Manipulator kinematic error calibration model

Theoretical background

1)T_N- end position; dT_N - error matrix

$$\mathbf{T}_N + d\mathbf{T}_N = (\mathbf{A}_1 + d\mathbf{A}_1)(\mathbf{A}_2 + d\mathbf{A}_2)\cdots(\mathbf{A}_N + d\mathbf{A}_N) = \prod_{n=1}^{N} (\mathbf{A}_n + d\mathbf{A}_n)$$

2)$U_N = \prod A_i$

$$d\mathbf{T}_N = \mathbf{T}_N \left[\sum_{n=1}^{N} \mathbf{U}_{n+1}^{-1} \delta\mathbf{A}_n \mathbf{U}_{n+1} \right]$$

3)Where T :

$$\mathbf{T} = \begin{bmatrix} \mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4)After simple math transformations:

$$dx_N = \sum_{n=1}^{N} [(\mathbf{n}_{n+1}^u \cdot \mathbf{k}_n^1) + (\mathbf{p}_{n+1}^u \times \mathbf{n}_{n+1}^u) \cdot \mathbf{k}_n^2]\Delta\theta_n + (\mathbf{n}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta r_n + (\mathbf{n}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta l_n + [(\mathbf{p}_{n+1}^u \times \mathbf{n}_{n+1}^u) \cdot \mathbf{k}_n^3]\Delta\alpha_n$$

$$dy_N = \sum_{n=1}^{N} [(\mathbf{o}_{n+1}^u \cdot \mathbf{k}_n^1) + (\mathbf{p}_{n+1}^u \times \mathbf{o}_{n+1}^u) \cdot \mathbf{k}_n^2]\Delta\theta_n + (\mathbf{o}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta r_n + (\mathbf{o}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta l_n + [(\mathbf{p}_{n+1}^u \times \mathbf{o}_{n+1}^u) \cdot \mathbf{k}_n^3]\Delta\alpha_n$$

$$dz_N = \sum_{n=1}^{N} [(\mathbf{a}_{n+1}^u \cdot \mathbf{k}_n^1) + (\mathbf{p}_{n+1}^u \times \mathbf{a}_{n+1}^u) \cdot \mathbf{k}_n^2]\Delta\theta_n + (\mathbf{a}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta r_n + (\mathbf{a}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta l_n + [(\mathbf{p}_{n+1}^u \times \mathbf{a}_{n+1}^u) \cdot \mathbf{k}_n^3]\Delta\alpha_n$$

$$\delta x_N = \sum_{n=1}^{N} [(\mathbf{n}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta\theta_n + (\mathbf{n}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta\alpha_n]$$

$$\delta y_N = \sum_{n=1}^{N} [(\mathbf{o}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta\theta_n + (\mathbf{o}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta\alpha_n]$$

$$\delta z_N = \sum_{n=1}^{N} [(\mathbf{a}_{n+1}^u \cdot \mathbf{k}_n^2)\Delta\theta_n + (\mathbf{a}_{n+1}^u \cdot \mathbf{k}_n^3)\Delta\alpha_n]$$

Matrix form and calibration steps

Matrix form:

$$\mathbf{d}_N = \mathbf{m}_1\Delta\boldsymbol{\theta} + \mathbf{m}_2\Delta\mathbf{r} + \mathbf{m}_3\Delta\mathbf{l} + \mathbf{m}_4\Delta\boldsymbol{\alpha}$$

$$\boldsymbol{\delta}_N = \mathbf{m}_2\Delta\boldsymbol{\theta} + \mathbf{m}_3\Delta\boldsymbol{\alpha}$$

$$\begin{bmatrix} \mathbf{d}_N \\ \boldsymbol{\delta}_N \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 & \mathbf{m}_2 & \mathbf{m}_3 & \mathbf{m}_4 \\ \mathbf{m}_2 & 0 & 0 & \mathbf{m}_3 \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\mathbf{r} \\ \Delta\mathbf{l} \\ \Delta\boldsymbol{\alpha} \end{bmatrix}$$

$$\delta T_N = J\Delta_{DH}$$

$$\boxed{\boldsymbol{\Delta_{DH} = J^{+}\delta T_N}}$$

Calibration steps:

1. Setting reference DH parameters and slightly modified DH parameters
2. Direct kinematics calculation
3. Jacobian calculation
4. Calculation of corrections to DH parameters
5. Error calculation

Simulation

DH-parameters:

Reference:

```
In [3]: DH_ref
Out[4]:
           d       a     alpha
q1  0.08946  0.0000  1.570796
q2  0.00000 -0.4250  0.000000
q3  0.00000 -0.3922  0.000000
q4  0.10910  0.0000  1.570796
q5  0.09465  0.0000 -1.570796
q6  0.08230  0.0000  0.000000
```

