Assignment 1 Report

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1.1 Intersection

To check for intersection was used angle between normal vectors. If this angle is zero, that could mean that planes are same or parallel to each other (figure 1). Otherwise, they are intersected (figure 2).

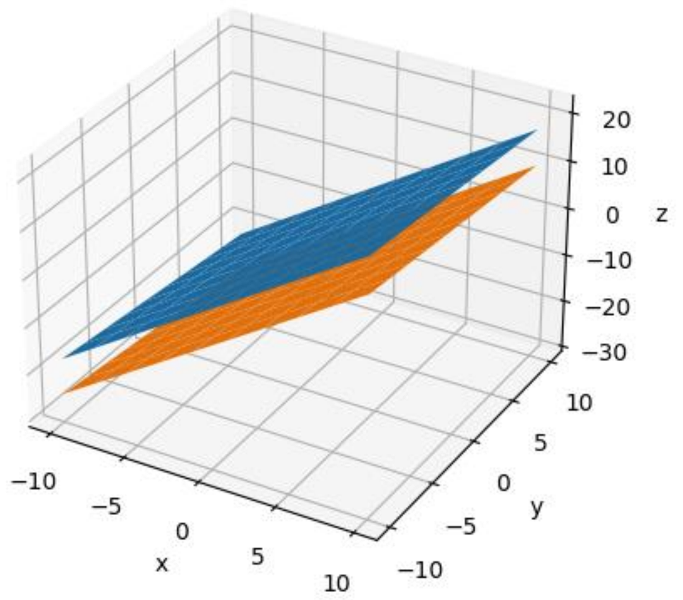

Figure 1. Parallel planes

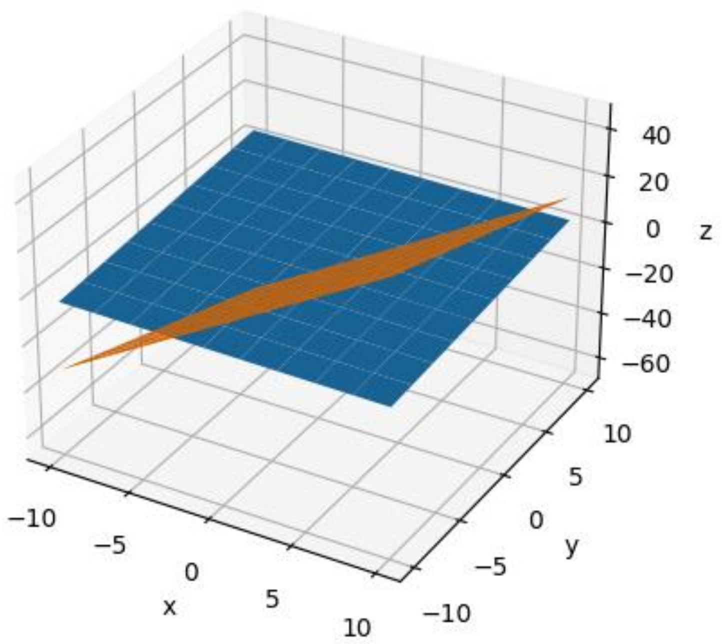

Figure 2. Intersected planes

1.2 Representation

In my solution, in form $n * (r - r_0) = 0$:

n is normal vector of plane, that is perpendicular to v and w.

r_0 is any point at the plane so it could be just p.

