MGSC 662 Group Project Report

COVID-19 Vaccine Allocation

Group 1: Vahid Hedley (261026933), Oleg Kartavtsev (261047411), Mohamad Khalili (260746712), Hirad Nourbakhsh (260715962)

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1 Introduction

As the global population rises and continues to expand at exponential rates, the supporting social and ecological systems requisite to manage and maintain this population seems to be unable to sustain the current rate of growth. There are a plethora of consequences that arise from humanity exceeding the carrying capacity of the earth's biological constraints, but one challenge that we are now much more aware of as an international community is the threat of future mass pandemics. The World Health Organization has stipulated that an influenza pandemic, along with other high-threat biological illnesses such as dengue and Ebola among the top 10 biggest threats to public health (WHO, 2021).

The coronavirus pandemic has highlighted the importance of preventing pulmonary infections and the efficacy of implementing national solutions centered around mass vaccination. Since the initial beginnings of the disease outbreak, there have been various policies employed ranging from measures involving social distance, border closure, remaining at home, to widespread testing which have collectively generated positive outcomes for reducing transmission (WHO, 2021).

There are currently over 100 companies investing in developing a COVID-19 vaccine across the globe, with many of them already moving forward to phase III of their clinical trials (Degeling et al., 2020). During the early stages of the pandemic demand for vaccines will far outweigh the available supply, which will then be regularized as firms experience economies of scale and cost centers become reduced for vaccine production. As such, the effective distribution of vaccines will be of great import moving forward as we look towards getting back to a life of normalcy. Within this context, we developed a mixed-integer linear programming model which aims to determine a strategy for finding the optimal allocation of the Pfizer vaccine across Canada subject to constraints ranging from budget considerations to shipping capacity for each distribution center. Real-world data is used to demonstrate the efficiency and effectiveness of the mathematical programming approach proposed in this investigation. This report will divulge the details of the problem formulation, solution, and generate inferences for other future model extensions which could add a greater dimension of depth to this problem.

2 Problem Description and Formulation

2.1 Literature Review

There is already an existing body of knowledge on various subtypes of problems with similar scopes attempted by different groups. To contextualize our solution, it may be prudent to examine the details of other linear programming applications employed in the similar space to understand the limitations and gaps within the current research and where we could fill some gaps. While the formulation and objective function may vary across these studies, there are underlying concepts and gleanings that can be used as reinforcement for proposals on possible problem extensions and further aid in providing a broader perspective for what has currently been done.

One group of researchers out of BMC Public Health generated an MIP model for re-designing a cold supply chain for distribution through observing location limitations and considerations. A hybrid heuristic algorithm was created for large-scale problem solving which was then validated using the data from various African countries (Straetemans et al., 2007). Elhedhli & Saif (2016) created a cold supply chain for the dissemination and stock management of vaccines while incorporating environmental considerations. To aid in their goal, they had proposed an MIP model using a Lagrangian decomposition algorithm to minimize total costs for all locales within their supply chain. Another research team was able to produce a stochastic inventory model for assessing vaccine supply for pediatric patients (Enayati & Özaltın, 2020). Within their formulation, they had attempted to examine the impact inventory of pediatric vaccines has to combat production disruptions within the USA. The results indicated that if the length of the disruption was greater than 6 months there would likely ensue shortages which would further speed up the transmission of the virus. As we can observe, there is a wealth of information and existing models with a range of research scopes and project outcomes. Elements of this information, alongside our own estimations and forecasting insights, will be further explored within *Section 5 – Problem Extensions* as we seek to enrich our result findings and build upon our initial conceptual framework.

2.2 Problem Formulation

Given the gravity of the problem and the wide scope within it is presented, we can begin to understand the vast implications that an effective vaccine distribution strategy would have on governments and planning agencies around the world. For our case study, we will be exploring the reality and intricacies involved in the dissemination of vaccines from a unary vaccine distribution lens within the local context of Canada. The results of the optimal solution would provide a vaccine roll-out strategy through observing the allocated amounts shipped from each distribution center to province. These results would provide answers to the following questions:

-What is the distribution schema that would ensure each group gets the vaccines they need, to **meet the coverage rate**?

