

# Лабораторная работа №4

## Исследование нерекурсивной цепи второго порядка



### 1. Исследование частотных характеристик

$$H(\omega, a1, a2) := 1 + a1 \cdot e^{-i \cdot \omega} + a2 \cdot e^{-2 \cdot i \cdot \omega}$$

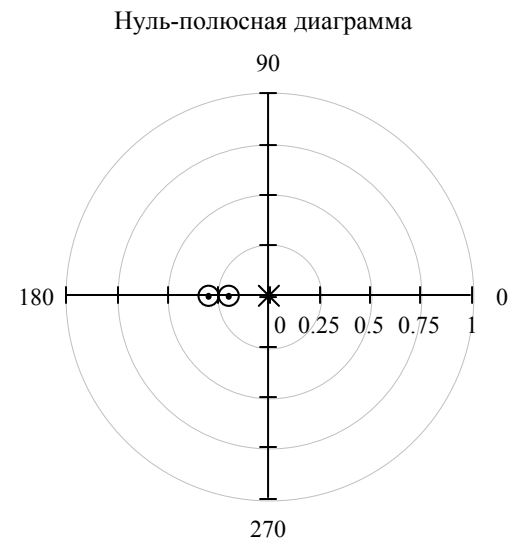
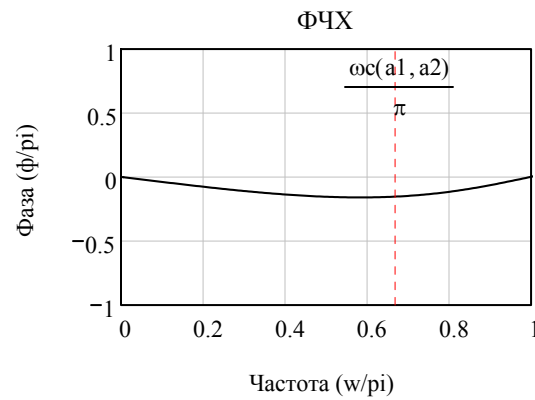
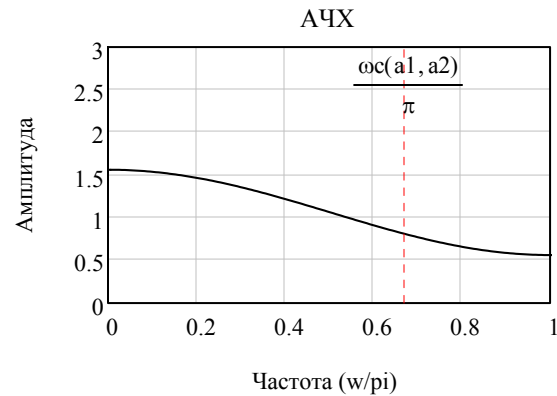
#### 1.1 ФНЧ

$$\omega c(a1, a2) := \begin{cases} \infty & \text{if } |a2| < 0.001 \\ \arccos \left[ \frac{-2 \cdot a1 \cdot (1 + a2) + \text{sign}(a1) \sqrt{[2 \cdot a1 \cdot (1 + a2)]^2 - 2 \cdot 4 \cdot a2 \cdot [2 \cdot a1^2 + (1 - a2)^2] - [a2 + 1 + (|a1|)^2]}}{2 \cdot (4 \cdot a2)} \right] & \text{otherwise} \end{cases}$$

$$a1 := 0.5$$

$$a2 := 0.06$$

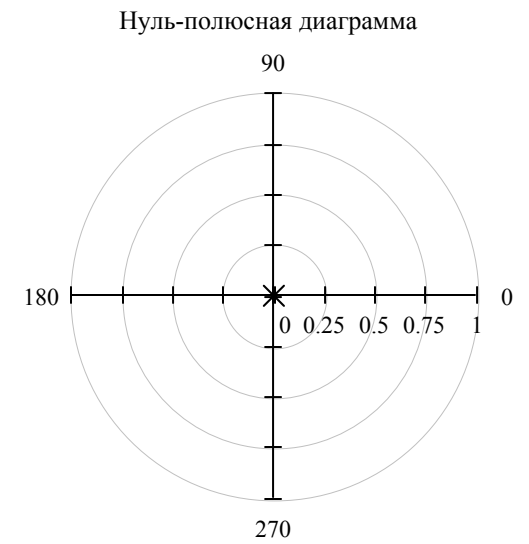
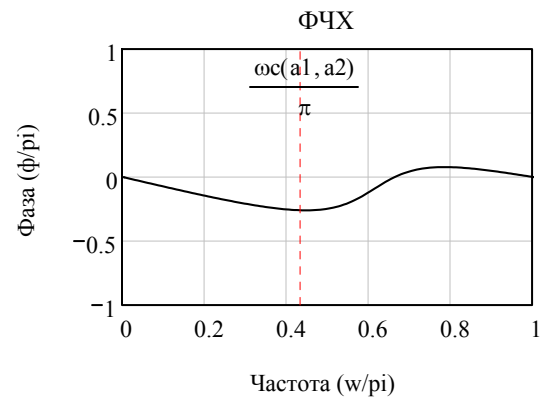
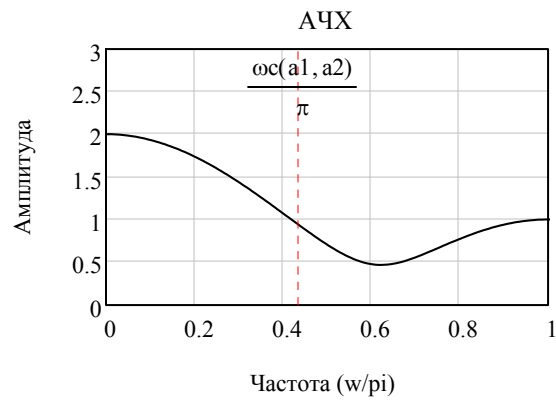
$$\omega c(a1, a2) = 2.107 \text{ rad}$$



$$a1 := 0.5$$

$$a2 := 0.5$$

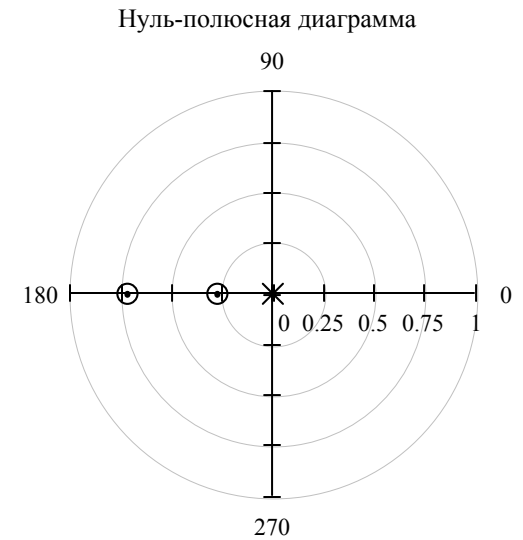
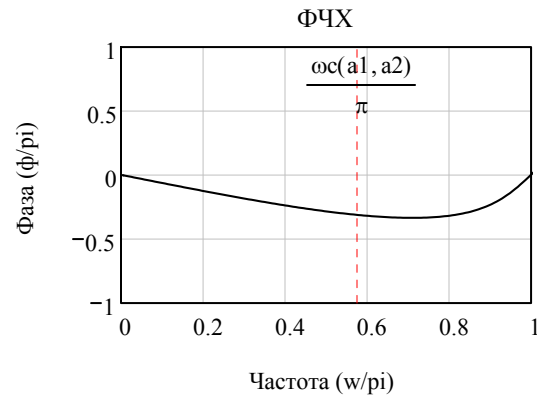
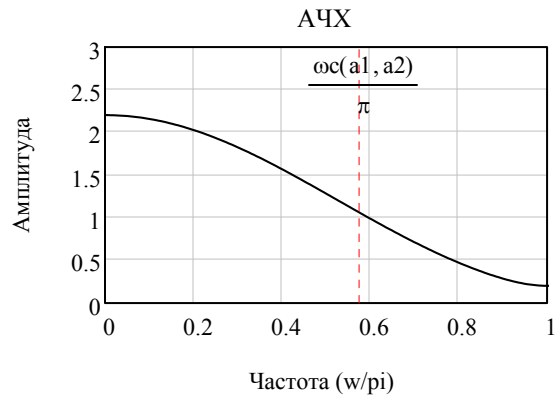
$$\omega c(a1, a2) = 1.372 \text{ rad}$$



$a1 := 1$

$a2 := 0.2$

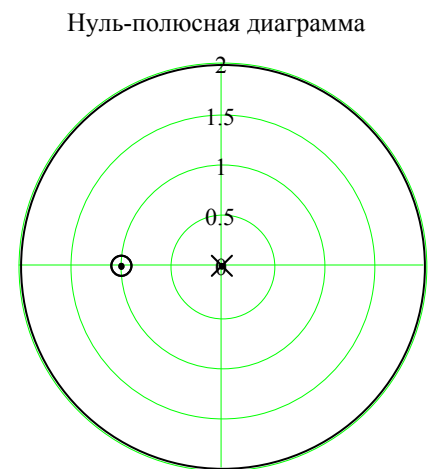
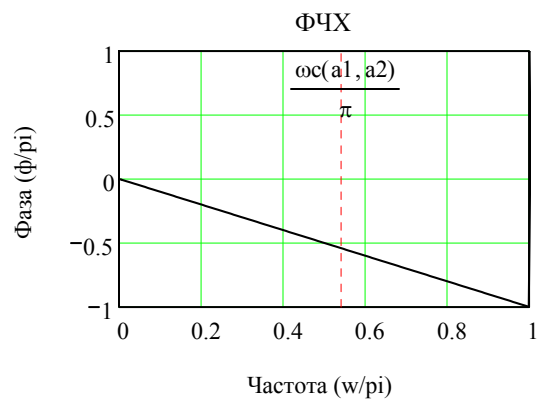
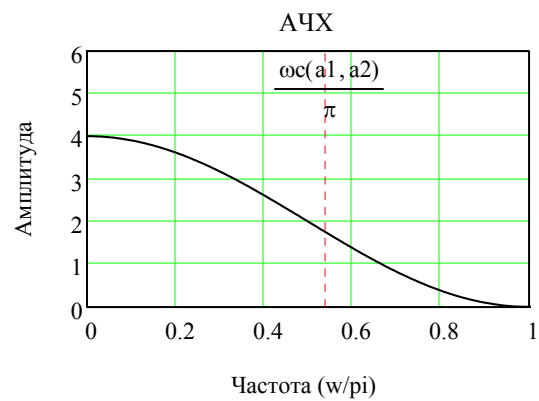
$\omega_c(a1, a2) = 1.818 \text{ rad}$



