Лабораторная работа №5

Исследование рекурсивной цепи второго порядка

1. Исследование частотных характеристик

$i := 0, 1 .. 2$

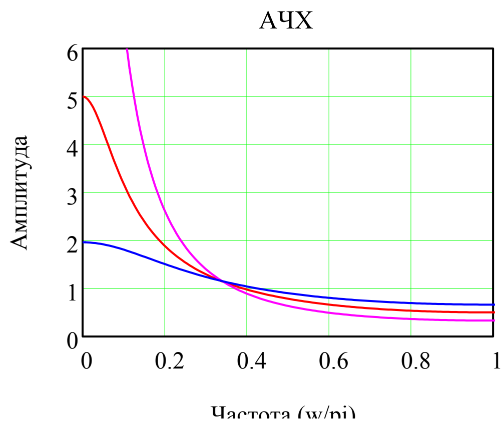
$$H(\omega, b1, b2) := \frac{1}{1 + b1 \cdot e^{-i\cdot\omega} + b2 \cdot e^{-2\cdot i\cdot\omega}}$$

$$\omega c(b1, b2) := \left| \begin{array}{l} arg \leftarrow \dfrac{-2\cdot b1\cdot(1+b2) + \mathrm{sign}(b1)\sqrt{[2\cdot b1\cdot(1+b2)]^2 - 4\cdot 4\cdot b2\cdot\left[b1^2 + (1-b2)^2 - 2\cdot(b2+1-|b1|)^2\right]}}{2\cdot(4\cdot b2)} \\ -\infty \quad \text{if } |arg| > 1 \\ \mathrm{acos}(arg) \quad \text{otherwise} \end{array} \right.$$

1.1 ФНЧ

$b1 := (-0.9 \;\; -1.5 \;\; -0.5)^T$

$b2 := (0.10 \;\; 0.53 \;\; 0.01)^T \qquad W_i := \omega c(b1_i, b2_i)$



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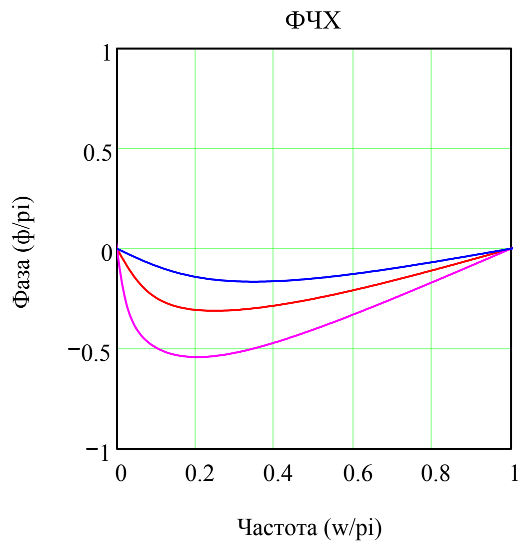
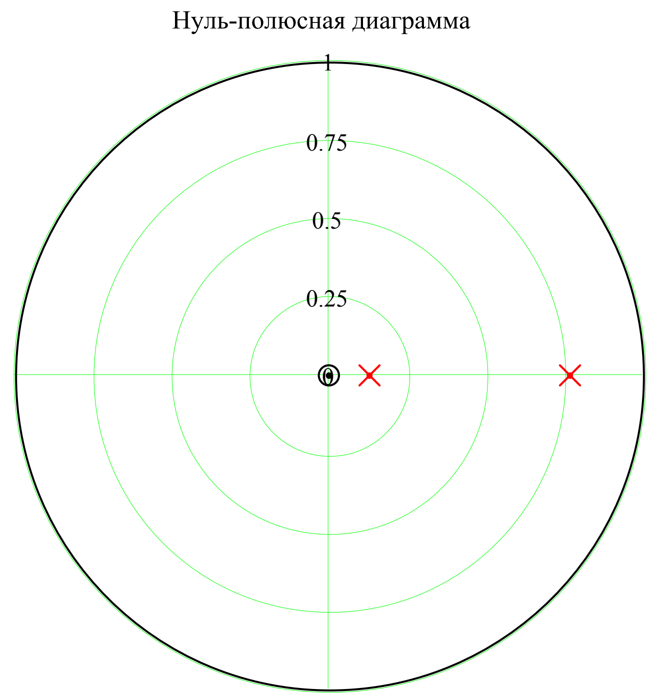
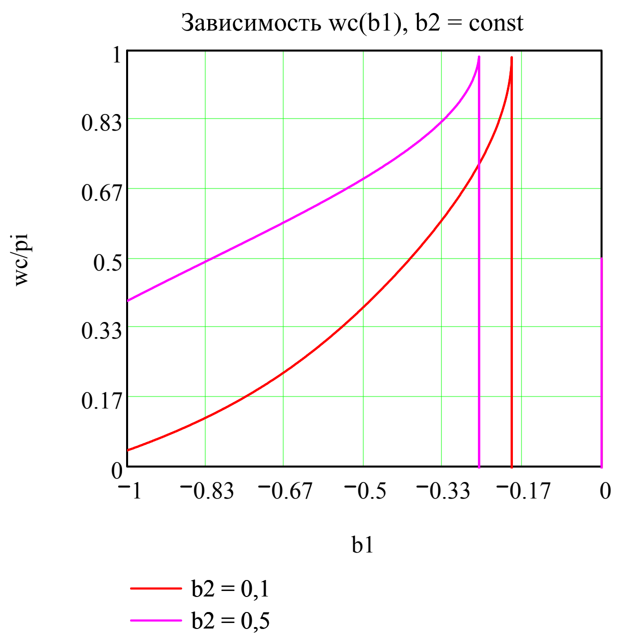
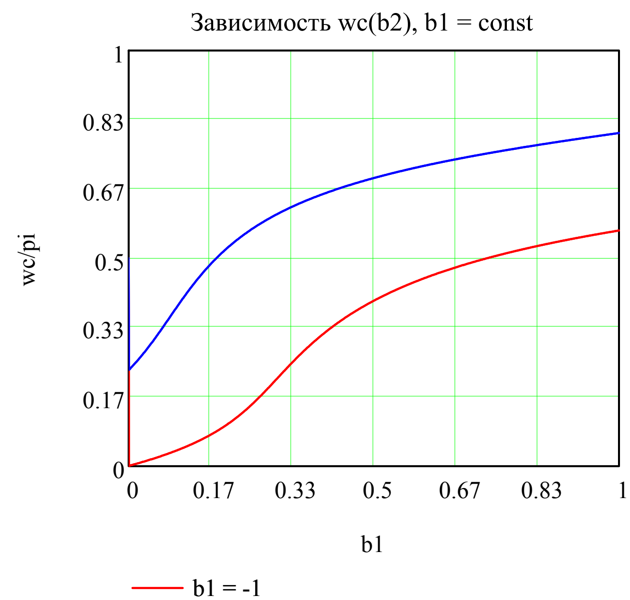




