

There Is No Largest Prime number

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Outline

There Is No
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number

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Motivation

The Basic Problem That
We Studied

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What are Prime Numbers

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Definition

A **prime number** is a number that has exactly two divisors.

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A **prime number** is a number that has exactly two divisors.

Example

- ▶ 2 is prime (two divisors: 1 and 2).

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A **prime number** is a number that has exactly two divisors.

Example

- ▶ 2 is prime (two divisors: 1 and 2).
- ▶ 3 is prime (two divisors: 1 and 3).

What are Prime Numbers

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Definition

A **prime number** is a number that has exactly two divisors.

Example

- ▶ 2 is prime (two divisors: 1 and 2).
- ▶ 3 is prime (two divisors: 1 and 3).
- ▶ 4 is not prime (**three** divisors: 1, 2, and 4).

There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

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Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.
2. Consider the number $q = p + 1$.
3. But q is greater than 1, thus divisible by some prime number not in the first p numbers.



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The proof uses *reductio ad absurdum*.

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Theorem

There is no largest prime number.

Proof.

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
4. But $q + 1$ is greater than 1, thus divisible by some prime number not in the first p numbers.



There Is No Largest Prime Number

The proof uses *reductio ad absurdum*.

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Proof.

1. Suppose p were the largest prime number.
2. Let q be the product of the first p numbers.
3. Then $q + 1$ is not divisible by any of them.
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Largest Prime
number

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We Studied

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What's Still To Do

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Answered Questions

How many primes are there?

Open Questions

Is every even number the sum
of two primes?

An Algorithm For Finding Primes Numbers.

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```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)

        return 0;
}
```

An Algorithm For Finding Primes Numbers.

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```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)
        if (is_prime[i])
        {

        }
    return 0;
}
```

An Algorithm For Finding Primes Numbers.

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```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)
        if (is_prime[i])
        {
            std::cout << i << " ";
            for (int j = i; j < 100; j+=i)
                is_prime[j] = false;
        }
    return 0;
}
```

An Algorithm For Finding Primes Numbers.

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```
int main (void)
{
    std::vector<bool> is_prime (100, true);
    for (int i = 2; i < 100; i++)
        if (is_prime[i])
        {
            std::cout << i << " ";
            for (int j = i; j < 100; j+=i)
                is_prime[j] = false;
        }
    return 0;
}
```

Note the use of `std::`.