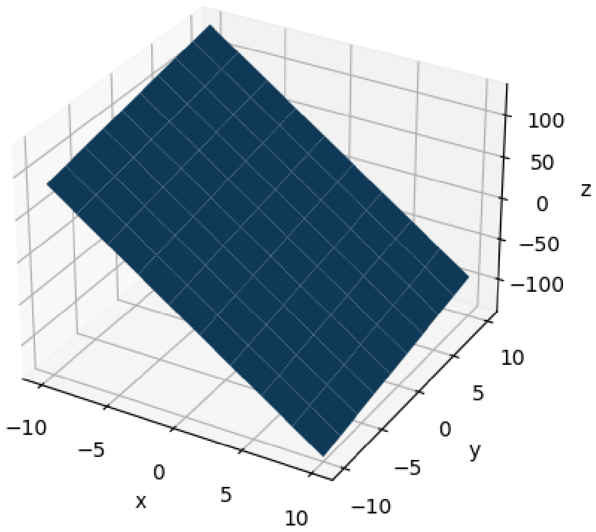

Figure 3. First plane [-1.0, 0.2, -0.1] * ([x, y, z] - [0, 1, 0] = 0

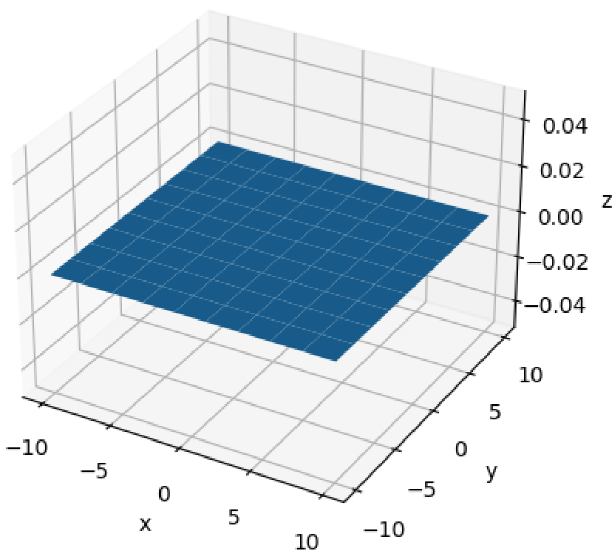

Figure 4. Second plane [0.7, -0.7, 0.0] * ([x, y, z] - [-1, 0, 0] = 0

1.3 Projection

Using the normal vector, the parameters of line were obtained:

x: -0.9 y: -0.35 z: -0.25

Projection of the point on line was found using dot product of coordinates of g and normal vector of plane.

1.4 Symmetry

Here was used same idea as in task 1.3. Projection of point g was found on plane and then sinus between this vector and normal vector of plane was calculated. As we know distance to plane from point g, it is possible to find coordinates of symmetrical point.

2.1 Basis in V

Two vectors were found using null space of given matrix

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Basis:
Vector 1 | Vector 2
[[ 0.4264  0.    ]
 [-0.6396 -0.7071]
 [-0.6396  0.7071]]
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2.2 Projections

Orthogonal projection onto V: dot product of vector g and normal vector of plane

Orthogonal compliment of V: difference between given vector g and obtained perpendicular projection

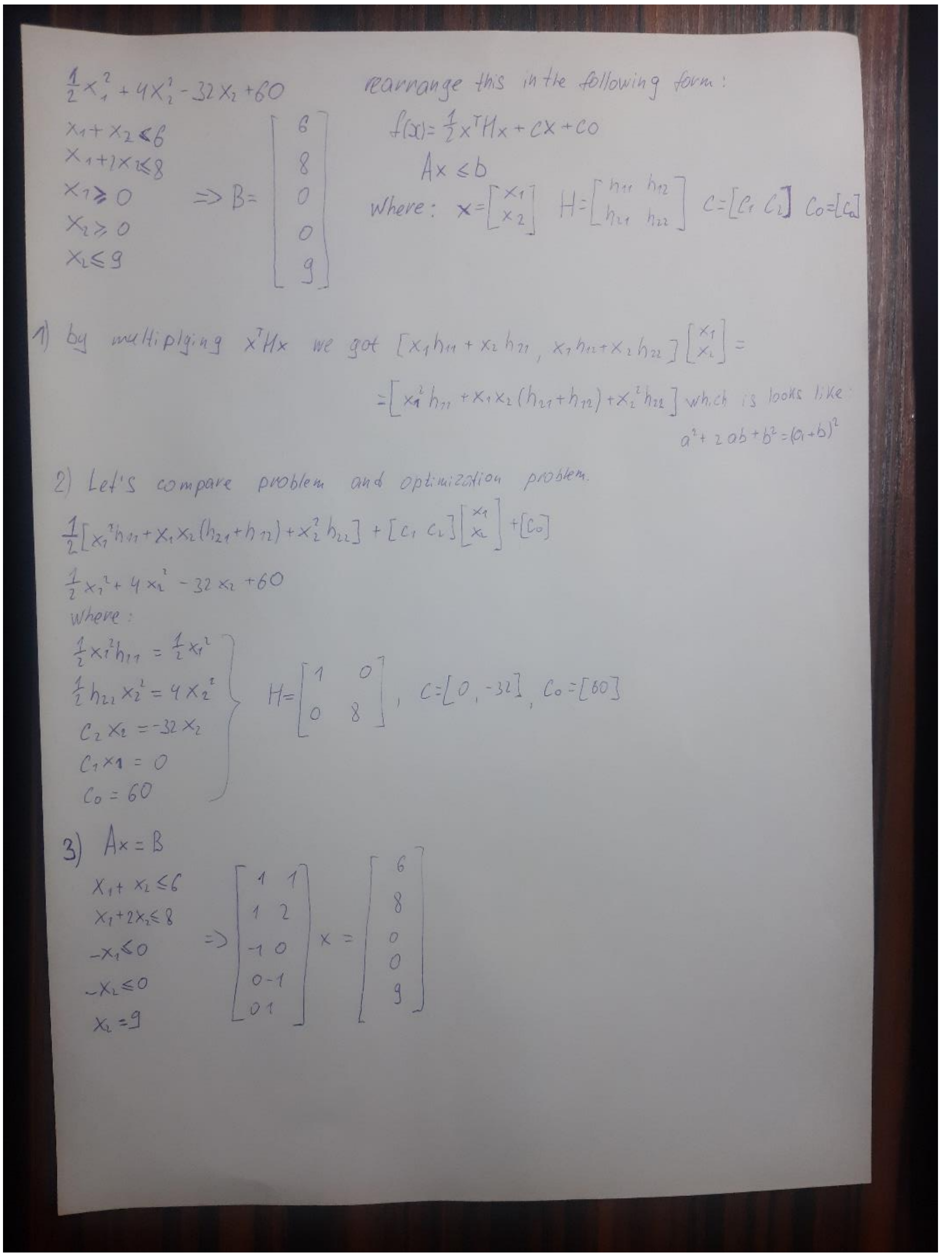
To prove this, we should be able to obtain initial vector g using recently obtained projections, which is possible if dot product between vectors is zero and sum of vectors that are orthogonal to each other is equal to initial

