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Excitation of Electromagnetic Waves by a Nonsymmetric Antenna Located On the Surface of a Semi-Infinite Gyrotropic Cylinder

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Abstract

Electromagnetic wave excitation by a given nonsymmetric source in the presence of a semi-infinite gyrotropic cylinder located in free space is considered. The total source-excited field and the fields scattered by the endface of the cylinder are expanded in terms of eigenwaves of an infinite open gyrotropic waveguide and free space. The partial powers going to the discrete- and continuous-spectrum waves are numerically calculated.

1 Introduction

In the past decades, an enhanced attention has been paid to the theoretical study of the problem of the electromagnetic wave excitation in systems including open gyrotropic guiding structures and given electric currents. In the case of employing a magnetized plasma as a gyrotropic medium, such a model can be used for describing antennas operated in laboratory devices – helicon plasma sources, which are of importance for numerous experimental researches [1–3]. In most studies devoted to such sources, consideration is limited to the excitation of only the discrete-spectrum waves (eigenmodes), which play a major role in forming the plasma discharge. Many works considering the complete spectrum of the excited waves deal with the case where an antenna is placed far from the ends of a plasma column so that it can be represented as an infinite structure. For an infinite plasma-filled cylindrical waveguide located in free space, the full-wave approach presented in [4] was applied for the analysis of the energy characteristics of a helicon-type source [5]. However, in that type of radiating systems, it is often important to take into account the longitudinal boundedness of a plasma and consider the diffraction effects on the edges of a discharge [3].

In this work, the main features of wave excitation by an antenna with the nonsymmetric electric-current distribution in the presence of a semi-infinite cylinder filled with a magnetoplasma are considered. For finding the energy characteristics of such a source, an expansion of the excited field and fields of waves scattered by the endface of the cylinder over the set of eigenwaves of an infinite open gyrotropic waveguide [4] and free space is used.

2 Formulation of the Problem

Consider a semi-infinite cylinder of radius a that is located in free space and has the axis of symmetry coinciding with the z axis of the cylindrical coordinate system (ρ, ϕ, z). The structure is placed in the region $z < 0$, and its endface lies in the plane $z = 0$ as is shown in Fig. 1. The field is excited

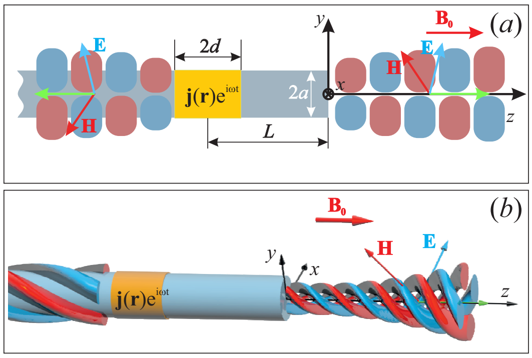

Figure 1. Geometry of the problem.

by a given electric current specified on the side surface of the cylinder between planes $z = -L - d$ and $z = -L + d$ (see Fig. 1) with the density written, with the $\exp(i\omega t)$ time dependence dropped, as

$$\mathbf{J}(\mathbf{r}) = (\boldsymbol{\phi}_0 j_\phi + \mathbf{z}_0 j_z)\delta(\rho - a) \times \exp(-im\phi - ik_0\tilde{p}z). \tag{1}$$

Here, d is the half-length of the antenna (the condition $d < L$ is fulfilled), integer m and the constant $\tilde{p}$ determine the current dependence on the azimuthal and longitudinal coordinates, respectively, and $k_0 = \omega/c$ is the wave number in free space (c is the speed of light in free space). The total amplitude of the current can be found as $|I_0|^2 = |I_\phi|^2 + |I_z|^2$, where $I_\phi = 2d\, j_\phi$ and $I_z = 2\pi a\, j_z$. The cylinder is aligned with an external static magnetic field $\mathbf{B}_0 = B_0\mathbf{z}_0$ and filled with a cold collisionless magnetoplasma, which is described by the dielectric permittivity tensor

