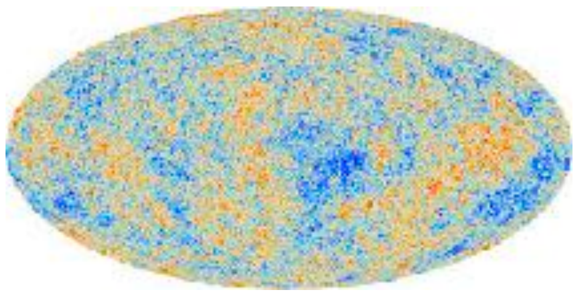
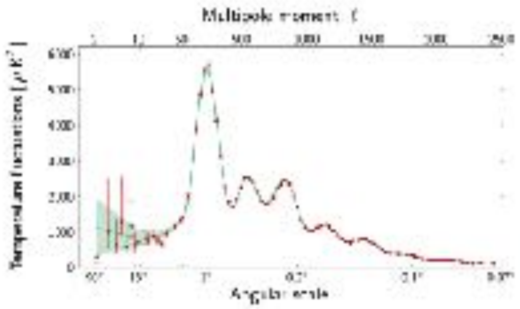
CMB Theory





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ICF - UNAM

Taller

July 30, 2018

VI TALLER DE GRAVITACIÓN Y COSMOLOGÍA
ICF-UNAM, Cuernavaca, Morelos, 2 y 3 de agosto, 2018
Segunda Circular

26/05/2018

El objetivo del presente taller es servir como foro para discutir características y consecuencias de los trabajos recientes en los temas de gravitación, cosmología y áreas afines. Este taller también pretende establecer nuevas colaboraciones entre los participantes que trabajan en dichas áreas ampliando con ello las redes de investigación dentro de la comunidad científica mexicana.

PARTICIPANTES

La inscripción al evento sigue abierta para investigadores y estudiantes que contribuyan al taller. Por el momento tenemos confirmada la participación de los siguientes investigadores.

MIGUEL ASPEITIA
NORA BRETON
KAREN CABALLERO MORA
JOSÉ ANTONIO GONZÁLEZ CERVERA
FRANCISCO S. GUZMÁN
ALFREDO HERRERA AGUILAR
GERMAN IZQUIERDO
ANDRÉS PLAZAS

ESTUDIANTES

Los estudiantes interesados en participar deberán estar inscritos en un programa de posgrado, ser estudiantes regulares y enviar carta de apoyo (recomendación) de su supervisor.

Enviar resumen de plática (dado el caso) y carta de recomendación a más tardar el día viernes 8 de junio de 2018 a Juan Carlos Hidalgo al correo hidalgo@fis.unam.mx

HOSPEDAJE PARA ESTUDIANTES Y POSTDOCS

Habrá apoyo en forma de hospedaje y alimentos para estudiantes y postdocs. Se dará preferencia a quienes presenten plática corta.

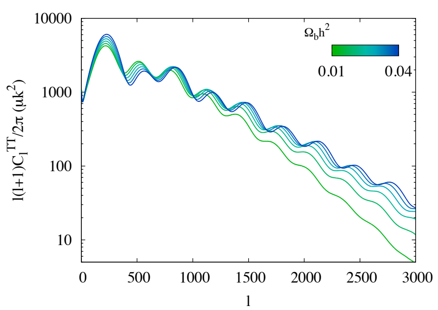
SEDE DEL EVENTO

La sede del Taller será el auditorio del Instituto de Ciencias Físicas de la Universidad Nacional Autónoma de México, ubicado en el Campus de la Universidad Autónoma del Estado de Morelos, en la ciudad de Cuernavaca, Morelos.

