Consider stochastic system

$$\boxed{\dot{x} = f(x) + \varepsilon\, \sigma(x)\, \xi(t)}$$

I-й абзац

Matrix of random disturbances

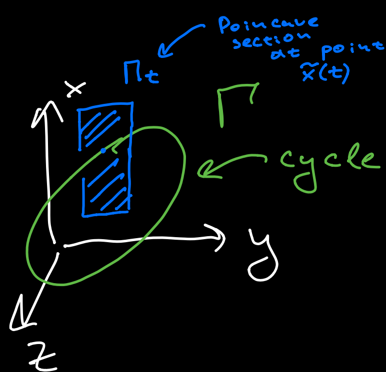
Gaussian noise

Take case $\varepsilon = 0$:

II-й Абзац

$\dot{x} = f(x)$ - deterministic system

Assume, we have exponentionally stable limit cycle Γ



$x = \tilde{x}(t)$

solution with period T

Method:

Deterministic $\longmapsto$ Stochastic

III-й Абзац

Let $p(x,\varepsilon)$ - Distribution of random states of system around the deterministic attractor

Hard to calculate !!!

Make approximation:

- Approximation based on quasi-potential

$$V(x) = -\lim_{\varepsilon\to 0} \varepsilon^2 \log(p(x,\varepsilon))$$

$$\Downarrow$$

$$p(x,\varepsilon) \approx K \cdot \exp\left(-\frac{V(x)}{\varepsilon^2}\right)$$

IV - Абзац

In poincare section Π_t (plane on the graphs):

limit cycle

$$V_t(x) \approx \frac{1}{2}\left(x-\tilde{x}(t),\ W^+(t)\,[x-\tilde{x}(t)]\right)$$

$$p_t(x,\varepsilon) \approx k \exp\left(-\frac{V_t(x)}{\varepsilon^2}\right)$$

Gaussian Approximation (как в предыдущем абзац, только конкретно в сечение)

Расшифровка:

We have covariance matrix

$$\mathrm{cov}_t = D(t, \varepsilon) = \varepsilon^2 W(t) \rightsquigarrow W^{+}(t) - \text{pseudo inverse}$$

Analogue of „$W^{-1}(t)$" for degenerate matrix

How to calculate $W(t)$?

$W(t)$ - solution of Lyapunov equation:

$$\begin{cases} \dot{W} = F \cdot W + W \cdot F^T + QSQ \\ W(0) = W(T), \quad W(t)v(t) = 0 \end{cases}$$

← начальные условие

Расшифровка

$F(t) = \frac{\partial f}{\partial x}(\tilde{x}(t))$, $S(t) = \sigma(\tilde{x}(t))\,\sigma^T(\tilde{x}(t))$

$v(t) = f(\tilde{x}(t))$, $Q(t)$ - symmetric matrix of orthogonal projection Π_t

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For 3-dimensional case can solve by using singular decomposition