$a1 := 2$

$a2 := 1$

$\omega_c(a1, a2) = 1.705 \text{ rad}$

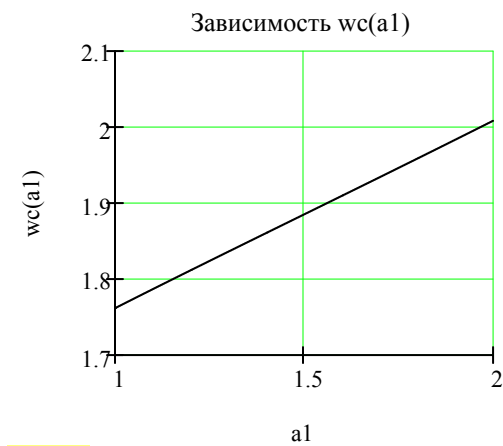
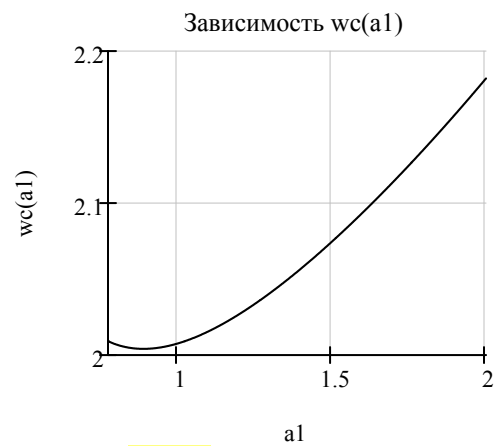
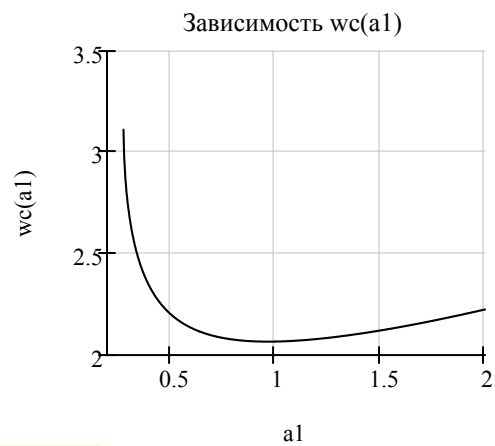


### Зависимость $w_c$ от параметров $a1, a2$

$a2 := 0.02$

$a2 := 0.06$

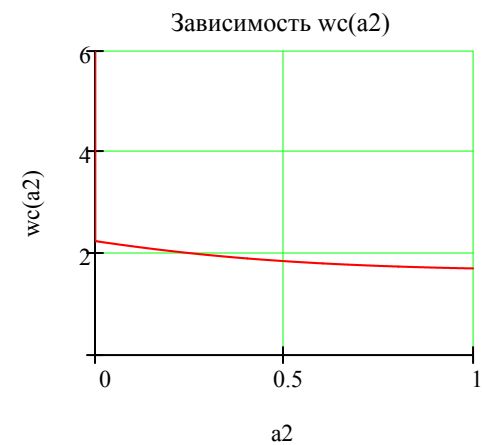
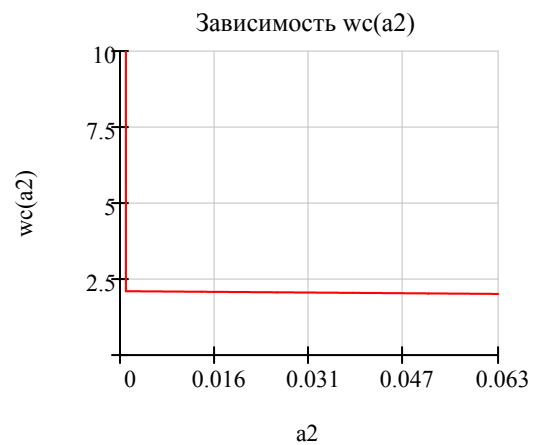
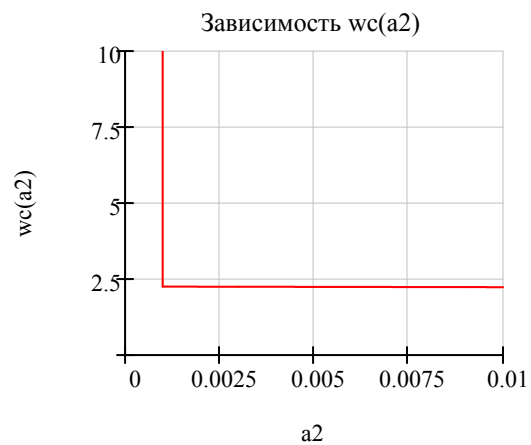
$a2 := 0.25$



$a1 := 0.5$

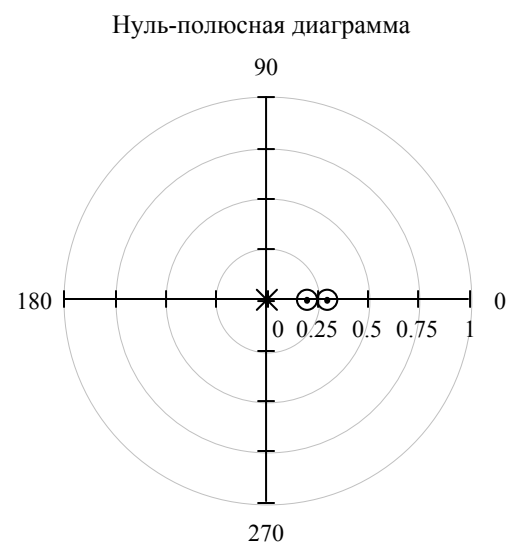
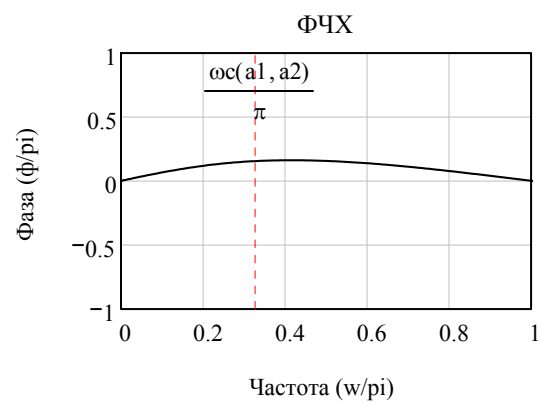
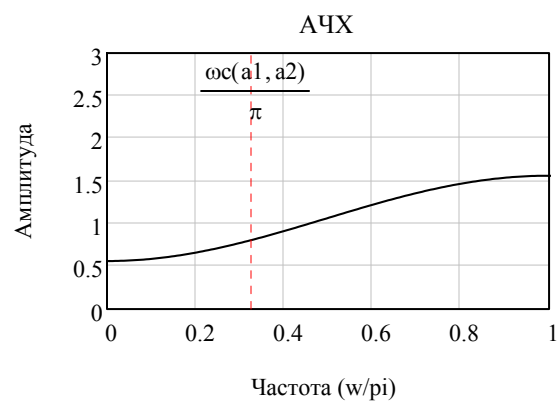
$a1 := 1$

$a1 := 2$

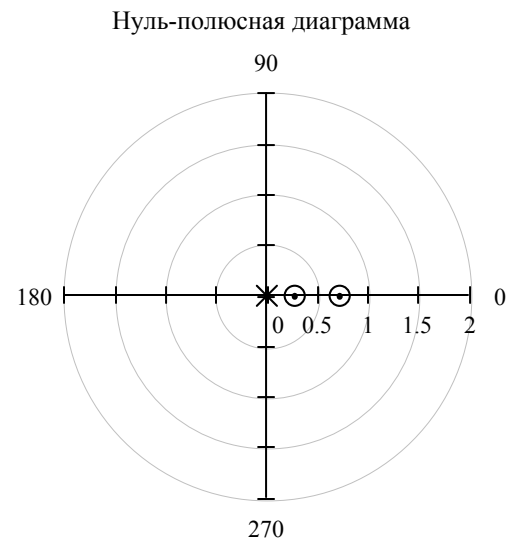
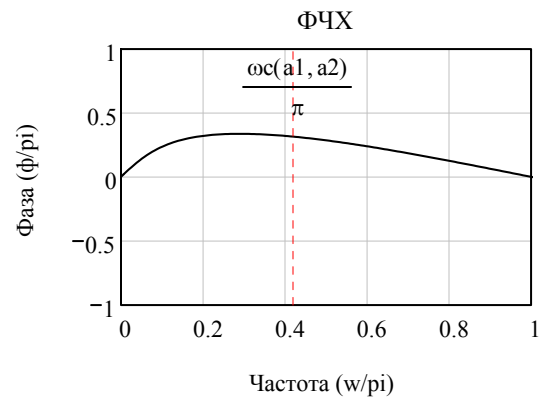
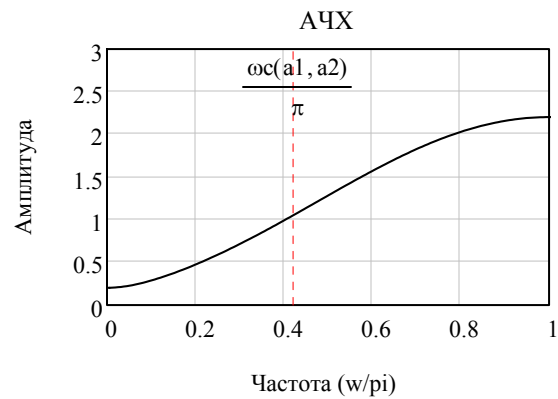


## 1.2 ФВЧ

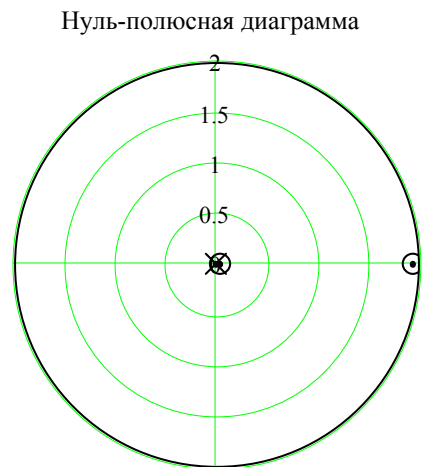
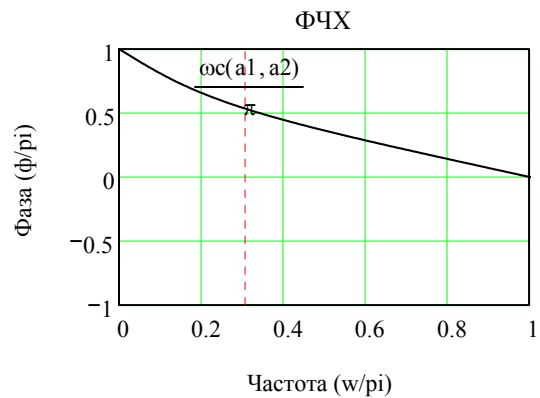
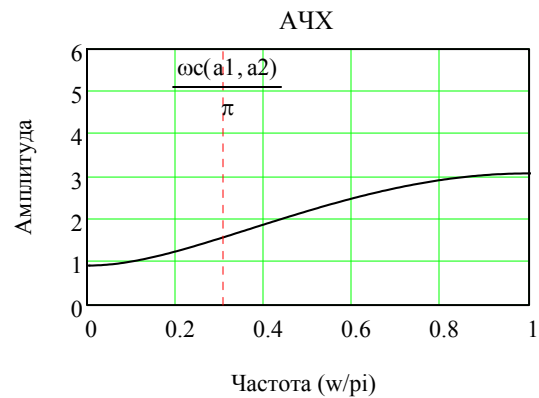
$a_1 := -0.5$     $a_2 := 0.06$     $\omega_c(a_1, a_2) = 1.035 \text{ rad}$