Motivation

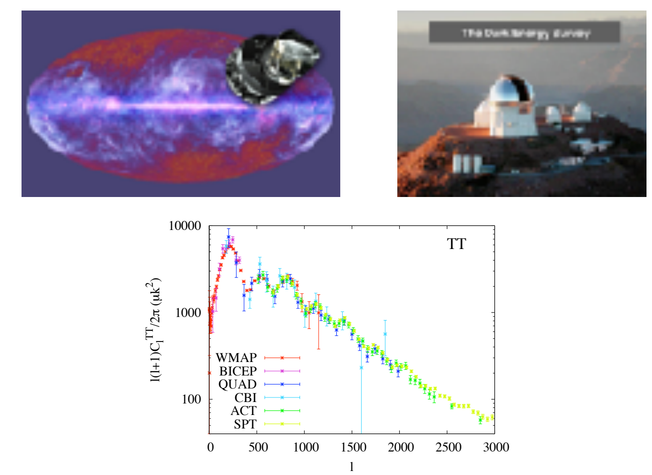
Theory

$$\int d^4x \sqrt{-g}\, R$$

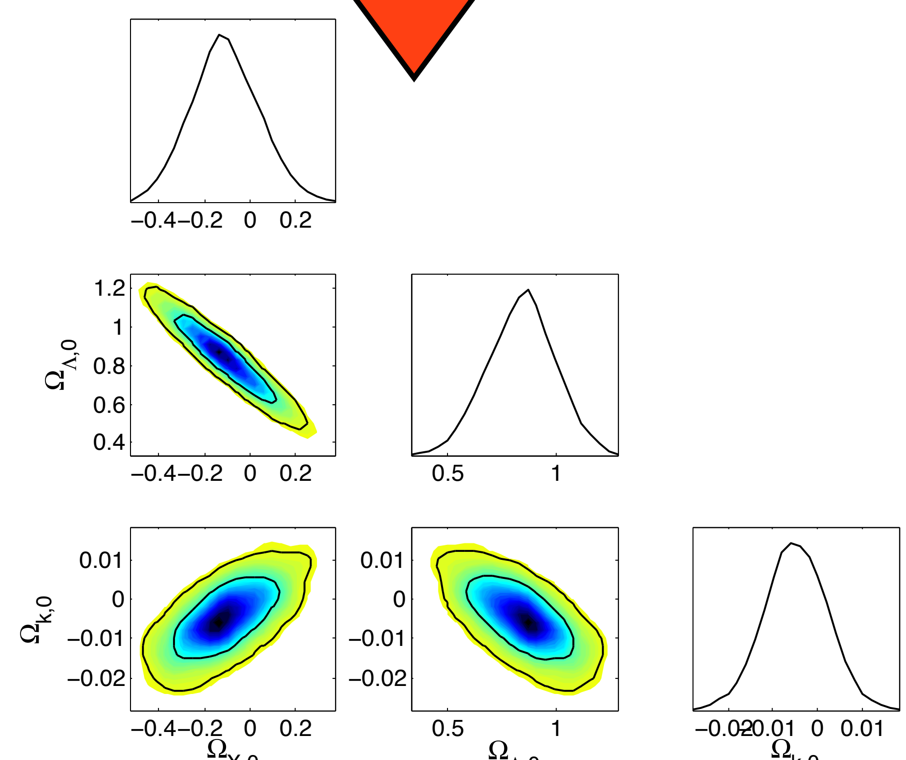


Compare

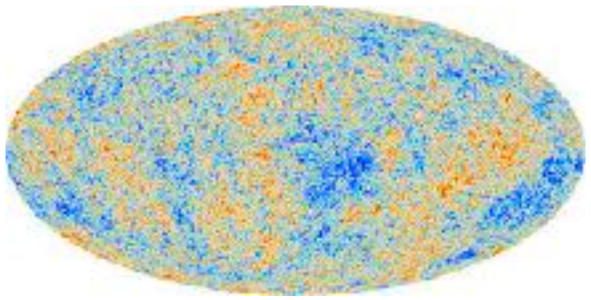
Observations



Analysis



Motivation

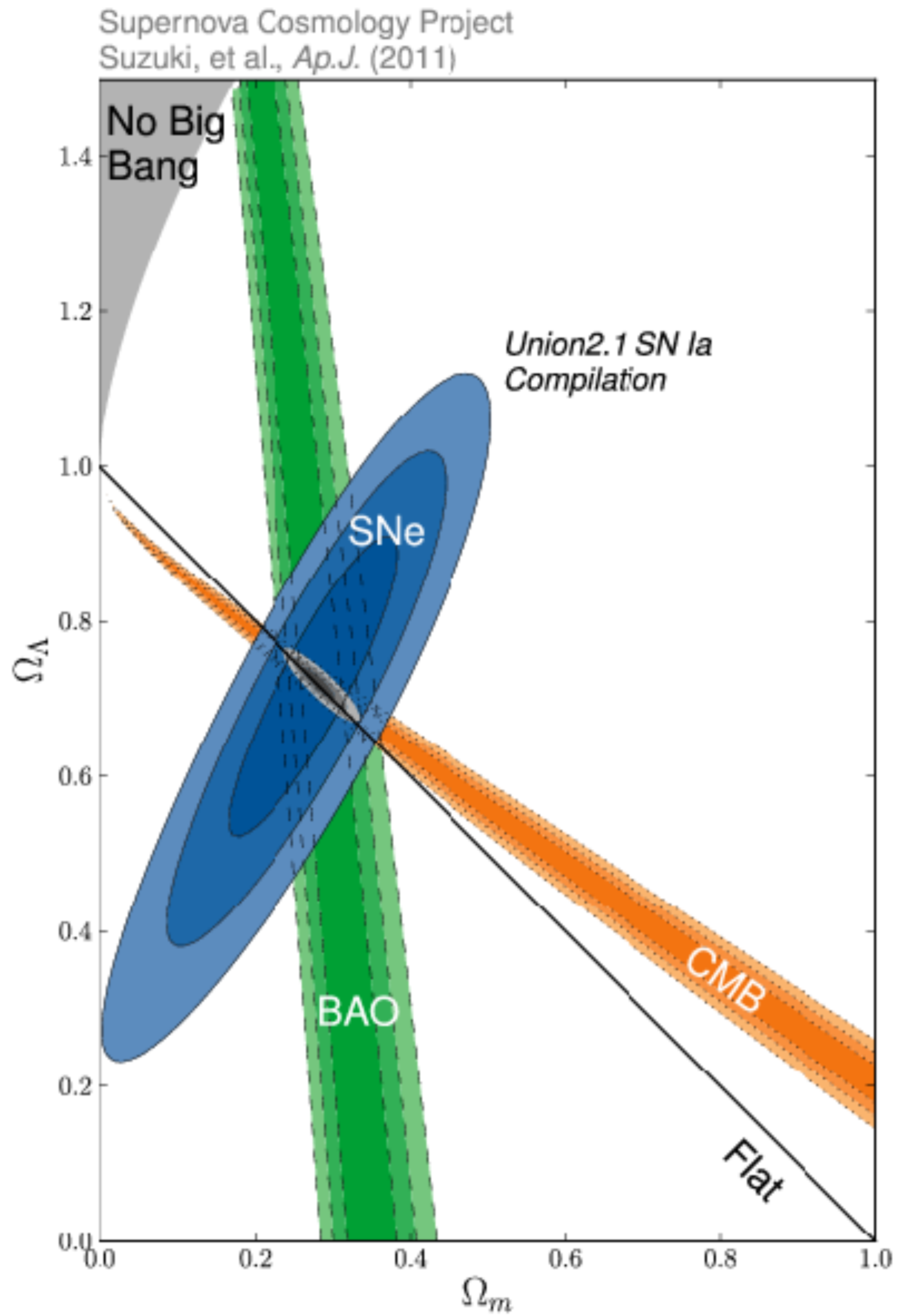


- The cosmic microwave background (CMB) is the **thermal radiation left over** from the "Big Bang", also known as **"relic radiation"**.

- The CMB is a **snapshot of the oldest light** in our Universe, imprinted on the sky when the Universe was just **380,000 years old,** dating to the epoch of **recombination**.

- It shows **tiny temperature fluctuations** that correspond to regions of slightly different densities, **representing the seeds of all future structure**: the stars and galaxies of today.

Motivation



Supernova Cosmology Project
Suzuki, et al., Ap.J. (2011)
No Big Bang
Union2.1 SN Ia Compilation
SNe
BAO
CMB
Flat
Ω_Λ
Ω_m
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

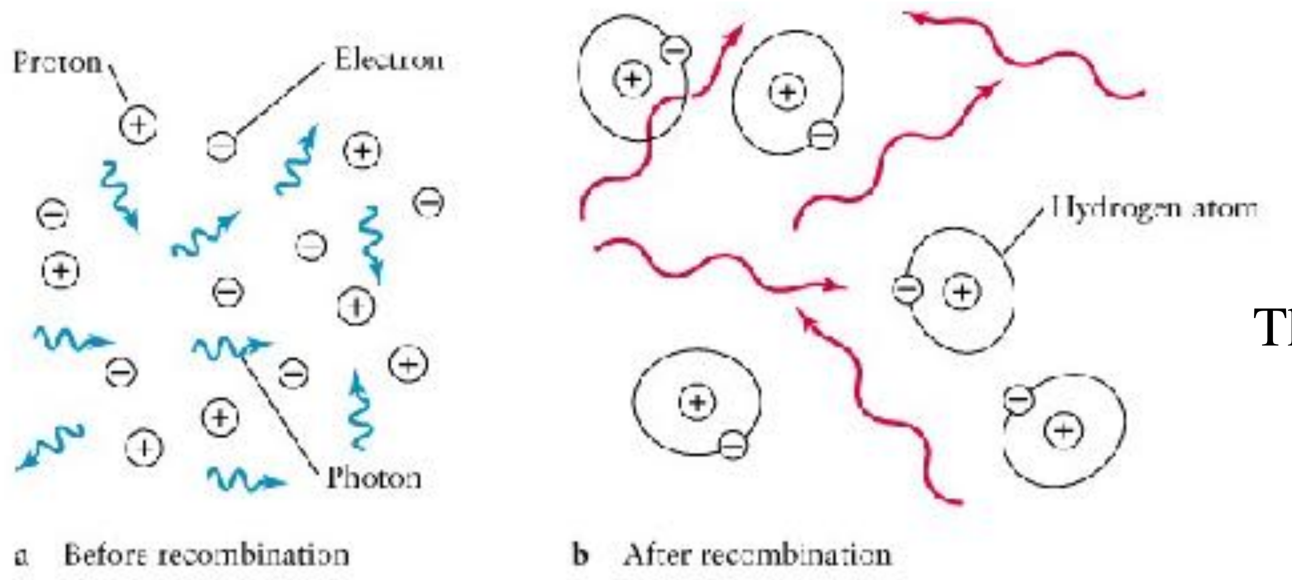
The Hot Big Bang

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Recombination

Photons were tightly **coupled to the electrons** via Compton scattering, which in turn strongly interacted with protons via Coulomb scattering.

When the temperature became low enough, **the electrons and nuclei combined**



The **redshift of recombination**:

$$\boxed{z_{\rm rec} \approx 1320}$$

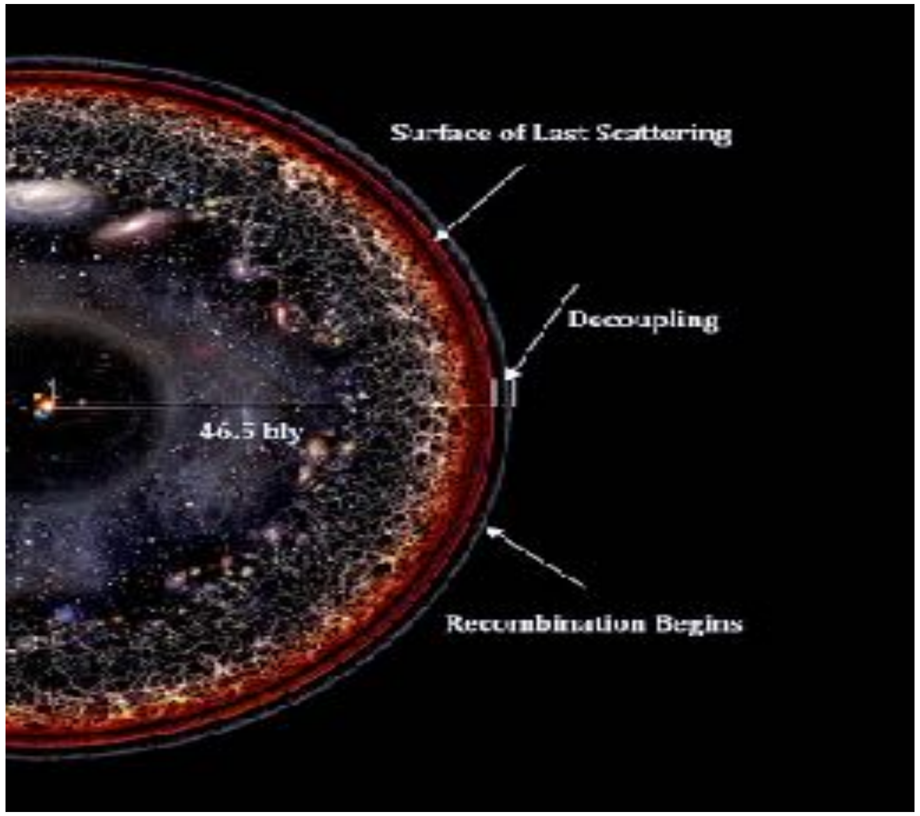
Photon Decoupling

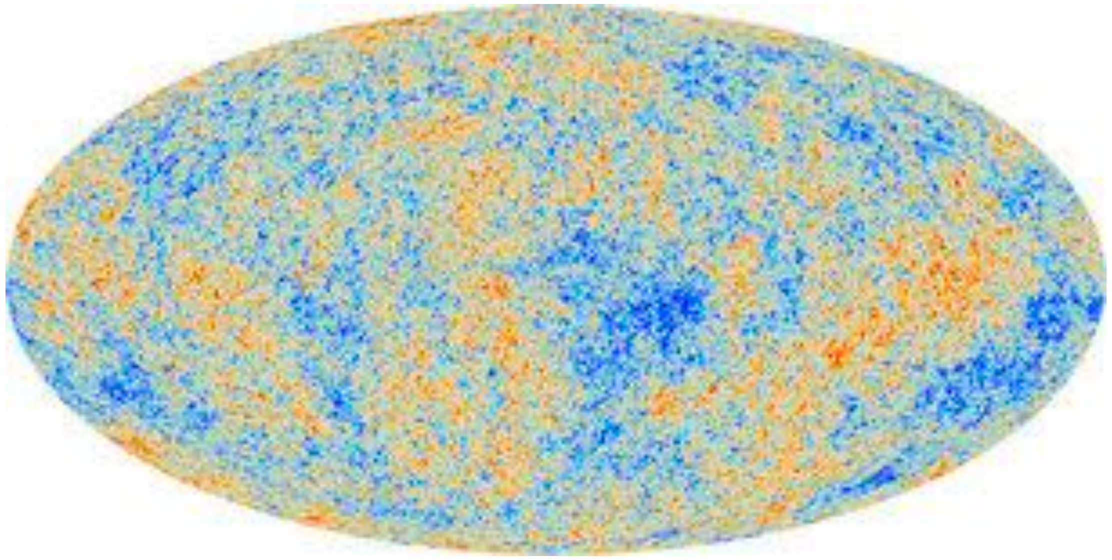
$$\boxed{z_{\rm dec} \sim 1100, \qquad t_{\rm dec} \sim 380\,000 \text{ yrs.}}$$

Last Scattering Surface

After their last scattering off an electron, **photons were able to travel unimpeded through the Universe**. These are the Cosmic Microwave Background **photons we receive today**, still with their blackbody distribution, now redshifted by a factor of 1100.