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{2}$$

where

$$\varepsilon = 1 + \frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{H}}^2 - \omega^2} + \frac{\Omega_{\mathrm{p}}^2}{\Omega_{\mathrm{H}}^2 - \omega^2}, \quad \eta = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} - \frac{\Omega_{\mathrm{p}}^2}{\omega^2},$$
$$g = -\frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{H}}}{(\omega_{\mathrm{H}}^2 - \omega^2)\omega} + \frac{\Omega_{\mathrm{p}}^2 \Omega_{\mathrm{H}}}{(\Omega_{\mathrm{H}}^2 - \omega^2)\omega}. \tag{3}$$

Here, ω_{p} and Ω_{p} are the electron and ion plasma frequencies, and ω_{H} and Ω_{H} are the gyrofrequencies of the corresponding particles, respectively. It should be noted that the Gaussian system of units is used throughout the work.

3 The Source-Excited and Diffracted Fields

On the basis of the cylindrical symmetry of the problem, the longitudinal components of the excited and diffracted wave fields can be represented as follows:

$$\begin{bmatrix} E_{z;s,m}(\mathbf{r},q) \\ H_{z;s,m}(\mathbf{r},q) \end{bmatrix} = \begin{bmatrix} E_{z;s,m}(\rho,q) \\ H_{z;s,m}(\rho,q) \end{bmatrix} e^{-im\phi - ik_0 p_s(q) z}. \tag{4}$$

Here, the azimuthal index m is the same as in (1), the subscript s denotes the wave propagation direction, either positive ($s = +$) or negative ($s = -$), q and $p_s(q)$ are the normalized (to k_0) transverse and longitudinal wave numbers in free space, respectively. The function $p_s(q)$ obeys the relation $p_+(q) \equiv p(q) = -p_-(q)$, where $p(q) = (1-q^2)^{1/2}$. It is assumed that $\mathrm{Re}[p(q)] > 0$ if q is real and less than unity, and $\mathrm{Im}[p(q)] < 0$ otherwise. The transverse field components $E_{\rho;s,m}$, $E_{\phi;s,m}$, $H_{\rho;s,m}$, and $H_{\phi;s,m}$ can be found from the longitudinal components presented in (4). In what follows, we will assume that only the waves with the azimuthal index m are excited by the source.

As the given sources are regarded, it is supposed that the antenna current is not affected by the radiated and diffracted wave fields. In the half-space $z < 0$, the field excited by the current (1), is represented as an expansion in terms of eigenwaves of an open waveguide and is written in the source-free regions as

$$\begin{bmatrix} E_z^{(\mathrm{exc})}(\mathbf{r}) \\ H_z^{(\mathrm{exc})}(\mathbf{r}) \end{bmatrix} = \sum_n a_{s,m,n} \begin{bmatrix} E_{z;s,m,n}(\rho) \\ H_{z;s,m,n}(\rho) \end{bmatrix} e^{-im\phi - ik_0 p_{s,m,n} z}$$
$$+ \sum_{\alpha=1}^{2} \int_0^\infty a_{s,m,\alpha}(q) \begin{bmatrix} E_{z;s,m,\alpha}(\rho,q) \\ H_{z;s,m,\alpha}(\rho,q) \end{bmatrix} e^{-im\phi - ik_0 p_s z} dq. \tag{5}$$

Here, $s = +$ for $z > -L+d$ and $s = -$ for $z < -L-d$; the functions $E_{z;s,m,n}(\rho)$ and $H_{z;s,m,n}(\rho)$ describe the radial distributions of the fields of the discrete-spectrum waves (eigenmodes) with the radial index n and the longitudinal wave numbers $p_{s,m,n}$ ($p_{+,m,n} = -p_{-,m,n} = p_{m,n}$); the subscript α corresponds to two kinds of the continuous-spectrum waves described by the functions $E_{z;s,m,\alpha}(\rho,q)$ and $H_{z;s,m,\alpha}(\rho,q)$. Detailed expressions for these fields, which constitute the complete set of eigenwaves of an open gyrotropic cylindrical waveguide, can be found in [4]. The quantities $a_{s,m,n}$ and $a_{s,m,\alpha}(q)$ are obtained using the well-known technique developed for finding the excitation coefficients of the modes of open waveguides [6] and are given by the expressions