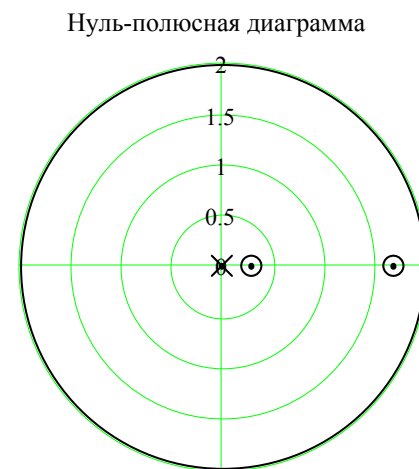
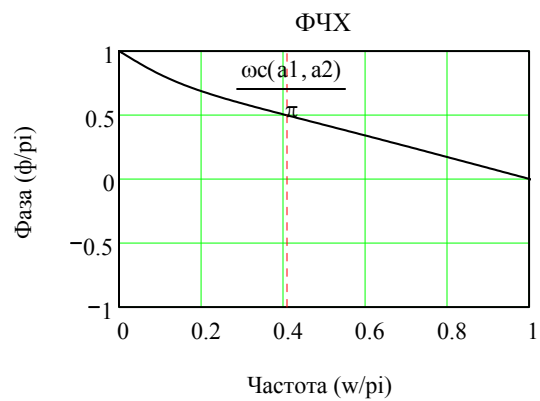
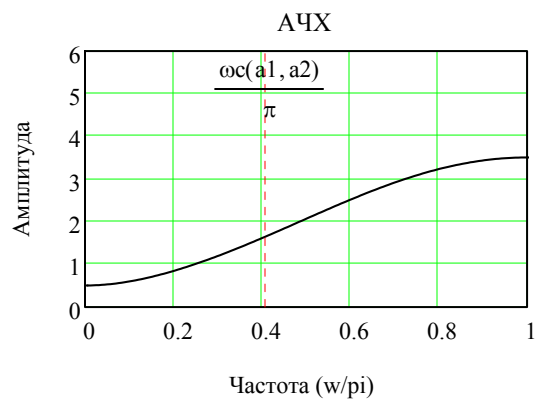
$a_1 := -1$     $a_2 := 0.2$     $\omega_c(a_1, a_2) = 1.323 \text{ rad}$



$a1 := -2$      $a2 := 0.08$      $\omega_c(a1, a2) = 0.98 \text{ rad}$

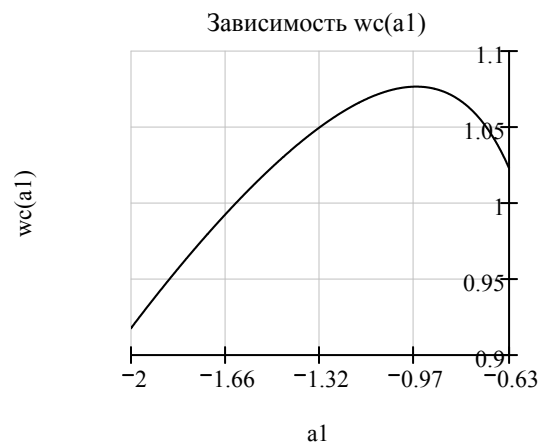


$a1 := -2$      $a2 := 0.5$      $\omega_c(a1, a2) = 1.292 \text{ rad}$



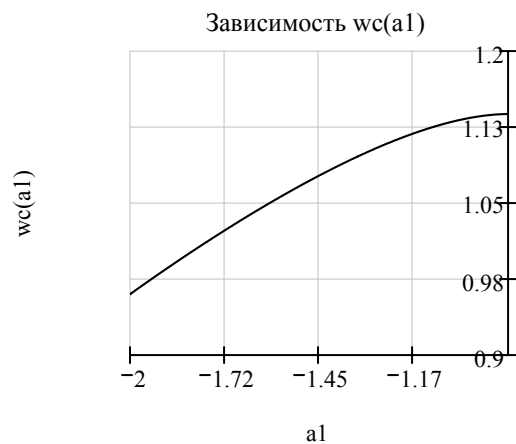
### Зависимость $\omega_c$ от параметров $a1, a2$

$a2 := 0.02$



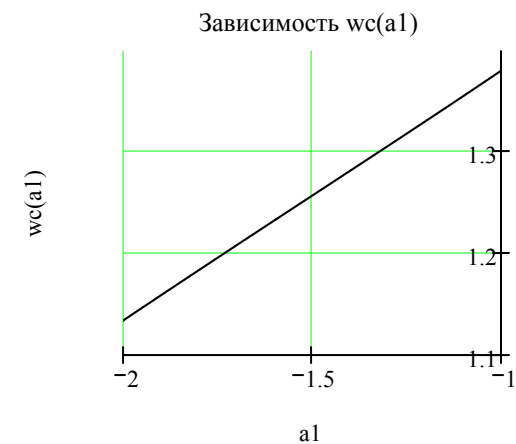
$a1 := -0.5$

$a2 := 0.06$

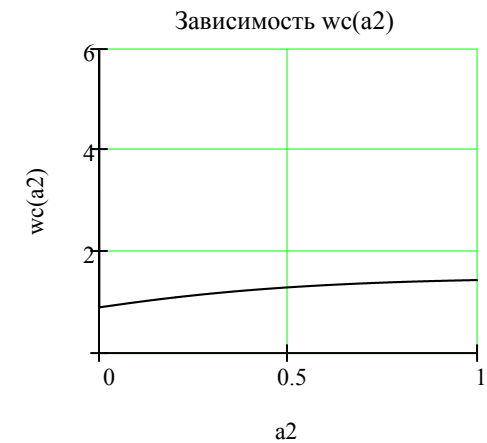
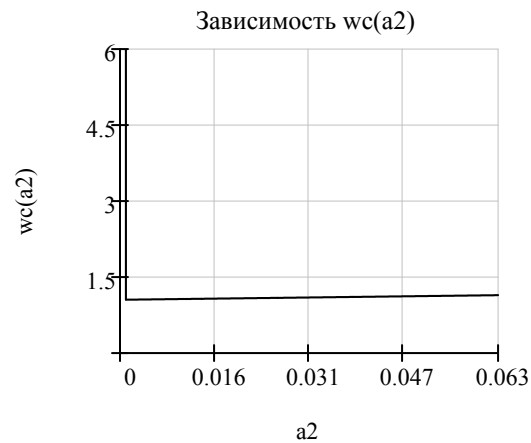
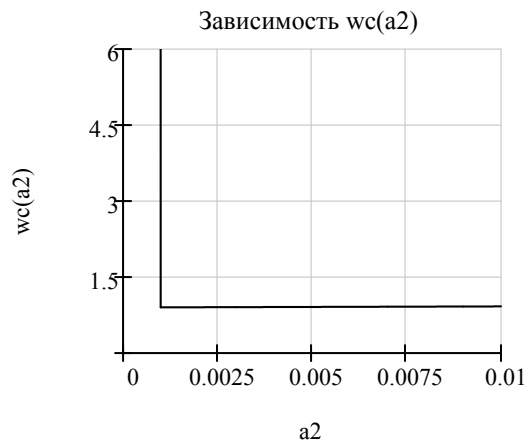


$a1 := -1$

$a2 := 0.25$



$a1 := -2$



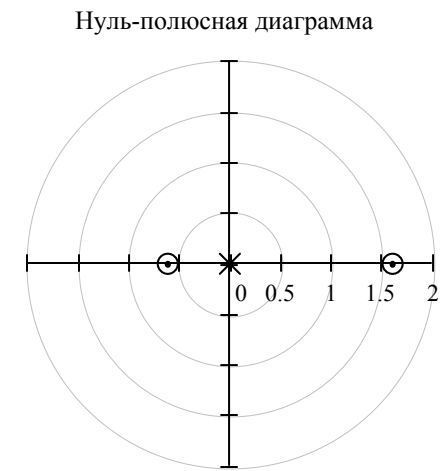
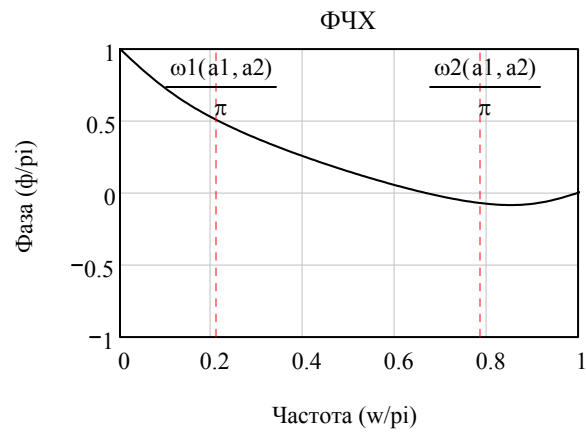
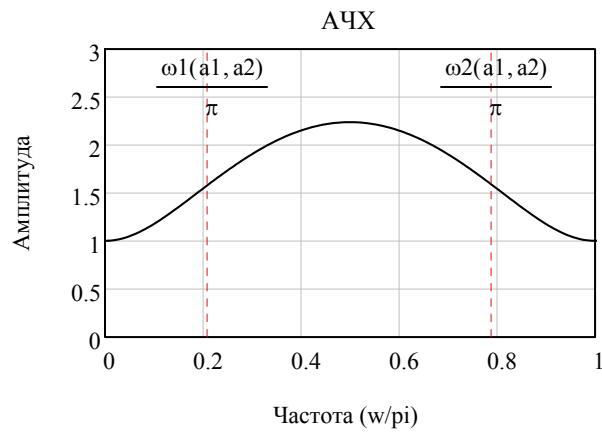
### 1.3 Полосовой фильтр

$$b1(a1, a2) := \frac{-2 \cdot a1 \cdot (1 + a2) - \sqrt{\frac{[2 \cdot a1 \cdot (1 + a2)]^2 - 4 \cdot 4 \cdot a2 \cdot [a1^2 + (1 - a2)^2]}{2}}}{2 \cdot 4 \cdot a2}$$

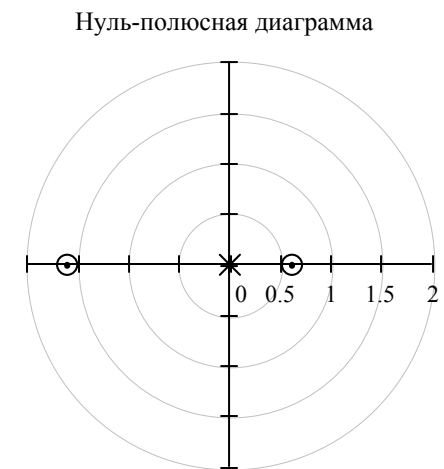
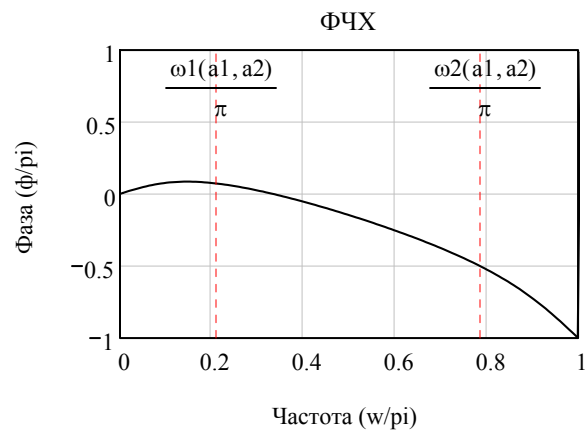
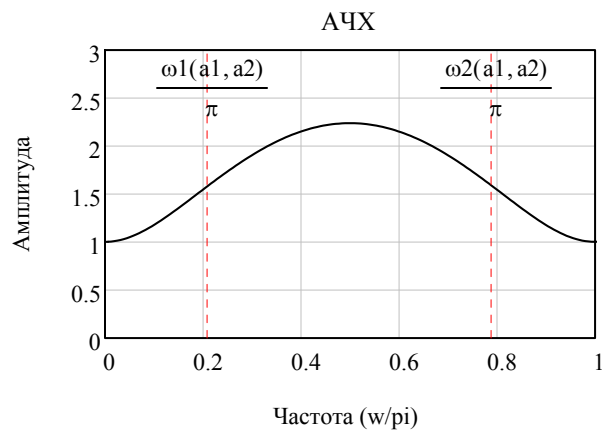
$$b2(a1, a2) := \frac{-2 \cdot a1 \cdot (1 + a2) + \sqrt{\frac{[2 \cdot a1 \cdot (1 + a2)]^2 - 4 \cdot 4 \cdot a2 \cdot [a1^2 + (1 - a2)^2]}{2}}}{2 \cdot 4 \cdot a2}$$

$$\omega_p(a1, a2) := \arccos\left[\frac{-a1(1 + a2)}{4 \cdot a2}\right] \quad \omega_2(a1, a2) := \arccos(b2(a1, a2)) \quad \omega_1(a1, a2) := \arccos(b1(a1, a2)) \quad Q(a1, a2) := \frac{\omega_p(a1, a2)}{\omega_2(a1, a2) - \omega_1(a1, a2)}$$

$$\underline{a1} := -1 \quad \underline{a2} := -1 \quad \omega_1(a1, a2) = 0.659 \text{ rad} \quad \omega_p(a1, a2) = 1.571 \text{ rad} \quad \omega_2(a1, a2) = 2.483 \text{ rad} \quad Q(a1, a2) = 0.861$$