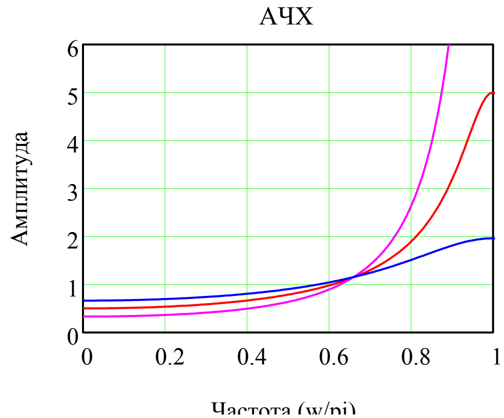




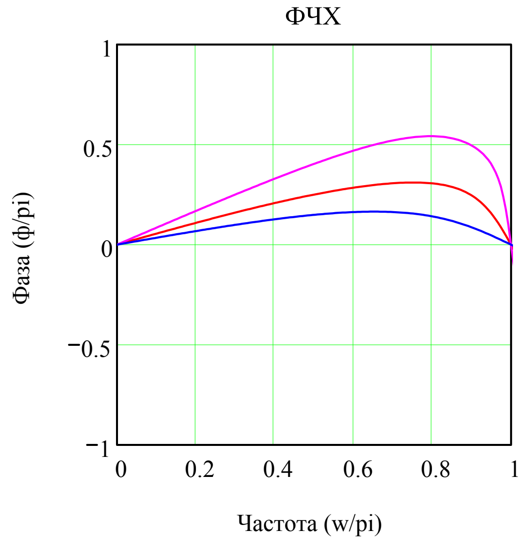
1.2 ФВЧ

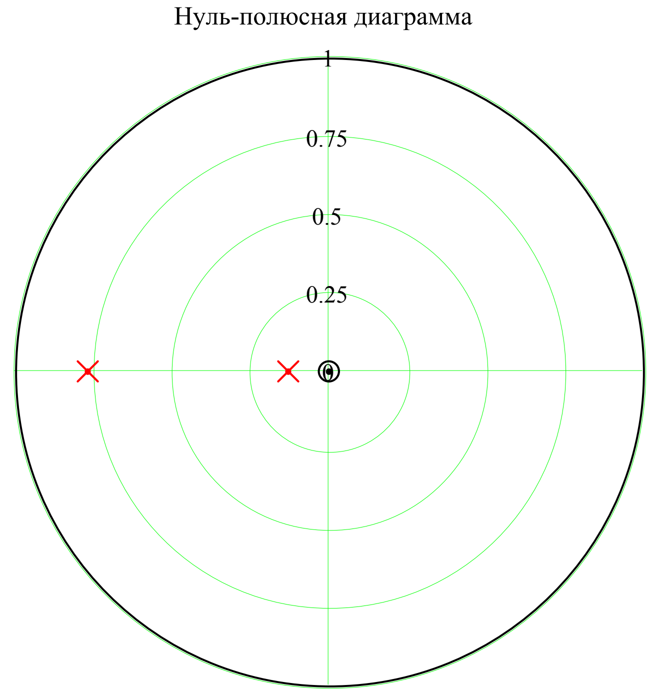
$$b1 := \left(0.9 \quad 1.5 \quad 0.5\right)^T$$

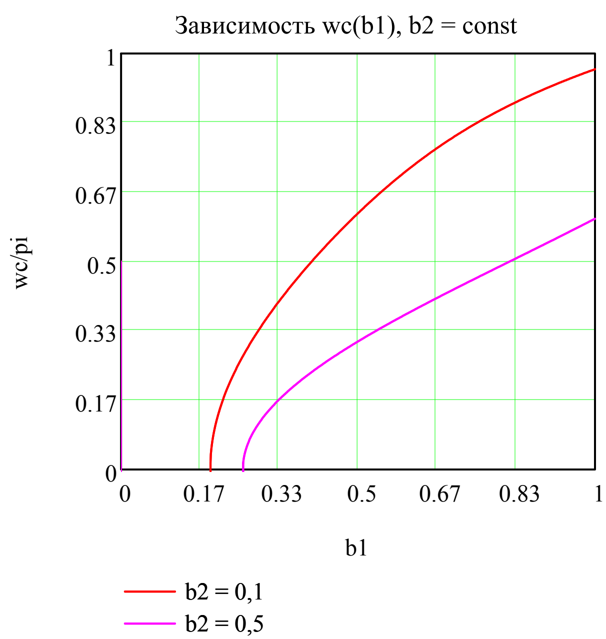
$$b2 := \left(0.10 \quad 0.53 \quad 0.01\right)^T \qquad W_i := \omega c\left(b1_i, b2_i\right)$$

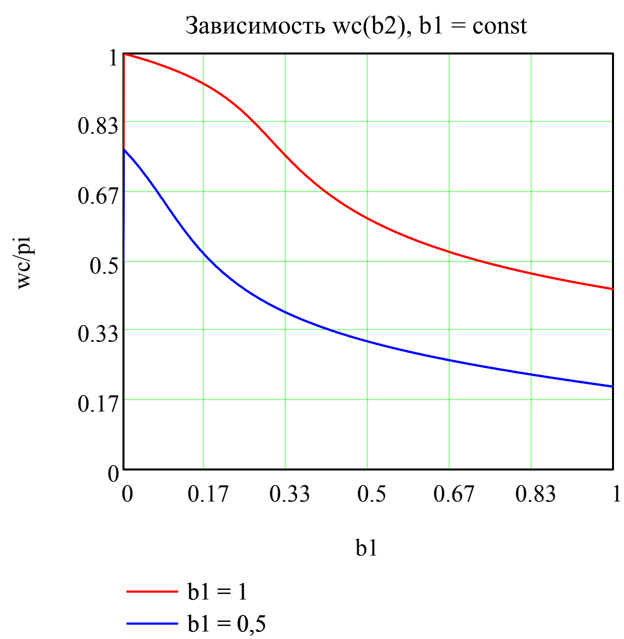

АЧХ
Амплитуда
Частота (w/pi)

[redacted]


ФЧХ
Фаза (ф/pi)
Частота (w/pi)


Нуль-полюсная диаграмма


Зависимость wc(b1), b2 = const
wc/pi
b1
b2 = 0,1
b2 = 0,5


Зависимость wc(b2), b1 = const
wc/pi
b1
b1 = 1
b1 = 0,5

1.3 ПФ

$$\omega r(b1, b2) := \begin{array}{|l} arg \leftarrow \dfrac{-b1\cdot(1 + b2)}{4\cdot b2} \\ \infty \ \ \text{if} \ \ |arg| > 1 \\ acos(arg) \ \ \text{otherwise} \end{array}$$