$$a_{s,m,n} = \frac{1}{N_{s,m,n}} \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{E}_{-s,-m,n}^{(\mathrm{T})}(\mathbf{r}) d\mathbf{r}$$
$$= \left[j_\phi E_{\phi;-s,-m,n}^{(\mathrm{T})}(a) + j_z E_{z;-s,-m,n}^{(\mathrm{T})}(a) \right]$$
$$\times \frac{4\pi a}{N_{s,m,n}} \frac{\sin[k_0(\tilde{p} - p_{s,m,n})d]}{k_0(\tilde{p} - p_{s,m,n})} e^{-ik_0 p_{s,m,n} L},$$
$$a_{s,m,\alpha}(q) = \frac{1}{N_{s,m,\alpha}(q)} \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{E}_{-s,-m,\alpha}^{(\mathrm{T})}(\mathbf{r},q) d\mathbf{r}$$
$$= \left[j_\phi E_{\phi;-s,-m,\alpha}^{(\mathrm{T})}(a,q) + j_z E_{z;-s,-m,\alpha}^{(\mathrm{T})}(a,q) \right]$$
$$\times \frac{4\pi a}{N_{s,m,\alpha}(q)} \frac{\sin[k_0(\tilde{p} - p_s(q))d]}{k_0(\tilde{p} - p_s(q))} e^{-ik_0 p_s L}, \tag{6}$$

where integration is performed over the region occupied by the current (1), the superscript (T) denotes fields taken in an auxiliary medium that is described by the transposed dielectric permittivity tensor $\hat{\varepsilon}^{\mathrm{T}}$, and $N_{s,m,n}$ and $N_{s,m,\alpha}(q)$ are the norms of the discrete- and continuous-spectrum waves, respectively, which are given by the orthogonality relations presented in [4].

The fields reflected from the cylinder endface to the region $z < 0$ are also expanded in terms of eigenwaves of an infinite open gyrotropic waveguide. The longitudinal components of the reflected field, hereafter marked by the superscript (r), are represented as

$$\begin{bmatrix} E_z^{(\mathrm{r})}(\mathbf{r}) \\ H_z^{(\mathrm{r})}(\mathbf{r}) \end{bmatrix} = \sum_n b_{-,m,n} \begin{bmatrix} E_{z;-,m,n}(\rho) \\ H_{z;-,m,n}(\rho) \end{bmatrix} e^{-im\phi + ik_0 p_{m,n} z}$$
$$+ \sum_{\alpha=1}^{2} \int_0^\infty b_{-,m,\alpha}(q) \begin{bmatrix} E_{z;-,m,\alpha}(\rho,q) \\ H_{z;-,m,\alpha}(\rho,q) \end{bmatrix} e^{-im\phi + ik_0 p z} dq, \tag{7}$$

where $b_{-,m,n}$ and $b_{-,m,\alpha}(q)$ are the expansion coefficients to be found. The longitudinal components of the field in the region $z > 0$ are expanded in terms of the continuous-spectrum waves of free space as

$$\begin{bmatrix} E_z^{(\mathrm{sc})} \\ H_z^{(\mathrm{sc})} \end{bmatrix} = \sum_{\alpha=1}^{2} \int_0^\infty c_{+,m,\gamma}(q) \begin{bmatrix} E_{z;-,m,\gamma}(\rho,q) \\ H_{z;-,m,\gamma}(\rho,q) \end{bmatrix} e^{-im\phi - ik_0 p z} dq, \tag{8}$$

where the superscript (sc) denotes the fields scattered to free space, $\gamma = 1$ and $\gamma = 2$ correspond to the E- and H-polarized waves, respectively, and $c_{+,m,\gamma}(q)$ are unknown coefficients depending on q. The eigenwaves of free space are written as

$$E_{z;s,m,\gamma}(\rho,q) = q J_m(k_0 q \rho) \delta_{\gamma,1},$$
$$H_{z;s,m,\gamma}(\rho,q) = q J_m(k_0 q \rho) \delta_{\gamma,2}, \tag{9}$$

where J_m is the Bessel function of the first kind of order m and $\delta_{\alpha,\beta}$ is the Kronecker delta.