2.3 Recovering

Proof from task 2.2 gives opportunity to recover initial vector g. To prove this we can simply compare initial and obtained vectors. They are the same.

3.1 Rearranging

On figure 5 my calculations were described



$\frac{1}{2}x_1^2 + 4x_2^2 - 32x_2 + 60$

rearrange this in the following form:

$$f(x) = \frac{1}{2}x^T H x + cx + co$$

$$Ax \leq b$$

$x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 8$

$x_1 \geq 0$

$x_2 \geq 0$

$x_2 \leq 9$

$$\Rightarrow B = \begin{bmatrix} 6 \\ 8 \\ 0 \\ 0 \\ 9 \end{bmatrix}$$

Where: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$ $c = [c_1\ c_2]$ $C_0 = [c_0]$

1) by multiplying $x^T H x$ we got $[x_1 h_{11} + x_2 h_{21},\ x_1 h_{12} + x_2 h_{22}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$

$= [x_1^2 h_{11} + x_1 x_2 (h_{21} + h_{12}) + x_2^2 h_{22}]$ which is looks like

$a^2 + 2ab + b^2 = (a+b)^2$

2) Let's compare problem and optimization problem.

$$\frac{1}{2}[x_1^2 h_{11} + x_1 x_2 (h_{21} + h_{12}) + x_2^2 h_{22}] + [c_1\ c_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [c_0]$$

$$\frac{1}{2}x_1^2 + 4x_2^2 - 32x_2 + 60$$

where:

$$\left.\begin{aligned} \frac{1}{2}x_1^2 h_{11} &= \frac{1}{2}x_1^2 \\ \frac{1}{2}h_{22}x_2^2 &= 4x_2^2 \\ c_2 x_2 &= -32x_2 \\ c_1 x_1 &= 0 \\ c_0 &= 60 \end{aligned}\right\} H = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix},\ c = [0, -32],\ c_0 = [60]$$

3) $Ax = B$

$x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 8$

$-x_1 \leq 0$

$-x_2 \leq 0$

$x_2 = 9$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ -1 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 6 \\ 8 \\ 0 \\ 0 \\ 9 \end{bmatrix}$$

Figure 5. Rearranging the problem in the needed form

3.2 CVXPY

Code describing this task could be found in Assignment1.ipynb

3.3 Visualization

Domain of the function is on figure 6 and its cost function and its solution are on figure 7.