They constitute a **last scattering surface**, or more appropriately a **last scattering layer**





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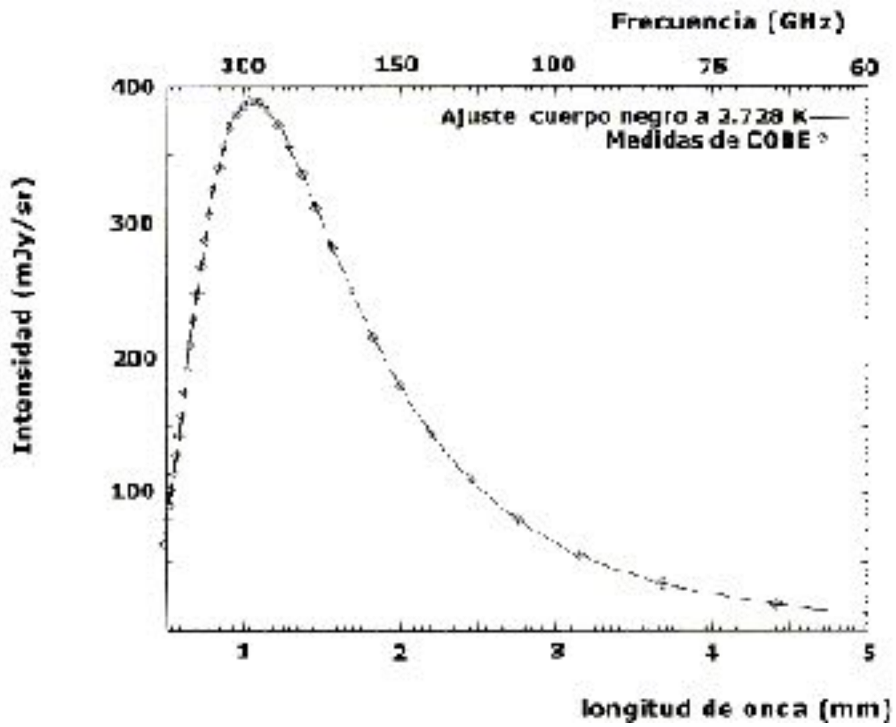
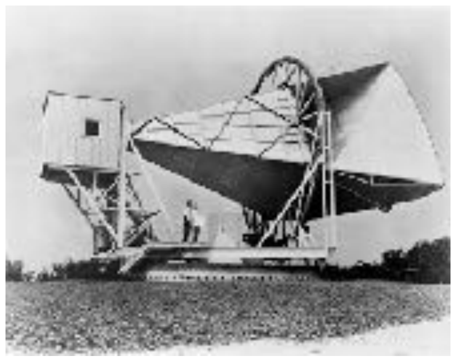




The **original detection** by Penzias and Wilson was at a wavelength of 73.5 mm, this being the wavelength of the telecommunication signals they were working with; this wavelength is **two orders of magnitude longer** than λpeak = 1.1mm of a T = 2.7255K blackbody.

$$\langle T \rangle = \frac{1}{4\pi}\int T(\theta,\phi)\sin\theta d\theta d\phi = 2.7255 \pm 0.0006K$$

The **deviations from this mean temperature** from point to point on the sky are tiny.

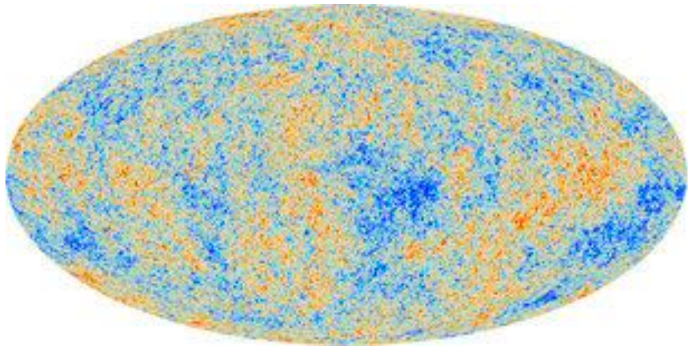
$$\frac{\delta T}{T}(\theta,\phi) = \frac{T(\theta,\phi) - \langle T \rangle}{\langle T \rangle}$$

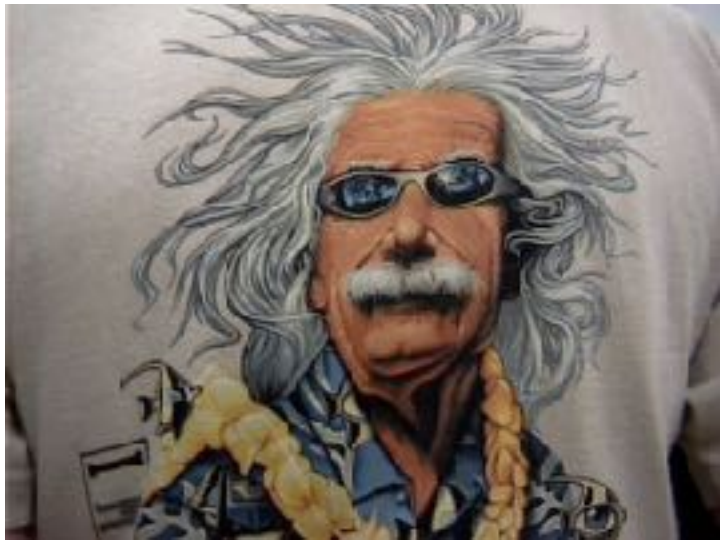
WMAP and Planck

$$\left\langle \left(\frac{\delta T}{T}\right) \right\rangle^{1/2} = 1.1 \times 10^{-5}$$

J.C. Hidalgo

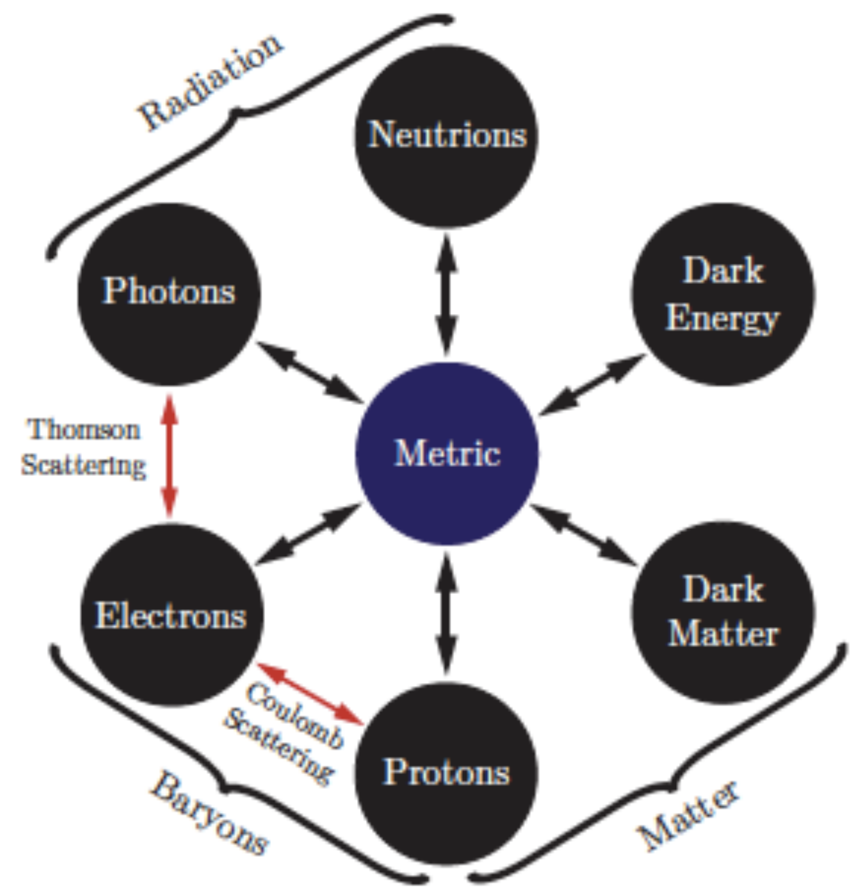
Linear Perturbations





Unperturbed **Perturbed**

The Boltzmann equation



Describes the statistical behaviour of a thermodynamic system not in a state of equilibrium

$$\frac{df}{d\eta} = C[f]$$

Perturbed temperature

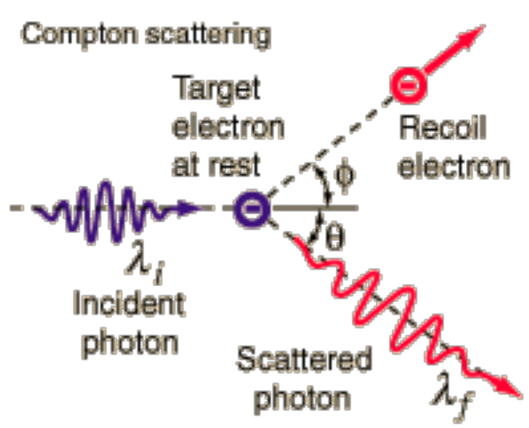
$$\boxed{T(\eta, \mathbf{x}, \mathbf{n}) = \bar{T}(\eta)[1 + \Delta(\eta, \mathbf{x}, \mathbf{n})],} \tag{3.13}$$

The Boltzmann equation

The **evolution of perturbations** in the universe is quantified by the Boltzmann equation:

$$\left(\frac{\partial f}{\partial \eta}\right)_P + \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \eta} + \frac{\partial f}{\partial P}\frac{\partial P}{\partial \eta} + \frac{\partial f}{\partial n^i}\frac{\partial n^i}{\partial \eta} = C[f, G],$$

Relates the **effects of gravity** on the photon distribution function f to the **rate of interactions with other species**, given by the collision term C[f,G].