- **How many vaccines does each province need** at this moment?

-What is the vaccine roll-out strategy that would ensure that each group receives the required vaccine doses **without going over budget**?

-Each distribution centre **has limitations when it comes to shipping capacity**. How many vaccines are going be shipped and from which distribution centre?

In the process of formulating the problem, there are also several assumptions that must be made to run the model and engender meaningful insights.

-This is a national strategy in which the federal government allocates vaccines according to levels of perceived demand by the provinces.

-The budget for the government is fixed prior to fluctuations in demand caused by the prevalence of a rise or rapid spike in cases.

-The location of the distribution centers is fixed across several cities within Canada. The location of these cities was set to maximize the geographical coverage that each of these locales may offer its respective citizens.

-Individuals are only to be vaccinated once and there will only be one vaccine offered to them.

-The approximation that around 25 percent of people will have a pre-existing condition across all group types.

With the above points in mind, we can now proceed to get a better understanding of the various components within our problem formulation.

2.2.1 Indices

Table 1: Indexes

d:	Distribution Center
p:	Province
g:	Group

2.2.2 Model Parameters

Table 2: Model Parameters

Dgp	Demand of group g in province p
PC	Purchasing cost of one dose of vaccine
TC$d,p$	Transportation costs from distribution center d to province p
ϕg	Coverage rate: % of the group g we aim to vaccinate
βd	Vaccine shipping capacity of distribution center d
B:	Available Budget

2.2.3 Decision Variables

Table 3: Decision Variables

x*gp:*	Number of doses of vaccine allocated to group g of province p
s*dp:*	Number of doses of vaccine shipped from distribution center d to province p

2.2.4 Objective Function

To achieve the goal of ensuring that all group types across the various distribution centers and provinces receive their dose of vaccine, our objective function seeks to maximize the minimum ratio between the total amount of allocated vaccine doses and the demand. This can be represented by defining function Max f equal to our minimization optimization as the following:

$$\text{Max} f = \text{minimum}(\frac{x_{\text{gp}}}{D_{\text{gp}}}) \tag{1}$$

As we can see, this objective function is non-linear, and we can linearize by introducing a new free variable term into the objective function. After this conversion is complete, we are then left with the following:

$$\text{Max}\{\text{f} = \mu\} \tag{2}$$

With such an approach, there will be an additional constraint that is generated to ensure this transformation is accurately captured which will be further discussed in *Section 2.2.5.*

2.2.5 Constraints

Meet the coverage rate:

$$x_{gp} \geq \phi_g D_{gp}$$

Respect the shipping capacity of the distribution center:

$$\sum_{p=1}^{13} s_{dp} \leq \beta_d$$

For all distribution centers d

Cannot exceed the federal budget allocation for provinces:

$$\sum_{p=1;d=1}^{13;6} TC_{dp} \times s_{dp} + \sum_{p=1;d=1}^{13;6} PC \times s_{dp} \leq B$$

We cannot allocate more vaccines than the demand:

$$x_{gp} \leq D_{gp}$$

Constraint that arises from the linearizing the objective function:

$$\mu \leq (\frac{x_{gp}}{D_{gp}})$$

3 Data

Coronavirus vaccines have only recently been discovered, and as such, veritable data on their distribution and logistics is not yet fully accessible. In this section we will go over the data sources used to create the model and discuss the logic behind selecting certain features over others. To begin, the entire Canadian population was broken up into 8 groups. The order and number value assigned to each group is not indicative of any potential order of preference but rather used to segment a smaller subset of the population to be able to produce results that internalize the principle of equity for the optimal distribution strategy. The breakdown outlining our methodology can be followed as such:

Group 1: Infants between the age of 6 and 48 months

Group 2: Pregnant women with pre-existing medical conditions

Group 3: Adults aged 65 and over with pre-existing medical conditions

Group 4: Physicians & nurses

Group 5: Pregnant women without pre-existing medical conditions

Group 6: Adults aged 65 and over without pre-existing medical conditions

Group 7: Adults between the ages of 18 and 64 with pre-existing medical conditions

Group 8: Adults between the ages of 18 and 64 without pre-existing medical conditions