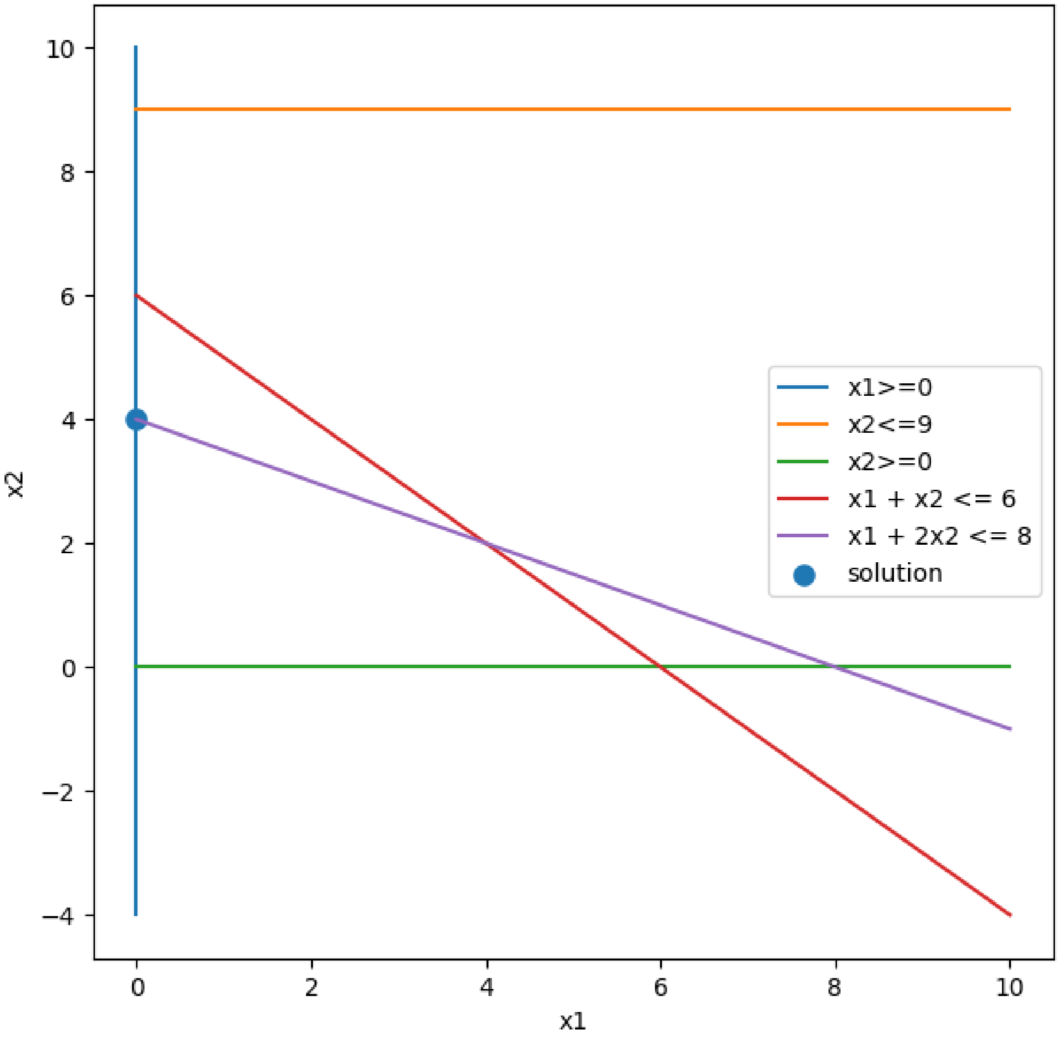


Figure 6. Domain of the function

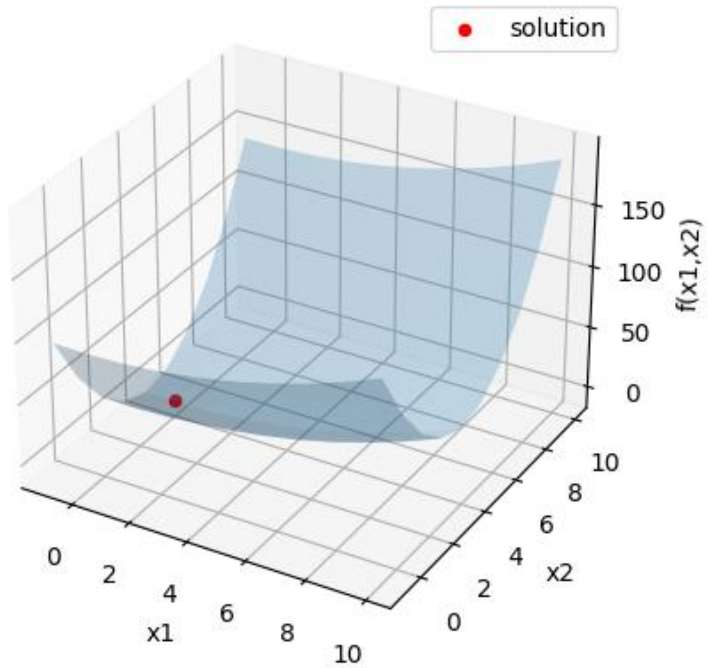


Figure 7. Cost function and solution