The Boltzmann equation thus yields to the evolution equation of temperature perturbations

$$\boxed{\Delta' + ik\mu\Delta + \kappa'\Delta = -i\mu k[\Phi + \Psi] - 2\Phi' + \kappa'\left\{\frac{1}{4}\delta_\gamma - \Phi - i\mu v_b + \frac{1}{10}P_2(\mu)[\sqrt{6}E_2 - \Delta_2]\right\}}$$

$\mathcal{M} = \Delta + 2\Phi$

Solving ...

$$\mathcal{M}' + ik\mu\mathcal{M} + \kappa'\mathcal{M} = i\mu k[\Phi - \Psi] + \kappa' \left\{ \frac{1}{4} D_g^\gamma - i\mu v_b + \frac{1}{10} P_2(\mu) \left[\sqrt{6} E_2 - \mathcal{M}_2 \right] \right\}.$$

The procedure is as follows: For each Legendre polynomials P_l

$$\begin{aligned}
\mathcal{M}_0' &= -\frac{k}{3} V_\gamma, \\
\mathcal{M}_1' &= \kappa'(V_b - V_\gamma) + k(\Psi - \Phi) + k\left(\mathcal{M}_0 - \frac{2}{5}\mathcal{M}_2\right), \\
\mathcal{M}_2' &= -\kappa'(\mathcal{M}_2 - \mathcal{C}) + k\left(\frac{2}{3} V_\gamma - \frac{3}{7}\mathcal{M}_3\right), \\
\mathcal{M}_l' &= -\kappa'\mathcal{M}_l + k\left(\frac{l}{2l-1}\mathcal{M}_{l-1} - \frac{l+1}{2l+3}\mathcal{M}_{l+1}\right), \quad l > 2,
\end{aligned}$$

The Line of Sight Strategy

$$\mathcal{M}(\mu, \eta_0) = \int_0^{\eta_0} e^{\imath\mu k(\eta - \eta_0)} S_T(k, \eta) d\eta$$

$$\begin{aligned}
S_T &= -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi' - \Psi'] + g'\left[\frac{V_b}{k} + \frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C} \\
&+ g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi - \Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right],
\end{aligned}$$

$$S_T = -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi' - \Psi'] + g'\left[\frac{V_b}{k} + \frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C} + g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi - \Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right],$$

The **density contrast D_g^γ** is the main contribution, driving the spectrum towards the **oscillatory behaviour**.

The **$(\Phi - \Psi)$ term** arises from the **gravitational redshift** when climbing out of the potential well at last scattering.

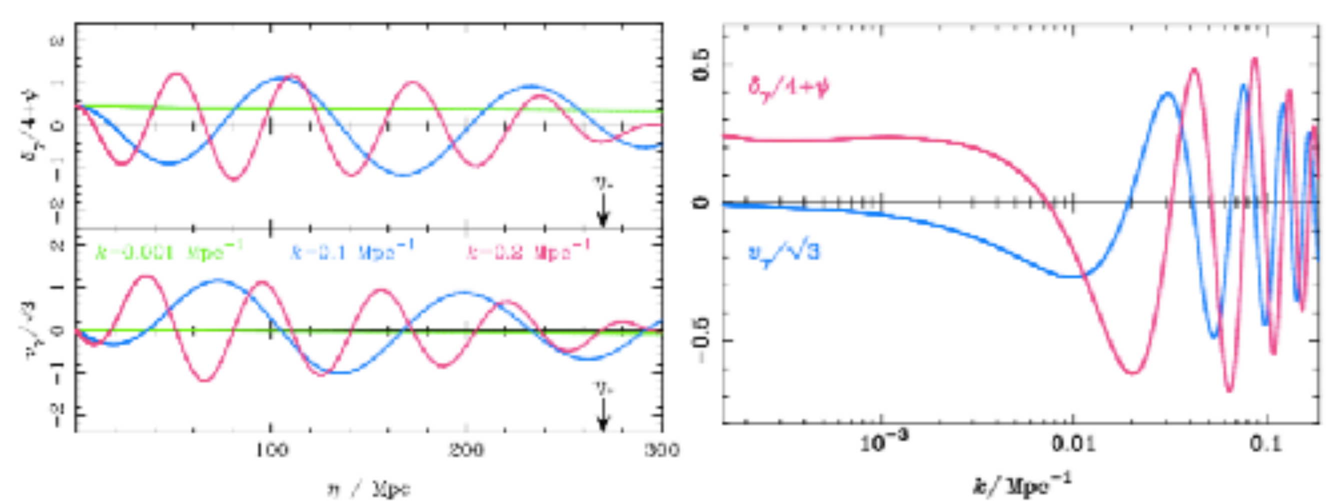
The **combination $D_g^\gamma/4 - (\Phi - \Psi)$** is known as the **ordinary Sachs-Wolfe effect (SW)**.

This gives the main contribution on scales that at decoupling were well outside the horizon

The **Doppler shift, V_b-term,** describes the blueshift caused by **last scattering electrons moving towards** the observer.

The term involving time derivatives of the potentials, **$(\Phi' - \Psi')$**, the **integrated Sachs-Wolfe effect (ISW)**.

It describes the **change of the CMB photon energy** due to the **evolution of the potentials** along the line of sight.



CMB power spectrum

Define the **power spectrum, P_f (k),** of a homogeneous and isotropic field,

$$\langle f(\mathbf{k}) f^*(\mathbf{k}')\rangle = \frac{2\pi^2}{k^3}\mathcal{P}_f(k)\delta(\mathbf{k}-\mathbf{k}')\,.$$

$$\frac{\Delta T}{\bar{T}}(\eta_0, \mathbf{x_0}, \mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}),$$

The angular **CMB power spectrum** C^{TT} is computed through the **two-point correlation function**

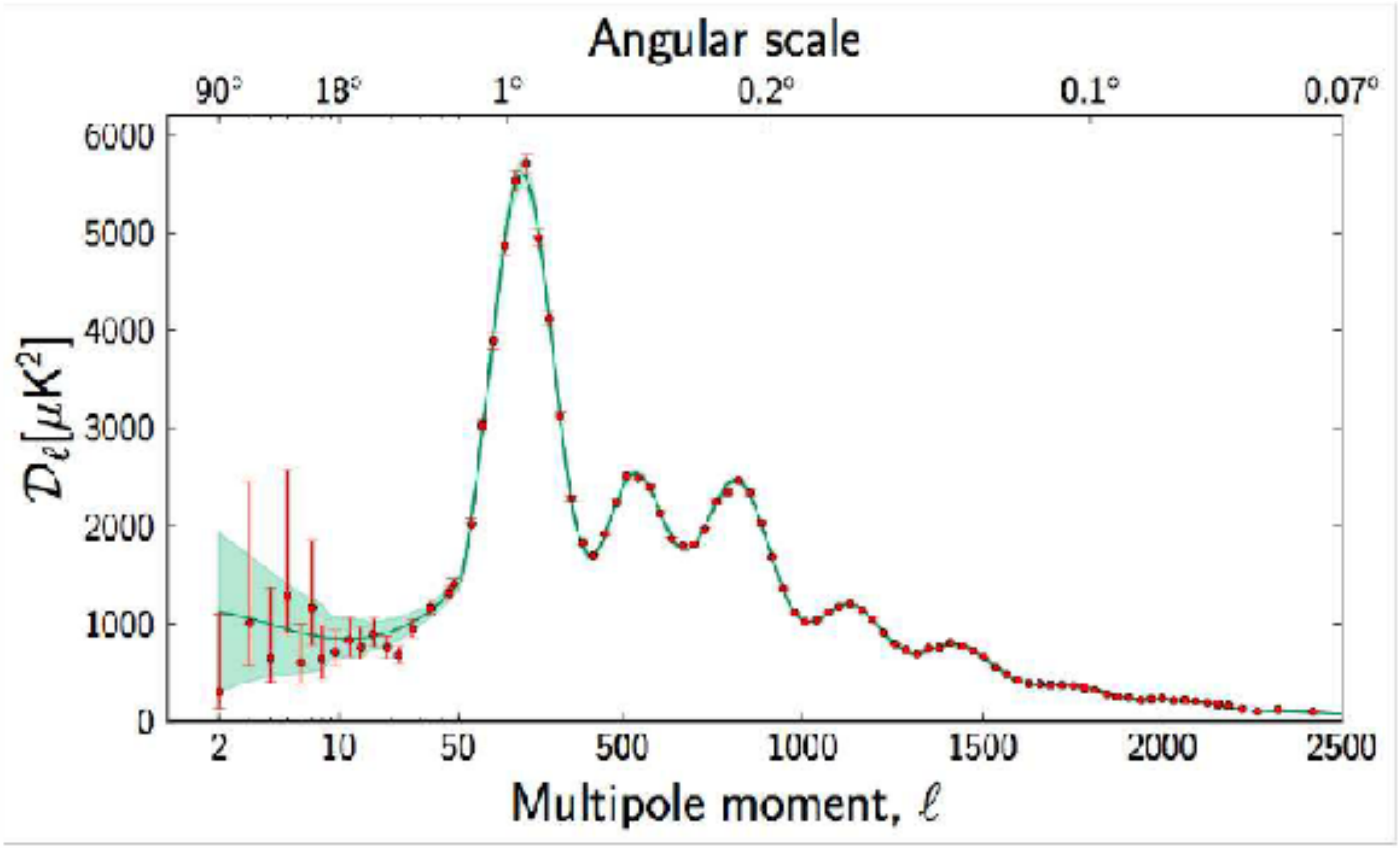
$$\boxed{C_l^{XY} = \frac{4\pi}{(2l+1)^2}\int \frac{d^3k}{(2\pi)^3}\,\mathcal{P}_{\mathcal{R}}(k)\,\Delta_l^X(k)\Delta_l^Y(k),}$$