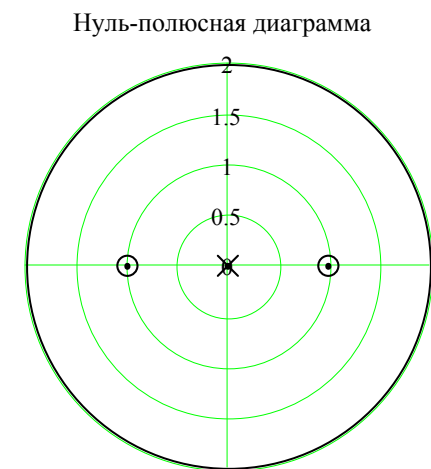
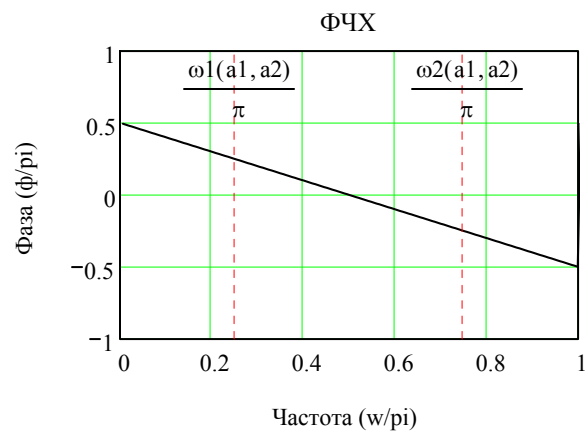
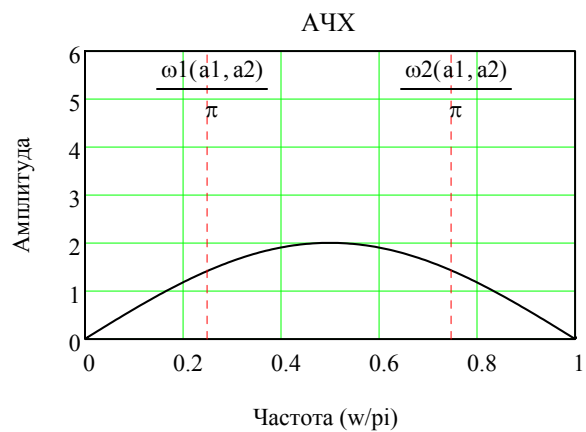


$a_1 := 1$   $a_2 := -1$   $\omega_1(a_1, a_2) = 0.659 \text{ rad}$   $\omega_p(a_1, a_2) = 1.571 \text{ rad}$   $\omega_2(a_1, a_2) = 2.483 \text{ rad}$   $Q(a_1, a_2) = 0.861$



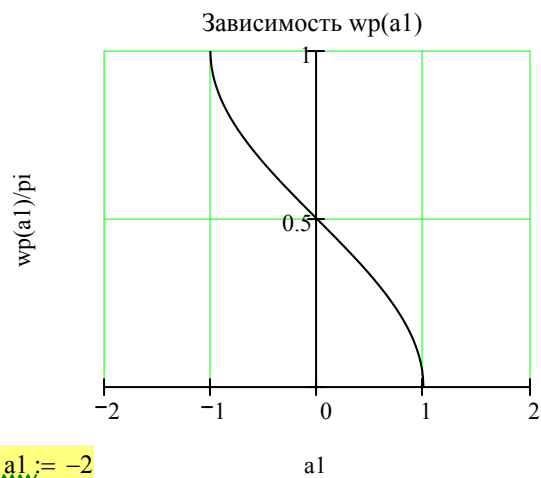
$a_1 := 0$   $a_2 := -1$   $\omega_1(a_1, a_2) = 0.785 \text{ rad}$   $\omega_p(a_1, a_2) = 1.571 \text{ rad}$   $\omega_2(a_1, a_2) = 2.356 \text{ rad}$   $Q(a_1, a_2) = 1$



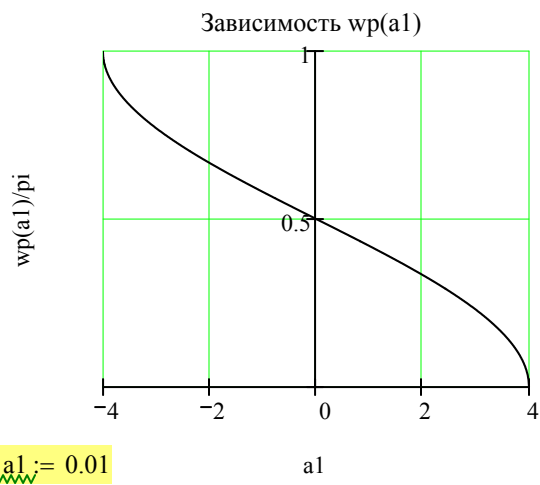


### Зависимости $w_p(a_1), w_p(a_2), Q(a_1), Q(a_2)$

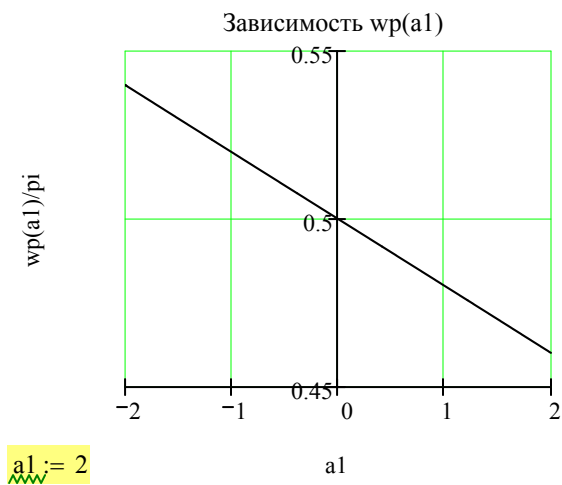
$a_2 := -0.2$

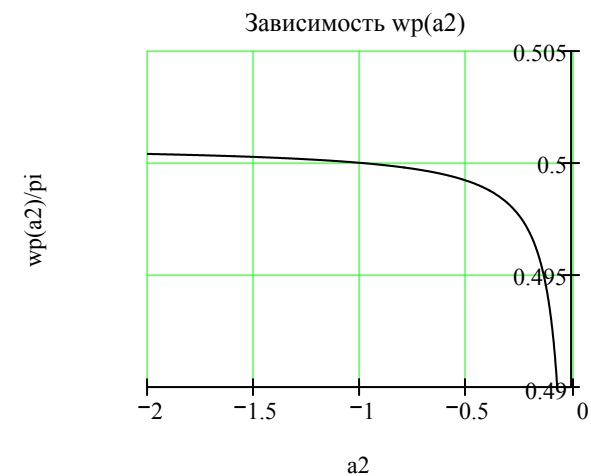
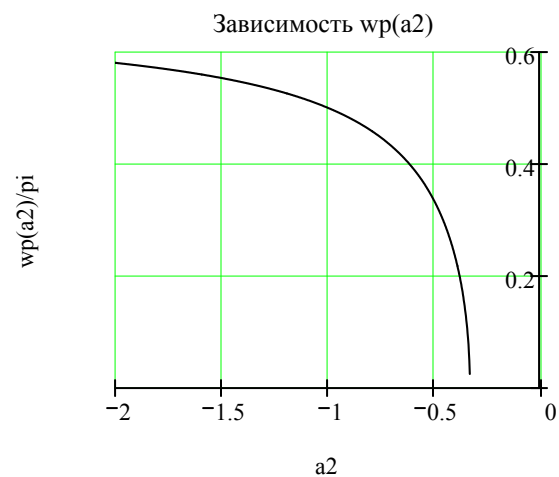
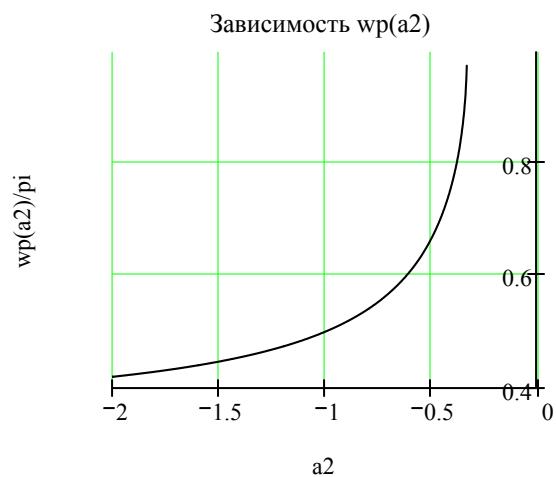


$a_2 := -0.5$



$a_2 := -0.8$

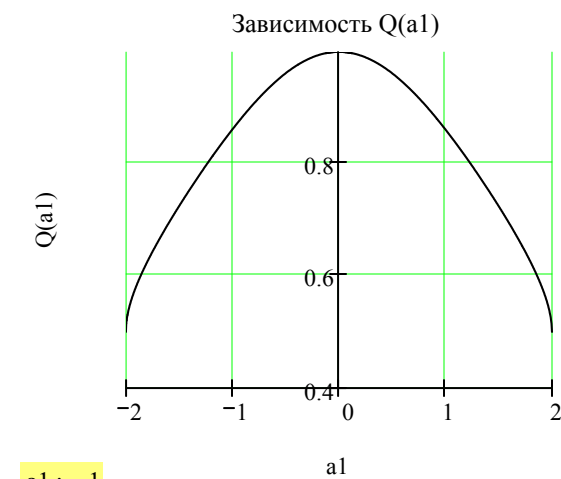
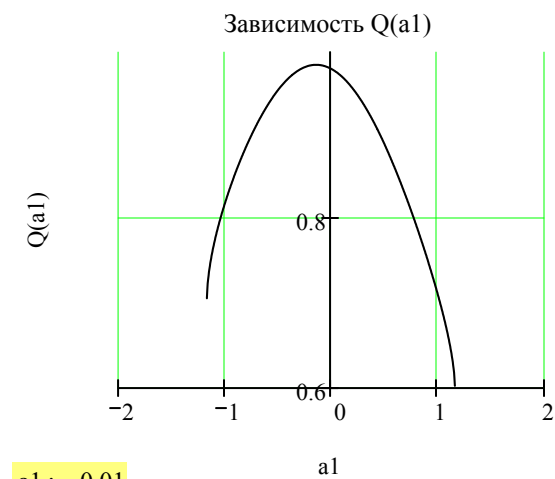
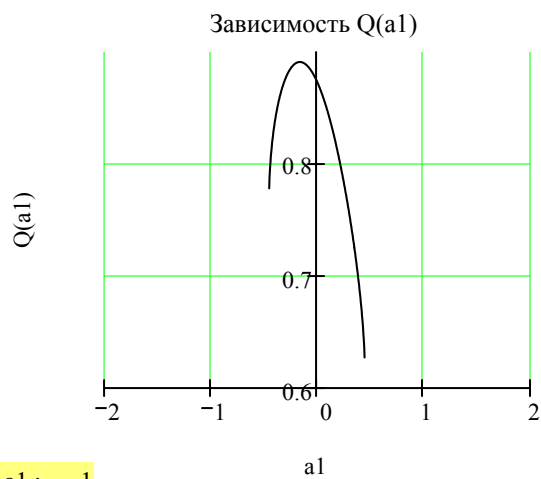