Satisfying the boundary conditions for the tangential field components at plane $z = 0$

$$E_\phi^{(\mathrm{exc})} + E_\phi^{(\mathrm{r})} = E_\phi^{(\mathrm{sc})}, \quad H_\phi^{(\mathrm{exc})} + H_\phi^{(\mathrm{r})} = H_\phi^{(\mathrm{sc})},$$
$$E_\rho^{(\mathrm{exc})} + E_\rho^{(\mathrm{r})} = E_\rho^{(\mathrm{sc})}, \quad H_\rho^{(\mathrm{exc})} + H_\rho^{(\mathrm{r})} = H_\rho^{(\mathrm{sc})}, \tag{10}$$

and taking into account the orthogonality relations for the eigenwaves in both the regions $z<0$ and $z>0$, the system of integral equations for the expansion coefficients $b_{-,m,n}$, $b_{-,m,\alpha}(q)$, and $c_{+,m,\gamma}(q)$ is derived in the following form:

$$
\begin{aligned}
c_{+,m,\gamma}(q)\tilde{N}_{+,m,\gamma}(q) &= \sum_n a_{+,m,n}K^{+,m,n}_{-,-m,\gamma}(q) \\
&+ \sum_{\alpha=1}^{2}\int_0^\infty a_{+,m,\alpha}(\tilde{q})K^{+,m,\alpha}_{-,-m,\gamma}(\tilde{q},q)d\tilde{q} + \sum_n b_{-,m,n}K^{-,m,n}_{-,-m,\gamma}(q) \\
&+ \sum_{\alpha=1}^{2}\int_0^\infty b_{-,m,\alpha}(\tilde{q})K^{-,m,\alpha}_{-,-m,\gamma}(\tilde{q},q)d\tilde{q}, \qquad (11)
\end{aligned}
$$

$$
b_{-,m,n}N_{-,m,n} = \sum_{\gamma=1}^{2}\int_0^\infty c_{+,m,\gamma}(\tilde{q})M^{+,-m,n}_{+,m,\gamma}(\tilde{q})d\tilde{q}, \qquad (12)
$$

$$
b_{-,m,\alpha}(q)N_{-,m,\alpha}(q) = \sum_{\gamma=1}^{2}\int_0^\infty c_{+,m,\gamma}(\tilde{q})M^{+,-m,\alpha}_{+,m,\gamma}(q,\tilde{q})d\tilde{q}. \qquad (13)
$$