$$\Delta_l^T = (2l+1)\int d\eta j_l(k[\eta-\eta_0])S_T(k,\eta),$$

$$\begin{aligned} S_T &= -e^{\kappa(\eta)-\kappa(\eta_0)}[\Phi'-\Psi'] + g'\left[\frac{V_b}{k}+\frac{3}{k^2}\mathcal{C}'\right] + g''\frac{3}{2k^2}\mathcal{C} \\ &+ g\left[\frac{1}{4}D_g^\gamma + \frac{V_b'}{k} - (\Phi-\Psi) + \frac{\mathcal{C}}{2} + \frac{3}{2k^2}\mathcal{C}''\right], \end{aligned}$$

Voilà!

CMB Spectrum



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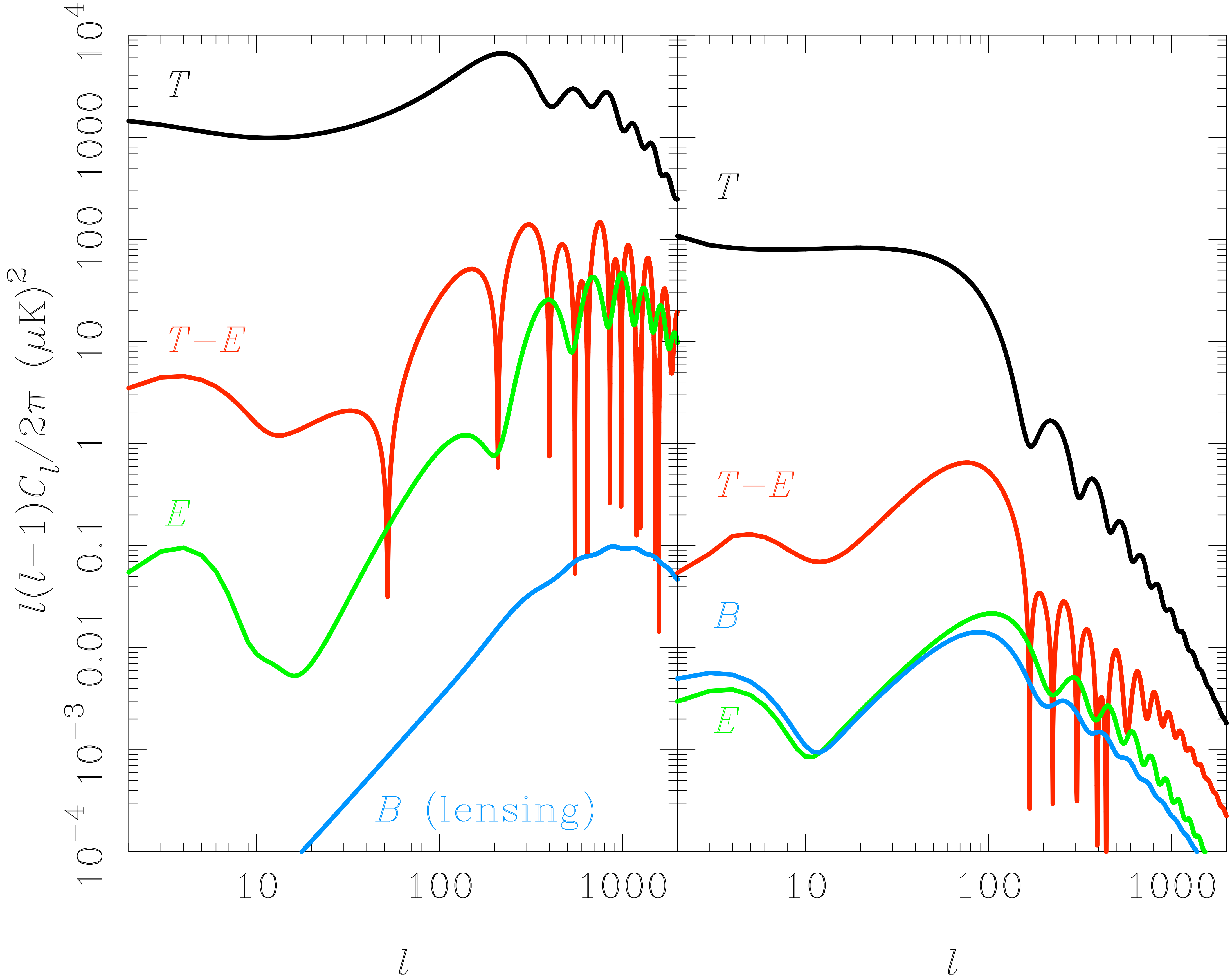

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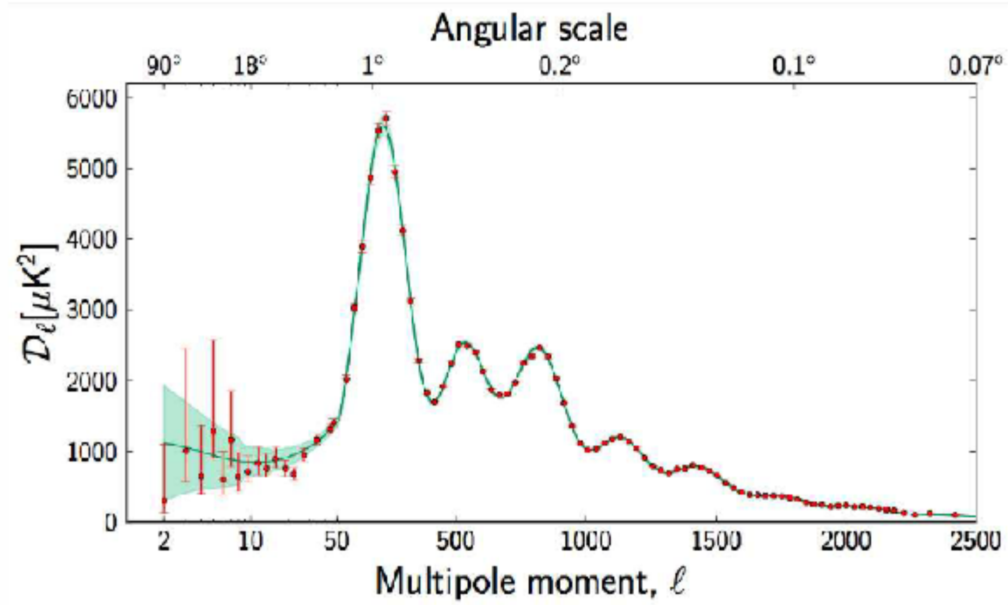
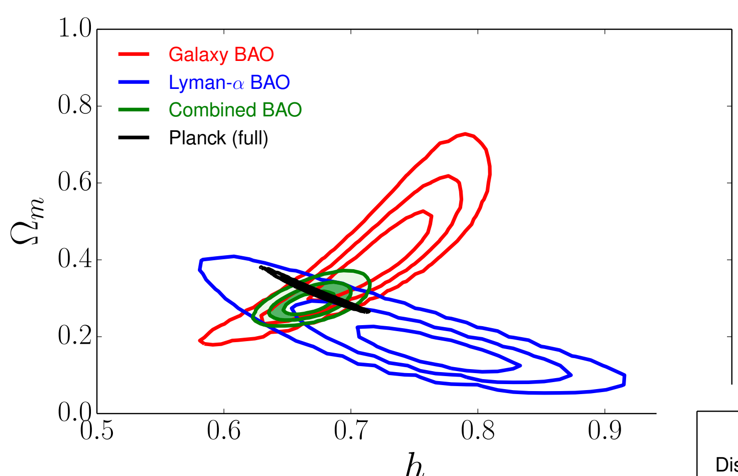
TABLE I. Comparison between CMB Codes [a]