For a more detailed breakdown of the demand for vaccines as a function of the provinces and groups, refer to Fig 13. Information on the Canadian population was obtained from Statistics Canada and data pertaining to the number of physicians per province was obtained from the Canadian Institute of Health Information (CIHI). In aggregating the data, several assumptions were made which allowed us to further dichotomize the information into parcels that we could work with and create significant inferences regarding vaccine sourcing strategies. These assumptions include:

-27% of all Canadian adults between the ages of 18 and 64 have pre-existing medical conditions

-40% of all Canadian adults above the age of 65 have a pre-existing medical condition

-The number of pregnant women in 2021 was supposed equal to the number of births in 2020

-Statistically, in Canada, there are about 6 nurses for 1 physician.

Other important pieces of data include the following, much of which was gathered from government websites or simulated based on distribution models.

-Distribution Centers: Montreal, Calgary, Toronto, Vancouver, Saskatoon, Winnipeg

The logic for establishing these locations as the central distribution centers was mainly predicated on the fact that they serve as the most central regions within their respective provinces and have the capacity and infrastructure to manage high loads of vaccine shipments.

-Shipping Capacities: How many vaccines can a distribution center ship?

-Budget: How much money do we have at our disposal to buy and ship vaccines to the vaccination centers across Canada?

-Transportation costs: How much money does it cost to ship our vaccines?

For transportation costs, we determined the total distance from each of the distribution center to and from the other distribution center location and then normalized the values into workable values with more reasonable ranges.

-Purchasing costs: How much does one dose of vaccine cost?

4 Numerical Implementation and Results

The following section explains in detail how the problem was implemented. Please refer to Appendix A for screenshots of the Gurobi code.

With the problem being mathematically formulated, and the necessary data at our disposal, the problem can now be numerically implemented. The numerical implementation of this problem has been done using Python, and the optimization was done using Python's module Gurobi.First, we begin by importing the necessary packages and libraries. Pandas allows us to import data and to manipulate dataframes. Numpy allows us to create arrays and perform other operations using 2D arrays. Please refer to Appendix - Figure 1 for a screenshot.

Next, we import the data that we have. The first dataset to import contains the transportation costs (CAD/dose) from each distribution center, to each province. The "Province" column has been dropped because it isn't necessary to the analysis: an index is attributed to each province. Please refer to Appendix A - Figure 2 for a screenshot.

The second dataset to import contains the demand for vaccines for each group in each province. It is imported similarly to the transportation costs dataset. Please refer to Appendix A - Figure 3 for a screenshot.

Now that the datasets are imported into Python, we create the other parameters and some useful variables pertaining to the problem, such as the coverage rates (minimal % of the demand we must satisfy), the purchasing cost of one dose of vaccine, the location of each distribution centre, their shipping capacity as well as each distribution centre's budget. Please refer to Appendix A - Figure 4 for a screenshot.

We can now hop into the optimization part. First, we begin by defining the problem in Gurobni. Please refer to Appendix A - Figure 5 for a screenshot.

Then, we define our decision variables. Let us remember that x stands for the amount of doses that have been **allocated** to each group in each province. For instance, x[1,2] stands for the amount of doses that have been allocated to pregnant women with a pre-existing medical condition (Group 2) in Ontario (Province #1). As such, a set of 13*8 variables with a lower bound of 0 has been created using addVars, since we took into consideration 13 provinces (10 provinces + 3 territories), with 8 population groups per province. Since x represents a vaccine quantity, it is an integer variable (we cannot allocate a fraction of a vaccine).

As for s, it stands for the amount of doses **to ship** to a province from a specific distribution centre. For instance, s[1,2] stands for the amount of doses that have to be shipped to Ontario (Province #1) from Toronto (Distribution centre #2). As such, a set of 13*6 variables with a lower bound of 0 has been created using addVars, since we took into consideration 13 provinces, served by 6 distribution centres across Canada. Since s represents a vaccine quantity, it is an integer variable (we cannot ship a fraction of a vaccine)

Finally, µ is the variable we created to linearize the objective function. It has a lower bound of 0, since the lowest possible count of allocated vaccines is 0. Please refer to Appendix A - Figure 6 for a screenshot of how the decision variables have been defined.