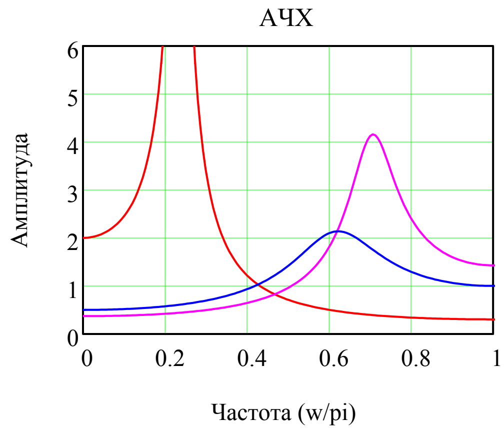
$$Q(b1, b2) := \frac{\omega r(b1, b2)}{\omega 2(b1, b2) - \omega 1(b1, b2)}$$

$$\omega 1(b1, b2) := \begin{array}{|l} arg \leftarrow \dfrac{-2\cdot b1\cdot(1 + b2) + \sqrt{4\cdot 4\cdot b2\cdot\left[b1^2 + (1 - b2)^2\right] - [2\cdot b1\cdot(1 + b2)]^2}}{2\cdot(4\cdot b2)} \\ \infty \ \ \text{if} \ \ |arg| > 1 \\ acos(arg) \ \ \text{otherwise} \end{array}$$

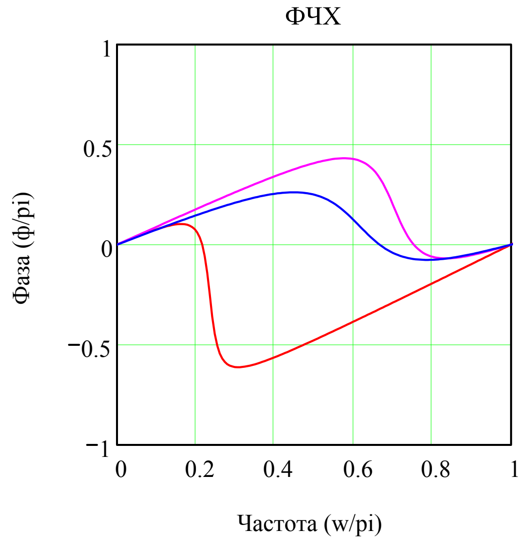
$$\omega 2(b1, b2) := \begin{array}{|l} arg \leftarrow \dfrac{-2\cdot b1\cdot(1 + b2) - \sqrt{4\cdot 4\cdot b2\cdot\left[b1^2 + (1 - b2)^2\right] - [2\cdot b1\cdot(1 + b2)]^2}}{2\cdot(4\cdot b2)} \\ \infty \ \ \text{if} \ \ |arg| > 1 \\ acos(arg) \ \ \text{otherwise} \end{array}$$

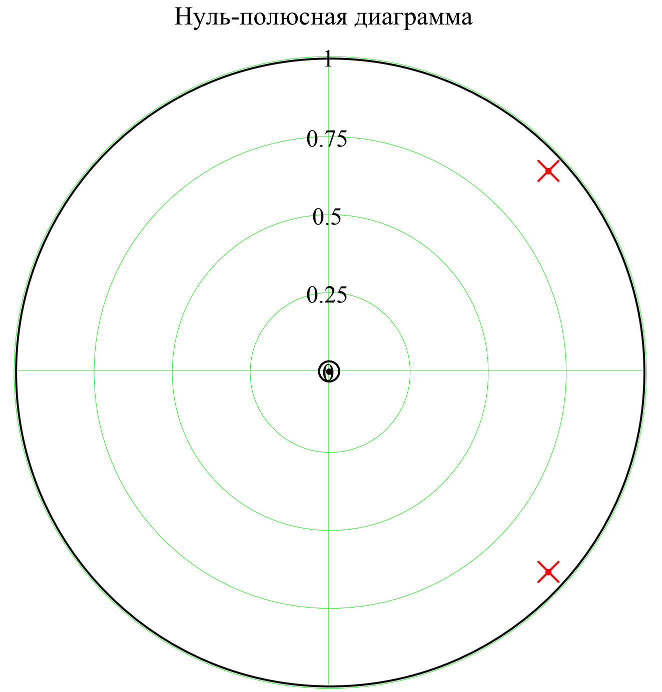
$$b1 := (-1.4 \ \ 1 \ \ 0.5)^T$$

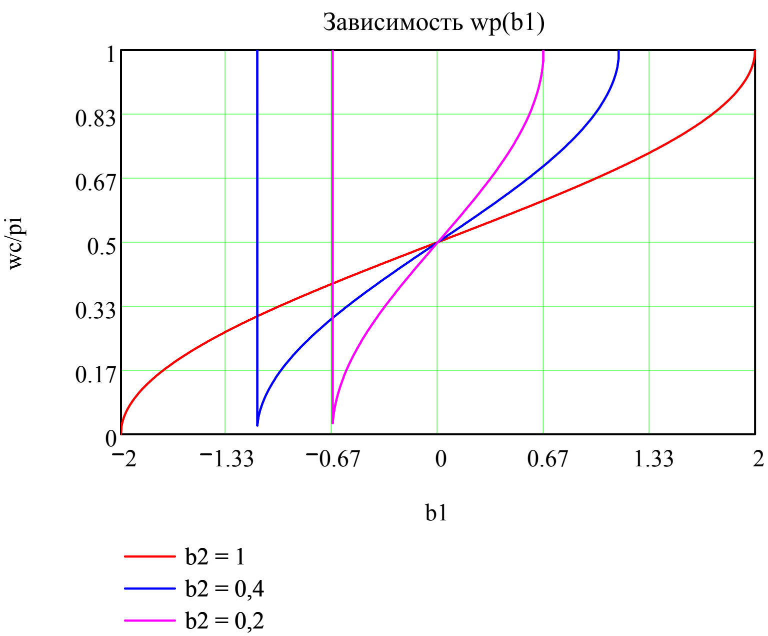
$$b2 := (0.9 \ \ 0.7 \ \ 0.5)^T \qquad W_i := \omega r\left(b1_i, b2_i\right) \qquad Qu_i := Q\left(b1_i, b2_i\right)$$