The norm of the eigenwaves of free space in (11) is given by the formula

$$
\tilde{N}_{s,m,\gamma}(q) = (-1)^{m+\gamma}cp_s/(k_0^2q), \qquad (14)
$$

and the kernels of integral equations (11)–(13) are written as

$$
\begin{aligned}
K^{s,m,n}_{-,-m,\gamma}(q) &= \frac{c}{4\pi}\int_0^{2\pi}d\phi\int_0^\infty\big[\mathbf{E}_{s,m,n}(\mathbf{r})\times\mathbf{H}_{-,-m,\gamma}(\mathbf{r},q) \\
&\quad -\mathbf{E}_{-,-m,\gamma}(\mathbf{r},q)\times\mathbf{H}_{s,m,n}(\mathbf{r})\big]\cdot\mathbf{z}_0\rho d\rho, \\
K^{s,m,\alpha}_{-,-m,\gamma}(\tilde{q},q) &= \frac{c}{4\pi}\int_0^{2\pi}d\phi\int_0^\infty\big[\mathbf{E}_{s,m,\alpha}(\mathbf{r},\tilde{q})\times\mathbf{H}_{-,-m,\gamma}(\mathbf{r},q) \\
&\quad -\mathbf{E}_{-,-m,\gamma}(\mathbf{r},q)\times\mathbf{H}_{s,m,\alpha}(\mathbf{r},\tilde{q})\big]\cdot\mathbf{z}_0\rho d\rho, \\
M^{+,-m,n}_{+,m,\gamma}(\tilde{q}) &= \frac{c}{4\pi}\int_0^{2\pi}d\phi\int_0^\infty\Big[\mathbf{E}_{+,m,\gamma}(\mathbf{r},\tilde{q})\times\mathbf{H}^{(\mathrm{T})}_{+,-m,n}(\mathbf{r}) \\
&\quad -\mathbf{E}^{(\mathrm{T})}_{+,-m,n}(\mathbf{r})\times\mathbf{H}_{+,m,\gamma}(\mathbf{r},\tilde{q})\Big]\cdot\mathbf{z}_0\rho d\rho, \\
M^{+,-m,\alpha}_{+,m,\gamma}(q,\tilde{q}) &= \frac{c}{4\pi}\int_0^{2\pi}d\phi\int_0^\infty\Big[\mathbf{E}_{+,m,\gamma}(\mathbf{r},\tilde{q})\times\mathbf{H}^{(\mathrm{T})}_{+,-m,\alpha}(\mathbf{r},q) \\
&\quad -\mathbf{E}^{(\mathrm{T})}_{+,-m,\alpha}(\mathbf{r},\tilde{q})\times\mathbf{H}_{+,m,\gamma}(\mathbf{r},q)\Big]\cdot\mathbf{z}_0\rho d\rho. \qquad (15)
\end{aligned}
$$

The power carried by the wave with the azimuthal index m to the negative direction of the z-axis is determined via the expansion coefficients of the source-excited and backscattered fields as

$$
P_{\mathrm{mod}} = \sum_n P_n = \sum_n |a_{-,m,n}+b_{-,m,n}|^2\mathscr{P}_{+,m,n} \qquad (16)
$$

for the discrete-spectrum waves (P_n is the partial power going to the n-th eigenmode) and

$$
P_{\mathrm{cs}} = \sum_{\alpha=1}^{2}\int_0^1 |a_{-,m,\alpha}(q)+b_{-,m,\alpha}(q)|^2\mathscr{P}_{+,m,\alpha}(q)dq \qquad (17)
$$

for the continuous-spectrum waves, where $\mathscr{P}_{s,m,n}$ and $\mathscr{P}_{s,m,\alpha}$ are defined by the power orthogonality relations presented in [5]. In obtaining (16) and (17), the relations $\mathscr{P}_{+,m,n} = -\mathscr{P}_{-,m,n}$ and $\mathscr{P}_{+,m,\alpha}(q) = -\mathscr{P}_{-,m,\alpha}(q)$ have been used. The power scattered from the cylinder endface to the region $z>0$ is written via the expansion coefficients for waves in free space:

$$
P_{\mathrm{fs}} = \sum_{\gamma=1}^{2}\int_0^1 |c_{+,m,\gamma}(q)|^2\frac{cp(q)}{4k_0^2q}dq. \qquad (18)
$$

4 Numerical results

The integral equations (11)–(13) have numerically been solved using the Simpson's integration method. The powers going to the eigenwaves of the cylinder and free space have been calculated for the values of parameters in (1) $a=2.5\,\mathrm{cm}$, $d=4a$, $\tilde{p}=0$, and $m=1$, which are typical of helical antennas used in gaseous plasma tubes for producing helicon waves. It is assumed that the frequency of the source belongs to the resonant interval $\omega_{\mathrm{LH}}<\omega<\omega_{\mathrm{H}}$ of the whistler range, where $\omega_{\mathrm{LH}}=(\omega_{\mathrm{H}}\Omega_{\mathrm{H}})^{1/2}$ is the lower-hybrid frequency. The values of dimensionless parameters $\omega_p/\omega_H=12.7$, $\omega_{\mathrm{LH}}/\omega_{\mathrm{H}}=3.7\times10^{-3}$ and $\omega_H a/c=1.17$ of a plasma medium have been used for calculations. Note that, due to a resonance character of a plasma in the considered frequency range, the cylinder supports an infinite number of the propagating eigenmodes. Figure 2 shows the powers P_{mod} and P_{cs} as functions of L that varies in the range $5a<L<2\lambda_0$ ($\lambda_0=2\pi/k_0\gg a$ is the free-space wavelength). All the values presented in this and forthcoming figures are normalized to the total power P_0 radiated from the same source located in free space. As can be seen in Fig. 2, the inequality $P_{\mathrm{mod}}\gg P_{\mathrm{cs}}$ holds for the powers. The oscillations in the dependence $P_{\mathrm{mod}}(L)$ are related to the interference of the small-scaled eigenmodes with the different propagation constants. The power P_{cs} carried by the continuous-spectrum waves increases significantly at certain values of L/λ_0, remaining to be less than P_{mod}. As was shown by the calculations, the relation $P_{\mathrm{fs}}\simeq P_{\mathrm{mod}}$ is