$a_2 := -0.4$

$a_2 := -0.7$

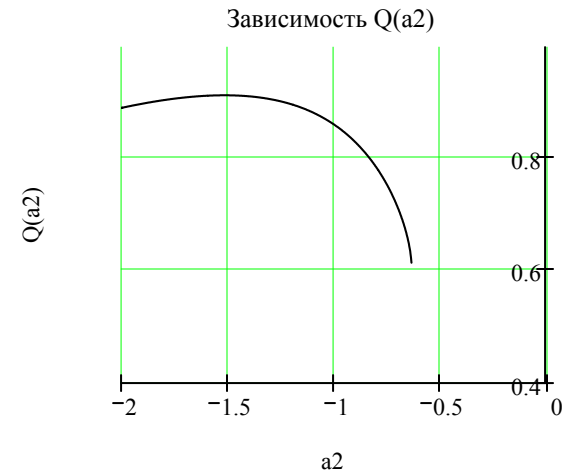
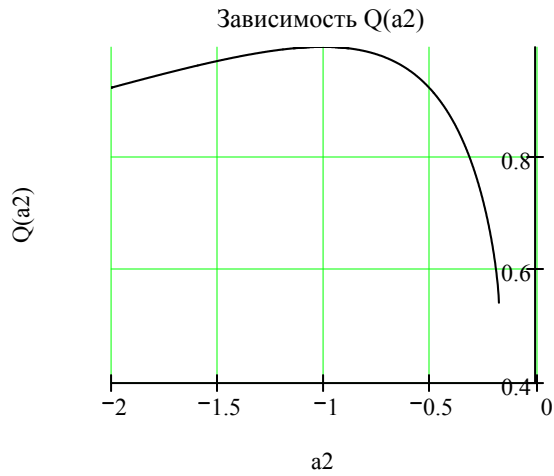
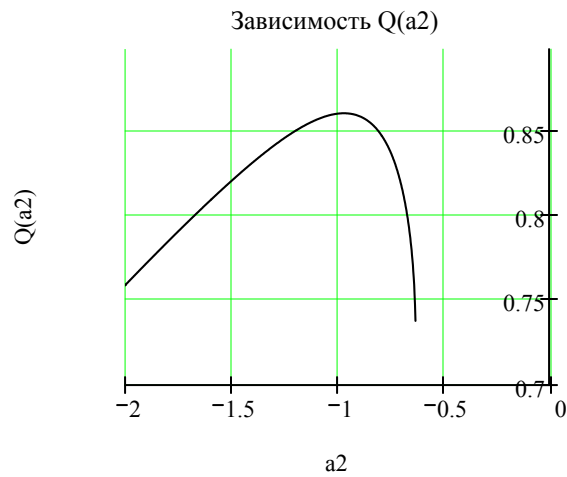
$a_2 := -1$



$a_1 := -1$

$a_1 := 0.01$

$a_1 := 1$



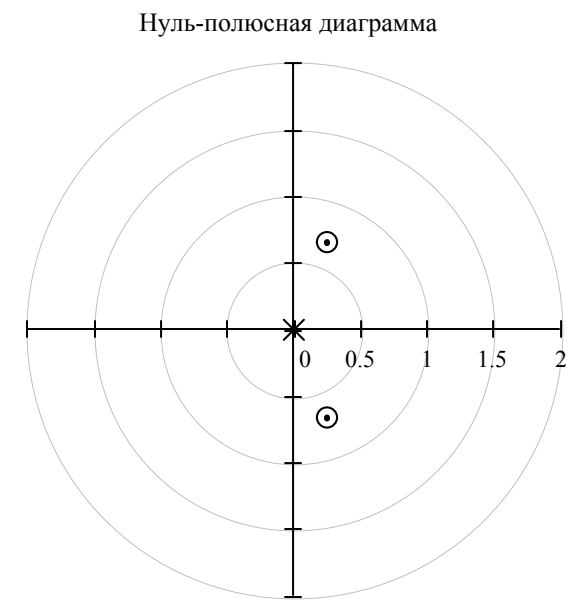
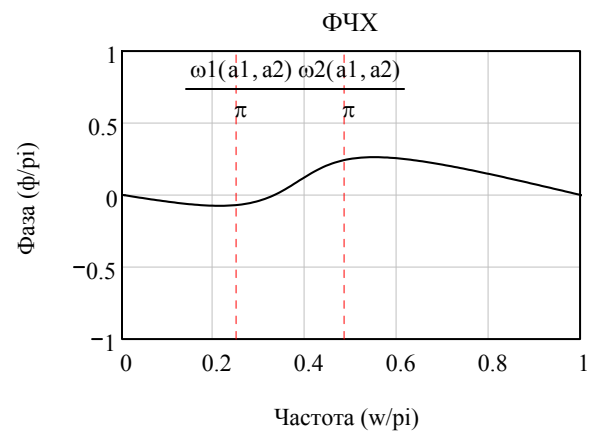
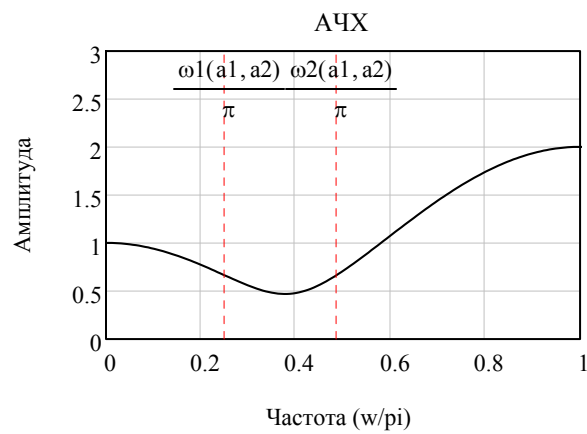
#### 1.4 Режекторный фильтр

$$b1(a1, a2) := \frac{-2 \cdot a1 \cdot (1 + a2) - \sqrt{[2 \cdot a1 \cdot (1 + a2)]^2 + 4 \cdot 4 \cdot a2 \cdot [a1^2 + (1 - a2)^2]}}{2 \cdot 4 \cdot a2}$$

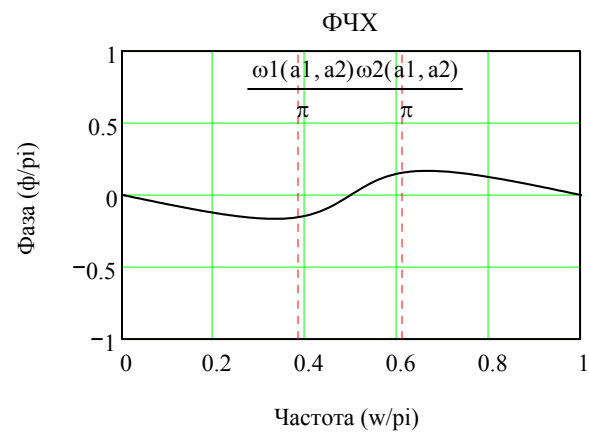
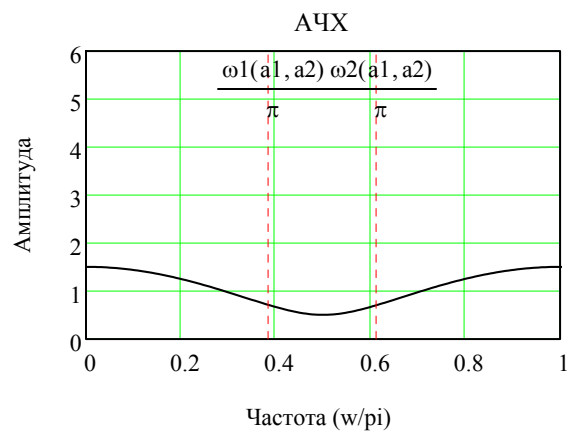
$$b2(a1, a2) := \frac{-2 \cdot a1 \cdot (1 + a2) + \sqrt{[2 \cdot a1 \cdot (1 + a2)]^2 + 4 \cdot 4 \cdot a2 \cdot [a1^2 + (1 - a2)^2]}}{2 \cdot 4 \cdot a2}$$

$$\omega p(a1, a2) := \arccos\left[\frac{-a1(1 + a2)}{4 \cdot a2}\right] \quad \omega 1(a1, a2) := \arccos(b2(a1, a2)) \quad \omega 2(a1, a2) := \arccos(b1(a1, a2)) \quad Q(a1, a2) := \frac{\omega p(a1, a2)}{\omega 2(a1, a2) - \omega 1(a1, a2)}$$

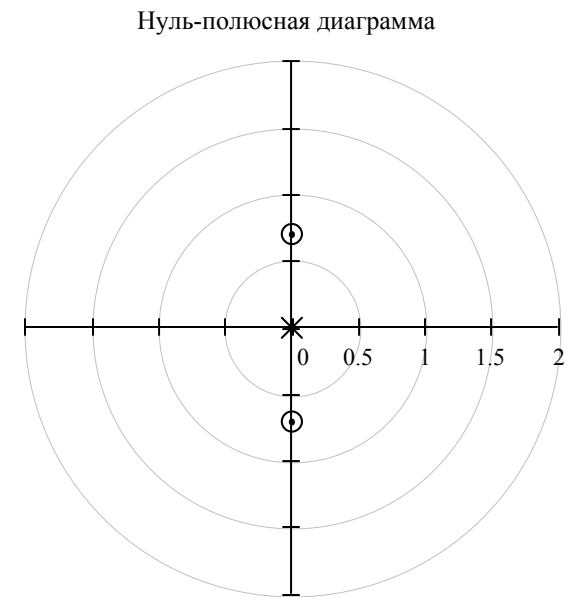
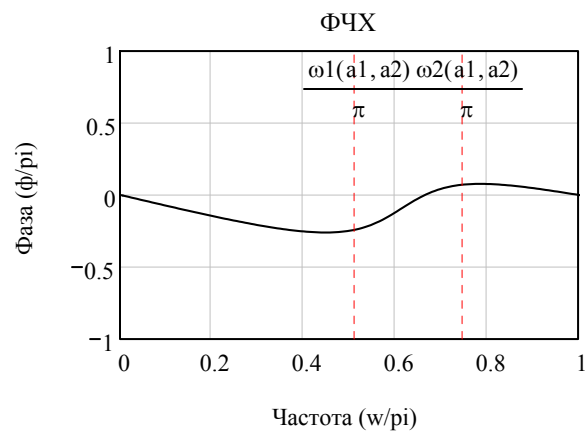
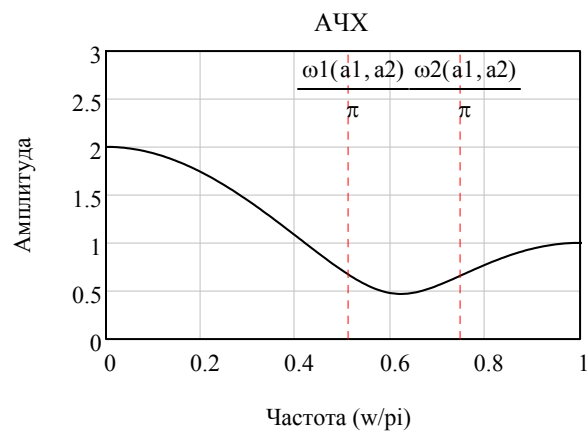
$$a1 := -0.5 \quad a2 := 0.5 \quad \omega 1(a1, a2) = 0.787 \text{ rad} \quad \omega p(a1, a2) = 1.186 \text{ rad} \quad \omega 2(a1, a2) = 1.527 \text{ rad} \quad Q(a1, a2) = 1.605$$