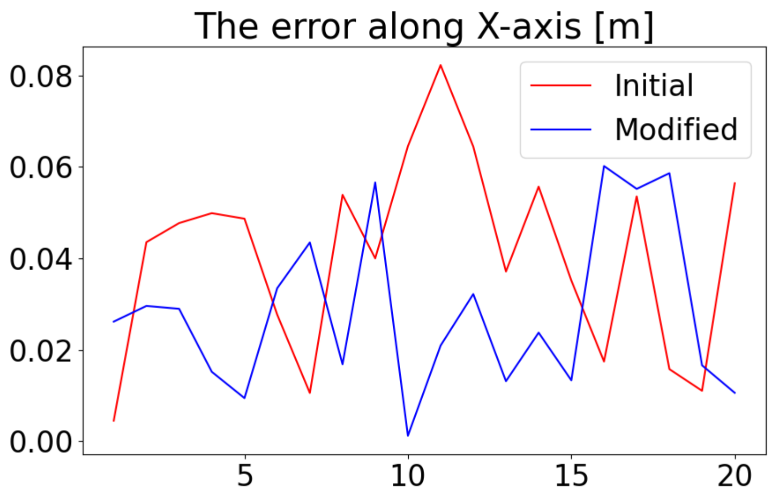
Actual:

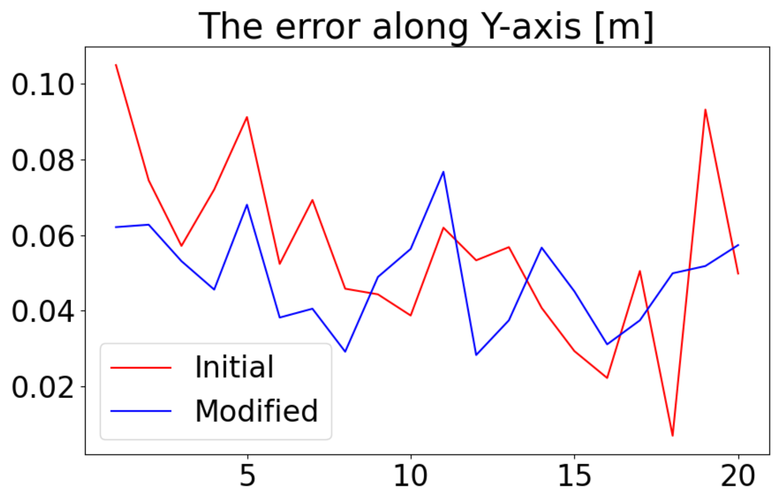
```
In [4]: DH_act
Out[5]:
        d     a     alpha
q1  0.09  0.00  1.727876
q2  0.00 -0.43  0.000000
q3  0.00 -0.40  0.000000
q4  0.10  0.00  1.413717
q5  0.10  0.00 -1.413717
q6  0.10  0.00  0.000000
```

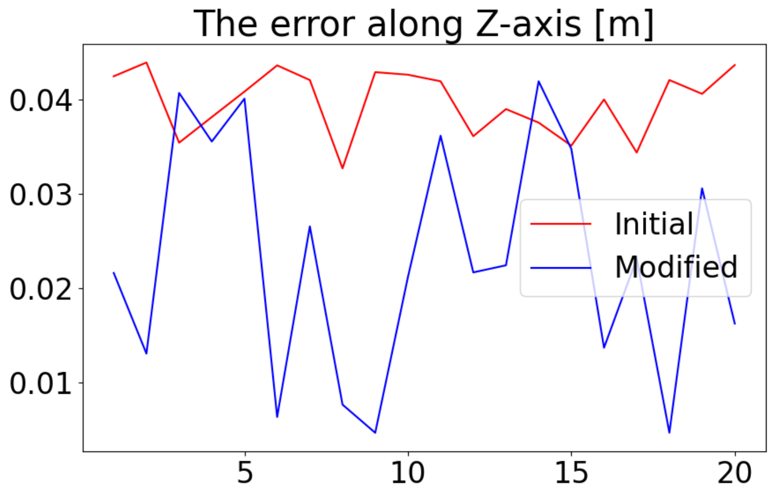
Modified:

```
In [2]: DH_mod
Out[3]:
            d         a     alpha
q1   0.074249  0.020235  1.694634
q2  -0.008041 -0.406155 -0.014565
q3  -0.001166 -0.391109 -0.045612
q4   0.091079  0.020648  1.415255
q5   0.114261  0.021299 -1.398770
q6   0.089561  0.021299  0.014947
```

Error graphs







	Before calibration [m]	After calibration [m]
maximum change	0.104921	0.076716
average error	0.045500	0.033416

Result: 26% accuracy increasing