1(x) = 0, l = \overline{1, k}; g_i(x) \leq 0, i = \overline{1, m},$$

$$f_1(x), g_i(x)$$

,
 ,
 — ,

.

.

, . .

(),

(). ,

. ,

,

. ,

, ,

, .

,

.

, , ,

. ,

,

, .

.

.

Q

, .

2.

2.1.

() –

,

.

,

,

..

,

.

()

,

,

,

.

:

$$\max f(x_1, \dots, x_n)$$

m

$$G_1(x_1, \dots, x_n) \leq B_1,$$

.....

.....

$$G_m(x_1, \dots, x_n) \leq B_m,$$

$f -$ (: ,);

$x_1, \dots, x_n -$.

,

,

.

2.2.

.

.

2.1

: $M1$ 2.

.

	()		
$M1$	6	4	24
2	1	2	6
	5	4	

2 (−),

,

()

.

()

,

:

,

1.

,

.

2.

,

.

3.

,

.

—

.

.

.

:

x_1 —

;

x_2 —

.

,

.

,

,

,

.

z

,

$$z = 5x_1 + 4x_2.$$

:

$$z = 5x_1 + 4x_2.$$

. , $_1 = 3 \quad _2 = 1$,
 ,
 , $_1 = 3 \quad _2 = 1$,

.
 $z = 5 \times 3 + 4 \times 1 = 19.$

, ,
 ,
 . ,
 (,) .

, . .
 . ,

1. .
 2. ,

.
2.3.
 ,
 .
 (,
), ,
 ,
 (-).

1. ,
 .
 2. .

2.2

2.1,

$$\begin{aligned}
 z &= 5x_1 + 4x_2 \\
 6x_1 + 4x_2 &\leq 24, \\
 1x_1 + 2x_2 &\leq 6, \\
 -x_1 + x_2 &\leq 1, \\
 x_2 &\leq 2, \\
 x_1 \geq 0, \quad x_2 &\geq 0.
 \end{aligned}$$

1.

x_1 , x_2 : $-x_2 \leq 1$ (2.1).
 $x_1 \geq 0, \quad x_2 \geq 0$.
 (2.1) x_1 x_2).

$$6x_1 + 4x_2 \leq 24$$

$$6x_1 + 4x_2 = 24.$$

$$\begin{aligned}
 x_1 = 0, \quad x_2 = 24 / 4 = 6. \quad x_2 = 0 \\
 x_1 = 24 / 6 = 4. \quad (0, 6) \quad (4, 0).
 \end{aligned}$$

2.1 1.

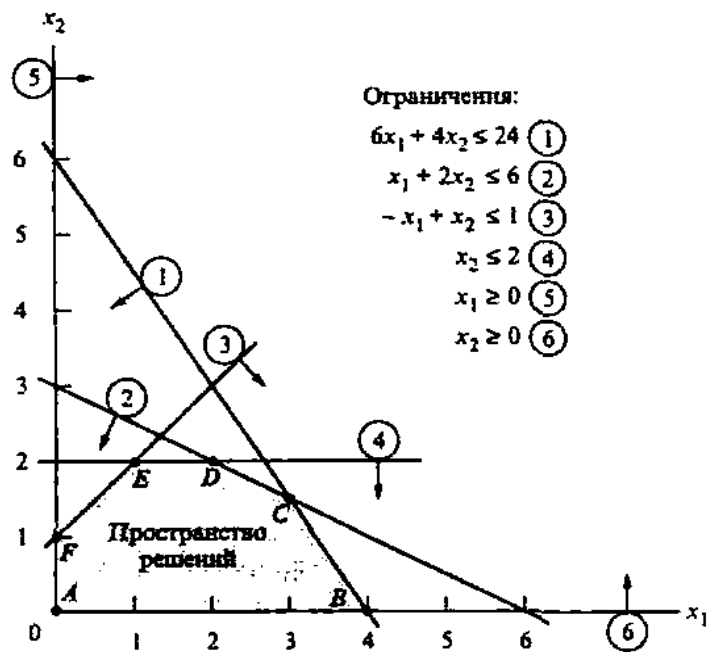
$$(x_1, x_2)$$

" " , , (0, 0).
 , - .

$$(6 \cdot 0 + 4 \cdot 0 = 0 \leq 24). \quad 6x_1 + 4x_2 \leq 24$$

$$(0, 0),$$

2.1



. 2.1

(0, 0)

2.

. 2.1,

, B,

, D, E F.

ABCDEF,

$$z = 5x_1 + 4x_2 \quad (z).$$

10 15.

z

10 15

$$5x_1 + 4x_2 = 10 \quad 5x_1 + 4x_2 = 15.$$

. 2.2

2.4.

" " (\leq) " " (\geq).
 .
 (),
 "≤" "≥" .
 ,
 (, , 0).
 ,
 — —
 (.
),
 " ", —
 " " .

"≤"
 .
 ().

2.1 $6x_1 + 4x_2 \leq 24$
 $M1$.
 $6x_1 + 4x_2 + s_1 = 24, \quad s_1 \geq 0.$
 $(s_1 = 24 - 6x_1 - 4x_2)$ $M1$.
 "≥"
 " — "

, $x_1 + x_2 \geq 800$,
 800 .
 $x_1 + x_2 - s_1 = 800,$
 $s_1 \geq 0.$
 800
 .

.
 ,
 ,

2.3

.
 0,25 ,
 — 0,2 . 200
 , 25
 . ,
 " " . 20
 15 —
 . ,
 900 .
 (. .)
 , ?
 . x_1 2
 ,
 .
 200 .
 $0,25x_1 + 0,2x_2 \leq 200,$ —
 $0,25x_1 + 0,2x_2 \geq 200.$,
 .
 ,
 $0,25x_1 + 0,2x_2 + x_3 = 200,$ $x_3 \leq s_1$.
 x_3
 , .
 .
 . ,
 ,
 .
 3 , . . .
 " " 3,
 ,
 :
 $x_3 = x_3^+ - x_3^- , \quad x_3^+ , x_3^- \geq 0$
 $x_3^+ > 0 \quad x_3^- = 0,$ x_3 . ,

$, x_3^- > 0 \quad x_3^+ = 0,$ x_3 $.($
 $. \quad 3$ $,$
 $x_3^+, \quad x_3^-.)$ $,$

$$0,25x_1 + 0,2 \quad x_3^+ - x_3^- = 200.$$

$$z = 0,20x_1 + 0,15 \quad x_3^- - 0,25x_3^-.$$

3. —

,

().

—

.

—

.

,

.

,

(

,

).

(

)

(

)

.

—

.

3.1.

,

,

.

3.1.1.

.

1.

(

)

.

2.

.

3.

,

.

1.

.

(

 \leq \geq)

—

(.

. 2).

" \leq ".

$$x_1 + 2x_2 \leq 3$$

$$x_1 + 2x_2 + s_1 = 3,$$

$s_1 \geq 0$.

$$s_1 \geq 0.$$

" \geq ".

$$3x_1 + 2x_2 \geq 5$$

$$3x_1 + 2x_2 - S_1 = 5,$$

$S_1 \geq 0$.

$$S_1 \geq 0.$$

-1.

,

,

" \leq "

" \geq "

-1.

,

$$2 < 4$$

-1

$$-2 > -4.$$

2.

.

j (. .

,

,

)

:

$$x_j = x_j^+ - x_j^-, \quad x_j^+, x_j^- \geq 0.$$

,

$$x_j = -5$$

$$x_j^+ = 0$$

$$x_j^- = 5.$$

$$x_j = +5,$$

$$x_j^+ = 5 \quad x_j^- = 0.$$

$$x_j^+ \quad x_j^-$$

.

.

$$x_j^+ \quad x_j^-$$

.

3.

.

$$f(x_1, x_2, \dots, x_n)$$

$$-f(x_1, x_2, \dots, x_n),$$

$$1, 2, \dots, n.$$

3.1

.

$$z = 2x_1 + 3x_2 + 5x_3$$

$$x_1 + x_2 - x_3 \geq -5,$$

$$-6x_1 + 7x_2 - 9x_3 \leq 4,$$

$$x_1 + x_2 + 4x_3 = 10,$$

$$x_1, x_2 \geq 0,$$

3 —

.

1.

$$\left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 1,$$

:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

"≤"

"≥",

$$\left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

2.

$$\left(\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} \right) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + x_4 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + x_5 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

3.

,

4.

$$x_3 = x_3^+ - x_3^-, \quad x_3^+, x_3^- \geq 0,$$

$$z = 2x_1 + 3x_2 + 5x_3^+ - 5x_3^-$$

:

$$-x_1 - x_2 + x_3^+ - x_3^- + x_4 = 5,$$

$$-6x_1 + 7x_2 - 9x_3^+ + 9x_3^- + x_5 = 4,$$

$$x_1 + x_2 + 4x_3^+ - 4x_3^- = 10,$$

$$x_1, x_2, x_3^+, x_3^-, x_4, x_5 \geq 0.$$

3.1.2.

 m n

,

,

 $(m < n).$: 1) $n - m$

,

; 2)

 m

,

 m

.

,

 m

,

 $n - m$

—

.

.

,

.

—

.

,

,

m n

$$\binom{n}{m} = \frac{n!}{m!(n-m)!},$$

$$\binom{n}{m}$$

$$C_m^n.$$

3.2

 $(m = 2, n = 5):$

$$\begin{aligned}x_1 + x_2 + 4x_3 + 2x_4 + 3x_5 &= 8, \\4x_1 + 2x_2 + 2x_3 + x_4 + 6x_5 &= 4.\end{aligned}$$

$$\frac{5!}{3!2!} = 10.$$

 $(\quad m),$ $(\quad n - m).$

1.

 $(\quad) : x_2, x_4, x_5.$

$$\begin{aligned}: x_1 + 4x_3 &= 8, \\4x_1 + 2x_3 &= 4.\end{aligned}$$

$$: x_1 = 0, x_3 = 2.$$

$$\text{-----} : \quad , \quad x_1, x_3 \geq 0.$$

2.

 $(\quad) : x_3, x_4, x_5.$

$$\begin{aligned}: x_1 + x_2 &= 8, \\4x_1 + 2x_2 &= 4.\end{aligned}$$

$$: x_1 = -6, x_3 = 14.$$

$$\text{-----} : \quad , \quad x_1 < 0.$$

3.

 $(\quad) : x_1, x_2, x_5.$

$$\begin{aligned}: 4x_3 + 2x_4 &= 8, \\2x_3 + x_4 &= 4.\end{aligned}$$

:

 $(\quad 2,$

).

$$\frac{\quad}{4.}:$$

() : $x_1, \quad_3 \quad x_4$.

: $_2 + 3 \quad_5 = 8$,

$2x_2 + 6x_5 = 4$.

