

National Research University Higher School of Economics
Faculty of Computer Science
HSE and University of London Double
Degree Programme in Data Science and
Business Analytics

BACHELOR's THESIS
"Deep-learning Scenarios: Neurodifferential
Equations and their Parameters"

Appendices

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Appendix A

We devote this section for listing several approximations of $\mathbf{f}(\mathbf{x}_n)$ using Maclaurin series:

$$\begin{aligned}
\mathcal{M}_0 &= \sum_{q=0}^{2n} \Phi_q \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] + \mathcal{O}(\varepsilon) \\
\mathcal{M}_1 &= \mathcal{M}_0 + \varepsilon \cdot \sum_{q=0}^{2n} \left[\Phi_q^{(1)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \cdot q \left(\sum_{p=1}^n \alpha_p \varphi^{(1)\varepsilon}(x_p) \right) \right] + \mathcal{O}(\varepsilon^2) \\
\mathcal{M}_2 &= \mathcal{M}_1 + \frac{\varepsilon^2}{2} \sum_{q=0}^{2n} q^2 \left[\Phi_q^{(1)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n \alpha_p \varphi^{(2)\varepsilon}(x_p) \right) + \right. \\
&\quad \left. + \Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^2 \right] + \mathcal{O}(\varepsilon^3) \\
\mathcal{M}_3 &= \mathcal{M}_2 + \frac{\varepsilon^3}{6} \sum_{q=0}^{2n} q^3 \left[\Phi_q^{(1)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n \alpha_p \varphi^{(3)\varepsilon}(x_p) \right) + \right. \\
&\quad + 3\Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n \alpha_p \varphi^{(1)\varepsilon}(x_p) \right) \left(\sum_{p=1}^n \alpha_p \varphi^{(2)\varepsilon}(x_p) \right) + \\
&\quad \left. + \Phi_q^{(3)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^3 \right] + \mathcal{O}(\varepsilon^4) \\
\mathcal{M}_4 &= \mathcal{M}_3 + \frac{\varepsilon^4}{24} \sum_{q=0}^{2n} \left[\left(\sum_{p=1}^n q^4 \alpha_p \varphi^{(4)\varepsilon}(x_p) \right) \Phi_q^{(1)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] + \right. \\
&\quad + 3 \left(\sum_{p=1}^n q^2 \alpha_p \varphi^{(2)\varepsilon}(x_p) \right)^2 \Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] + \\
&\quad + 4 \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right) \left(\sum_{p=1}^n q^3 \alpha_p \varphi^{(3)\varepsilon}(x_p) \right) \Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] + \\
&\quad + 6 \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^2 \left(\sum_{p=1}^n q^2 \alpha_p \varphi^{(2)\varepsilon}(x_p) \right) \Phi_q^{(3)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] + \\
&\quad \left. + \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^4 \Phi_q^{(4)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \right] + \mathcal{O}(\varepsilon^5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{M}_5 = \mathcal{M}_4 &+ \frac{\varepsilon^5}{120} \sum_{q=0}^{2n} \left[\Phi_q^{(1)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q^5 \alpha_p \varphi^{(5)\varepsilon}(x_p) \right) + \right. \\
&+ 10 \Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q^2 \alpha_p \varphi^{(2)\varepsilon}(x_p) \right) \left(\sum_{p=1}^n q^3 \alpha_p \varphi^{(3)\varepsilon}(x_p) \right) + \\
&+ 5 \Phi_q^{(2)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right) \left(\sum_{p=1}^n q^4 \alpha_p \varphi^{(4)\varepsilon}(x_p) \right) + \\
&+ 15 \Phi_q^{(3)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right) \left(\sum_{p=1}^n q^2 \alpha_p \varphi^{(2)\varepsilon}(x_p) \right)^2 + \\
&+ 10 \Phi_q^{(3)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^2 \left(\sum_{p=1}^n q^3 \alpha_p \varphi^{(3)\varepsilon}(x_p) \right) + \\
&+ 10 \Phi_q^{(4)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^3 \left(\sum_{p=1}^n q^2 \alpha_p \varphi^{(2)\varepsilon}(x_p) \right) + \\
&\left. + \Phi_q^{(5)\varepsilon} \left[\sum_{p=1}^n \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^n q \alpha_p \varphi^{(1)\varepsilon}(x_p) \right)^5 \right]
\end{aligned}$$

Appendix B