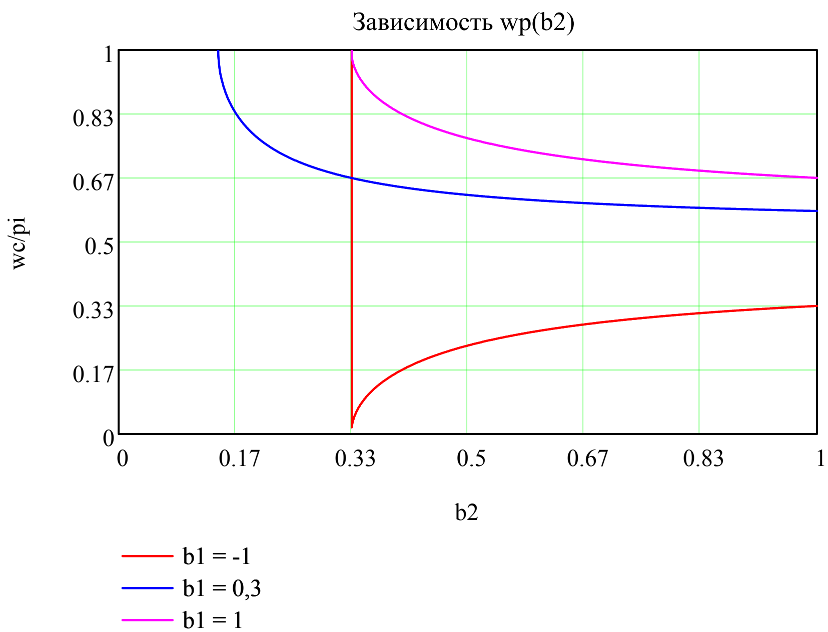

АЧХ
Амплитуда
Частота (w/pi)
0
1
2
3
4
5
6
0.2
0.4
0.6
0.8

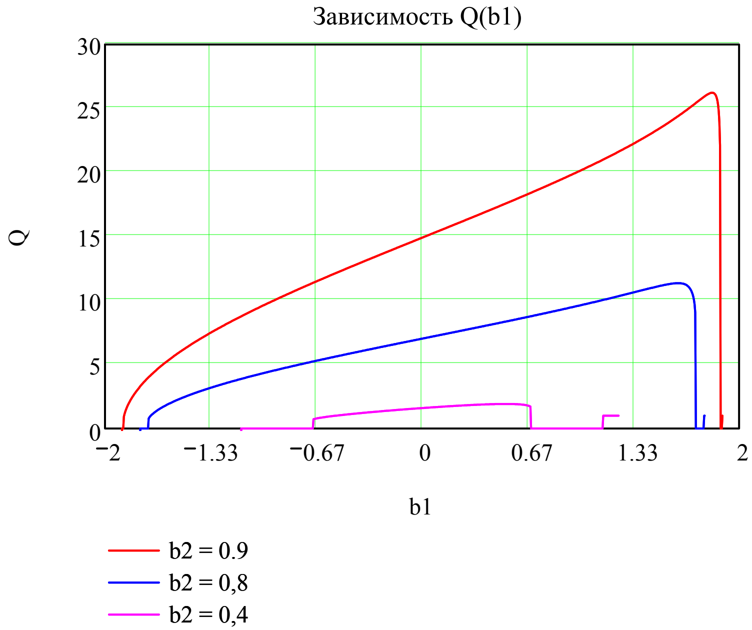
[redacted]

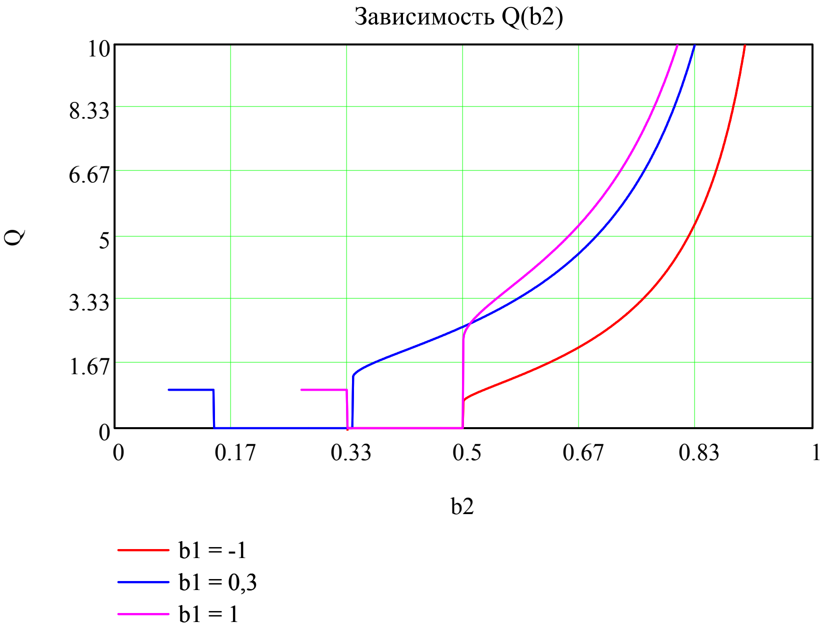

ФЧХ
Фаза (ф/pi)
Частота (w/pi)
1
0.5
0
−0.5
−1
0.2
0.4
0.6
0.8


Нуль-полюсная диаграмма
1
0.75
0.5
0.25


Зависимость wp(b1)
wc/pi
b1
1
0.83
0.67
0.5
0.33
0.17
0
−2
−1.33
−0.67
0.67
1.33
2
b2 = 1
b2 = 0,4
b2 = 0,2


Зависимость wp(b2)
wc/pi
b2
1
0.83
0.67
0.5
0.33
0.17
0
b1 = -1
b1 = 0,3
b1 = 1





1.4 РФ

$$\omega r(b1,b2) := \left| \begin{array}{l} arg \leftarrow \dfrac{-b1\cdot(1+b2)}{4\cdot b2} \\ \infty \ \text{ if } \ |arg| > 1 \\ \text{acos}(arg) \ \text{ otherwise} \end{array} \right.$$

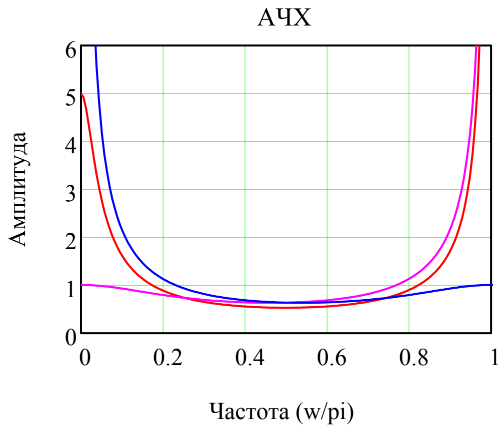
$$\omega 1(b1,b2) := \left| \begin{array}{l} arg \leftarrow \dfrac{-2\cdot b1\cdot(1+b2) - \sqrt{\dfrac{[2\cdot b1\cdot(1+b2)]^2 - 4\cdot 4\cdot b2\cdot\left[b1^2 + (1-b2)^2\right]}{2}}}{2\cdot(4\cdot b2)} \\ \infty \ \text{ if } \ |arg| > 1 \lor arg \neq \text{Re}(arg) \\ \text{acos}(arg) \ \text{ otherwise} \end{array} \right.$$

$$\omega 2(b1,b2) := \left| \begin{array}{l} arg \leftarrow \dfrac{-2\cdot b1\cdot(1+b2) + \sqrt{\dfrac{[2\cdot b1\cdot(1+b2)]^2 - 4\cdot 4\cdot b2\cdot\left[b1^2 + (1-b2)^2\right]}{2}}}{2\cdot(4\cdot b2)} \\ \infty \ \text{ if } \ |arg| > 1 \lor arg \neq \text{Re}(arg) \\ \text{acos}(arg) \ \text{ otherwise} \end{array} \right.$$

$$Q(b1,b2) := \frac{\omega r(b1,b2)}{\omega 2(b1,b2) - \omega 1(b1,b2)}$$

$$b1 := (0.1 \quad 0.5 \quad -0.5)^T$$

$$b2 := (-0.9 \quad -0.5 \quad -0.5)^T \quad W_i := \omega r(b1_i, b2_i) \quad Qu_i := Q(b1_i, b2_i)$$