:

$$\frac{\quad}{3.1.3.}:$$

3.1.1 , x_j ,

:

$$x_j = x_j^+ - x_j^-, x_j^+, x_j^- \geq 0$$

. 3.1.2,

, , $x_j^+ \quad x_j^-$

,

x_j^+ , x_j^- .

, ,

$x_j^+ \quad x_j^-$, . . .

3.2. —

. 3.1,

,

()

. , , .

— ,

—

—

"

, , — ()

.

,

, . . .

, m
 . (, m)
 —
 (), — ()
).
 (—)
 ,
 ,
 ,
 ,
 ,
 " "

$$\max z = - \min (-z).$$

$z -$ — ,
 ,
 $z -$.

3.3

$$(\quad 2.1)$$

— .

:

$$z = 5x_1 + 4x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$$

$$6x_1 + 4x_2 + s_1 = 24,$$

$$1x_1 + 2x_2 + s_2 = 6,$$

$$-x_1 + x_2 + s_3 = 1,$$

$$x_2 + s_4 = 2,$$

$$x_1, x_2, s_1, s_2, s_3, s_4 \geq 0.$$

$$s_1, s_2, s_3, s_4 - \quad (\quad) \quad ,$$

	z	x_1	x_2	s_1	s_2	s_3	s_4	
z	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	24
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2

$z -$
 $s_1 -$
 $s_2 -$
 $s_3 -$
 $s_4 -$

;

" $z - 5x_1 - 4x_2 = 0$.

s_1, s_2, s_3, s_4

,

x_1^2 ,

" $s_1 = 24, s_2 = 6, s_3 = 1, s_4 = 2$.

.

?

x_1^2 ,

$z = 5x_1 + 4x_2 - 5 (x_1) - 4 (x_2)$.

x_1

x_1

(

,

$z -$

.

1

. 3.1 , , $(x_1 = 0, x_2 = 0)$,

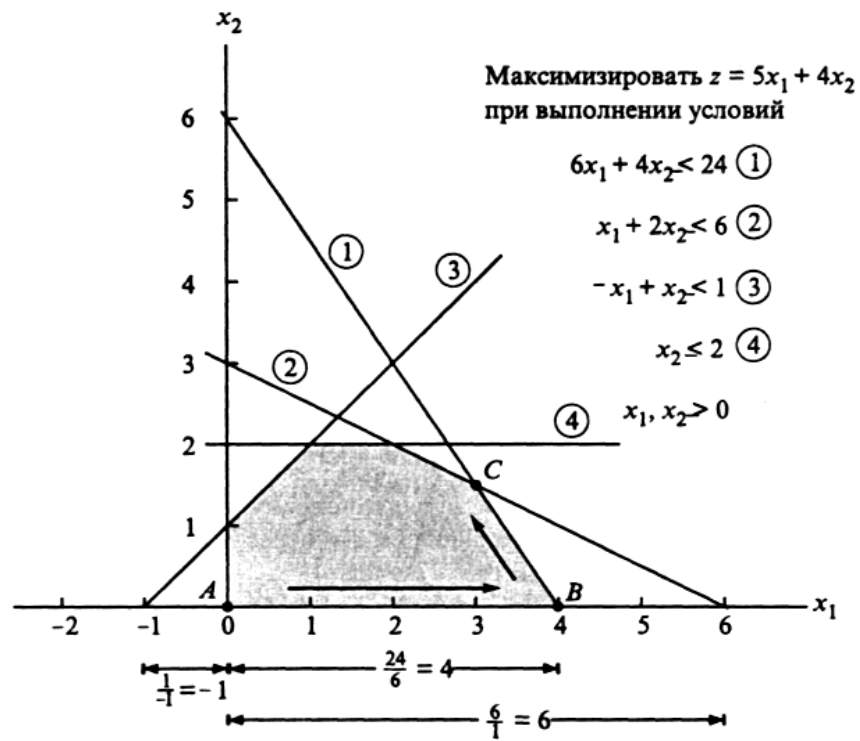
1 (

), 4,

$(x_1 = 4, x_2 = 0)$.

.

—



. 3.1

. 3.1

	x_1	
s_1	6	24
s_2	1	6
s_3	-1	1
s_4	0	2

()

$24 / 6 = 4$ ()

$6 / 1 = 6$

$1 / (-1) = -1$ ()

$2 / 0 = \infty$ ()

(

1),

x_1 .

"

"

,

,

.

,

,

, .

$x_1 = 4$ (. 3.1).

$20 (5 \times 4).$

s_1, s_2, s_3, s_4

,
 $1 \cdot ($,

4 .)

() $s_1,$

, s_1

4. s_1

$(x_1, s_2, s_3, s_4).$

—

. ,

,

—

.

.

() ,

() $z -$.

$z -$,

.

,

$z -$

().

.

,

,

,

.

,

.

(—) . ,

,

,

,

,

.

,

, .

	z	x_1	x_2	s_1	s_2	s_3	s_4	
z	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	24
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2

1. .
 $\frac{24}{6} = 4$ / .
 2. , $z -$.
 $= -$
 \times .
 $(s_1 -)$
 $(= 6)$,
 $()$ x_1
 s_1 . " " $x_1 = 4$.

	z	x_1	x_2	s_1	s_2	s_3	s_4	
z								
x_1	0	1	4/6	1/6	0	0	0	24/6 = 4
s_2								
s_3								
s_4								

.

1. $z -$.
 $z -$: $(1 \ -5 \ -4 \ 0 \ 0 \ 0 \ 0 \ | \ 0)$
 $-(-5) \times$: $(0 \ 5 \ 10/3 \ 5/6 \ 0 \ 0 \ 0 \ | \ 20)$

$$= z - : (1 \ 0 \ -2/3 \ 5/6 \ 0 \ 0 \ 0 \ | 20)$$

2. $s_2 -$

$$s_2 - : (0 \ 1 \ 2 \ 0 \ 1 \ 0 \ 0 \ | 6)$$

$$- (1) \times : (0 \ -1 \ -2/3 \ -1/6 \ 0 \ 0 \ 0 \ | -4)$$

$$= s_2 - : (0 \ 0 \ 4/3 \ 1/6 \ 1 \ 0 \ 0 \ | 2)$$

3. $s_3 -$

$$s_3 - : (0 \ -1 \ 1 \ 0 \ 0 \ 1 \ 0 \ | 1)$$

$$- (1) \times : (0 \ 1 \ 2/3 \ 1/6 \ 0 \ 0 \ 0 \ | 4)$$

$$= s_3 - : (0 \ 0 \ 5/3 \ 1/6 \ 0 \ 1 \ 0 \ | 5)$$

4. $s_4 -$. $s_4 -$ $s_4 -$,

.

,

— .

0. .

1.

. , .

2.

.

3. — .

1.

—

, ,

— ,

.

— .

— ,

(x_1, s_2, s_3, s_4), .

	z	x_1	x_2	s_1	s_2	s_3	s_4	
z	1	0	$-2/3$	$5/6$	0	0	0	20
x_1	0	1	$2/3$	$1/6$	0	0	0	4
s_2	0	0	$4/3$	$-1/6$	1	0	0	2
s_3	0	0	$5/3$	$1/6$	0	1	0	5
s_4	0	0	1	0	0	0	1	2

\vdots
 s_1
 $(s_1 = 4, s_2 = 2, s_3 = 5, s_4 = 2)$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

$z = \frac{2}{3}x_2 - \frac{5}{6}s_1 + 20.$
 $z = 20.$

3.3.

“ \leq ” (),
()
.
:
“ \geq ”?

.
.
.
:
—

3.3.1. —

i ,
 R_i ,
(
),
 M ,
 R_i ,
 $-MR_i$
 $+ MR_i$ —
 R_i .

3.4

$$\begin{aligned} z &= 4x_1 + 3x_2 \\ 3x_1 + 2x_2 &= 3, \\ 4x_1 + 3x_2 &\geq 6, \\ x_1 + 2x_2 &\leq 4, \\ x_1, x_2 &\geq 0. \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix}.$$

$$\begin{aligned} z &= 4x_1 + x_2 \\ 3x_1 + x_2 &= 3, \\ 4x_1 + 3x_2 - x_3 &= 6, \\ x_1 + 2x_2 + x_4 &= 4, \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$R_1 \quad R_2, \qquad MR_1 + R_2.$$

$$\begin{aligned} z &= 4x_1 + x_2 + MR_1 + R_2 \\ 3x_1 + x_2 + R_1 &= 3, \\ 4x_1 + 3x_2 - x_3 + R_2 &= 6, \\ x_1 + 2x_2 + x_4 &= 4, \\ x_1, x_2, x_3, x_4, R_1, R_2 &\geq 0. \end{aligned}$$

$$R_1, R_2 \quad 4$$

$$-$$

	x_1	x_2	x_3	R_1	R_2	x_4	
z	-4	-1	0	$-M$	$-M$	0	0
R_1	3	1	0	1	0	0	3
R_2	4	3	-1	0	1	0	6
x_4	1	2	0	0	0	1	4

$$-$$

$$z - \qquad, \qquad z, \\ R_1 = 3, R_2 = 6 \quad x_4 = 4,$$

$$3M + 6M + 0 = 9M, \qquad 0,$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 4 \\ 0 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

3.3,

$z -$
).
 R_1
 R_2
 M
 $z -$
. (
"
"
 $-$
,
.)
 $z -$
 $=$
 $z -$
 $+ M \times R_1 -$
 $+ M \times R_2 -$
.
 $z -$
: $(-4$
 -1
 0
 $-M$
 $-M$
 $|$
 $0)$
 $M \times R_1 -$
: $(3M$
 M
 0
 M
 0
 $|$
 $3M)$
 $M \times R_2 -$
: $(4M$
 $3M$
 $-M$
 0
 M
 $|$
 $6M)$
 $z -$
: $(-4 + 7M$
 $-1 + 4M$
 $-M$
 0
 0
 $|$
 $9M)$
 $-$
.

	x_1	x_2	x_3	R_1	R_2	x_4	
z	$-4 + 7M$	$-1 + 4M$	$-M$	0	0	0	$9M$
R_1	3	1	0	1	0	0	3
R_2	4	3	-1	0	1	0	6
x_4	1	2	0	0	0	1	4

,
 $z = 9M,$
 $R_1 = 3, R_2 = 6$
 $x_4 = 4.$
 $-$
.
,
 $z -$
.
 $-4 + 7M$
 $x_1,$
 R_1
,
 $-$
 $-$
.
,
 $z -$
,
 $M,$
 $z -$
 $- (-4 + 1)$
 $z -$
.
,

$R_1,$

.

	x_1	x_2	x_3	R_1	R_2	x_4	
x_1	0	$(1 + 5M)/3$	$-M$	$(4 - 7M)/3$	0	0	$4 + 2M$
R_1	1	$1/3$	0	$1/3$	0	0	1
R_2	0	$5/3$	-1	$-4/3$	1	0	2
x_4	0	$5/3$	0	$-1/3$	0	1	3

,

,

,

$x_2 \quad R_2$

.