Now that we have the decision variables, we declare the objective function. As we've shown in the mathematical formulation, the objective function will be defined with µ. Since it's a maximization problem, we use GRB.MAXIMIZE to define the problem as a maximization problem.

Next, we define the constraints necessary to the problem. Constraint 0 sets a maximum to the amount of doses that can be allocated: we cannot allocate more doses than the demand. Constraint 1 defines µ according to the change of variables that was made to linearize the objective function. Constraint 2 ensures that the allocated vaccines meet the coverage rate for each group in each province. As for Constraint 3 ensures that the total amount of shipped vaccines from each distribution centre does not exceed the shipping capacity of that same distribution centre. Constraint 4 sets a maximum to the money that can be spent: the total cost of buying the vaccine doses and transporting them cannot exceed the budget. Finally, Constraint 5 ensures that we do not ship more vaccines than the amount allocated. Please refer to Appendix A - Figure 8 for a screenshot of how the constraints have been defined. The problem is now formulated, and we use prob.optimize() to solve the maximization problem, under the defined constraints.

Unfortunately, the current model returns an error when running the code. As such, a numerical solution is not available. However, the answer to this problem would provide many valuable insights. The solution would provide a vaccine roll-out strategy, to supply the necessary dose count to each province. Simply put, in theory, the model should tell us how many doses of vaccines must be shipped to each province, from each distribution centre. The solution given provides a vaccine distribution strategy respecting the global budget, as well as the shipping capacities of each distribution centre. It is worth noting that the model **tells us how many vaccines can be shipped for one shipping period**: shipping periods can be daily, weekly, or monthly. In the previous code, the shipping capacity can be chosen to be on a weekly basis, for example. As such, the results this model gives us would allow us to know how much time would be required to complete a vaccination campaign that would ensure coverage rates are met. However, the model is not perfect. First, the data used for the shipping capacities, as well as the budgets, is hypothetical. It is not real data and could therefore hinder the performance of the model and the veracity of the results. Next, many considerations have been left out to simplify the problem. As one can expect, this can be an extremely complicated supply-chain problem, with various implications that have not been considered in the current model. For instance, COVID19 vaccines need to be stored at a specific temperature, and must be used within a specific period of time.

5 Problem extensions

While the results gathered from our research proposal and optimization model were interesting by themselves, there are several key areas in which a deeper exploration of the topic could add valuable insights from stakeholders ranging from policymakers to non-profit community-based groups. The first and most obvious extension would be to include several vaccine types. Each vaccine has its own respective costs, refrigeration requirements, and levels of efficacy. With this additional consideration, there would be a range of new constraints and decision variables, such as ensuring that a particular distribution center should only be set up for vaccines which require a certain type of cold storage. For many vaccines there are a few gradients of cold storage requirement, and in the literature on vaccine logistics and supply chain management it is widely referenced that vaccines fall into either cold, very cold, or ultra-cold storage. Incorporating metrics for different storage temperatures among multiple vaccines would necessitate creating new constraints. For example, if we want to take into consideration building storage facilities capable of receiving cold and very cold vaccine doses, the budget constraint would be modelled as follows:

$$\sum_{p=1;d=1}^{13;D} TC_{dp} \times s_{dp} + \sum_{p=1;d=1}^{13;D} \times PCs_{dp} + \sum_{d=1}^{D} SC_{d,cold} \times \omega_d + \sum_{d=1}^{D} SC_{d,verycold} \times \omega_d \leq B$$

Where: SC*d,cold* and SC*d,very cold* are the costs to set-up a distribution centre d with cold or very cold refrigeration amenities, and wd is a binary variable representing whether the distribution center d is already set up (0) or not set up (1). This constraint is a modification to the previous budget constraint, where we now account for the cost of setting up cold or very cold storage centres for vaccines that require them. Please refer to Appendix A - Figures 10 to 12 to see how the constraint has been implemented in Gurobi.