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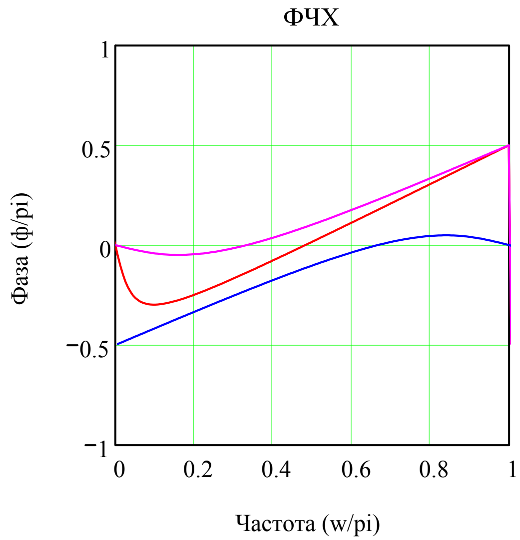
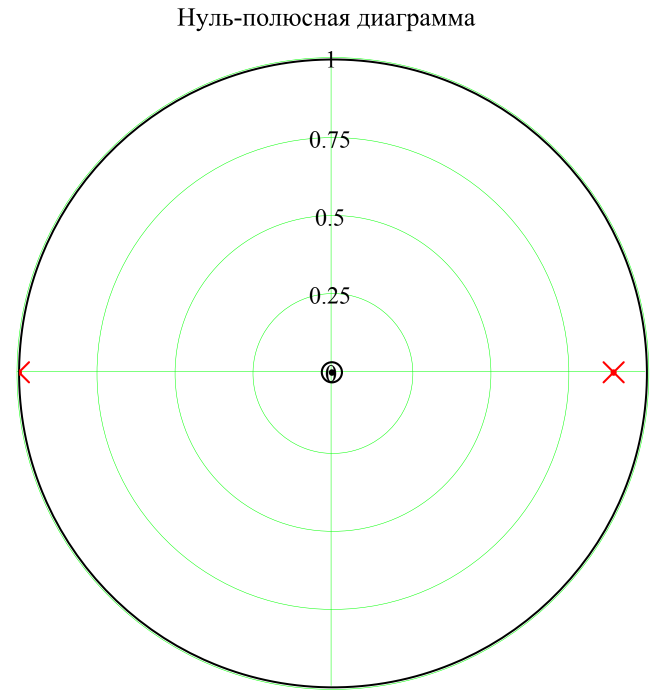
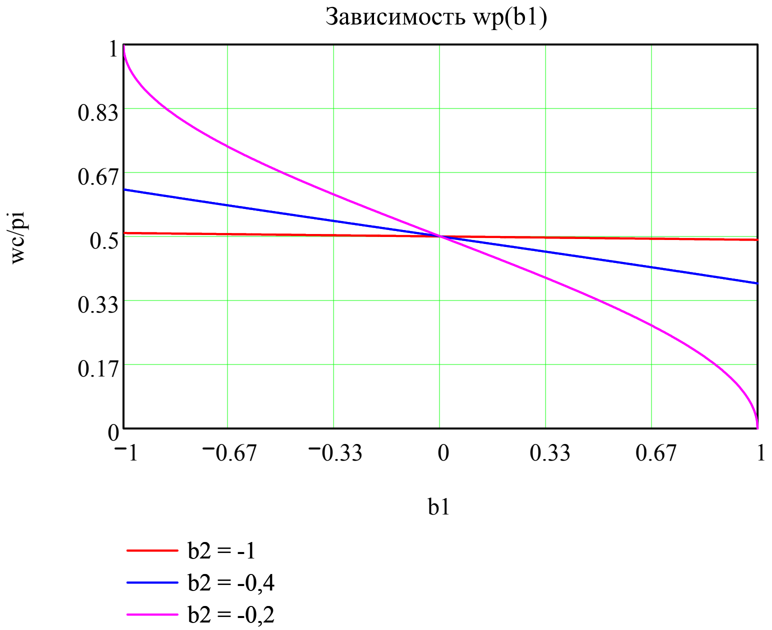
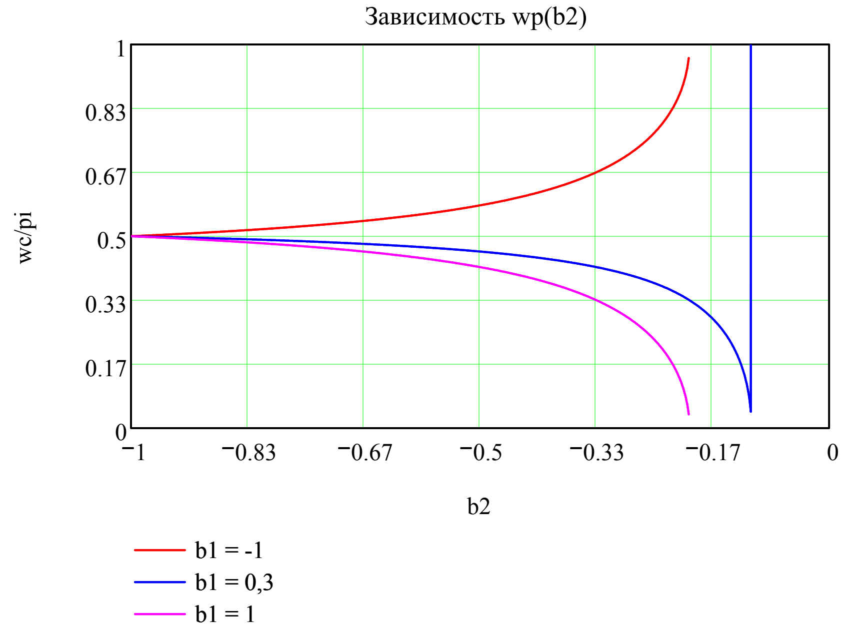
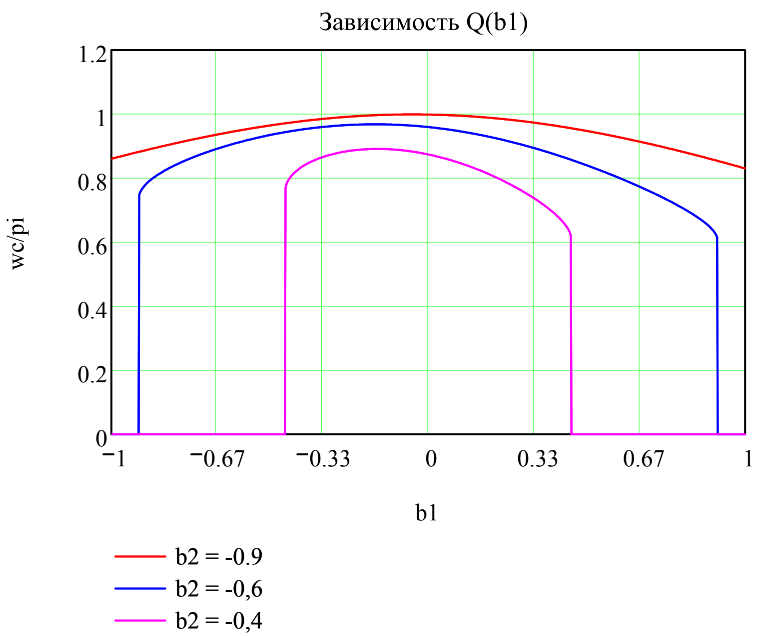
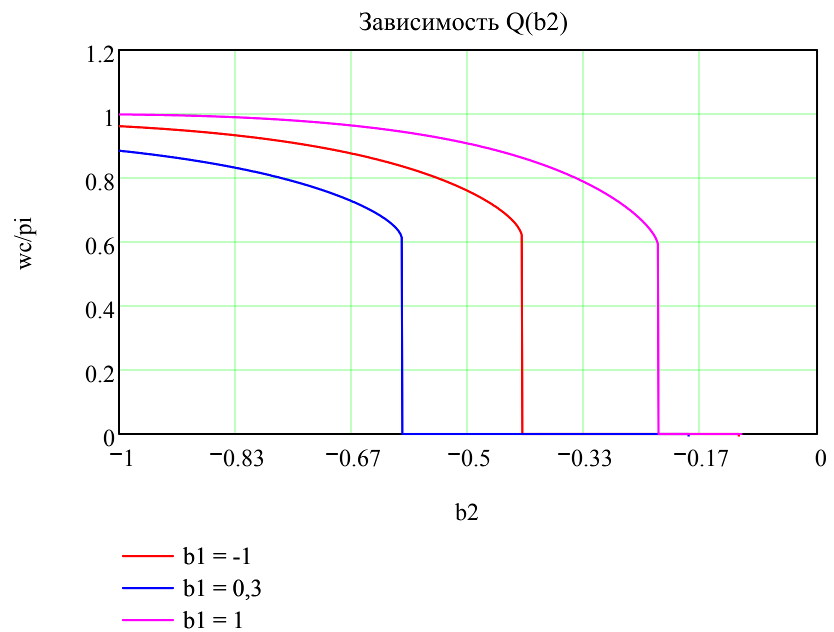