,

.

$_1 = 2/5, \quad _2 = 9/5, \, x_3$

$= 1 \quad z = 17/5.$

$M-$

.

1.

M

—

.

($\quad,$ \quad),

—

,

,

"

"

,

.

.

2.

$M-$

,

$M \rightarrow \infty.$

,

,

.

M

"

",

,

,

.

,

,

.

3.3.2.

,

$M-$

.

$M-$

.

,

,

.

1.

2.

3.5

3.4

1.

$$r = R_1 + R_2$$

$$3x_1 + x_2 + R_1 = 3,$$

$$4x_1 + 3x_2 - x_3 + R_2 = 6,$$

$$x_1 + 2x_2 + x_4 = 4,$$

$$x_1, x_2, x_3, x_4, R_1, R_2 \geq 0.$$

	x_1	x_2	x_3	R_1	R_2	x_4	
r	0	0	0	-1	-1	0	0
R_1	3	1	0	1	0	0	3
R_2	4	3	-1	0	1	0	6
x_4	1	2	0	0	0	1	4

M-

$r -$

$$\begin{array}{lcl} r - & : & (\quad 0 \quad 0 \quad 0 \quad -1 \quad -1 \quad 0 \quad | 0) \\ + 1 \times R_1 - & : & (\quad 3 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad | 3) \\ + 1 \times R_2 - & : & (\quad 4 \quad 3 \quad -1 \quad 0 \quad 1 \quad 0 \quad | 6) \\ = & r - & : (\quad 7 \quad 4 \quad -1 \quad 0 \quad 0 \quad 0 \quad | 9) \end{array}$$

$$r - \quad \quad \quad r + 7x_1 + 4x_2 - x_3 + 0R_1 + 0R_2 + 0x_4 = 9$$

	x_1	x_2	x_3	R_1	R_2	x_4	
r	0	0	0	-1	-1	0	0
x_1	1	0	1/5	3/5	-1/5	0	3/5
x_2	0	1	-3/5	-4/5	3/5	0	6/5
x_4	0	0	1	1	-1	1	1

$$r = 0, \quad ,$$

$$x_1 = 3/5, \quad x_2 = 6/5 \quad x_4 = 1.$$

$$"$$

2.

$$z = 4x_1 + x_2$$

$$x_1 + \frac{1}{5}x_3 = \frac{3}{5},$$

$$x_2 - \frac{3}{5}x_3 = \frac{6}{5},$$

$$x_3 + 2x_4 = 1,$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

	x_1	x_2	x_3	x_4	
z	-4	-1	0	0	0
x_1	1	0	1/5	0	3/5
x_2	0	1	-3/5	0	6/5
x_4	0	0	1	1	1

$$z - \quad ,$$

$$z - \quad : \quad (\quad -4 \quad -1 \quad 0 \quad 0 \mid \quad 0)$$

$$+ 4 \times x_1 - \quad : \quad (\quad 4 \quad 0 \quad 4/5 \quad 0 \mid 12/5)$$

$$+ 1 \times x_2 - \quad : \quad (\quad 0 \quad 1 \quad -3/5 \quad 0 \mid 6/5)$$

$$= \quad z - \quad : \quad (\quad 0 \quad 0 \quad 1/5 \quad 0 \mid 18/5)$$

4.

4.1.

. — ,
 .
 .
 ,
 (,
 (, . . .)
 (,
 . ,
 .
 . ,
 .

$$z = \sum_{j=1}^n c_j x_j$$

$$\sum_{j=1}^n a_{ij} x_j = b_j, i = 1, 2, \dots, m,$$

$$x_j \geq 0, j = 1, 2, \dots, n.$$

n x_j .

1. . (
2.).
3. .

.
 — . ,
 — ,
 . ,
 ,
 —

1. m

2. n

3. $-$

4.

	x_1	x_2	\dots	x_j	\dots	x_n	
	c_1	c_2	\dots	c_j	\dots	c_n	
y_1	a_{11}	a_{12}	\dots	a_{1j}	\dots	a_{1n}	b_1
y_2	a_{21}	a_{22}	\dots	a_{2j}	\dots	a_{2n}	b_2
\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot
\cdot	\cdot	\cdot	\dots	\cdot	\dots	\cdot	\cdot
\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot	\cdot
y_m	a_{m1}	a_{m2}	\dots	a_{mj}	\dots	a_{mn}	b_m
			$j -$				

		\geq	
		\leq	

.

4.1

$z = 5x_1 + 6x_2$ $x_1 + 2x_2 = 5,$ $-x_1 + 5x_2 \geq 8,$ $4x_1 + 7x_2 \leq 8,$ $x_1 -$ $x_2 \geq 0$	$x_1 = x_1^+ - x_1^-$ $:$ $z = 5x_1^+ - 5x_1^- + 6x_2$ $x_1^+ - x_1^- + 2x_2 = 5,$ $-x_1^+ + x_1^- + 5x_2 - x_3 = 3,$ $4x_1^+ - 4x_1^- + 7x_2 + x_4 = 8,$ $x_1^+, x_1^-, x_2 \geq 0$	y_1 y_2 y_3

$$w = 5y_1 + 3y_2 + 8 \quad_3$$

$$\left. \begin{array}{l} y_1 - y_2 + 4y_3 \geq 5 \\ -y_1 + y_2 - 4y_3 \geq -5 \end{array} \right\} \rightarrow y_1 - y_2 + 4y_3 = 5,$$

$$2 \quad_1 + 5y_2 + 7 \quad_3 \geq 6,$$

$$-y_2 \geq 0 \rightarrow y_2 \leq 0,$$

$$y_3 \geq 0,$$

$$1 -$$

$$y_2, y_3 - \quad_1 - \quad_3, \quad_2 \quad_3).$$

.

:

， ，

．

4.2.

，
(

) — ，

．

．

1. —

．

$$\left(\begin{array}{c} \\ z- \\ j- \end{array}\right)=\left(\begin{array}{c} j- \\ \\ \end{array}\right).$$

．

— ，

．

(

),

，

100

500

，

，

100

．

4.2

4.1.

$z=5x_1+12x_2+4x_3$ $x_1+2x_2+x_3\leq 10,$ $2x_1-x_2+3x_3=8,$ $x_1,x_2,x_3\geq 0$	$w=10y_1+8y_2$ $y_1+2y_2\geq 5,$ $2y_1-y_2\geq 12,$ $y_1+3y_3\geq 4,$ $y_1\geq 0,y_2\geq 0$

.

		x_1	x_2	x_3	x_4	R	
0	z	$-5 - 2M$	$-12 + M$	$-4 - 3M$	0	0	$-8M$
	x_4	1	2	1	1	0	10
	R	2	-1	3	0	1	8
1	z	$-7/3$	$-40/3$	0	0	$4/3 + M$	$32/3$
	x_4	$1/3$	$7/3$	0	1	$-1/3$	$22/3$
	x_3	$2/3$	$-1/3$	1	0	$1/3$	$8/3$
2	z	$-3/7$	0	0	$40/7$	$-4/3 + M$	$368/7$
	x_2	$1/7$	1	0	$3/7$	$-1/7$	$22/7$
	x_3	$5/7$	0	1	$1/7$	$2/7$	$26/7$
3	z	0	0	$3/5$	$29/5$	$-2/5 + M$	$274/5$
	x_2	0	1	$-1/5$	$2/5$	$-1/5$	$12/5$
	x_1	1	0	$7/5$	$1/5$	$2/5$	$26/5$

—

1

 $x_4 \quad R$

,

.

	x_4	R
$z - 3 -$ $y_1 \geq 0$ $y_1 - 0 = 29/5$ 1	$29/5$ $y_1 \geq 0$ $y_1 - 0 = 29/5$	$-2/5 +$ $y_2 \geq -M$ $2 - (-) = -2/5$ $+ M$

$$_1 = 29/5 \quad _2 = -2/5.$$

,

.

,

1,

—

: $x_1 = 26/5$, $x_2 = 12/5$ $x_3 = 0$. (

,

.)

1

,

.

,

$$\left(\quad , \quad x_1, \quad _2 \quad _3 \right.$$

)

,

.

,

1

,

$$x_1 \quad x_3:$$

$$y_1 + 2 \quad _2 - 5 = 0,$$

$$y_1 + 3 \quad _2 - 4 = 3/5.$$

$$_1 = 29/5 \quad _2 = -2/5.$$

,

$$_4 \quad R,$$

.

(

,

,

$$x_1, x_2, x_3, x_4, R,$$

.)

,

1

.

2.

$$\left(\quad \right) \leq \left(\quad \right)$$

.

4.3

4.2

(

),

$$_1 = 0, \quad _2 = 0, \quad _3 = 8/3 \quad y_1 = 6, y_2 = 0.$$

$$z = 32/3 \quad w = 60.$$

$$_1 = 26/5, \quad _2 = 12/5, \quad _3 = 0 \quad _1 = 29/5, \quad _2 = -2/5 \quad z = w = 54,8.$$

,

2.

2

,

.

,

,
 —
 .
 " " ,
 .
4.3. —
 —
 , , .
 (, " ")
 . ,
 , ()
 — , 3,
 ,
 .
 — — (. .
)
 — ,
 .
 ,
 ,
 ,
 ,
 ,
 ,
 ,
 ,
 :

$$\min_{x_j} \left\{ \left| \frac{z_j - c_j}{\alpha_{rj}} \right|, \alpha_{rj} < 0 \right\},$$

$$\alpha_{rj} -$$
 ,
 (x_j .
 , .

, — .3.

4.4

.

$$\begin{aligned} z &= 3x_1 + 2x_2 \\ 3x_1 + x_2 &\geq 3, \\ 4x_1 + 3x_2 &\geq 6, \\ x_1 + x_2 &\leq 3, \\ x_1, x_2 &\geq 0. \end{aligned}$$

— .

	x_1	x_2	x_3	x_4	x_5	
z	-3	-2	0	0	0	0
x_3	-3	-1	1	0	0	-3
x_4	-4	-3	0	1	0	-6
x_5	1	1	0	0	1	3

, x_3 x_4

, x_5 — .

, -1 ;

, $(x_3 = -3, x_4 = -6,$

$x_5 = 3)$.

— . $z_j - c_j \leq 0$ ($j = 1, \dots, 5,$

).

,

— , —

.

$x_4 = -6$

.

.

	x_1	x_2	x_3	x_4	x_5
$z - (z_j - c_j)$	-3	-2	0	0	0
$x_4 - \alpha_{4j}$	-4	-3	0	1	0
$\left \frac{z_j - c_j}{\alpha_{4j}} \right $	3/4	2/3	—	—	—

2. , j , α_{4j} , x_3 , 4 5 .

↓

	x_1	x_2	x_3	x_4	x_5	
z	-1/3	0	0	-2/3	0	4
← x_3	-5/3	0	1	-1/3	0	-1
x_2	4/3	1	0	-1/3	0	2
x_5	-1/3	0		1/3	1	1
	1/5	—	—	2	—	

3 x_i . — .