$a_1 := 0$   $a_2 := 0.5$   $\omega_1(a_1, a_2) = 1.209 \text{ rad}$   $\omega_p(a_1, a_2) = 1.571 \text{ rad}$   $\omega_2(a_1, a_2) = 1.932 \text{ rad}$   $Q(a_1, a_2) = 2.173$

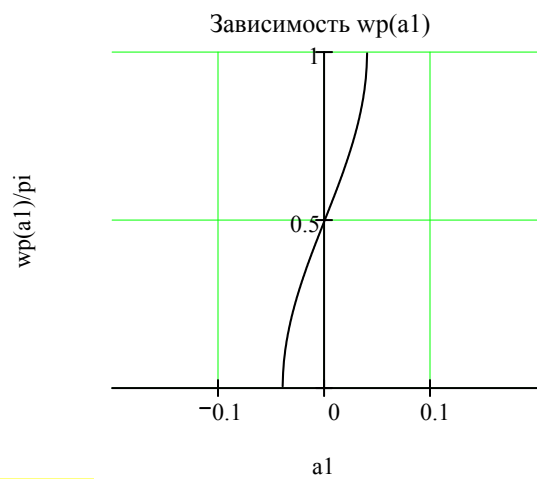


$a1 := 0.5$   $a2 := 0.5$   $\omega1(a1, a2) = 1.615 \text{ rad}$   $\omega p(a1, a2) = 1.955 \text{ rad}$   $\omega2(a1, a2) = 2.354 \text{ rad}$   $Q(a1, a2) = 2.645$



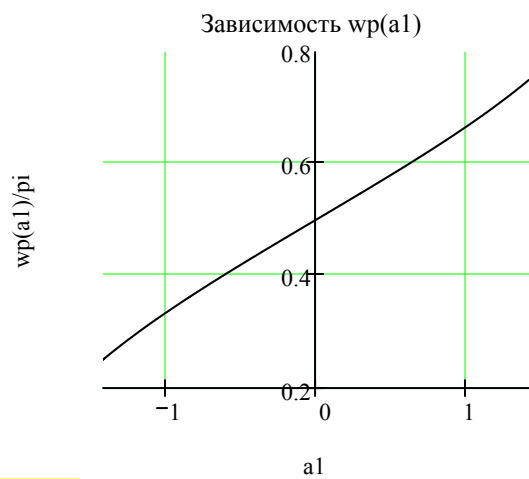
Зависимости  $\omega p(a1), \omega p(a2), Q(a1), Q(a2)$

$a2 := 0.01$



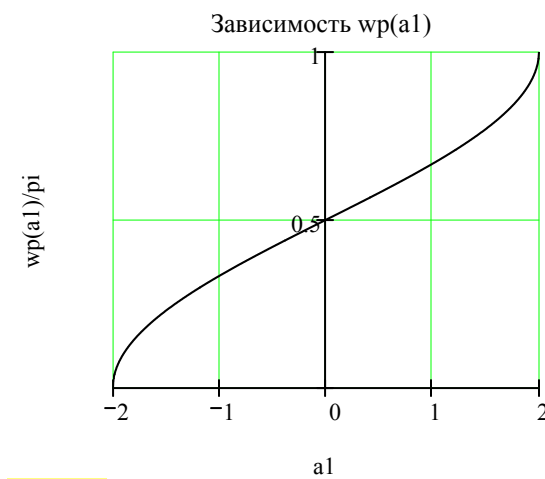
$a1 := -1$

$a2 := 0.5$

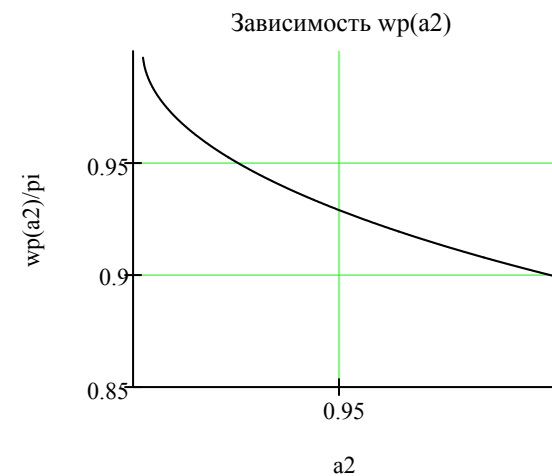
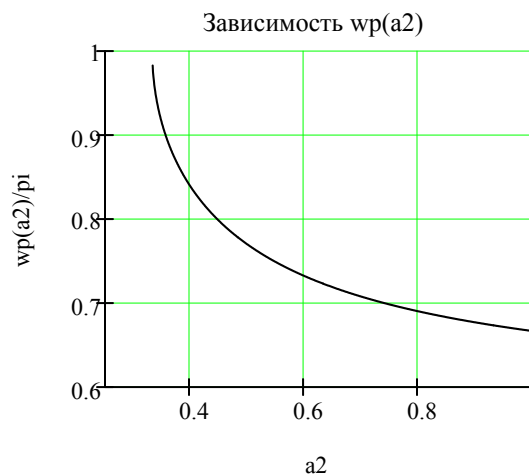
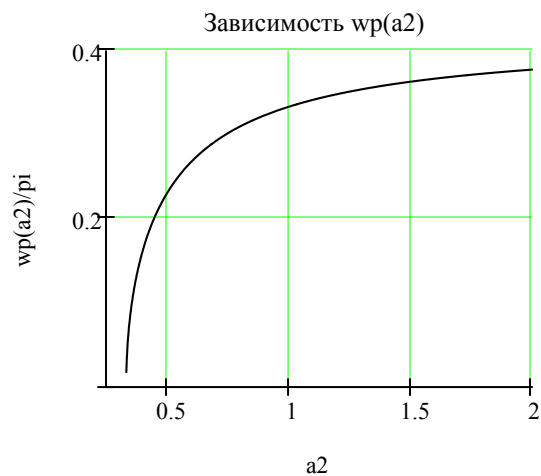


$a1 := 1$

$a2 := 1$



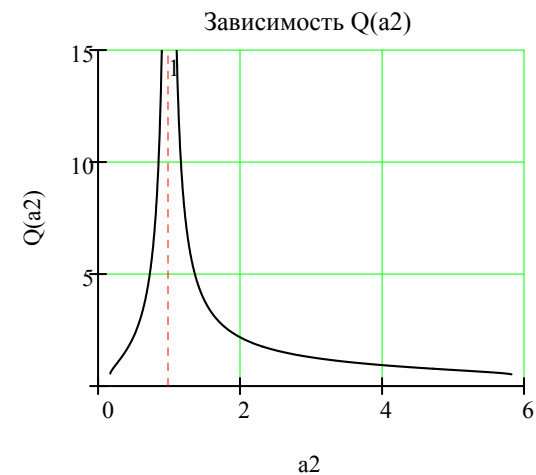
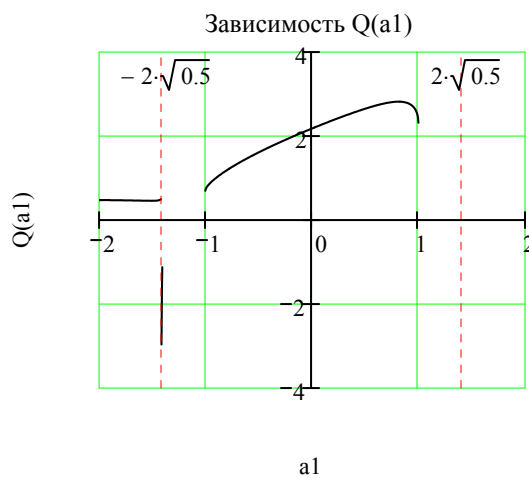
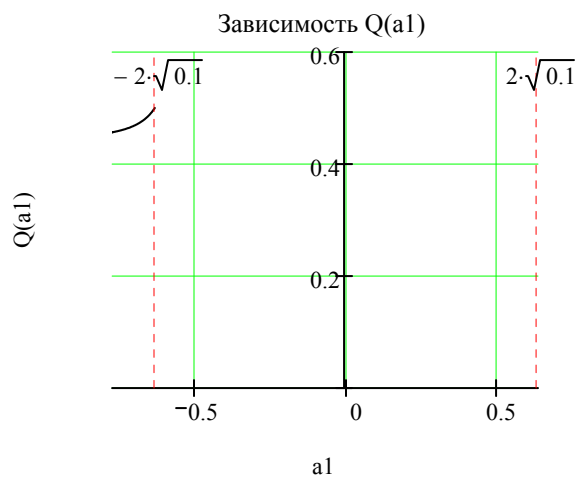
$a1 := 1.9$



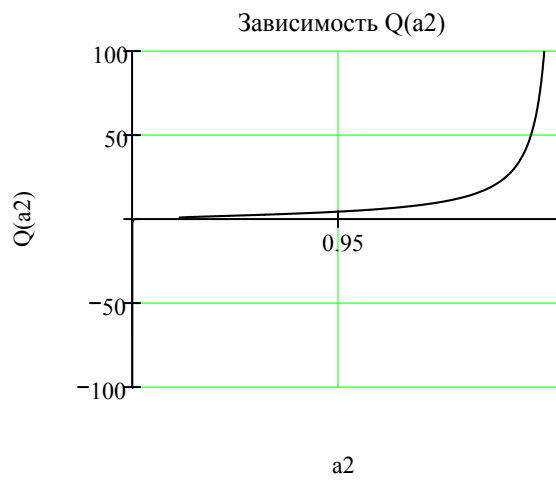
$a_2 := 0.1$

$a_2 := 0.5$

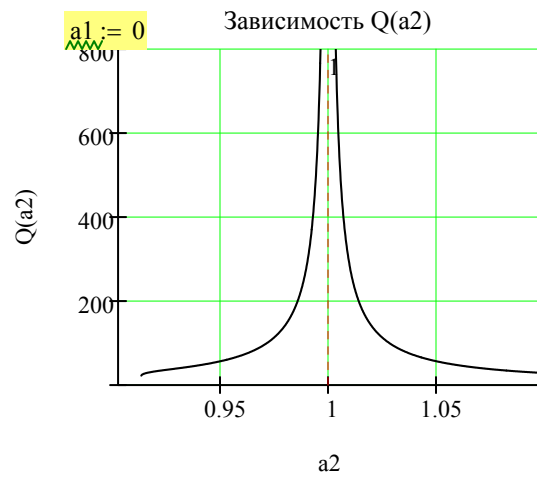
$a_2 := 0.99$



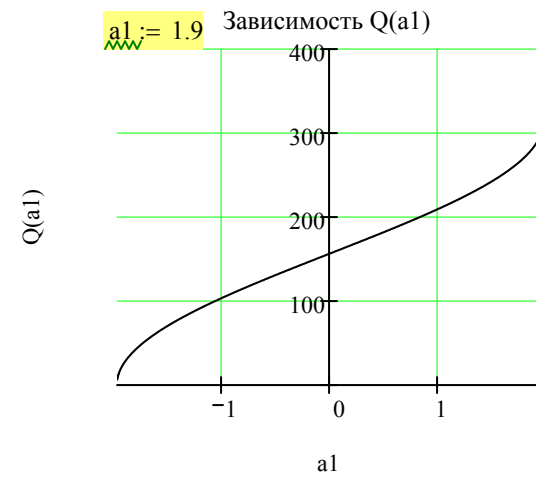
$a1 := -1.9$



$a1 := 0$



$a1 := 1.9$



## 2. Исследование временных характеристик

### 2.1. Фильтр нижних частот

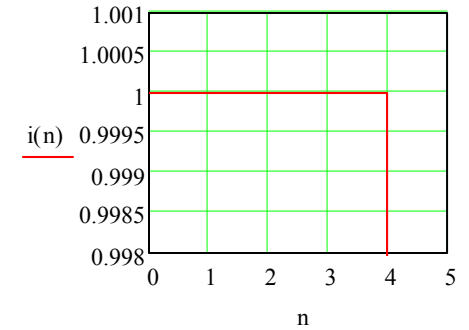
$$d(n) := \begin{cases} 0 & \text{if } n > 0 \\ 0 & \text{if } n < 0 \\ 1 & \text{otherwise} \end{cases}$$