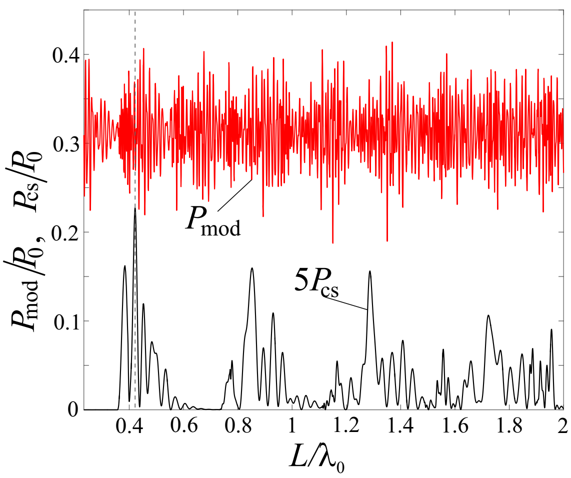

Figure 2. The normalized powers P_{mod} (red line) and P_{cs} (black line) as functions of the parameter L/λ_0 for $\omega/\omega_{\mathrm{H}}=2.5\cdot10^{-2}$, $\omega_p/\omega_H=12.7$, $\omega_{\mathrm{LH}}/\omega_{\mathrm{H}}=3.7\times10^{-3}$, $\omega_H a/c=1.17$, and $j_z=0.25j_\phi$. The value of P_{cs} is multiplied by 5.

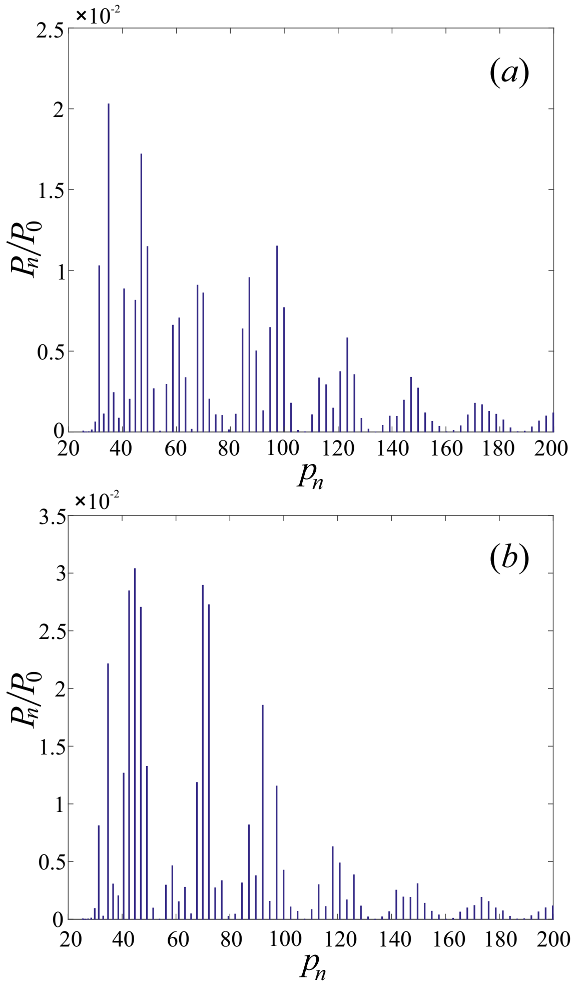

Figure 3. The normalized partial powers P_n for the individual eigenmodes with the radial indices n and the longitudinal wave numbers p_n for $L = 0.423\lambda_0$ (a) and $L = 0.697\lambda_0$ (b). The same parameters as in Fig. 2.

justified for the power scattered to free space for each value of L. Thus, most of the radiated power goes to the negative z-axis direction by means of the eigenmode waves.