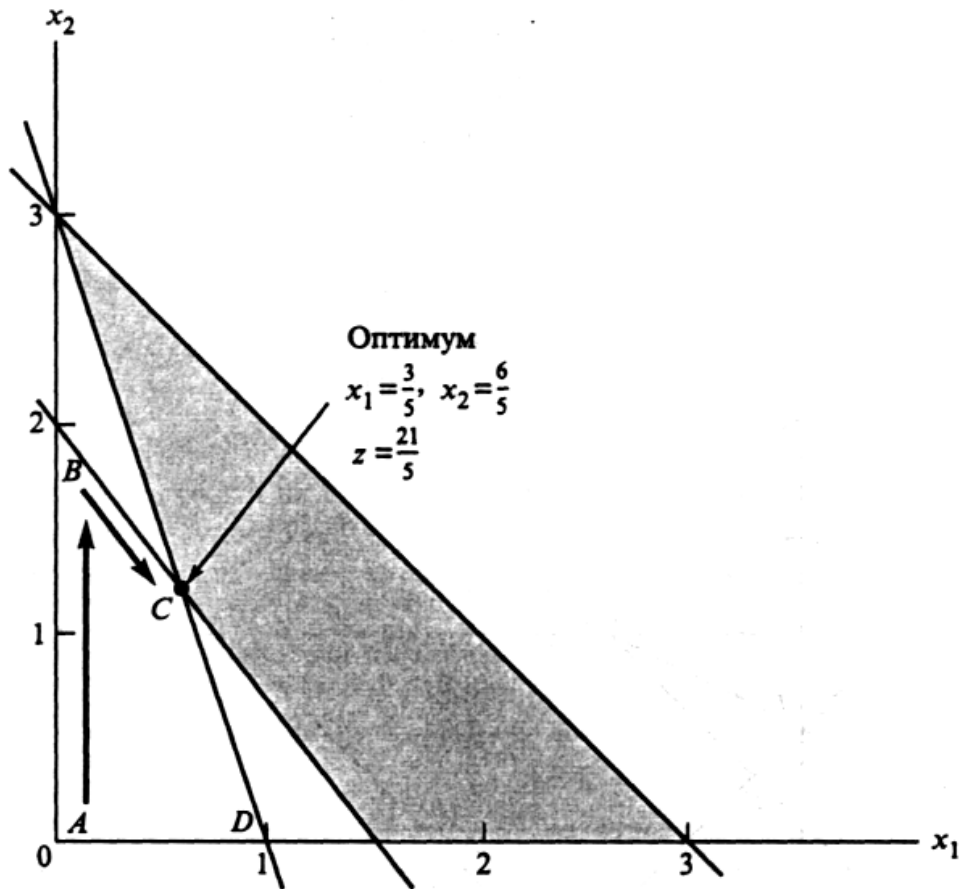
	x_1	x_2	x_3	x_4	x_5	
z	0	0	-1/5	-3/5	0	21/5
x_1	1	0	-3/5	1/5	0	3/5
x_2	0	1	4/5	-3/5	0	6/5
x_5	0	0	-1/5	2/5	1	6/5

), ($x_1 = 3/5$, $x_2 = 6/5$ $z = 21/5$.

. 4.1

4.4.

(
 "),
 , " ,
 ")
 ,



. 4.1

5.

5.1.

() —

()

—

(

,

)

(

,

).

—

.

,

,

(

),

,

().

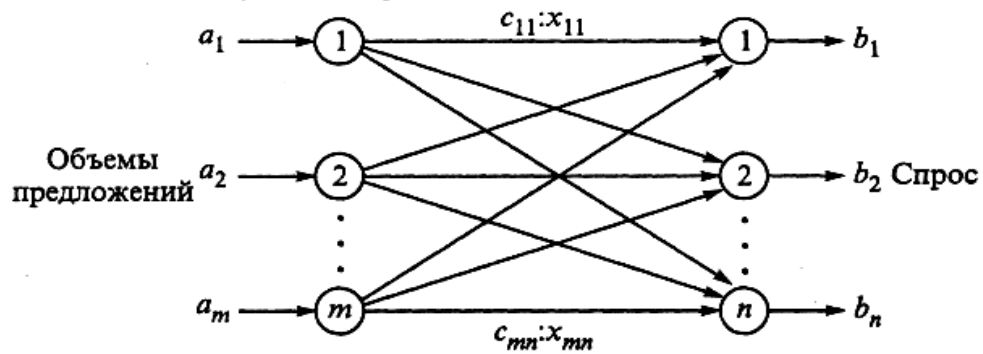
,

—

,

Пункты отправления

Пункты назначения



. 5.1

. 5.1

m

n

,

.

,

,

(i, j) ,

i

j ,

: 1)

ij

i

j ;

i

2)

a_i , а

$j — b_j$.

x_{ij} ,

\cdot
 ij $i-$ $j-$

\cdot

\cdot
1. ,

\cdot
2. —

\cdot
 \cdot ,
 \cdot ;

3. —
 \cdot \cdot

5.2.1.

m n
 $m + n$,
 \cdot
 (=),

\cdot ,
 $m + n - 1$,
 $m + n - 1$,
5.1 $3 + 4 - 1 = 6$

\cdot

(

—).

1. — \cdot
2. \cdot
3. \cdot

" " ,

11.

1.

11

2.

3.

5.2

5.1,

	1	2	3	4	Предложение
1	10 (5) → (10)	2	20	11	15
2	12	↓ 7 (5) → (15) → (5)	9	20	25
3	4	14	16	↓ 18 (10)	10
Спрос	5	15	15	15	

$$\begin{aligned}
 x_{11} &= 15, x_{12} = 10, \\
 x_{22} &= 5, x_{23} = 10, x_{24} = 5, \\
 x_{34} &= 10.
 \end{aligned}$$

$$z = 5 \times 10 + 10 \times 2 + 5 \times 7 + 15 \times 9 + 5 \times 20 + 10 \times 18 = 520.$$

— , , . , . , . (, .) , . , (, —). , . , .

5.3

5.1.

1. (1, 2) , 2. x_{12} , 15.
2. (3, 1). 31 5
3. 15, 14 — 0; $x_{34} = 5$ $x_{24} = 10$.

	1	2	3	4	Предложение
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
Спрос	5	15	15	15	

, (

6):

$$x_{12} = 15, x_{14} = 0,$$

$$x_{23} = 15, x_{24} = 10,$$

$$x_{31} = 5, x_{34} = 5.$$

$$z = 15 \times 2 + 0 \times 11 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = 475.$$

,

,

,

—

(

5.2).

.

.

.

1.

(),

(),

().

2.

.

,

.

,

,

,

.

.

,

,

.

,

,

()

,

().

3.)

,

.

)

()

(),

()

,

.

)

,

,

.

)

1.

5.4

5.1.

.

	1	2	3	4		
1	10	2	20	11	15	$10 - 2 = 8$
2	12	7	9	20	25	$9 - 7 = 2$
3	4	14	16	18	10	$14 - 4 = 10$
	5	15	15	15		
	$10 - 4 = 6 \quad 7 - 2 = 5 \quad 16 - 9 = 7 \quad 18 - 11 = 7$					

, 10
(3, 1),

 x_{31}

5.

,

.

.

	1	2	3	4		
1	10	2	20	11	15	$11 - 2 = 9$
		15				
2	12	7	9	20	25	$9 - 7 = 2$
3	4	14	16	18	5	$16 - 14 = 2$
	5					
	0	15	15	15		
	$\text{-----} \quad 5 \quad 7 \quad 7$					

, 9.

15

 x_{12} ,

.

,

.

,
 ,
 ,
 .
 ,
 ,
 (20 − 9 = 11),
 x₂₃
15.
 ,
 10
 .
 15
 .

x₁₄ = 0, x₃₄ = 5
 24 = 10.

z = 15 × 2 + 0 × 11 + 15 × 9 + 10 × 20 + 5 × 4 + 5 × 18 = 475.

5.2.2.

(
 . 5.2.1)
 ,

1.

,
 .
 ,

2.

.
 ,
 −
 , −
 .
 (. .)
 ,
 z−
 ,
 .
 i
 j

·

Θ

· ,

· (,

$(i, j))$.

(,

· ,

Θ .

·

Θ .

·

().

·

·

·

5.5

5.1,

5.2.

	1	2	3	4	
1	10 5	2 10	20	11	15
2	12	7 5	9 15	20 5	25
3	4	14	16	18 10	10
	5	15	15	15	

7

() 6 ,

·

(

$u_1 = 0$)

·

x_{11}	$u_1 + v_1 = 10$	$u_1 = 0 \rightarrow v_1 = 10$
x_{12}	$u_1 + v_2 = 2$	$u_1 = 0 \rightarrow v_2 = 2$
x_{22}	$u_2 + v_2 = 7$	$v_2 = 2 \rightarrow u_2 = 5$
x_{23}	$u_2 + v_3 = 9$	$u_2 = 5 \rightarrow v_3 = 4$
x_{24}	$u_2 + v_4 = 20$	$u_2 = 5 \rightarrow v_4 = 15$
x_{34}	$u_3 + v_4 = 18$	$v_4 = 15 \rightarrow u_3 = 3$

,

$$u_1 = 0, u_2 = 5, u_3 = 3,$$

$$v_1 = 10, v_2 = 2, v_3 = 4, v_4 = 15.$$

$$u_i + v_j$$

— ij .

.

	$u_i + v_j - c_{ij}$
x_{13}	$u_1 + v_3 - c_{13} = 0 + 4 - 20 = -16$
x_{14}	$u_1 + v_4 - c_{14} = 0 + 15 - 11 = 4$
x_{21}	$v_2 + u_1 - c_{21} = 5 + 10 - 12 = 3$
x_{31}	$u_3 + v_1 - c_{31} = 3 + 10 - 4 = 9$
x_{32}	$u_3 + v_2 - c_{32} = 3 + 2 - 14 = -9$
x_{33}	$v_3 + u_4 - c_{34} = 3 + 4 - 16 = -9$

.

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	
$u_1 = 0$	10 5	2 10	20 -16	11 4	15
$u_2 = 5$	12 3	7 5	9 15	20 5	25
$u_3 = 3$	4 9	14 -9	16 -9	18 10	10
	5	15	15	15	

z-

31.