$$p(n) := \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{otherwise} \end{cases}$$

$$i(n) := (p(n) - p(n - 4))$$

$$p(n, a1, a2) := p(n) + a1 \cdot p(n - 1) + a2 \cdot p(n - 2)$$

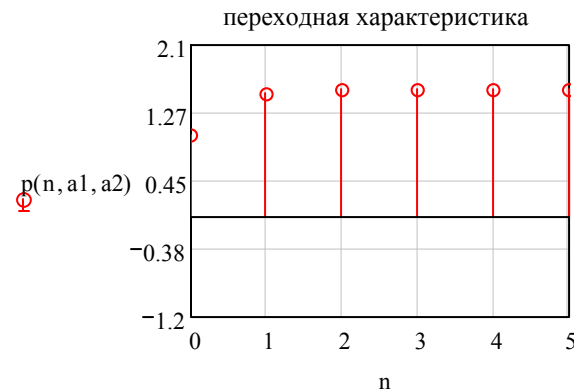
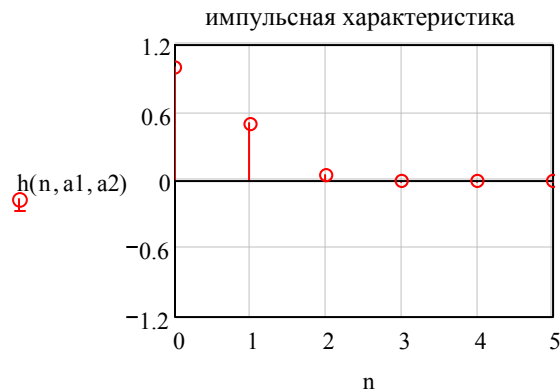
$$h(n, a1, a2) := d(n) + a1 \cdot d(n - 1) + a2 \cdot d(n - 2) \quad n := 0, 1 \dots 5$$



$$i(n, a1, a2) := i(n) + a1 \cdot i(n - 1) + a2 \cdot i(n - 2)$$

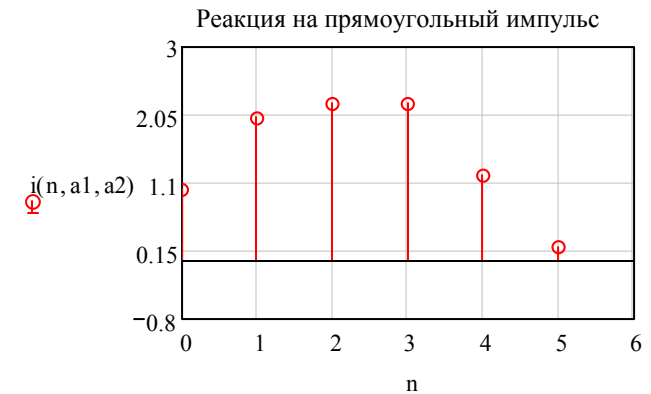
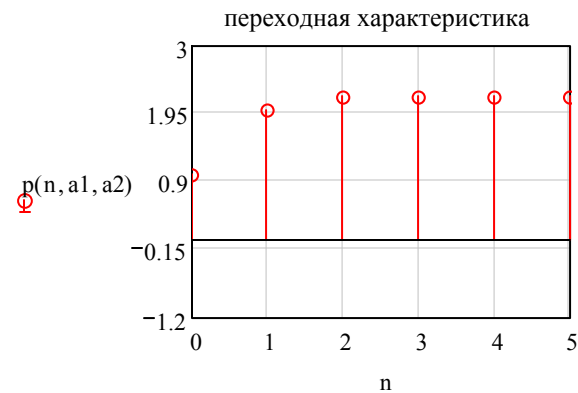
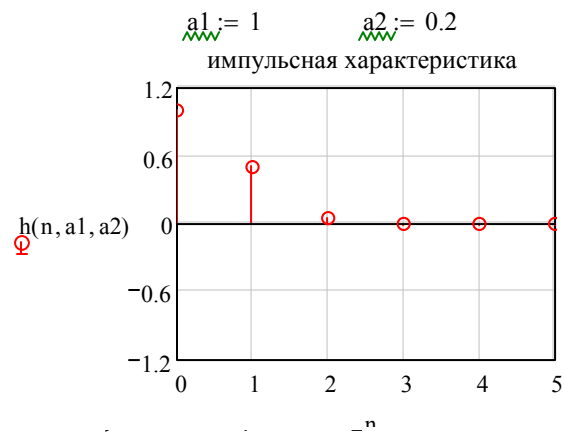
1)  $a_1=0.5, a_2=0.05$ ;

$$a1 := 0.5 \quad a2 := 0.05$$

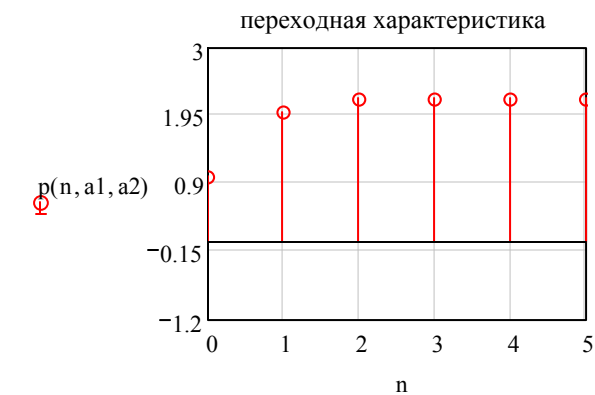
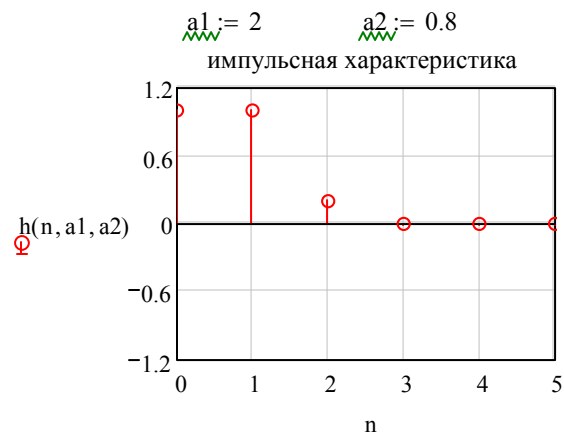


2)  $a_1=1, a_2=0.2$ ;

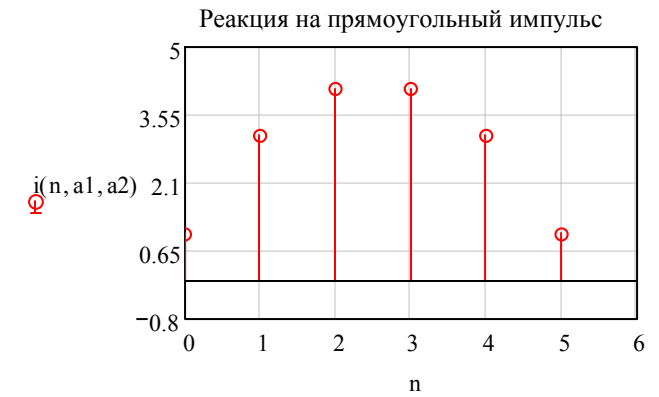
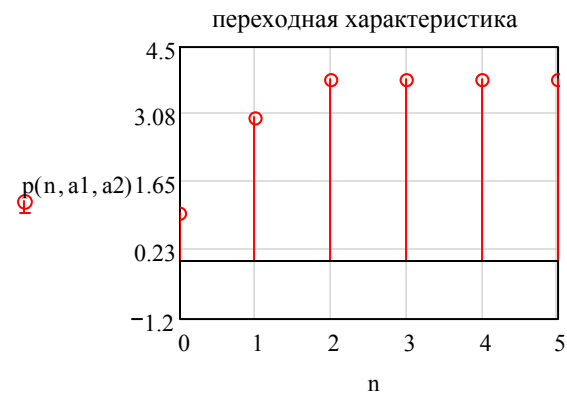
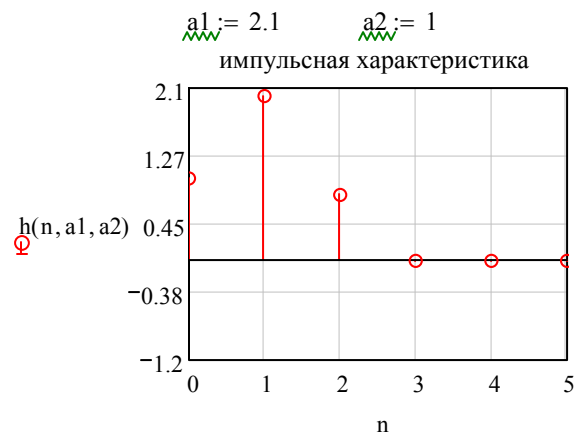




3)  $a_1=2$ ,  $a_2=0.8$ ;



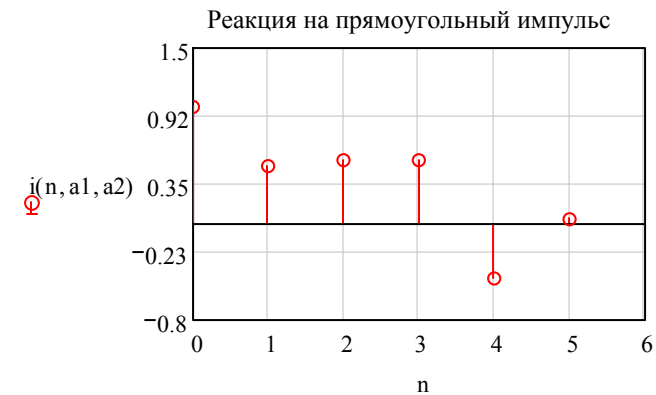
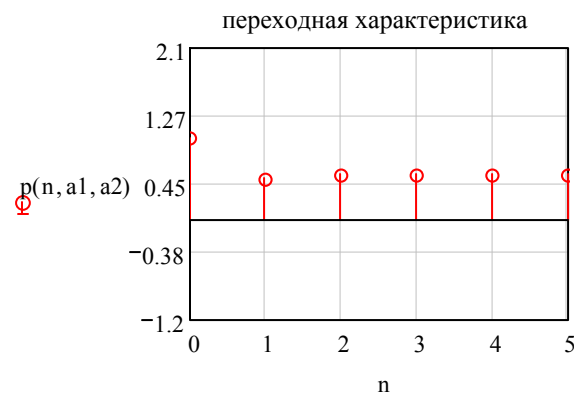
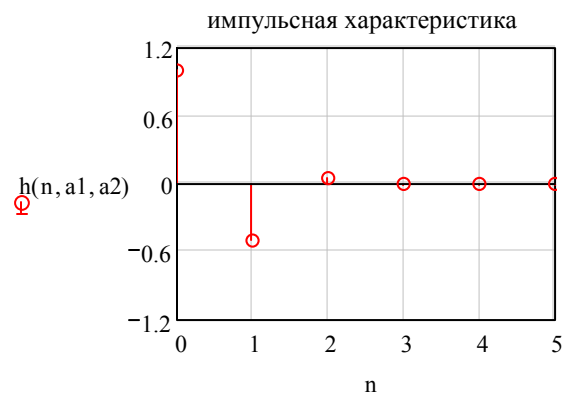
4)  $a_1=2.1$ ,  $a_2=1$ .