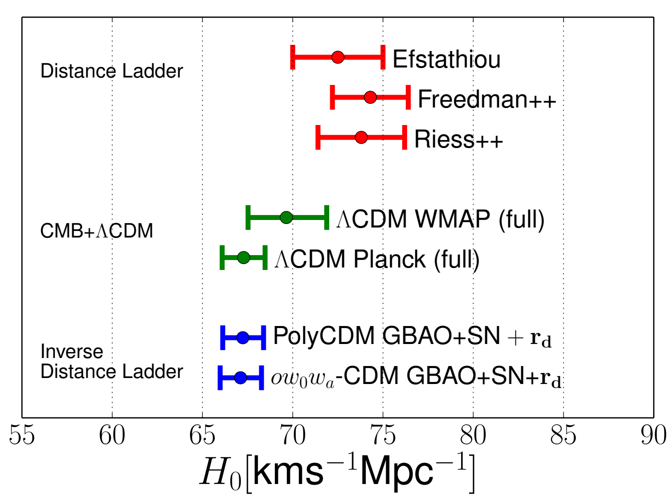
	CAMB	CLASS	CMBEASY	CMBquick	CosmoLib [b]
Language	F90	C	C++	Mathematica	F90[c]
gauge [d]	syn.	syn./Newt. [e]	syn./gauge-inv.	Newt.	Newt.
open/close universe	Yes	No	No	No	No
massive neutrinos	Yes	Yes	Yes	Yes	No
tensor perturb.	Yes	Yes	Yes	Yes	Yes
CDM isocurvature mode	Yes	Yes	Yes	Yes	Yes
dark energy perturb.	Yes	Yes	Yes	No	Yes
nonzero $c_{s,b}^2$	Yes	Yes	Yes	No	Yes
dark energy EOS.	constant	$w_0 + w_a(1-a)$	arbitrary	-1	arbitrary
non-smooth primordial power	No	No	No	No	Yes
MCMC driver	Yes	No	Yes	No	Yes
periodic proposal density	No	No	No	No	Yes
data simulation	No	No	No	No	Yes
second-order perturb. [f]	No	No	No	Yes	No [g]

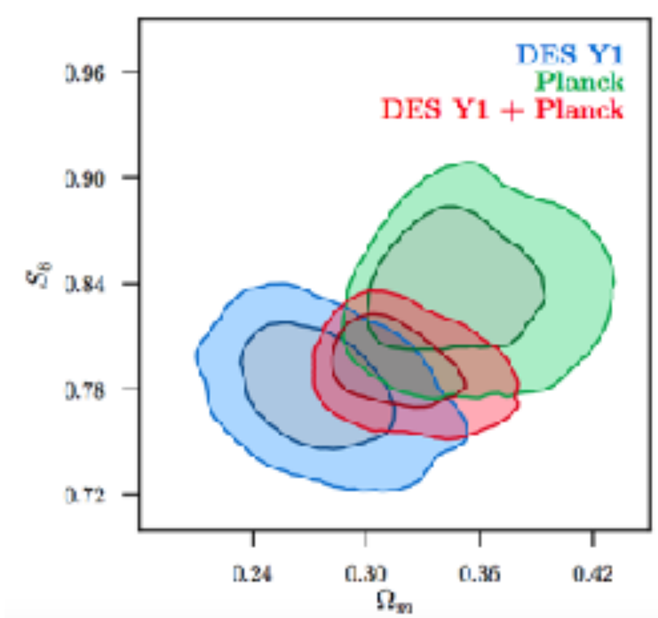
[a] Here we do not include CMBFast, which is no longer supported by its authors or available for download.

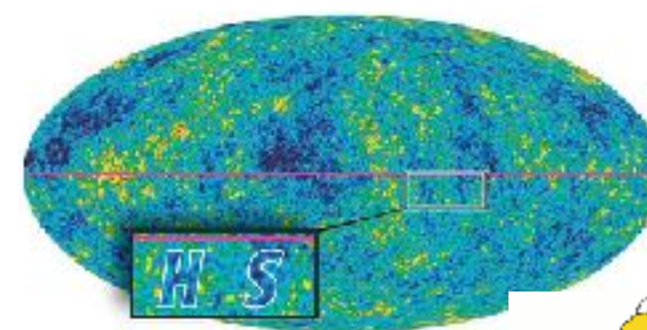
Theory











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COSMOLOGICAL PARAMETERS

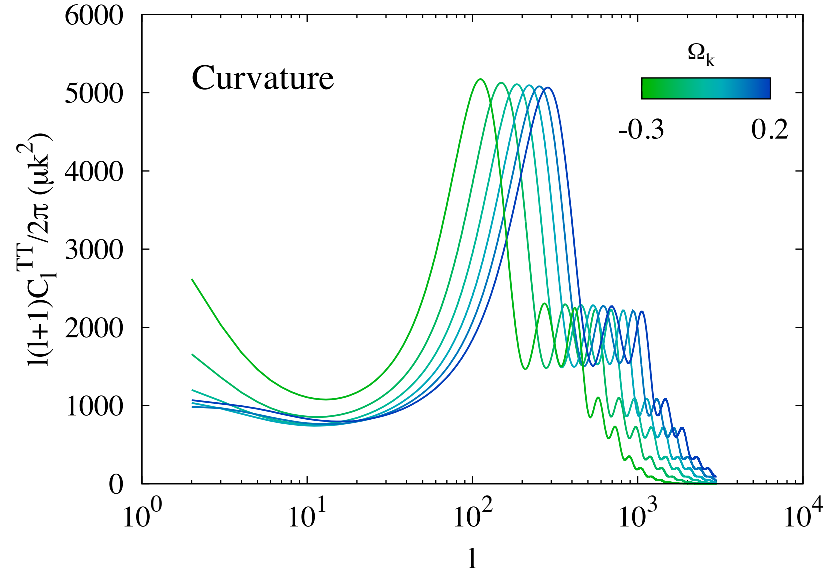
The **whole structure of the CMB** depends strongly on the initial conditions emerging from the inflationary era ($P_{R,T}$), on the matter-energy content ($\Omega_{i,0}$), and on the expansion rate history (H_0).

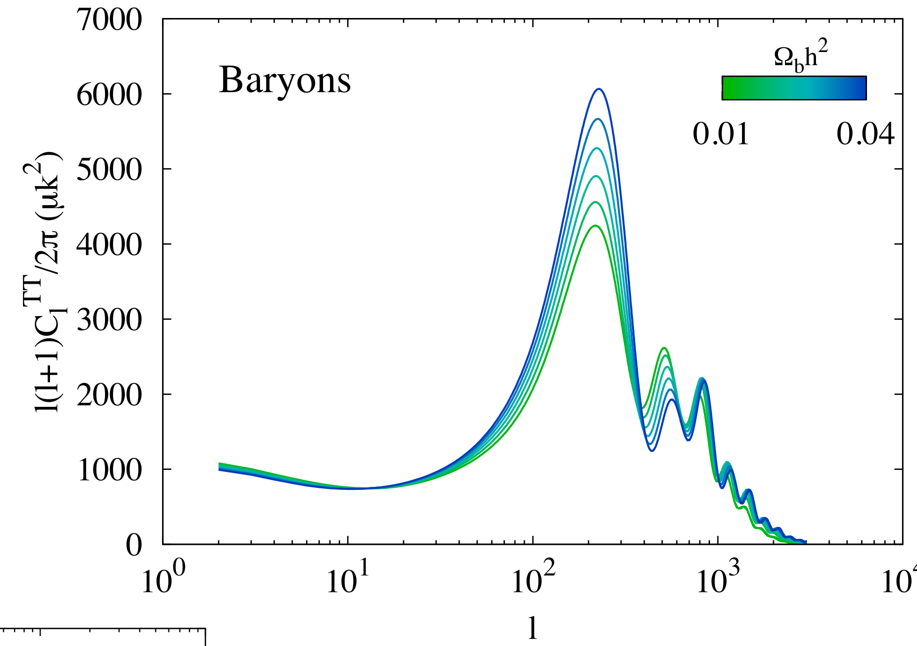
These parameters, commonly **called standard parameters**, are considered as the principal quantities used **describe the universe**.

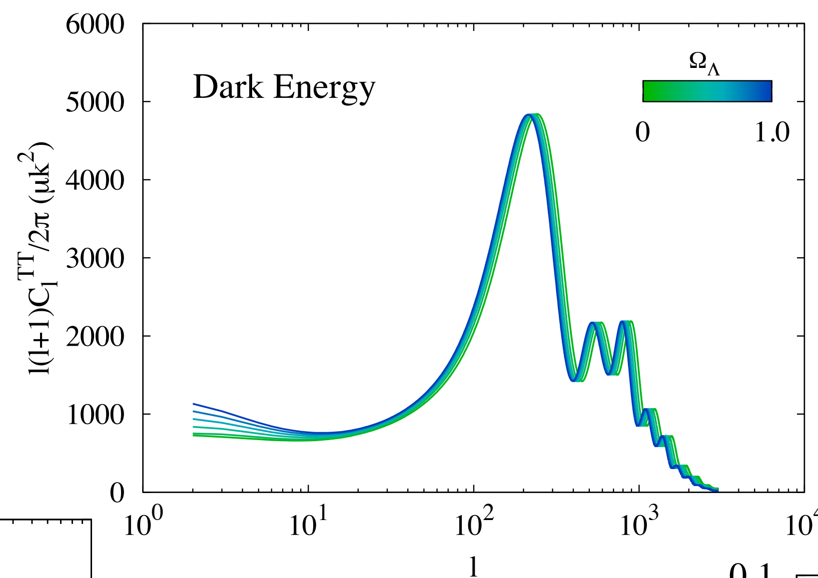
They are not, however, **predicted** by any fundamental theory, rather **we have to fit them by hand** in order to determine which **combination** best describes the current astrophysical observations