2. Исследование временны характеристик

2.1 ФНЧ

$$b1 := (-0.5 \quad -1)^T$$

$$b2 := (0.05 \quad 0.2)^T$$

$$ni := (4 \quad 10)^T$$

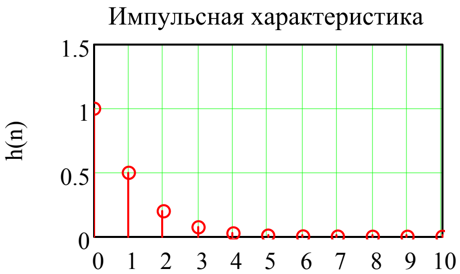
$$\nu := 0, 1 .. 20$$

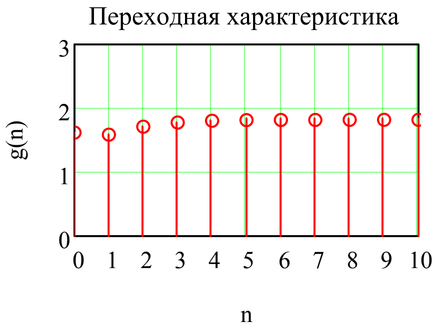
$$h(n, b1, b2) := \left| \begin{array}{l} p1 \leftarrow -\frac{b1}{2} - \sqrt{\frac{b1^2}{4} - b2} \\ p2 \leftarrow -\frac{b1}{2} + \sqrt{\frac{b1^2}{4} - b2} \\ \frac{p1^{n+1} - p2^{n+1}}{p1 - p2} \end{array} \right.$$

$$g(n, b1, b2) := \left| \begin{array}{l} p1 \leftarrow -\frac{b1}{2} - \sqrt{\frac{b1^2}{4} - b2} \\ p2 \leftarrow -\frac{b1}{2} + \sqrt{\frac{b1^2}{4} - b2} \\ \frac{1}{1 + b1 + b2} + \frac{p1^{n+1}}{(p1 - 1)\cdot(p1 - p2)} + \frac{p2^{n+2}}{(p2 - 1)\cdot(p2 - p1)} \end{array} \right.$$

$$y1(n, ni, b1, b2) := \left| \begin{array}{l} p1 \leftarrow -\frac{b1}{2} - \sqrt{\frac{b1^2}{4} - b2} \\ p2 \leftarrow -\frac{b1}{2} + \sqrt{\frac{b1^2}{4} - b2} \\ g(n, b1, b2)\cdot\Phi(n) - g(n - ni, b1, b2)\cdot\Phi(n - ni) \end{array} \right.$$

$$y2(n, ni, b1, b2) := \left| \begin{array}{l} \omega \leftarrow \omega r(b1, b2) \\ p1 \leftarrow -\frac{b1}{2} - \sqrt{\frac{b1^2}{4} - b2} \\ p2 \leftarrow -\frac{b1}{2} + \sqrt{\frac{b1^2}{4} - b2} \\ p3 \leftarrow e^{i\cdot\omega} \\ y3 \leftarrow \left[\frac{e^{i\cdot\omega\cdot(n+2)}}{p3^2 + p1\cdot b1 + b2} + \frac{p1^{n+2}}{\left(p1 - e^{i\cdot\omega}\right)\cdot(p1 - p2)} + \frac{p2^{n+2}}{\left(p2 - e^{i\cdot\omega}\right)\cdot(p2 - p1)}\right]\cdot\Phi(n) \\ y4 \leftarrow e^{i\cdot\omega\cdot ni}\cdot y3 \\ y2 \leftarrow y3 - y4 \\ y2 \end{array} \right.$$