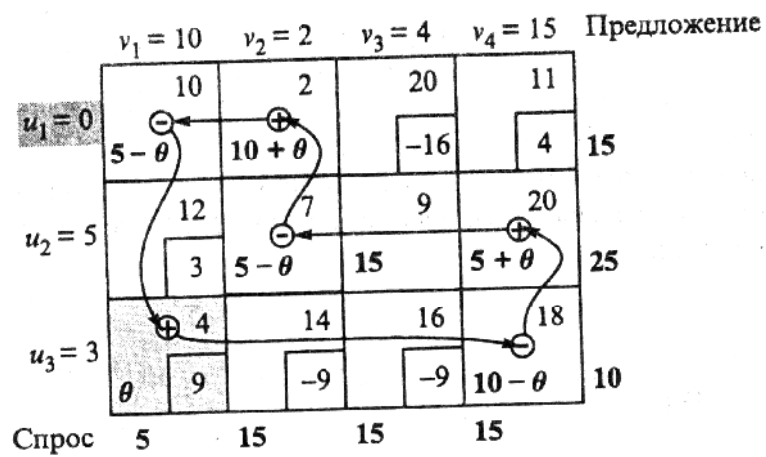
	x_{11}	x_{12}	x_{13}	x_{14}	x_{21}	x_{22}	x_{23}	x_{24}	x_{31}	x_{32}	x_{33}	x_{34}
z	0	0	-16	4	3	0	0	0	9	-9	-9	0

 Θ

(3, 1)

(. . $x_{31} = \Theta$).

(3, 1).

 x_{31} . Θ .

,

:

$$x_{11} = 5 - \Theta \geq 0,$$

$$x_{22} = 5 - \Theta \geq 0,$$

$$x_{34} = 10 - \Theta \geq 0.$$

,

,

 Θ ,

5,

 x_{11} x_{22}

.

,

 x_{11} , x_{22} . x_{11} .($x_{31} = 5$)

,

,

(3, 1)

$$\begin{aligned} &9 \qquad (u_3 + v_1 - c_{31} = 9), \\ 9 \times 5 = 45 \qquad , \end{aligned}$$

$$520 - 45 = 475.$$

,

.

,

,

.

1	2	5
1	4	10
2	2	10
2	3	15
3	1	5
3	4	5
435.		

5.2.3. —

—

(. 4.2).

,

.

$$z = \sum_{i=1}^m a_i u_i + \sum_{j=1}^n b_j v_j$$

$$u_i + v_j \leq c_{ij} \qquad i \quad j,$$

$$u_i \quad v_j \text{ — } ,$$

$$i \text{ — } (\quad) \qquad i,$$

$$b_j \text{ — } (\quad) \qquad j,$$

$$ij \text{ — } \qquad i$$

$$j,$$

$$u_i \text{ — } ,$$

$$i,$$

$$v_j \text{ — } ,$$

$$j.$$

(4.2) ,

ij

, . . . $u_i + v_j - ij.$,

,

. ,

$u_i + v_j = ij.$ $m + n - 1$

(-

, $u_1 = 0$), u_i

$v_j.$,

,

$u_i + v_j - ij.$

(, $u_i = 0$) ,

, (())

.

5.3.

"

" —

.

;

,

.

,

,

,

.

—

()

.

n

n

.

ij

i

j

$(i, j = 1, 2, ..., n).$,

,

,

.

.
 ,
 ()
 ,
 1 2
 ,
 .
 ()
 4. 1 2
 ,
)
 ,
)
 ,
)
 ,
 4, .
 3
 5.6
 , 4 4 .
 .

	1	2	3	4
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

$$\begin{matrix} & 1 & 2 \\ p_1 = 1, & p_2 = 7, & p_3 = 4, & p_4 = 5, & q_1 = 0, & q_2 = 0, & q_3 = 3 & q_4 = 0) \end{matrix}$$

.

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	1	4	3
4	3	2	0	0

· ,

1,

· ,

·

4,

·

	1	2	3	4
1	0	3	2	2
2	2	0	0	2
3	0	<u>1</u>	4	3
4	3	2	0	0

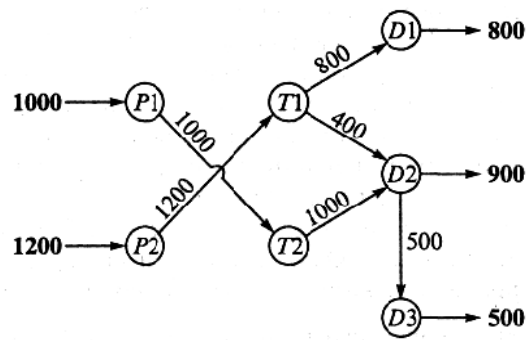
() 1.

, ·

, ·

	1	2	3	4
1	<u>0</u>	2	1	1
2	3	0	<u>0</u>	2
3	0	<u>0</u>	3	2
4	4	2	0	<u>0</u>

	$T1$	$T2$	$D1$	$D2$	$D3$	
$P1$	3	4	M	M	M	1000
$P2$	2	5	M	M	M	1200
$T1$	0	7	8	6	M	B
$T2$	M	0	M	4	9	B
$D1$	M	M	0	5	M	B
$D2$	M	M	M	0	3	B
	B	B	$800 + B$	$900 + B$	500	



. 5.3

6.

()

,

()

,

,

.

.

,

.

6.1.

,

,

,

,

,

,

,

—

—

6.1 ()

	(./)			(.)
	1—	2—	3—	
1	5	1	8	20
2	4	7	10	40
3	3	9	2	20
4	7	4	1	15
5	8	6	10	30
(.)	25	25	25	

. ,

" — "

j .

$$x_j = \begin{cases} 1, & j \\ 0, & j \end{cases}.$$

$$z = 20 \text{ }_1 + 40 \text{ }_2 + 20 \text{ }_3 + 15 \text{ }_4 + 30 \text{ }_5$$

$$5x_1 + 4 \text{ }_2 + 3 \text{ }_3 + 7 \text{ }_4 + 8 \text{ }_5 \leq 25,$$

$$\text{ }_1 + 7 \text{ }_2 + 9 \text{ }_3 + 4 \text{ }_4 + 6x_5 \leq 25,$$

$$8x_1 + 10 \text{ }_2 + 2 \text{ }_3 + x_4 + 10x_5 \leq 25,$$

$$\text{ }_1, \text{ }_2, \text{ }_3, \text{ }_4, x_5 = 0 \quad 1.$$

$$\text{ }_1 = \text{ }_2 = \text{ }_3 = \text{ }_4 = 1,$$

$$\text{ }_5 = 0 \quad z = 95$$

" "

$$j = 0 \quad 1$$

$$0 \leq j \leq 1 \quad j.$$

$$\text{ }_1 = 0,5789, \text{ }_2 = \text{ }_3 = \text{ }_4 = 1,$$

$$\text{ }_5 = 0,7368 \quad z = 108,68 \text{ (} .).$$

$$\text{ }_1 = \text{ }_5 = 1.$$

6.2.

1. " "

y

$$0 \leq \leq 1$$

.

.

2.

.

3.

()

,

,

,

,

,

,

.

3.

,

1.

.

2.

.

,

,

,

.

6.2.1.

.

6.2

.

$$z = 5x_1 + 4x_2$$

$$x_1 + x_2 \leq 5,$$

$$10x_1 + 6x_2 \leq 45,$$

$$x_1, x_2 \geq 0$$

.

. 6.1

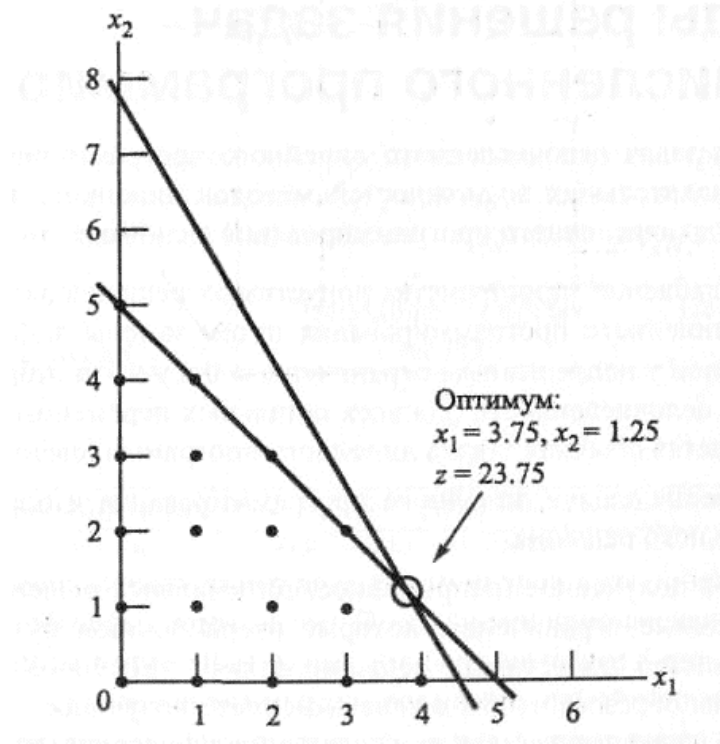
.

()

0)

.

$$x_1 = 3,75, x_2 = 1,25 \quad z = 23,75.$$



. 6.1

0

,

,

,

.

,

0

.

,

 $x_1 = 3.75,$

,

 $3 < x_1 < 4$

0

 x_1 ,

,

,

.

0

1

2,

:

 $x_1 =$ $0 + (x_1 \leq 3),$ $x_2 =$ $0 + (x_1 \geq 4).$

. 6.2

1

2.

.

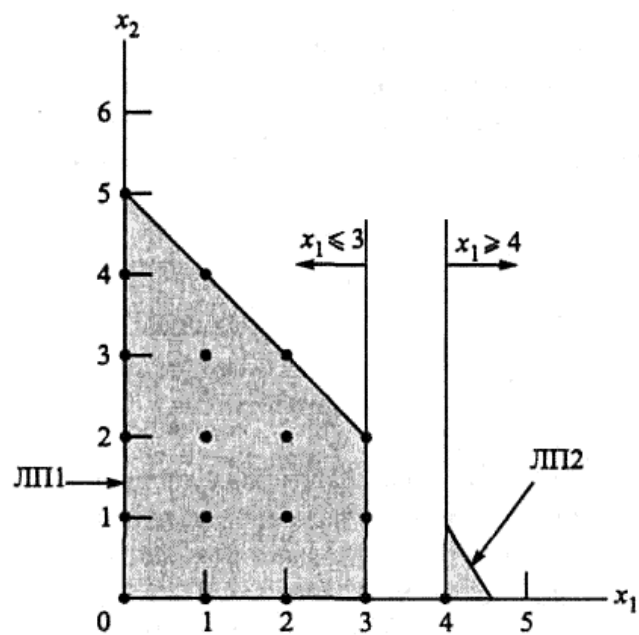
,

1

2 "

"

0.