Alongside having a distribution center equipped for handling cold vaccine storage, there is also the matter of ensuring that the transportation for said vaccine meets the cold storage requirement which would naturally increase the cost center for freight services and impose an additional requirement for the model. If one were wanting to get to a very granular level, one could also consider creating conditions in the model in which certain higher risk group types are delivered vaccine types with a higher overall rate of efficacy to increase their chance of survival. This detail is more of a privilege to incorporate as many countries around the world are still very behind in the vaccination campaign but is still a component that might be worth considering.

Another direction our base model could have expanded is to consider the setting up of a temporary facility in the case of shortages or overcapacity for the existing distribution centers. Within the same line of thinking, there could also be considerations around determining the optimal location for facilities across the provinces. As mentioned earlier, the cold storage requirements for each distribution center should also reflect with vaccines that are distributed to those locations. If we were to take things even one step further, we could also consider the costs of shipping vaccines from the manufacturer to the distribution center. Combining information for all stages of the supply chain could aid enriching our understanding of how optimal solutions could be manipulated the further down we go in the system. In most applications of MIP in the real world, the objective function comes down to minimizing cost and maximizing profit (Karimi, 2021). An interesting lens to this problem contrary to the predominating paradigm could involve creating a model which minimizes greenhouse gas emissions for the distribution of vaccines.

One aspect that was missing from our analysis was the inclusion of temporal considerations for vaccine delivery and scheduling. A natural problem extension would involve thinking about vaccine roll out in a sequential manner, with certain doses being delivered at a particular time and date to create greater efficiencies at all stages of the supply chain. This element would also ensure that there is not over or under allocation of vaccines and that there is a consistent stock of inventory for when we experience a potential rise in demand. On top of this, scheduling applications could also ensure that individuals who had received their first dose have their second dose ready at the distribution center in which they had first gotten their dose. This would add a great amount of complexity to the problem but would also create an interesting framework for modelling an approach to maximize total vaccinations across the country.

A sensitivity analysis could also be created in several budgeting scenarios can be employed to ascertain the performance and behavior of the model. One could infer that the total vaccine doses are expected to rise by increasing the budget and decrease by decreasing the budget. Manipulating these values with respect to actual government budgets could allow researchers to determine the upper and lower bounds for vaccine offerings.

6 Recommendations and Conclusions

The vaccine allocation problem can easily become complex if we take into account all of the possible variables affecting the processes surrounding its distribution. In the case of our project, we learned that it is crucial to correctly pinpoint the constraints that identify the lower and upper bounds of the optimization. Otherwise, the problem will become unbounded. It is also important to ensure that constraints are not contradictory, otherwise the problem will become infeasible.

Furthermore, we learned that it is important to keep in mind the additional constraints that have to be implemented in case of new decision variables or conditions we begin to consider. For instance, we had to decide against the addition of more distribution centers for the rest of Canada because it would involve developing potentially problematic constraints regarding the Euclidean distance and including it into the present model, while ensuring that they provide realistic and accurate optimal solution.

Additionally, we believe that another major issue for the vaccine allocation problem is finding the accurate data regarding the distribution centers' location and transportation costs, since some of this information might not be easily accessible or be private. In the scope of the COVID-19 vaccine, it proved extremely challenging to find realistic locations of the actual distribution centers (in addition to the constraints that need to be implemented after finding this data), because their numbers and geographic locations are constantly changing.

Another point to consider is the bottleneck that might possibly occur if we decide to update the model with more types of vaccines (discussed in problem extensions) and distribution centers, since it would result in thousands of new variables. This number will only increase with the approval of the additional vaccines (for example, the upcoming Omicron-focused Moderna vaccine) and new distribution centers.

To sum up, the overall recommendation would be to pay additional consideration to the scope of the project, especially regarding the additional distribution aspects that would result in additional data needs and the development of additional constraint that will ensure that Gurobi does not end up unbounded or presents an unrealistic solution. In other words, there needs to be a trade-off between the information that is being fed into the optimization solver and the output received.

A similar model could be developed by governments to plan an optimal vaccine distribution schema, when the demand highly surpasses the available supply.