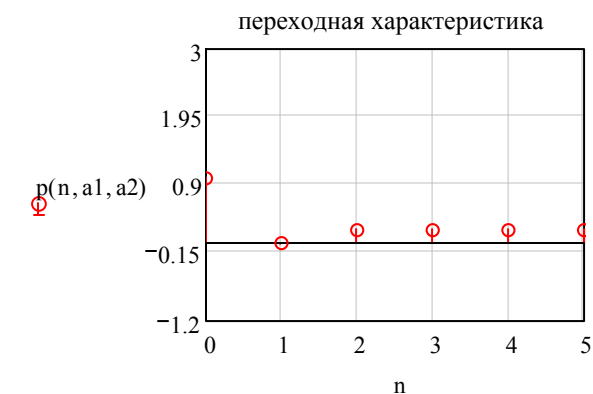
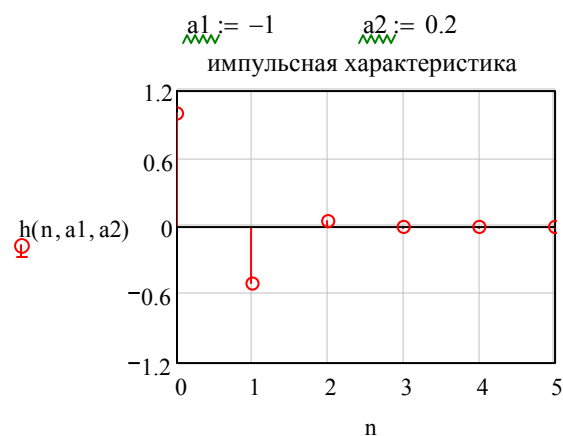
## 2.2. Фильтр верхних частот

1)  $a_1 = -0.5$ ,  $a_2 = 0.05$ ;

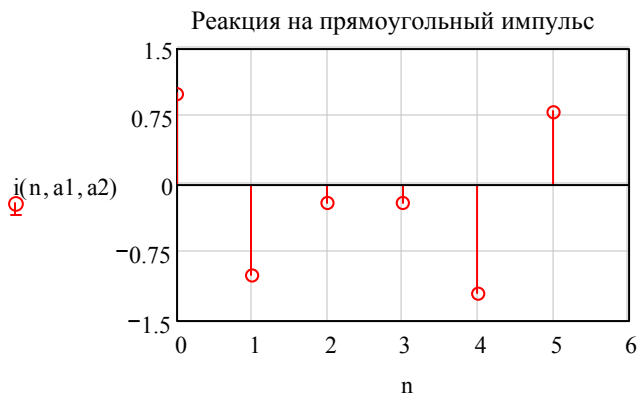
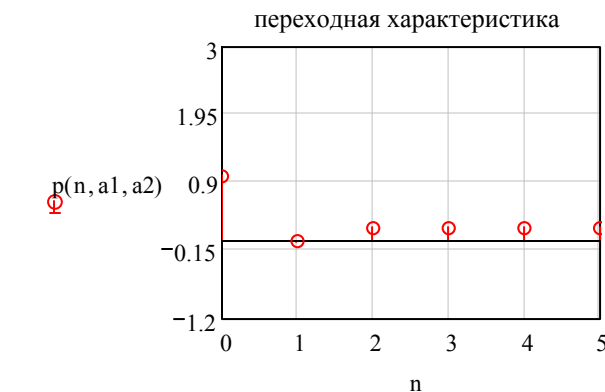
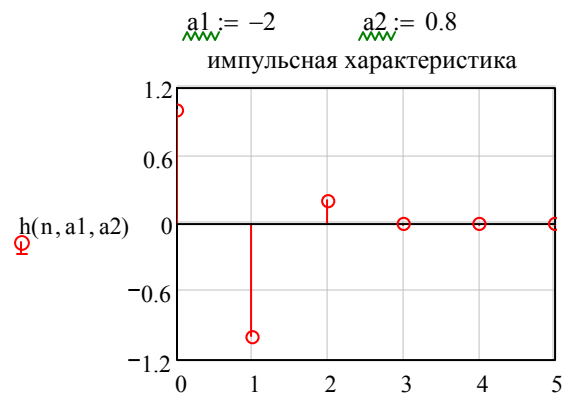
$a1 := -0.5$   $a2 := 0.05$



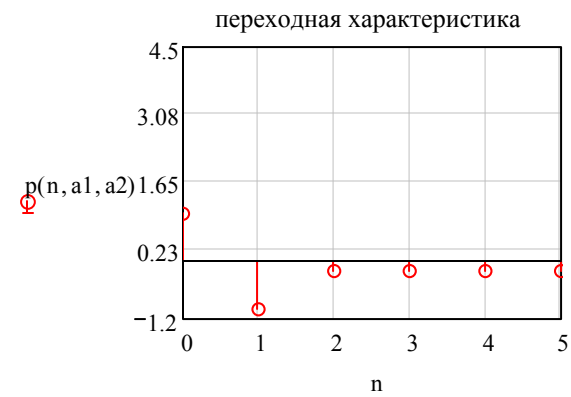
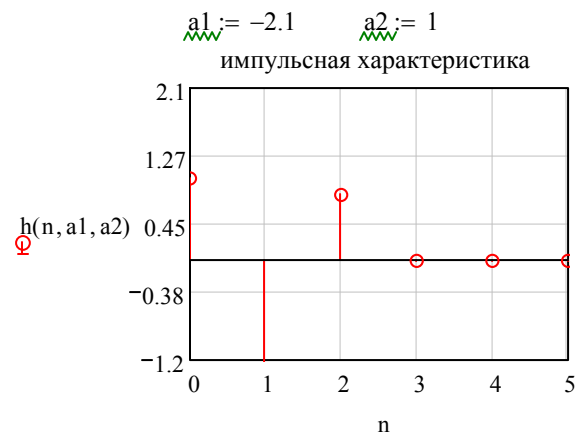
2)  $a_1 = -1$ ,  $a_2 = 0.2$ ;



3)  $a_1 = -2$ ,  $a_2 = 0.8$ ;



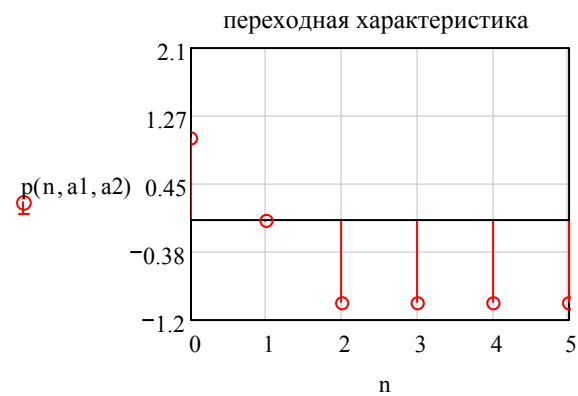
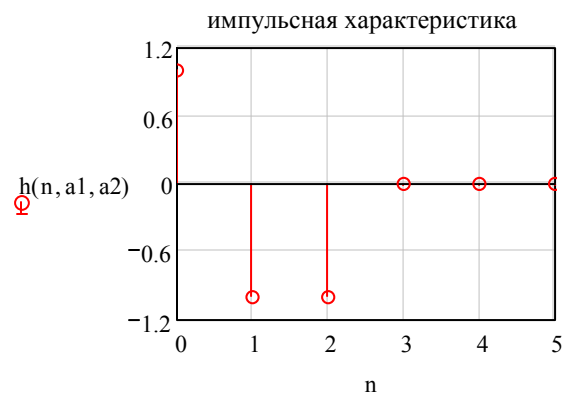
4)  $a_1 = -2.1$ ,  $a_2 = 1$ .



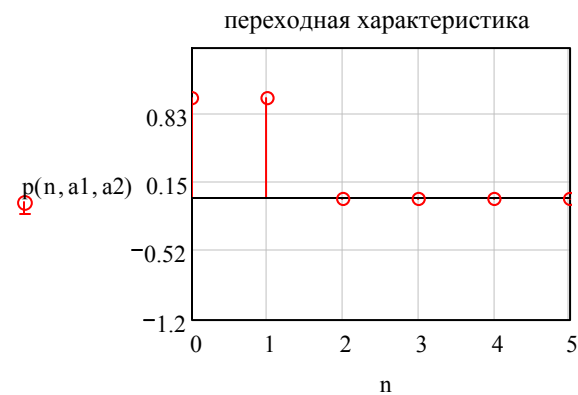
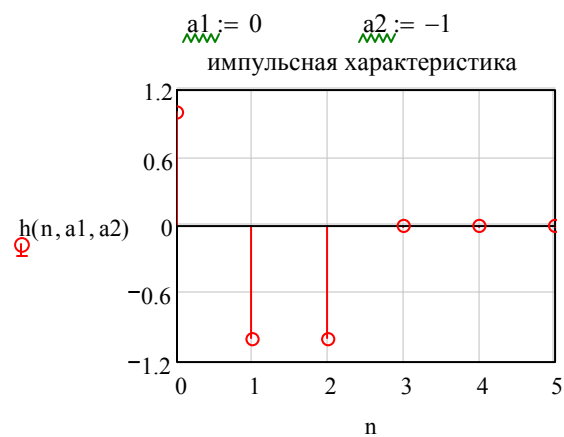
## 2.3. Полосовой фильтр

1)  $a_1 = -1$ ,  $a_2 = -1$ ;

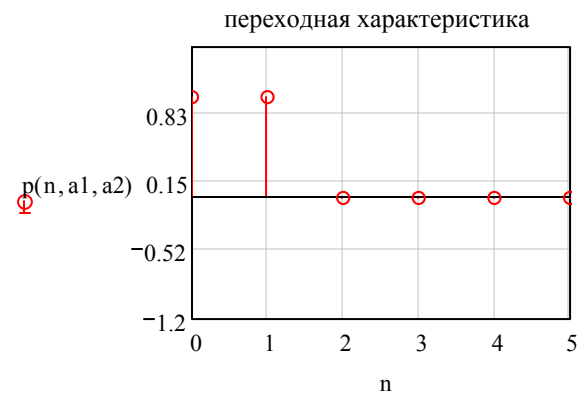
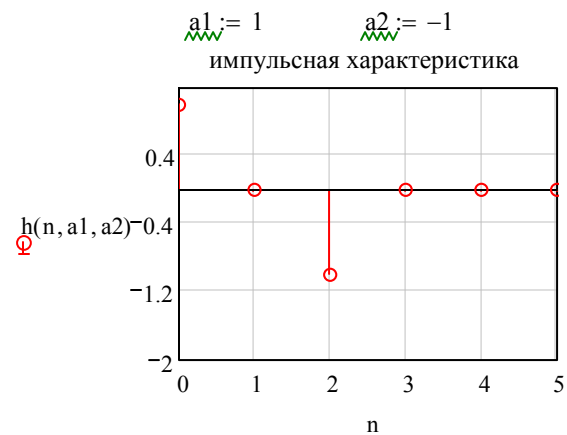
$a1 := -1$      $a2 := -1$



2)  $a_1=0, a_2=-1;$



3)  $a_1=1, a_2=-1.$



## 2.4. Резжесекторный фильтр

для  $a_1 \in \{-1; 0; 1\}$ ,  $a_2 \in \{0.75; 1\}$ ,