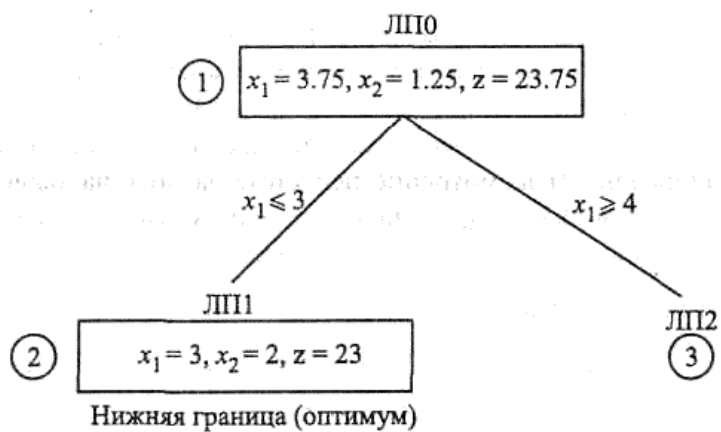
. 6.2

(, $3 < x_1 < 4$),

$$x_1 \leq 3 \quad x_1 \geq 4$$

1 2

. 6.3.



. 6.3

1, 2.

. 1 (

), $x_1 \leq 3$.

$$z = 5x_1 + 4x_2$$

$$x_1 + x_2 \leq 5,$$

$$10x_1 + 6x_2 \leq 45,$$

$$x_1 \leq 3,$$

$$x_1, x_2 \geq 0.$$

$$x_1 = 3, x_2 = 2 \quad z = 23.$$

1

x_1, x_2 , 1

, 1

,

.

, 1,

2 (

z).

$z = 23$ (

.

,

,

,

.

,

.

$z = 23$ 2

(

0 23,75

,

2 (

, 0),

.

2

.

,

1 2

(

1.

0

2

 x_1 ?

2.

2

 x_1 ?

2,

(. 6.4),

2

$$x_1 = 4, \quad x_2 = 0,83 \quad z = 23,33.$$



. 6.4

$$x_2 (= 0,83)$$

2

3

4,

$$x_2 \leq 0 \quad x_2 \geq 1$$

$$3 =$$

$$2 + (x_2 \leq 0) =$$

=

$$0 + (x_1 \geq 4) + (x_2 \leq 0),$$

$$4 =$$

$$2 + (x_2 \geq 1) =$$

$$= 0 + (\text{ }_1 \geq 4) + (\text{ }_2 \geq 1).$$

, — 1, 3 4. , , 4. , 3.

$$_1 = 4,5, x_2 = 0 \quad z = 22,5.$$

$$_1 = 4,5$$

$$_1 \leq 4$$

$$_1 \geq 5$$

$$5 \quad 6.$$

$$5 =$$

$$0 + (\text{ }_1 \geq 4) +$$

$$+ (\text{ }_2 \leq 0) + (\text{ }_1 \leq 4),$$

$$6 =$$

$$0 + (\text{ }_1 \geq 4) +$$

$$+ (\text{ }_2 \leq 0) + (\text{ }_1 \geq 5).$$

$$1, \quad 5 \quad 6.$$

$$6$$

$$5$$

$$(\text{ }_1 = 4, \text{ }_2 = 0, z = 20) \quad ,$$

$$(z = 20)$$

$$1,$$

$$(\text{ }_1 = 3, \text{ }_2 = 2, z = 23).$$

$$23.$$

$$, \quad _1 = 3, \text{ }_2 = 2 \quad z = 23.$$

$$2, \quad 4, \quad 3, \quad 6, \quad 5, \quad 1), \quad ; \quad . \quad 6.4 \quad (\quad 0,$$

.

:

?

,

$$. \quad 6.3,$$

$$1,$$

$$2$$

.

,

,

.

,

$$. \quad 6.4,$$

,

.

,

"

",

,
 ,
 .
 . , .
 z
 $-\infty$, $i = 0$.
 1. () $i-$
 i . i
 , .
 1. i
 .
 2. i ,
 .
 3. i .
 .
 1. i ,
 , .
 , :
 ,
 , $i = i + 1$ 1.
 2. i , 2
 .
 2. () j ,
 j^* i
 .
 $[j^*] < j < [j^*] + 1$ ($[j^*] -$
 $j^*)$,
 $j \leq [j^*] \quad j \geq [j^*] + 1$.
 $i = i + 1$ 1.
 .
 ($z = +\infty$).
 — (

).

,

.

,

.

6.2.2.

,

$$(\dots 0 \leq x \leq \dots),$$

$$= 2^0 \dots_0 + 2^1 \dots_1 + 2^2 \dots_2 + \dots + 2^k \dots,$$

k —

,

$$2^{k+1} - 1 \geq u,$$

$0, \dots, k$ —

.

,

$$(\dots)$$

,

.

,

.

,

,

1965 .,

.

,

;

.

.

,

.

,

.

1.

,

.

2.

" \leq ",

.

.

,

6.3

,

$$\begin{aligned}
 z &= 3 \;_1 - 5 \;_2 \\
 x_1 + \;_2 &= 5, \\
 4 \;_1 + 6 \;_2 &\geq 4, \\
 x_1, \;_2 &= 0 \qquad 1.
 \end{aligned}$$

- "≤".
1.
 $z \leq -1,$
 $w = -3 \;_1 + 5 \;_2,$
2.
 $\qquad \qquad \qquad - \qquad \qquad \qquad -$
- "≤",
 $x_1 + \;_2 \leq 5 \qquad - \;_1 - \;_2 \leq -5.$
3.
 $\qquad \qquad \qquad -1 \qquad \qquad \qquad -4x_1 - 6 \;_2 \leq -4.$
 $s_1, \;s_2 \qquad \qquad s_3,$

$$\begin{aligned}
 w &= -3 \;_1 + 5 \;_2 \\
 x_1 + \;_2 + s_1 &= 5, \\
 - \;_1 - \;_2 + s_2 &= -5, \\
 -4x_1 - 6 \;_2 + s_3 &= -4, \\
 x_1, \;_2 &= 0 \qquad 1, \\
 s_1, s_2, s_3 &\geq 0.
 \end{aligned}$$

$$\begin{aligned}
 & \qquad \qquad \qquad , \\
 & \qquad \qquad \qquad j = 1 - \;_j \qquad \qquad \qquad j \\
 & \qquad \qquad \qquad . \\
 & \qquad \qquad \qquad \;_1 = 1 - \;_1, \\
 & \qquad \qquad \qquad . \\
 & \qquad \qquad \qquad \;_1 \qquad \;_2. \\
 & \qquad \qquad \qquad , \\
 & \qquad \qquad \qquad j. \\
 & \qquad \qquad \qquad , \\
 & \qquad \qquad \qquad j = 0 \qquad \qquad j = 1, \qquad \qquad j \\
 & \qquad \qquad \qquad .
 \end{aligned}$$

,

, —

- 1.
- 2.
- 3.

6.4 ()

$$\begin{aligned}
 z &= 3x_1 + 2x_2 - 5y_3 - 2x_4 + 3x_5 \\
 x_1 + x_2 + y_3 + 2x_4 + x_5 &\leq 4, \\
 7x_1 + 3x_3 - 4x_4 + 3x_5 &\leq 8, \\
 11x_1 - 6x_2 + 3x_4 - 3x_5 &\geq 3, \\
 x_1, x_2, y_3, x_4, x_5 &= 0 \quad 1.
 \end{aligned}$$

- 1.
- 2.
- 3.
- 4.

$$-1.$$

$$-1.$$

$$s_1, s_2, s_3$$

$$x_1 = 1 - x_1, x_2 = 1 - x_2, x_5 = 1 - x_5, y_3 = x_3, y_4 = x_4.$$

$$z' = 3x_1 + 2x_2 + 5x_3 + 2x_4 + 3x_5 - 8.$$

$$-8$$

$$z' + 8 = z,$$

$$z = 3x_1 + 2x_2 + 5x_3 + 2x_4 + 3x_5$$

$$-x_1 - x_2 + x_3 + 2x_4 - x_5 + s_1 = 1,$$

$$-7x_1 + 3x_3 - 4x_4 - 3x_5 + s_2 = -2,$$

$$11x_1 - 6x_2 - 3x_4 - 3x_5 + s_3 = -1,$$

$x_1, x_2, x_3, x_4, x_5 = 0 \qquad 1.$

, , .

,

.

.

	x_1	x_2	x_3	x_4	x_5	s_1	s_2	s_3	
s_1	-1	-1	1	2	-1	1	0	0	1
s_2	-7	0	3	-4	-3	0	1	0	-2
s_3	11	-6	0	-3	-3	0	0	1	-1
	3	2	5	2	3				

,

.

,

:

$(s_1, s_2, s_3) = (1, -2, -1),$

$z = 0.$

,

,

,

,

.

$(\qquad \qquad \qquad),$

,

(

,

).

.

,

.

$(\qquad \qquad \qquad)$

.

$$91$$

$$\begin{array}{c} , \\ , \end{array} \qquad \qquad \qquad 3$$

$$\left(\begin{array}{c} \cdot \\ \cdot \end{array} \right) \qquad \qquad \qquad , \qquad \qquad \qquad 3 = 1,$$

$$s_2 \quad s_3.$$

$$\begin{array}{c} , \\ , \\ \cdot \end{array} \qquad \qquad \qquad ,$$

$$\cdot \qquad \qquad \qquad , \qquad \qquad \qquad x_1, \quad 2, \quad 4 \quad 5 \qquad \qquad \qquad .$$

$$\cdot$$

$$, \qquad \qquad \qquad , \qquad \qquad \qquad j$$

$$1,$$

$$I_j = \sum_i \min\{0,s_i-a_{ij}\},$$

$$s_i - \qquad \qquad \qquad , \quad ij -$$

$$j \quad i - \qquad \qquad \qquad .$$

$$I_j \qquad \qquad \qquad , \qquad \qquad \qquad ,$$

$$\begin{array}{c} j \quad 1. \\ \vdots \end{array}$$

$$I_j = \sum_i (\qquad \qquad \qquad s_i \qquad \qquad \qquad x_j = 1).$$

$$, \qquad \qquad \qquad 1 = 1, \qquad \qquad \qquad s_1 = 1 - (-1) = 2,$$

$$s_2 = -2 - (-7) = 5 \quad s_3 = -1 - 11 = -12. \qquad \qquad \qquad , I_1 = -12. \qquad \qquad \qquad ,$$

$$I_2 = -2, I_4 = -1 \quad I_5 = 0 \left(\qquad \qquad \qquad , \qquad \qquad \qquad 3 \right). \qquad \qquad \qquad I_5$$

$$, \qquad \qquad \qquad 5$$

$$\cdot \qquad \qquad \qquad \cdot \quad 6.5 \qquad \qquad \qquad ,$$

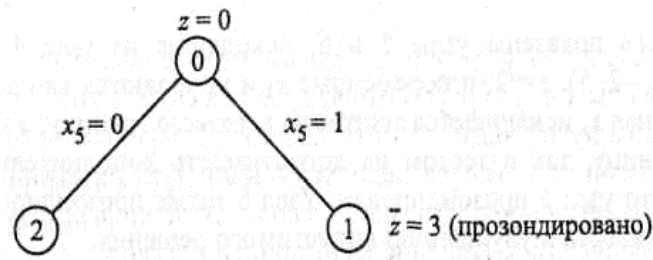
$$5 = 1 \quad 5 = 0, \qquad \qquad \qquad 1 \quad 2.$$

$$1$$

$$(s_1, s_2, s_3) = (2, 1, 2) \quad z = 3. \qquad \qquad \qquad , \quad 1 \qquad \qquad \qquad ,$$

$$\overline{z} = 3$$

$$\cdot$$



. 6.5

1,

2,

 $s_5 = 0$.

$$(s_1, s_2, s_3) = (1, -2, -1) \quad z = 0, \quad \dots$$

.