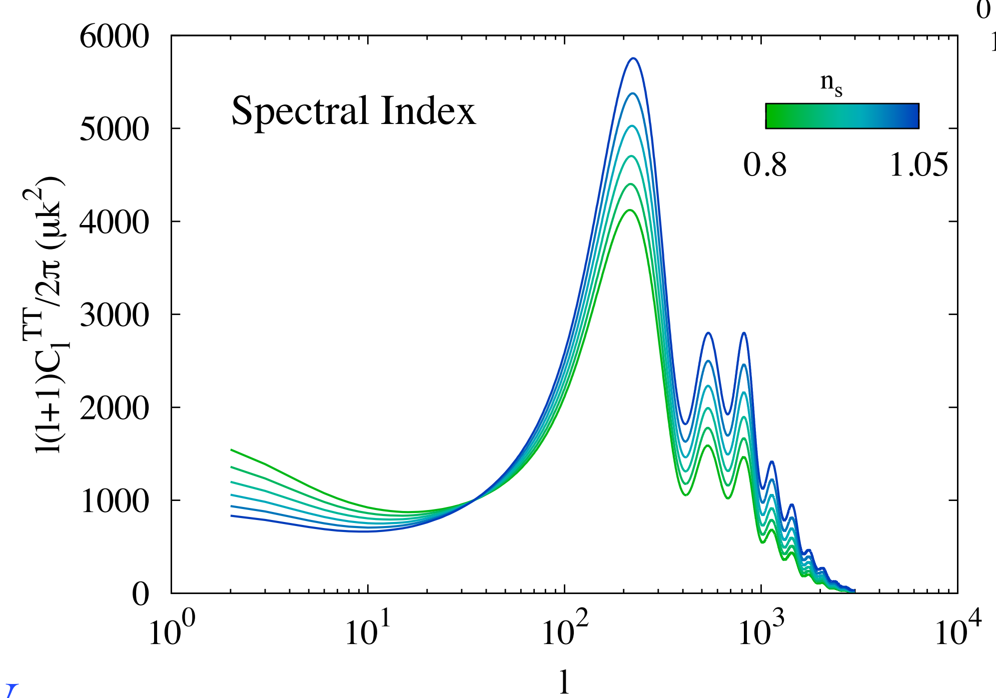
Model	Abbreviation	Parameters
Cosmological constant	ΛCDM	Ω_k, Ω_m
Constant w	wCDM	Ω_k, Ω_m, w
Varying w (CPL)	CPL	Ω_k, Ω_m, w_0, w_a
Generalized Chaplygin Gas	GCG	Ω_k, A_s, α
Dvali-Gabadadze-Porrati	DGP	Ω_k, Ω_m
Modified Polytropic Cardassian	MPC	Ω_k, Ω_m, q, n
Interacting Dark Energy	IDE	Ω_k, Ω_m, w_x, δ
Early Dark Energy	EDE	Ω_k, Ω_m, Ω_e, w_0