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8 Appendix

Figure 1: Importing the packages

```
#Importing packages and libraries:
import gurobipy as gp
from gurobipy import *
import pandas as pd
import numpy as np
```

Figure 2: Importing Transportation Costs

```
#Importing the data: Importing transportation costs
transportation_costs = pd.read_excel('TransportationCosts.xlsx')
transportation_costs = transportation_costs.drop(columns = 'Province')
transportation_costs
tca = np.asarray(transportation_costs)
transportation_costs
```

	Montreal	Toronto	Winnipeg	Saskatoon	Calgary	Vancouver
0	0.55	0.20	0.55	1.75	2.00	2.75
1	0.20	0.35	2.00	2.00	2.50	3.20
2	0.40	0.55	2.50	2.50	2.75	3.50
3	0.40	0.50	2.25	2.25	2.75	3.75
4	0.80	0.65	0.20	0.50	1.25	1.75
5	3.00	2.80	1.75	0.75	0.50	0.20
6	0.75	1.55	2.50	2.60	2.25	3.00
7	2.00	2.00	0.50	0.20	0.50	1.25
8	2.25	2.25	0.75	0.50	0.20	0.50
9	0.30	1.90	2.25	2.80	2.50	2.75
10	2.50	2.35	1.25	0.70	1.25	1.50
11	3.25	3.10	1.75	1.25	1.25	0.75
12	2.90	2.70	1.50	1.25	1.25	1.00

Figure 3: Importing the demand for each group

```
#Importing the data: Importing the demand for vaccines of each province
dem = pd.read_excel('Demand.xlsx', index_col = 1)
dem = dem.drop(columns = 'Province')
demand = np.asarray(dem)
n = len(demand)
dem
```

p	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
1	712821	35685	949684	236810	107054	1424526	3085914	8343398
2	421670	21100	634896	154266	63300	952344	1736757	4695675
3	41451	2048	77044	19033	6145	115566	199186	538539
4	33048	1579	63296	13643	4737	94944	156270	422507
5	82260	4209	82680	20909	12628	124020	289918	783853
6	224047	10749	360072	91511	32247	540108	1070783	2895081
7	7095	338	11764	2254	1015	17646	33471	90496
8	72825	3609	70968	17745	10826	106452	245709	664323
9	258669	12999	219376	78379	38997	329064	957172	2587908
10	19556	929	43796	9527	2786	65694	93242	252099
11	2850	144	1376	364	432	2064	10377	28056
12	2204	121	1964	595	362	2946	9407	25433
13	4075	212	592	175	635	888	9099	24602

Figure 4: Defining parameters

```
# Defining the parameters and some useful variables
groups = ['1','2','3','4','5','6','7','8']

coverage_rate = [0.1, 0.9, 0.9, 1, 0.7, 0.7, 0.75, 0.8]

purchasing_cost = 19.5

distribution_centres = ['Montreal', 'Toronto', 'Saskatoon', 'Winnipeg', 'Calgary', 'Vancouver']

shipping_capacity = [100000, 100000, 250000, 250000, 750000, 750000]

budgets = [150000,200000,10000,65000,120000,100000]

provinces = []
for i in range(1,14,1):
    provinces.append('p_'+str(i))

g = len(groups)
p = len(provinces)
d = len(distribution_centres)
```

Figure 5: Defining the problem

```
#Defining the problem:
prob = Model('Vaccine Allocation Problem')
```

Figure 6: Defining the decision variables

```
#Defining the decision variables: The allocated and shipped vaccines are integer variables.
x = prob.addVars(p, g, lb=0, vtype = GRB.INTEGER, name = ['x_' + str(i) + '_' + str(j) for i in range(p) for j in ra
s = prob.addVars(p, d, lb=0, vtype = GRB.INTEGER, name = ['s_' + '_' + str(i) + '_' + str(j) for i in range(p) for j
μ = prob.addVar(lb = 0, ub=1, vtype = GRB.CONTINUOUS, name = 'mu')
```

Figure 7: Defining the maximization problem

```
#Defining the objective function: We want to maximize μ, which is the minimum ratio between the amount of allocated
prob.setObjective(μ, GRB.MAXIMIZE)
```