The distributions of the partial powers P_n over the discrete-spectrum waves (for only the first 75 eigenmodes for which $p_n < 200$) are shown in Figs. 3(a) and 3(b) for two values of L/λ_0. In the case presented in Fig. 3(a), the ratio of the power going to the discrete-spectrum waves to the power going to the continuous spectrum waves amounts its minimum $P_{\mathrm{mod}}/(P_{\mathrm{cs}} + P_{\mathrm{fs}}) = 3.028$ at $L = 0.423\lambda_0$. The corresponding value of L/λ_0 is indicated by the vertical dashed line in Fig. 2. The ratio of the powers changes periodically with increasing L and reaches one of its maximum values $P_{\mathrm{mod}}/(P_{\mathrm{cs}} + P_{\mathrm{fs}}) = 2 \cdot 10^3$ at $L = 0.696\lambda_0$. For this case, the distribution of P_n over the set of the eigenwaves is presented in Fig. 3(b). It follows from the comparison of the panels (a) and (b) in Fig. 3 that the notable redistribution of the partial powers over the eigenmode spectrum occurs when the parameter L/λ_0 changes.

5 Conclusions

In this work, the electromagnetic wave excitation has been studied in the case where the nonsymmetric given source is placed on the surface of a semi-infinite cylinder filled with a magnetoplasma. The integral equations for the field expansion coefficients of waves diffracted from the endface of the cylinder have been derived and numerically solved. The partial powers going to the discrete- and continuous-spectrum waves as functions of the distance from the antenna to the cylinder endface have been analysed. It has been demonstrated that in the considered frequency range almost all the radiated power goes to the eigenmodes, reflecting from the endface. It has also been established that the part of the power that goes to the continuous-specrtum waves noticeably depends on the source position.

6 Acknowledgements

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References

[1] F. F. Chen, "Helicon Discharges and Sources: A Review," *Plasma Sources Science & Technology*, **24**, p. 014001, January 2015, doi: 10.1088/0963-0252/24/1/014001.

[2] J.F. Caneses, B.D. Blackwell, and P. Piotrowicz, "Helicon Antenna Radiation Patterns in a High-Density Hydrogen Linear Plasma Device," *Physics of Plasmas*, **24**, 11, November 2017, p. 113513, doi: 10.1063/1.5000848.

[3] W.E. Amatucci, D.D. Blackwell, E.M. Tejero, C.D. Cothran, L.Rudakov, G.I. Ganguli, and D.N. Walker, "Whistler Wave Resonances in Laboratory Plasma," *IEEE Transactions on Plasma Science*, **39**, 2, February 2011, pp. 637–643, doi: 10.1109/TPS.2010.2096235.

[4] V.A. Es'kin and A.V. Kudrin, "A New Method for Constructing an Orthogonal System of Eigenwaves of an Open Cylindrical Waveguide Surrounded by an Isotropic Medium," *Proceedings of PIERS 2017*, May 2017, pp. 843–848, doi: 10.1109/PIERS.2017.8261860.

[5] O.M. Ostafiychuk, V. A. Es'kin, and A. V. Kudrin, "Electromagnetic Wave Excitation by a Nonsymmetric Antenna Located on the Surface of an Open Gyrotropic Cylindrical Waveguide," *Proceedings of 2019 URSI AP-RASC*, March 2019, pp. 1–4, doi: 10.23919/URSIAP-RASC.2019.8738518.

[6] I.G. Kondrat'ev, A.V. Kudrin, and T.M. Zaboronkova, *Electrodynamics of Density Ducts in Magnetized Plasmas*, Amsterdam, The Netherlands: Gordon and Breach, 1999.