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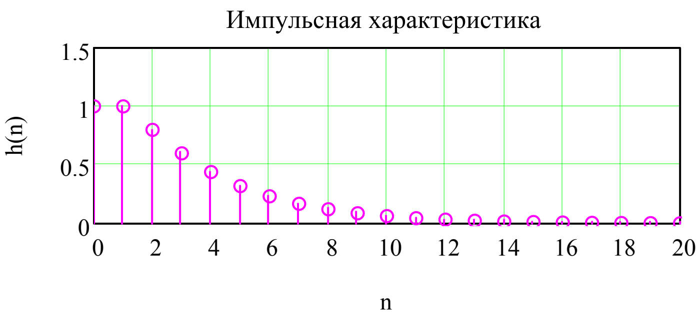
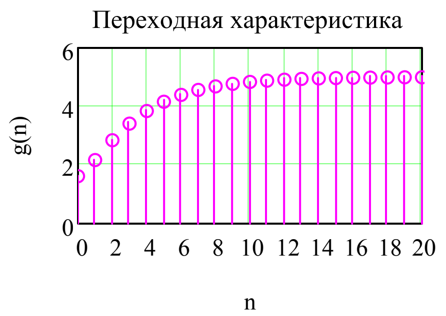
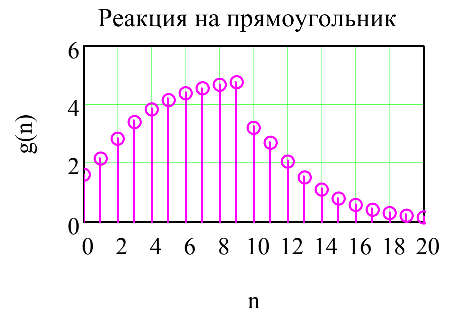


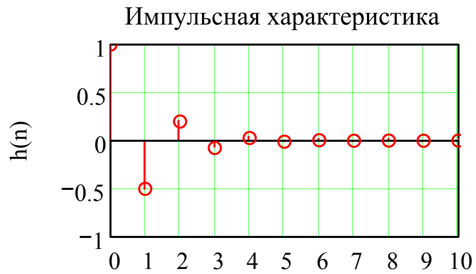


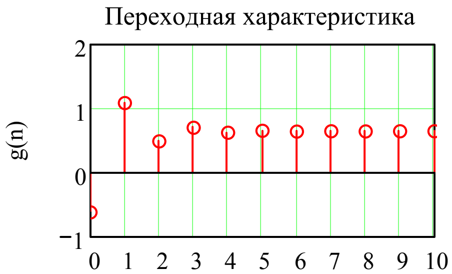


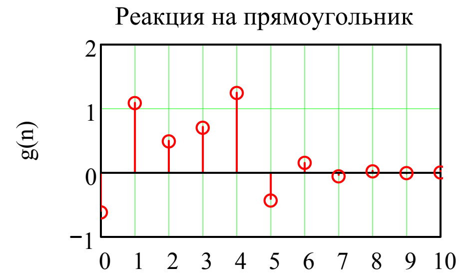
2.2 ФВЧ

$b1 := (0.5\ \ 1)^T \qquad ni := (4\ \ 10)^T$

$b2 := (0.05\ \ 0.2)^T$







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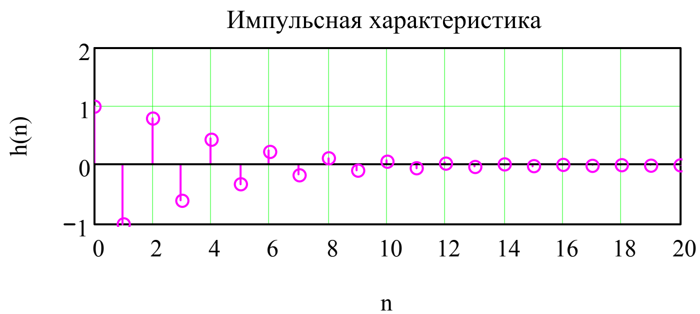
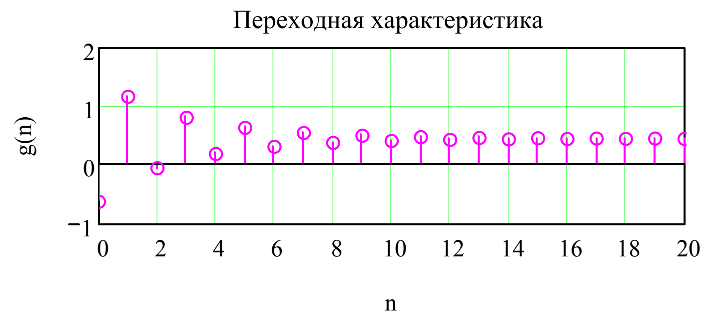
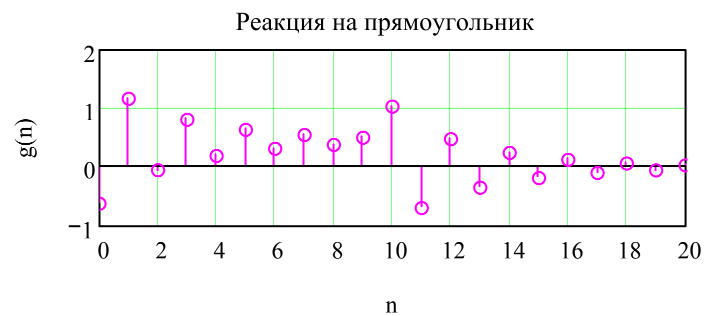