Figure 8: Defining the constraints

```
#Defining the constraints

#Constraint #0: Limiting the number of doses we can allocate:
prob.addConstrs(x[i,j]<=demand[i,j] for i in range(p) for j in range(g))

#Constraint #1: Defining μ following the linearization of the problem
prob.addConstrs(μ <= x[i,j]/demand[i,j] for i in range(p) for j in range(g))

#Constraint #2: Meet the coverage rate
prob.addConstrs(x[i,j] >= coverage_rate[j] * demand[i,j] for i in range(p) for j in range(g))

#Constraint #3: Distribution centres cannot ship more than their capacity
for j in range(d):
    prob.addConstr(sum(s[i,j] for i in range(p)) <= shipping_capacity[j]) #replace x by s

#Constraint #4: Expenses cannot exceed the budget:
for j in range(d):
    prob.addConstr(sum(19.5*s[i,j] for i in range(p)) + sum(tca[i,j]*s[i,j] for i in range(p)) <= budgets[j])

#Constraint #5: The total amount of shipped vaccines cannot exceed the total amount of allocated vaccines per
for i in range(p):
    prob.addConstr(sum(x[i,j] for j in range(g)) - sum(s[i,k] for k in range(d)) >= 0)
```

Figure 9: Optimizing

```
prob.optimize()
```

Figure 10: 10

```
SC_cold = [100000]

SC_very_cold = [200000]
```

Figure 11: 11

```
# decision variable omega: whether or not a distribution centre is set up

omega = prob.addVar(lb = 0, vtype = GRB.BINARY, name = 'set_up')
```

Figure 12: 12

```
# Constraint #6: Expenses cannot exceed the budget, accounting for storage costs for cold and very cold vaccines

for j in range(d):
    prob.addConstr(sum(SC_cold[j]*omega[j]) +
                   sum(SC_very_cold[j]*omega[j]) +
                   sum(19.5*s[i,j] for i in range(p)) +
                   sum(tca[i,j]*s[i,j] for i in range(p)) <= budgets[j])
```

Figure 13: Demand for each group in each province

Province	p	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8
Ontario	1	712821	35685	949684	236810	107054	1424526	3085914	8343398
Quebec	2	421670	21100	634896	154266	63300	952344	1736757	4695675
Nova Scotia	3	41451	2048	77044	19033	6145	115566	199186	538539
New Brunswick	4	33048	1579	63296	13643	4737	94944	156270	422507
Manitoba	5	82260	4209	82680	20909	12628	124020	289918	783853
British Columbia	6	224047	10749	360072	91511	32247	540108	1070783	2895081
Prince Edward Island	7	7095	338	11764	2254	1015	17646	33471	90496
Saskatchewan	8	72825	3609	70968	17745	10826	106452	245709	664323
Alberta	9	258669	12999	219376	78379	38997	329064	957172	2587908
Newfoundland & Labrador	10	19556	929	43796	9527	2786	65694	93242	252099
Northwest Territories	11	2850	144	1376	364	432	2064	10377	28056
Yukon	12	2204	121	1964	595	362	2946	9407	25433
Nunavut	13	4075	212	592	175	635	888	9099	24602

Figure 14: Transportation costs to each province from each distribution centre (Canadian Dollars per dose)

Province	Montreal	Toronto	Winnipeg	Saskatoon	Calgary	Vancouver
Ontario	0.55	0.2	0.55	1.75	2	2.75
Quebec	0.2	0.35	2	2	2.5	3.2
Nova Scotia	0.4	0.55	2.5	2.5	2.75	3.5
New Brunswick	0.4	0.5	2.25	2.25	2.75	3.75
Manitoba	0.8	0.65	0.2	0.5	1.25	1.75
British Columbia	3	2.8	1.75	0.75	0.5	0.2
Prince Edward Island	0.75	1.55	2.5	2.6	2.25	3
Saskatchewan	2	2	0.5	0.2	0.5	1.25
Alberta	2.25	2.25	0.75	0.5	0.2	0.5
Newfoundland & Labrador	0.3	1.9	2.25	2.8	2.5	2.75
Northwest Territories	2.5	2.35	1.25	0.7	1.25	1.5
Yukon	3.25	3.1	1.75	1.25	1.25	0.75
Nunavut	2.9	2.7	1.5	1.25	1.25	1