Mafs

September 22, 2021

1 Equilibrium Surface

We obtain formula for surface from CCM model with coordinate system (m_1, n_1, n_2), equation being: $n_2 = am_1^3 + bn_1m_1$. In order to accurately evaluate groundwater potential we have to apply the CCM model to indexes obtained from the PCA, them being N, S and the groundwater potential itself GP which in turn means that a proper conversion must be established from (m_1, n_1, n_2) to (GP, N, S).

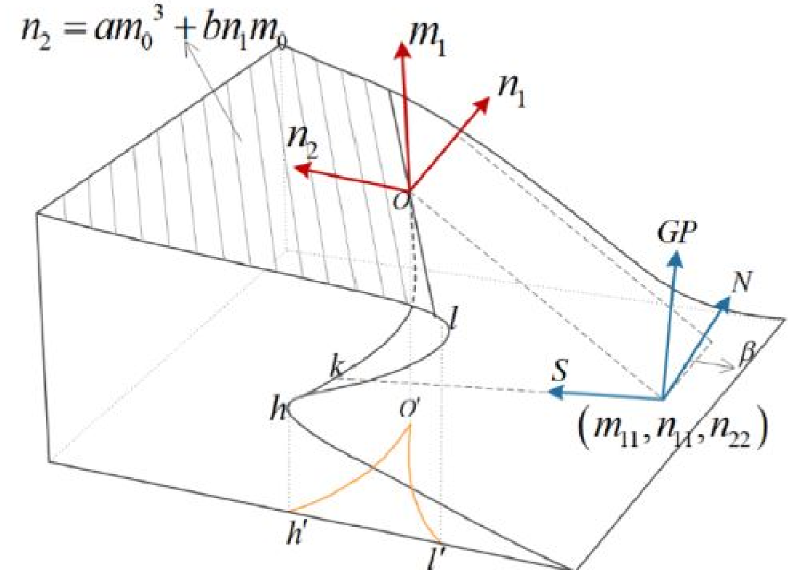

Figure 1: CCM Model

From the graph we can see that when $n_1 > 0$, m1 slowly increases with an increase in n_2. When $n_1 < 0$, m_1 continues to increase with

n_2. However, when m_1 reaches a specific range, a small increase in n_2 can lead to a sharp and discontinuous increase in m_1

2 Coordination Conversion

The original coordinate system (m_1, n_1, n_2) is transformed into the new coordinate system (GP, N, S) when the translation is set to (m_{11}, n_{11}, n_{22}), and the rotation angle is around the m_1 axis is β. In short we are rotating around the z axis.

Hence we can say that our set of transformation equations are:

$$\begin{cases} m_1 = GP + m_{11} \\ n_1 = Ncos\beta + Ssin\beta + n_{11} \\ n_2 = -Nsin\beta + Scos\beta + n_{22} \end{cases}$$

Using rotation matrix: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\beta & -sin\beta \\ 0 & cos\beta & sin\beta \end{bmatrix}$
where $sin\beta = \frac{bm_{11}}{\sqrt{1+b^2m_{11}^2}}$ and $cos\beta = \frac{1}{\sqrt{1+b^2m_{11}^2}}$

3 Construction of evaluation model

Using the transformation equations along with sin and cos values in our original surface equation, we get:

$$(GP + m_{11})^3 + \frac{b}{a}\left(\frac{N}{\sqrt{1+b^2{m_{11}}^2}} + \frac{Sbm_{11}}{\sqrt{1+b^2{m_{11}}^2}} + n_{11}\right)(GP + m_{11}) + \frac{1}{a}\left(\frac{bNm_{11}}{\sqrt{1+b^2{m_{11}}^2}} - \frac{S}{\sqrt{1+b^2{m_{11}}^2}} - n_{22}\right) = 0$$

This is now a cubic equation where $GP + m_{11}$ is the unknown. Now we use the Cardano Formula which is used to solve cubic equations of the form $t^3 + pt + q = 0$. In our case only the discriminant is sufficient, it being:

$$W = 9\left[\frac{1}{a}\left(\frac{bNm_{11}}{\sqrt{1+b^2{m_{11}}^2}} - \frac{S}{\sqrt{1+b^2{m_{11}}^2}} - n_{22}\right)\right]^2 + 4\left[\frac{b}{a}\left(\frac{N}{\sqrt{1+b^2{m_{11}}^2}} + \frac{Sbm_{11}}{\sqrt{1+b^2{m_{11}}^2}} + n_{11}\right)\right]^3$$

Finally we have an model to evaluate groundwater potential. The CCM is a cubic equation hence, it consists of three unreal roots. When W < 0, we get three unequal real roots, which implies that the groundwater potential corresponds to a "very poor" status. When W >= 0, the natural breaks classification method is used to classify the results into statuses of "poor," "moderate," "good," and "very good."