1, 2, 3 4

(, 0 2 , 2 , .)

5

0,

3

,

(

)

 s_2 s_3 .

,

$$x_3 = 1$$

,

5,

$$\bar{z} = 3.$$

1

,

3,

$$s_1 = 1$$

.

2 4

$$: I_2 = -2,$$

$$I_4 = -1.$$

,

2

4.

. 6.6

$$x_4 = 1 \quad x_1 = 0,$$

3 4

.

3 (

$$s_5 = 0$$

$$x_4 = 1)$$

$$(s_1, s_2, s_3) = (-1, 2, 2), z = 2,$$

.

1, 2

3.

,

1,

 z

$$(\bar{z} = 3).$$

,

,

3

.

4,

$$s_5 = s_4 = 0,$$

$$(s_1, s_2, s_3) = (1, -2, -1), z = 0.$$

1

3

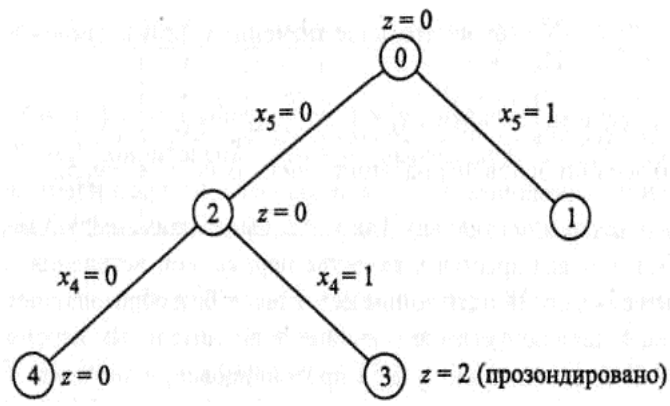
.

2

.

, 2

.



. 6.6

. 6.7

5 6,

4.

5

$$(s_1, s_2, s_3) = (2, -2, 5), z = 2,$$

1 3

1

3 —

,

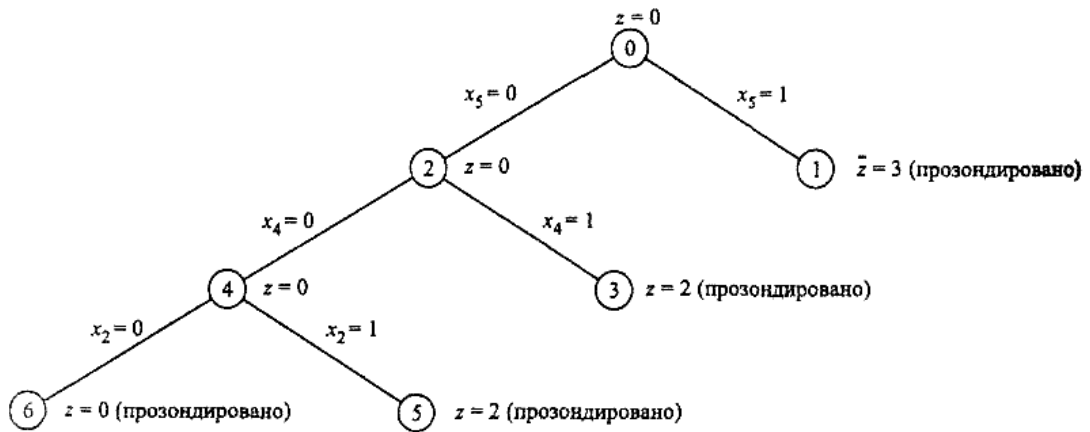
5

6

,

1, 3

.



. 6.7

(. 6.7)

,

1, . .

$$s_5 = 1, z = 3,$$

:

$$y_1 = y_2 = 1, y_3 = y_4 = y_5 = 0, w = 5.$$

. 6.7

,

,

,

.

,

1

$$(s_5 = 1),$$

$2^{5-1}=16$ (,
5 = 1). 3
 ,
 .
 ,
 6.4,

$$2^{5-2}=8$$

,
 , " "
 ,
 .
 , 6.4.
 .

6.2.3.

,
 " " ()
 .
 ,
 ()
).
 ,

6.5

.

$$\begin{aligned} z &= 7x_1 + 10x_2 \\ 12x_1 + 3x_2 &\leq 6, \\ 7x_1 + x_2 &\leq 35, \\ x_1, x_2 &\geq 0 \end{aligned}$$

()

,
 ,
 . 6.8 .

$$(x_1, x_2) = \left(4\frac{1}{2}, 3\frac{1}{2}\right) \quad z = 66\frac{1}{2}.$$

I,

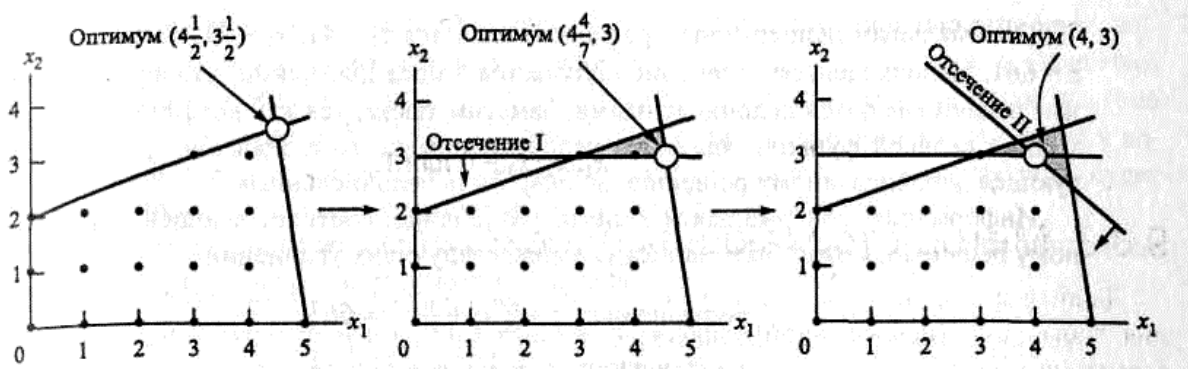
$$(x_1, x_2) = \left(4\frac{4}{7}, 3\right)$$

$$z = 62.$$

II,

I

$$(x_1, x_2) = (4, 3) \quad z = 58.$$



. 6.8

6.5

—

.

	x_1	x_2	x_3	x_4	
z	0	0	$63/22$	$31/22$	66,5
x_2	0	1	$7/22$	$1/22$	3,5
x_1	1	0	$-1/22$	$3/22$	4,5

$$x_1 = 4\frac{1}{2}, x_2 = 3\frac{1}{2}, x_3 = 0,$$

$$x_4 = 0 \quad z = 66\frac{1}{2}.$$

,

.

,

,

 z ,

,

.

,

—

,

,

:

$$z - \left(\frac{63}{22}x_3 + \frac{31}{22}x_4 \right) = 66\frac{1}{2},$$

$$x_2 - \left(\frac{7}{22}x_3 + \frac{1}{22}x_4 \right) = 3\frac{1}{2},$$

$$x_1 - \left(-\frac{1}{22}x_3 + \frac{3}{22}x_4 \right) = 4\frac{1}{2}.$$

$$z, \quad x_1, \quad x_2$$

—

,

. ()

 $z -$

$$z + \frac{63}{22}x_3 + \frac{31}{22}x_4 = 66\frac{1}{2} \quad \left(\quad z - \quad \right).$$

,

.

,

$$\frac{5}{2} = \left(2 + \frac{1}{2} \right),$$

$$-\frac{7}{3}=\left(-3+\frac{2}{3}\right).$$

$z-$

:

$$z+\left(2+\frac{19}{22}\right)x_3+\left(1+\frac{9}{22}\right)_4=\left(66+\frac{1}{2}\right).$$

,

$$z+2_3+_4-66=\frac{1}{2}-\frac{19}{22}x_3-\frac{9}{22}_4.$$

,

,

,

.

:

$$\frac{1}{2}-\frac{19}{22}x_3-\frac{9}{22}_4=\frac{1}{2}-\left(\frac{19}{22}x_3+\frac{9}{22}x_4\right).$$

$$_3,_4\geq 0,$$

.

$$\frac{1}{2}-\frac{19}{22}x_3-\frac{9}{22}_4,$$

,

1.

,

:

$$\frac{1}{2}-\frac{19}{22}x_3-\frac{9}{22}_4\leq 0.$$

,

$z-$

.

,

$$_1=4\frac{1}{2},_2=3\frac{1}{2},_3=0,$$

$$_4=0$$

,

$$_3=_4=0\left(\right.$$

$$\left.1\leq 0\right).$$

—

,

—

"

"

.

,

$1-$

x_2-

.

$1-$

.

$$x_1-\frac{1}{22}_3+\frac{3}{22}_4=4\frac{1}{2}\left(\right.1-\left.\right).$$

$$x_1+\left(-1+\frac{21}{22}\right)_3+\left(0+\frac{3}{22}\right)_4=\left(4+\frac{1}{2}\right).$$

$$-\frac{21}{22}x_3 - \frac{3}{22}x_4 + \frac{1}{2} \leq 0.$$

$$x_2 + \frac{7}{22}x_3 + \frac{1}{22}x_4 = 3\frac{1}{2} \quad (2^-)$$

$$x_2 + \left(0 + \frac{7}{22}\right)x_3 + \left(0 + \frac{1}{22}\right)x_4 = \left(3 + \frac{1}{2}\right).$$

,

$$-\frac{7}{22}x_3 - \frac{1}{22}x_4 + \frac{1}{2} \leq 0.$$

.

$$\left(\quad \right) \quad , \quad (2^-) \quad ,$$

.

$$-\frac{7}{22}x_3 - \frac{1}{22}x_4 + s_1 = -\frac{1}{2}, s_1 \geq 0 \quad (1).$$

—

.

	x_1	x_2	x_3	x_4	s_1	
z	0	0	63/22	31/22	0	66,5
x_2	0	1	7/22	1/22	0	3,5
x_1	1	0	-1/22	3/22	0	4,5
s_1	0	0	-7/22	-1/22	1	-0,5

,

.