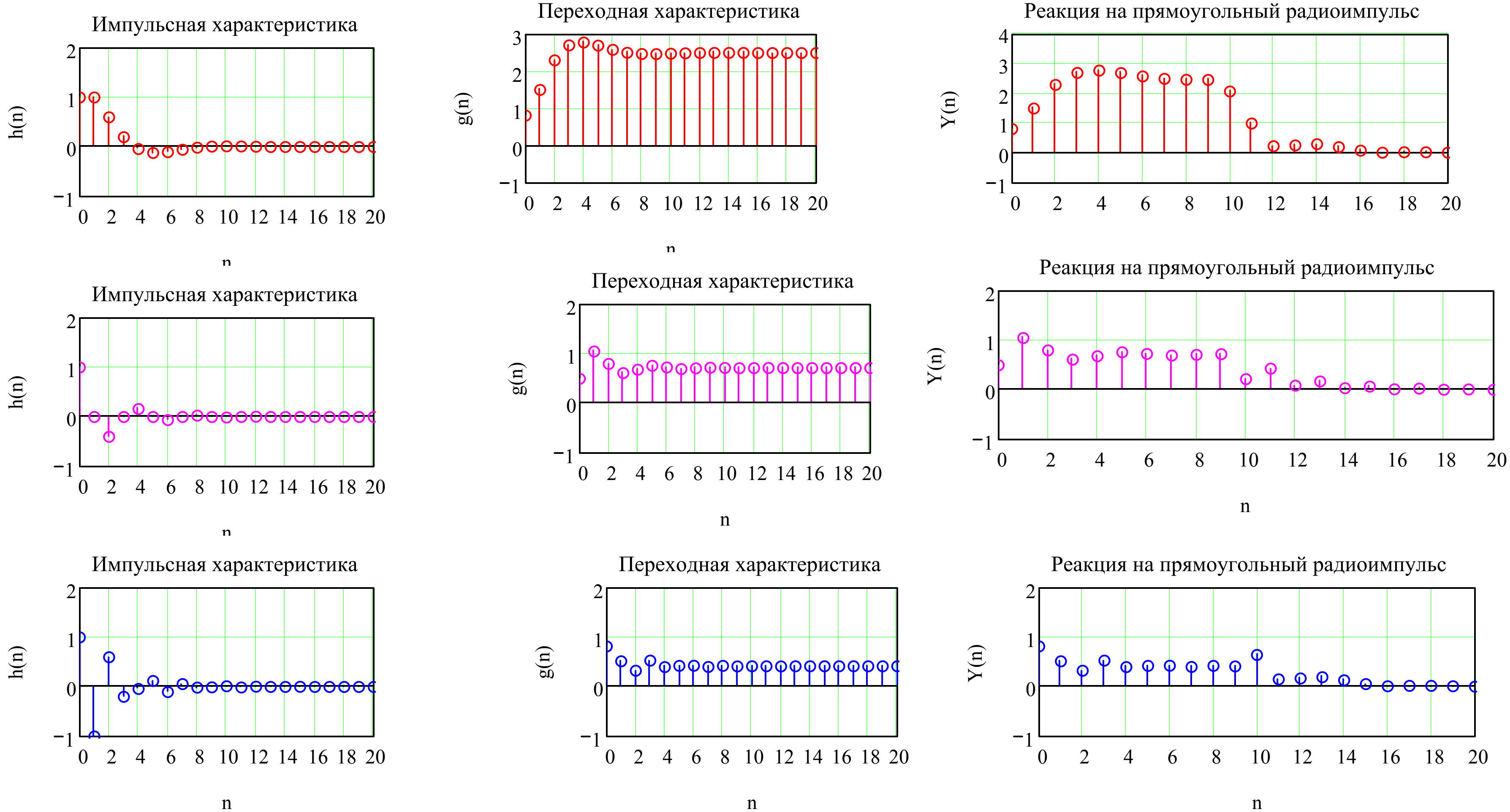




2.3 ПФ

$b1 := (-1 \quad 0 \quad 1)^T$ $\qquad$ $ni := (10 \quad 10 \quad 10)^T$ $\qquad$ $(0.5 \quad 1)^T$

$b2 := (0.4 \quad 0.4 \quad 0.4)^T$



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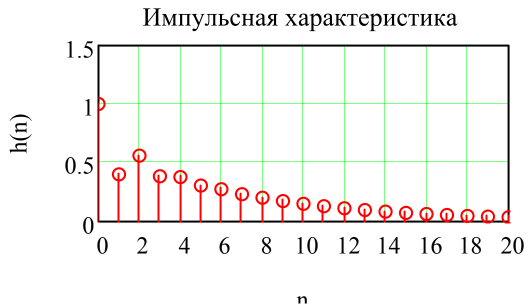
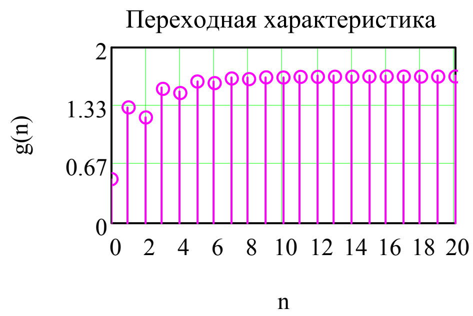
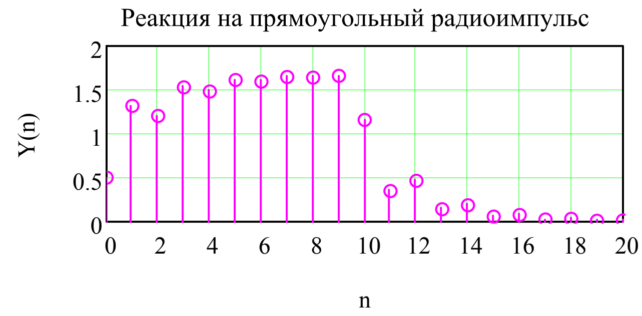
2.4 РФ

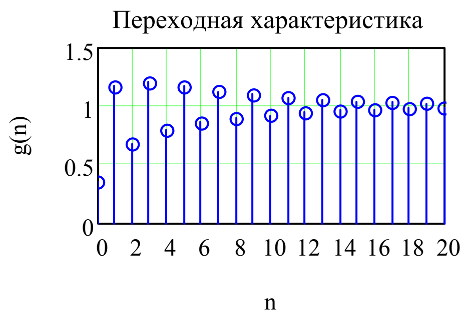
$b1 := (-0.4 \;\; 0 \;\; 0.4)^T$ $\qquad ni := (10 \;\; 10 \;\; 10)^T$ $\qquad (0.5 \;\; 1)^T$

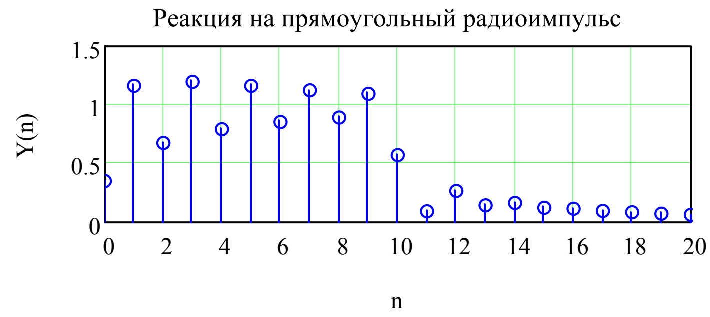
$$b2 := (-0.4\ \ -0.4\ \ -0.4)^{T}$$











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$g(\nu, b1_0, b2_0) =$

$0.5-0.645i$
$1.5-0.129i$
$2.3+0.129i$
$2.7+0.181i$
$2.78+0.129i$
$2.7+0.057i$
$2.588+5.164i\cdot 10^{-3}$
$2.508-0.018i$
$2.473-0.02i$
$2.47-0.013i$
$2.48-4.751i\cdot 10^{-3}$
$2.493+2.892i\cdot 10^{-4}$
$2.5+2.19i\cdot 10^{-3}$
$2.503+2.074i\cdot 10^{-3}$
$2.503+1.198i\cdot 10^{-3}$
$2.502+3.685i\cdot 10^{-4}$