National Research University Higher School of Economics
Faculty of Computer Science
HSE and University of London Double
Degree Programme in Data Science and
Business Analytics

BACHELOR's THESIS
"Deep-learning Scenarios: Neurodifferential Equations and their Parameters"

Appendices

Prepared by the student of Group 192, Year 4,
Malchenko Oleg Maksimovich

Thesis Supervisor:
Doctor of Physical and Mathematical Sciences, Full Professor,
Gromov Vasilii Aleksandrovich

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Appendix A

We devote this section for listing several approximations of $\mathbf{f}(\mathbf{x}_n)$ using Maclaurin series:

$$\mathcal{M}_0 = \sum_{q=0}^{2n} \Phi_q \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] + \mathcal{O}(\varepsilon)$$

$$\mathcal{M}_1 = \mathcal{M}_0 + \varepsilon \cdot \sum_{q=0}^{2n} \left[\Phi_q^{(1)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \cdot q \left(\sum_{p=1}^{n} \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right) \right] + \mathcal{O}(\varepsilon^2)$$

$$\begin{aligned} \mathcal{M}_2 = \mathcal{M}_1 + \frac{\varepsilon^2}{2} \sum_{q=0}^{2n} q^2 \Bigg[& \Phi_q^{(1)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^{n} \alpha_p \varphi^{(2)_\varepsilon}(x_p) \right) + \\ & + \Phi_q^{(2)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^{n} \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right)^2 \Bigg] \end{aligned}$$

$$\begin{aligned} \mathcal{M}_3 = \mathcal{M}_2 + \frac{\varepsilon^3}{6} \sum_{q=0}^{2n} q^3 \Bigg[& \Phi_q^{(1)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^{n} \alpha_p \varphi^{(3)}(x_p) \right) + \\ & + 3\Phi_q^{(2)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^{n} \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right) \left(\sum_{p=1}^{n} \alpha_p \varphi^{(2)_\varepsilon}(x_p) \right) + \\ & + \Phi_q^{(3)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \left(\sum_{p=1}^{n} \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right)^3 \Bigg] \end{aligned}$$

$$\begin{aligned} \mathcal{M}_4 = \mathcal{M}_3 + \frac{\varepsilon^4}{24} \sum_{q=0}^{2n} \Bigg[& \left(\sum_{p=1}^{n} q^4 \alpha_p \varphi^{(4)_\varepsilon}(x_p) \right) \Phi_q^{(1)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] + \\ & + 3 \left(\sum_{p=1}^{n} q^2 \alpha_p \varphi^{(2)_\varepsilon}(x_p) \right)^2 \Phi_q^{(2)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] + \\ & + 4 \left(\sum_{p=1}^{n} q \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right) \left(\sum_{p=1}^{n} q^3 \alpha_p \varphi^{(3)_\varepsilon}(x_p) \right) \Phi_q^{(2)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] + \\ & + 6 \left(\sum_{p=1}^{n} q \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right)^2 \left(\sum_{p=1}^{n} q^2 \alpha_p \varphi^{(2)_\varepsilon}(x_p) \right) \Phi_q^{(3)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] + \\ & + \left(\sum_{p=1}^{n} q \alpha_p \varphi^{(1)_\varepsilon}(x_p) \right)^4 \Phi_q^{(4)_\varepsilon} \left[\sum_{p=1}^{n} \alpha_p \varphi(x_p) \right] \Bigg] \end{aligned}$$

$$\mathcal{M}_5 = \mathcal{M}_4 + \frac{\varepsilon^5}{120}\sum_{q=0}^{2n}\left[\Phi_q^{(1)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q^5\alpha_p\varphi^{(5)_\varepsilon}(x_p)\right)+\right.$$
$$+10\Phi_q^{(2)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q^2\alpha_p\varphi^{(2)_\varepsilon}(x_p)\right)\left(\sum_{p=1}^{n}q^3\alpha_p\varphi^{(3)_\varepsilon}(x_p)\right)+$$
$$+5\Phi_q^{(2)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q\alpha_p\varphi^{(1)_\varepsilon}(x_p)\right)\left(\sum_{p=1}^{n}q^4\alpha_p\varphi^{(4)_\varepsilon}(x_p)\right)+$$
$$+15\Phi_q^{(3)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q\alpha_p\phi^{(1)_\varepsilon}(x_p)\right)\left(\sum_{p=1}^{n}q^2\alpha_p\varphi^{(2)_\varepsilon}(x_p)\right)^2+$$
$$+10\Phi_q^{(3)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q\alpha_p\varphi^{(1)_\varepsilon}(x_p)\right)^2\left(\sum_{p=1}^{n}q^3\alpha_p\varphi^{(3)_\varepsilon}(x_p)\right)+$$
$$+10\Phi_q^{(4)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q\alpha_p\varphi^{(1)_\varepsilon}(x_p)\right)^3\left(\sum_{p=1}^{n}q^2\alpha_p\varphi^{(2)_\varepsilon}(x_p)\right)+$$
$$\left.+\Phi_q^{(5)_\varepsilon}\left[\sum_{p=1}^{n}\alpha_p\varphi(x_p)\right]\left(\sum_{p=1}^{n}q\alpha_p\varphi^{(1)_\varepsilon}(x_p)\right)^5\right]$$

Appendix B