—

,

—

.

	x_1	x_2	x_3	x_4	s_1	
z	0	0	0	1	9	62
x_2	0	1	0	0	1	3
x_1	1	0	0	1/7	-1/7	$4\frac{4}{7}$
x_3	0	0	1	1/7	-22/7	$1\frac{4}{7}$

— x_1 3

— x_1 — , . .

$$x_1 + \left(0 + \frac{1}{7}\right)x_4 + \left(-1 + \frac{6}{7}\right)s_1 = \left(4 + \frac{4}{7}\right).$$

$$-\frac{1}{7}x_4 - \frac{6}{7}s_1 + s_2 = -\frac{4}{7}, s_2 \geq 0 \quad (2).$$

2

— ,

.

	x_1	x_2	x_3	x_4	s_1	s_2	
z	0	0	0	1	9	0	62
x_2	0	1	0	0	1	0	3
x_1	1	0	0	1/7	-1/7	0	$4\frac{4}{7}$
x_3	0	0	1	1/7	-22/7	0	$1\frac{4}{7}$
s_2	0	0	0	-1/7	-6/7	1	-4/7

—

— .

	x_1	x_2	x_3	x_4	s_1	s_2	
z	0	0	0	0	3	7	58
x_2	0	1	0	0	1	0	3
x_1	1	0	0	0	-1	1	4
x_3	0	0	1	0	-4	1	1
x_4	0	0	0	1	6	-7	4

$$(x_1 = 4, x_2 = 3, z = 58),$$

— , . , .

— , .

— , .

,

,

.

,

.

.

$$x_1 + \frac{1}{3} x_2 \leq \frac{13}{2},$$
$$x_1, x_2 \geq 0.$$

$$s_1, \dots$$

$$x_1 + \frac{1}{3} x_2 + s_1 = \frac{13}{2}.$$

,

$$x_1, x_2 \geq 0, s_1 \geq 0.$$

,

$$x_1, x_2 \geq 0, s_1 \geq 0,$$

$$s_1$$

.

$$x_1, x_2 \geq 0, s_1 \geq 0,$$

.

$$x_1, x_2 \geq 0.$$

.

1.

.

$$6x_1 + 2x_2 \leq 39.$$

$$x_1, x_2 \geq 0.$$

.

,

.

2.

,

—

.

,

,

$$(x_1, x_2)$$

.

.

6.4.

,

,

.

, . .

,

, ,

,

()

,

.

.

,

1.

.

2.

,

,

.

3.

.

,

.

,

.

,

.

$$\begin{aligned} &+s_4^+-s_4^-=2, \\ &\quad ,\quad ,\quad ,\quad 0, \\ &s_i^+,s_i^-\quad 0,i=1,2,3,4. \\ &\quad s_i^+\quad s_i^-\quad , \end{aligned}$$

$$\begin{aligned} &s_i^+\quad s_i^-\quad , \\ &\quad .\quad , \end{aligned}$$

$$i-\quad " \quad " \quad s_i^+>0,$$

$$. \quad s_i^->0,$$

$$\begin{aligned} &.\quad , \\ &s_i^+\quad s_i^-\quad i-\quad , \quad . \end{aligned}$$

,

$$.\quad ,$$

.

$$" \quad ", \quad \text{---} \quad " \quad " .$$

$$s_1^+,s_2^+,s_3^+\quad s_4^-$$

,

.

,

$$(\quad):$$

$$\begin{aligned} G_1&=s_1^+; \\ G_2&=s_2^+; \\ G_3&=s_3^+; \\ G_4&=s_4^-. \end{aligned}$$

,

?

$$:1)$$

2)

7.3.

.

.

.

.

,

"

"

.

,

.

,

;

,

.

7.3.1.

,

,

.

$G_i, i = 1, 2, \dots,$.

.

$$z = w_1 G_1 + w_2 G_2 + \dots + w_n G_n.$$

$w_i (i = 1, 2, \dots, n)$ —

,

,

.

,

$$w_1 = 1$$

i

.

.

,

.

7.2

,

10

,

.

.

,

,

.

.

(

),

6– . 100 000 ,

?

()	4	8
(. .)	8	24
	1	2

1 2 ,

.

$G_1 = s_1^+ ($
 $).$

$G_2 = s_1^- ($
 $)$

$4x_1 + 8x_2 + s_1^+ - s_1^- = 45 ($
 $),$
 $8x_1 + 24x_2 + s_2^+ - s_2^- = 100 ($
 $),$
 $x_1 + 2x_2 = 10 ($
 $),$
 $x_1 = 6 ($
 $),$
 $x_1, x_2, s_1^+, s_1^-, s_2^+, s_2^- \geq 0.$

,
,

.

.

$z = 2G_1 + G_2 = 2s_1^+ + s_2^-.$

$z = 10, s_1^+ = 5,$

$x_2 = 2,5, s_1^+ = 5.$

, ,

, , ,

, $s_1^+ = 5,$
 5

,

, $s_2^- = 0.$

,

,

.

, $n_1 = 6$ $n_2 = 2$

$(4 \times 6 + 8 \times 2 = 40)$,

$(8 \times 6 + 24 \times 2 = 96\,000)$.).

,

" "

" "

7.3.2.

,

, . . .

$G_{I=1} ($),

...

$G_{n= n} ($).

$i \longrightarrow$, . . . s_i^+

s_i^- , $i \longrightarrow$. ,

7.1 $s_1^+ p_2 = s_2^-$.

,

,

,

.

,

,

$z(G_i) \longrightarrow$

G_i , $i = 1$

$G_j \ (j > i)$

$z(G_i)$.

" "

— ,

.

,

— G_K

$$G_{k+1} \cdot \left(x_j - z_j - j \right) = 0, \quad x_j,$$

$$,$$

$$,$$

$$,$$

$$.$$

$$,$$

$$-$$

$$,$$

$$.$$

$$,$$

$$.$$

$$0.$$

$$: G_1 = _1 > G_2 = _2 > \dots > G_n = _n. \quad i = 1.$$

$$\mathbf{i}.$$

$$i-$$

$$G_i.$$

$$i^*$$

$$i = n,$$

$$,$$

$$n-$$

$$i = i^*,$$

$$i$$

$$i = i + 1$$

$$i.$$

$$i = i^*$$

$$"$$

$$",$$

$$.$$

$$,$$

$$.$$

$$,$$

$$, \dots$$

$$.$$

$$.$$

$$(\quad i = i^*),$$

$$,$$

$$i^*$$

$$p_i,$$

$$. ($$

$$i = i^*,$$

$$i \quad i^*.)$$

,

,

.

7.3

7.2.

0. $G_1 > G_2$.

$$\begin{aligned} G_1: & \quad s_1^+ (\\ G_2: & \quad s_2^- (\end{aligned} \quad),$$

1.

$$G_1 = s_1^+$$

$$4x_1 + 8x_2 + s_1^+ - s_1^- = 45 ($$

$$8x_1 + 24x_2 + s_2^+ - s_2^- = 100 ($$

$$x_1 + 2x_2 = 10 ($$

$$x_1 = 6 ($$

$$x_1, x_2, s_1^+, s_1^-, s_2^+, s_2^- = 0.$$

$$= 5, \quad s_1^+ = 5, \quad s_2^- = 2,5, \quad s_1^- = 5,$$

$$5, \quad s_1^+ = 5.$$

$$s_1^+ = 5.$$

2.

$$G_2 = s_2^-$$

$$s_1^+ = 5.$$

$$s_2^- = 0.$$

$$s_2^- = 0$$

$$s_1^+ = 5$$

$$5$$

$$s_1^+$$

$$45 \quad 40.$$

$$G_2 = s_2^-$$

$$4x_1 + 8x_2 - s_1^- = 40,$$

$$8x_1 + 24x_2 + s_2^+ - s_2^- = 100,$$

$$x_1 + 2x_2 \leq 10,$$

$$x_1 \leq 6,$$

$$x_1, x_2 \geq 0.$$

$$x_1 = 0, \quad x_2 = 5 \quad P_1 = 40.$$

2.

$$4x_1 + 8x_2 \geq 40,$$

$$x_2 = 8x_1 + 24$$

$$x_1 + 2x_2 \leq 10,$$

$$x_1 \leq 6,$$

$$4x_1 + 8x_2 \geq 40,$$

$$x_1, x_2 \geq 0.$$

$$: \quad x_2 = 96\,000,$$

$$x_1 = 6 \quad x_2 = 2$$

$$(P_1 = 40).$$

7.3.

P_1 ,

2.

2

$$x_1 = 0,$$

$$x_2 = 5 \quad P_1 = 40$$

—

j ,

$$z_j - c_j \neq 0.$$

	x_1	x_2	s_1	s_2	
P_1	-4	-8	0	0	0
P_2	-8	-24	0	0	0
s_1	1	2	1	0	10
s_2	1	0	0	1	6
P_1	0	0	4	0	40
P_2	4	0	12	0	120
x_2	1/2	1	1/2	0	5
s_2	1	0	0	1	6

(

).

s_1 ,

$z_1 - c_2 = 4.$

$2-$

$-$

,

s_1 ,

,

$2.$

(

$x_1 = \quad_2 = 0$),

,

$P_1 = 0$

$P_1 = 40$,

.

.

s_1

$1,$

$z_j - c_j,$

$4 (> 0),$

$2.$

$-$

.

$1-$

,

$2.$

(

$_1 = 6,$

$_2 = 2)$

$_1 = 40$

$_2 = 96$

,

.

,

$—$

,

,

,

.

.

		x_1	x_2	s_1	s_2	
1	1					40
	P_2	4	0		0	120
	x_2	1/2	1		0	5
	s_2	1	0		1	6
2	1					40
	P_2	0	0		-4	96
	x_2	0	1		-1/2	2
	x_1	1	0		1	6

, . , , , .

1. X. . - .: « », 2007.
2. . . . / . . . , 2003.
3. . . . - .: , 2007.
4. . . , . . : . - .: ; - , 2010.
5. . . : , , . - .: , 2010.
6. . . : : , 2010.
7. . . : . . - .: , 2005.
8. . . : . . - .: , 2010.

1.	3
1.1.	3
1.2.	5
1.3.	7
1.4.	9
1.5.	13
2.	18
2.1.	18
2.2.	18
2.3.	21
2.4.	25
3.	—	28
3.1.	28
3.1.1.	28
3.1.2.	30
3.1.3.	32
3.2.	—	32
3.3.	40
3.3.1.	—	40
3.3.2.	43
4.	47
4.1.	47
4.2.	50
4.3.	—	53
5.	57
5.1.	57
5.2.	58
5.2.1.	59
5.2.2.	64
5.2.3.	—	69
5.3.	70
5.3.1.	71
5.3.2.	—	74

5.4.	75
6.	78
6.1.	78
6.2.	79
6.2.1.	80
6.2.2.	87
6.2.3.	94
6.4.	100
7.	102
7.1.	102
7.2.	102
7.3.	104
7.3.1.	105
7.3.2.	107
	114

1

. . .
. . .
. . .

020565 23.06.1997 .
60 84 1/16.

. . . – 7,0. . – . . – 6,8.
300 .

« »

, , 44

, , 44