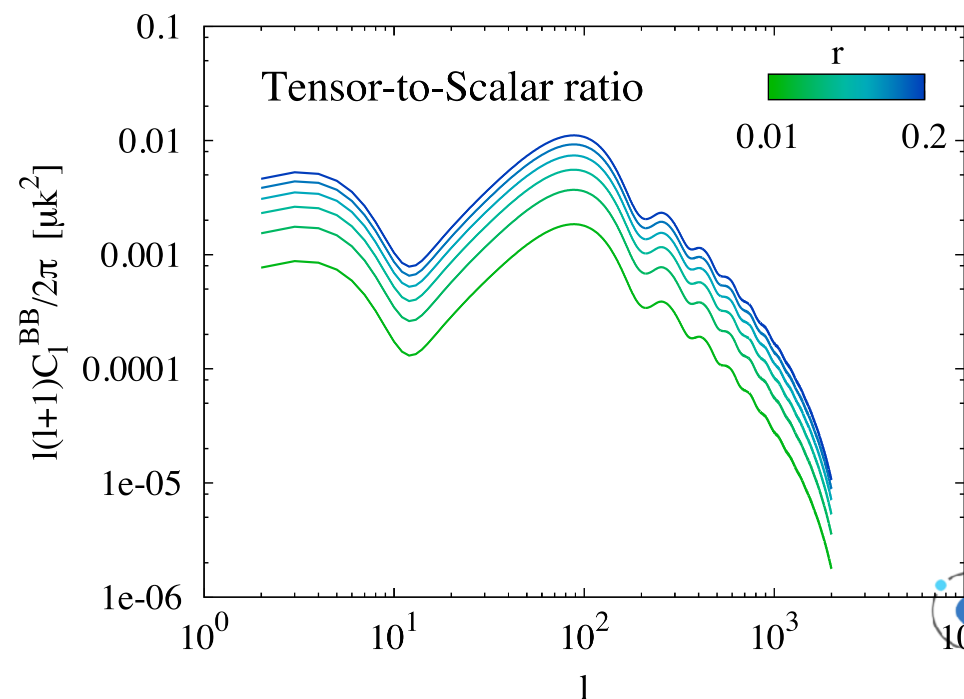
PARAMETERS


Curvature
Ω_k
-0.3
0.2
$l(l+1)C_l^{TT}/2\pi\ (\mu k^2)$
l


Baryons
$\Omega_b h^2$
0.01
0.04
$l(l+1)C_l^{TT}/2\pi\ (\mu k^2)$
l

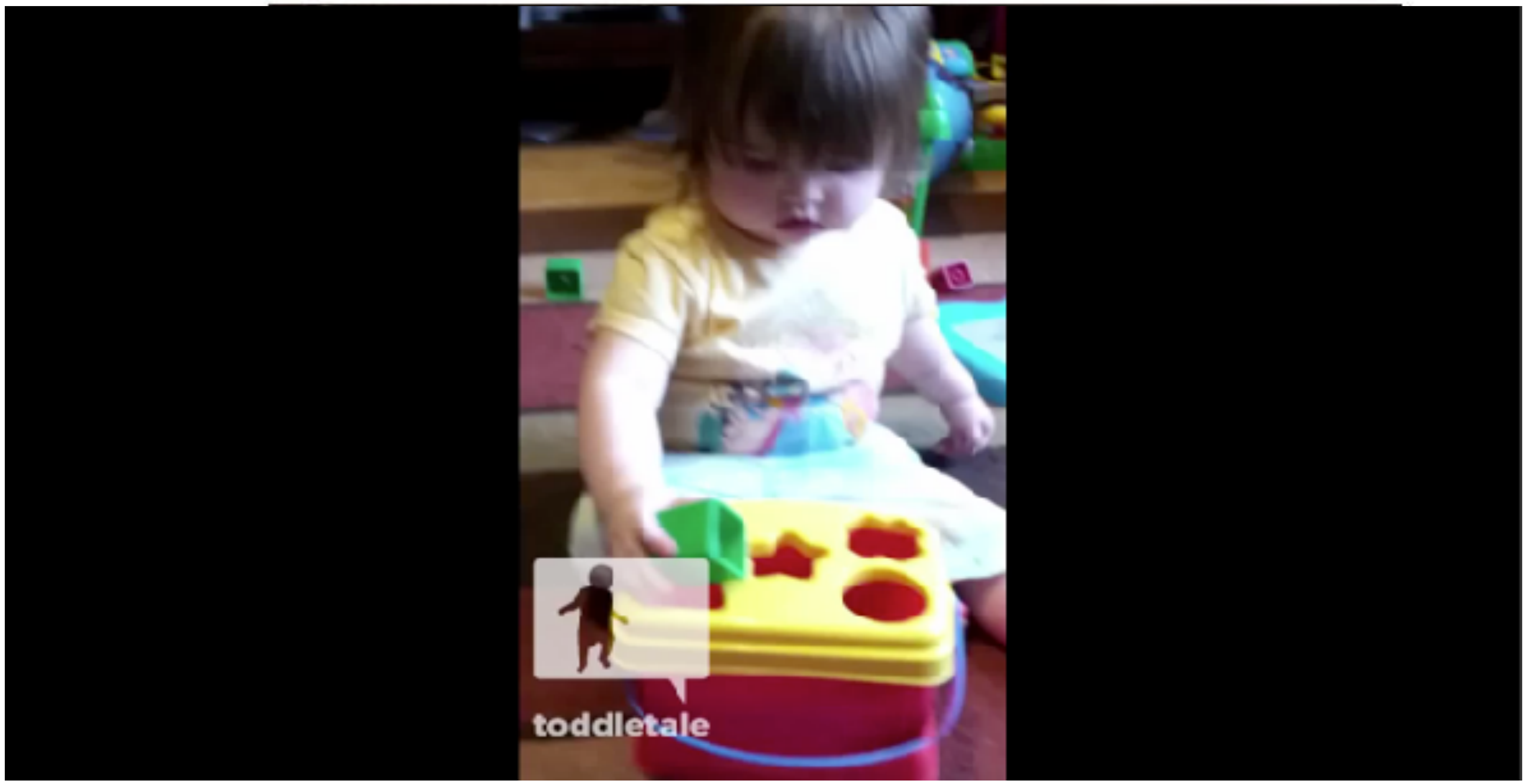

Dark Energy
Ω_Λ
0
1.0
$l(l+1)C_l^{TT}/2\pi\ (\mu k^2)$
l


Spectral Index
n_s
0.8
1.05
$l(l+1)C_l^{TT}/2\pi\ (\mu k^2)$
l


Tensor-to-Scalar ratio
r
0.01
0.2
$l(l+1)C_l^{BB}/2\pi\ [\mu k^2]$
l

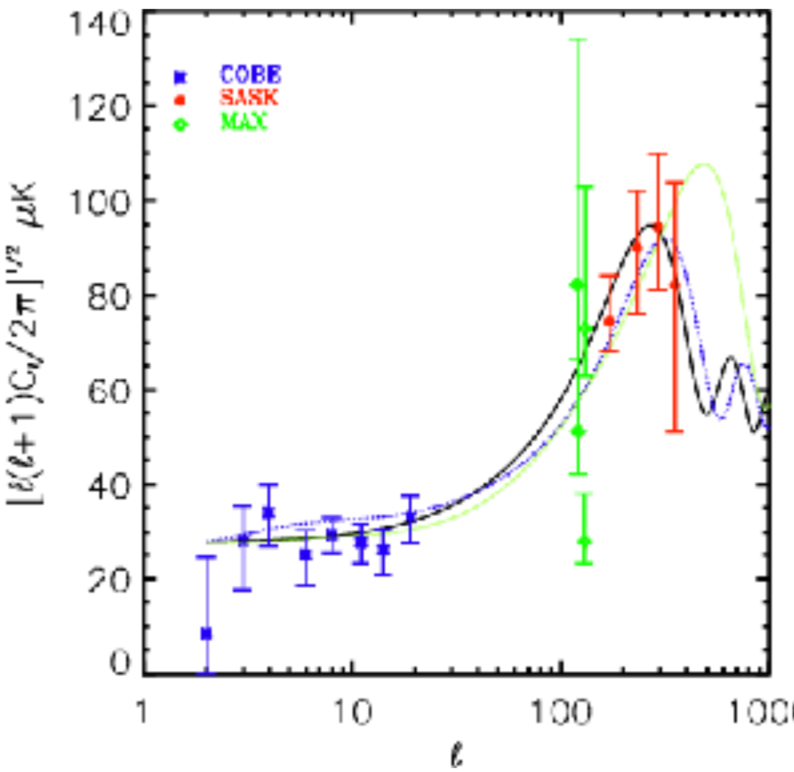
Observations

When you force your data to fit the constraints of your model



let the observations decide …

Observations



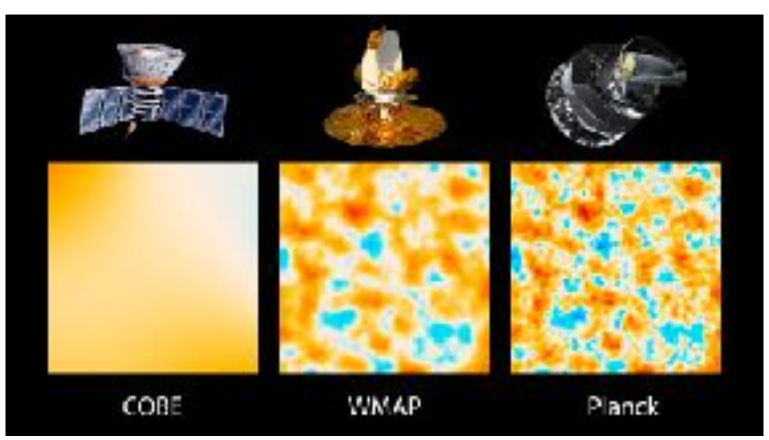
Rapid advance in the development of powerful observational-instruments has led to the establishment of **precision cosmology**.

Satellite experiments:

COBE

Wilkinson Microwave Anisotropy Probe

Planck

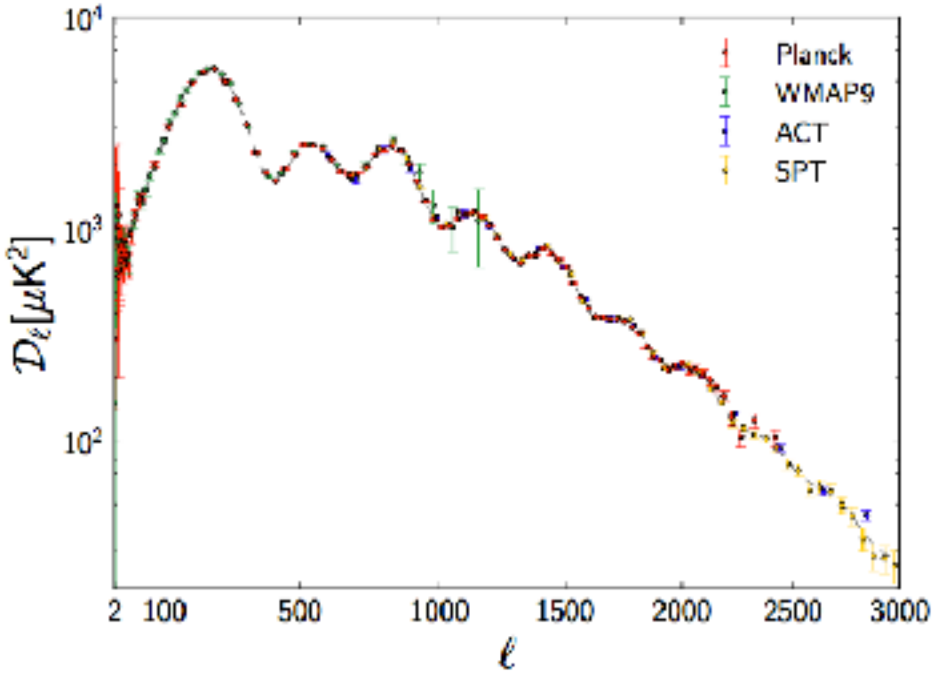


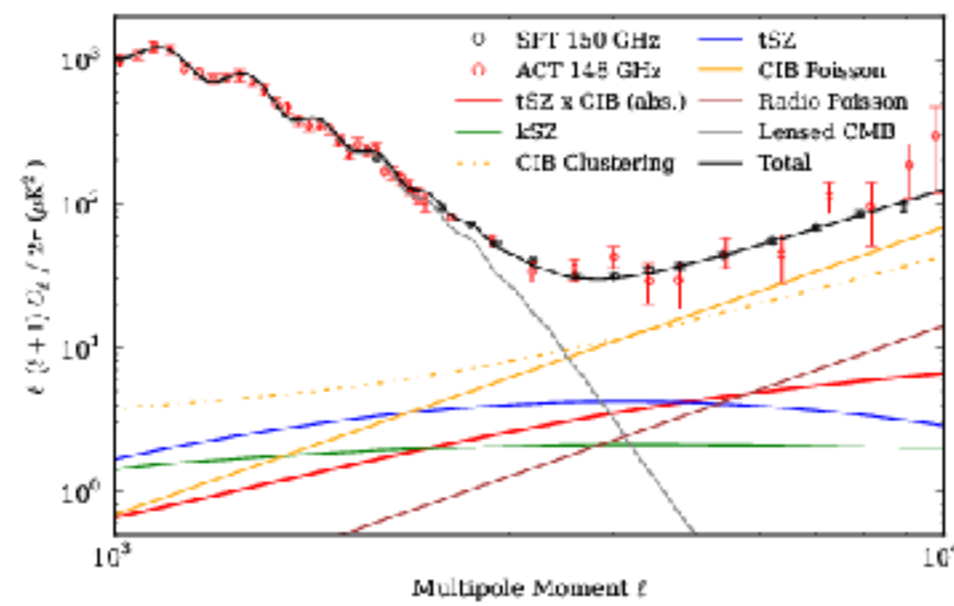
Ground-based telescopes:

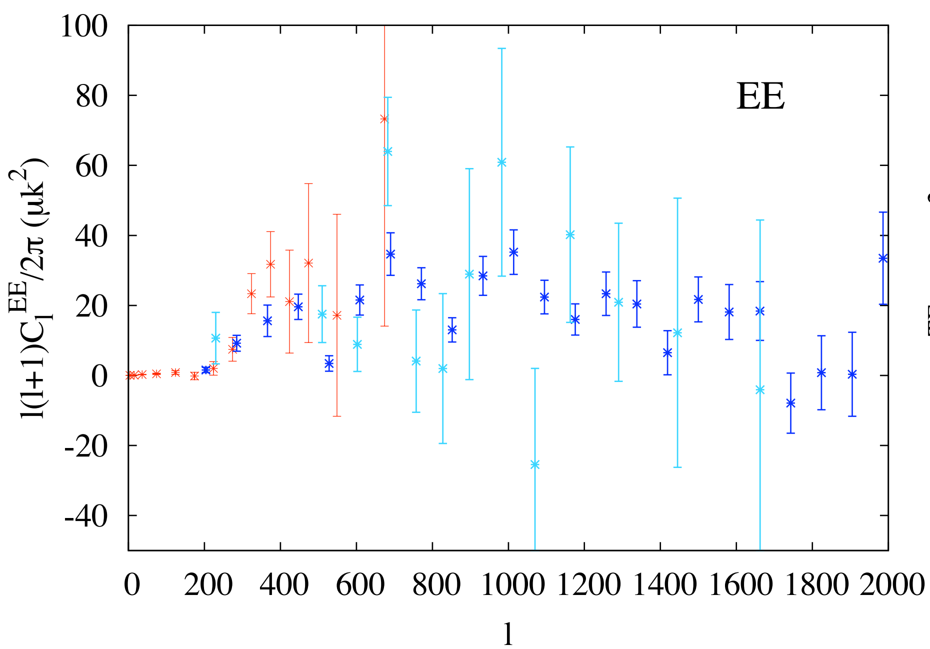
Ballon-borne experiments:

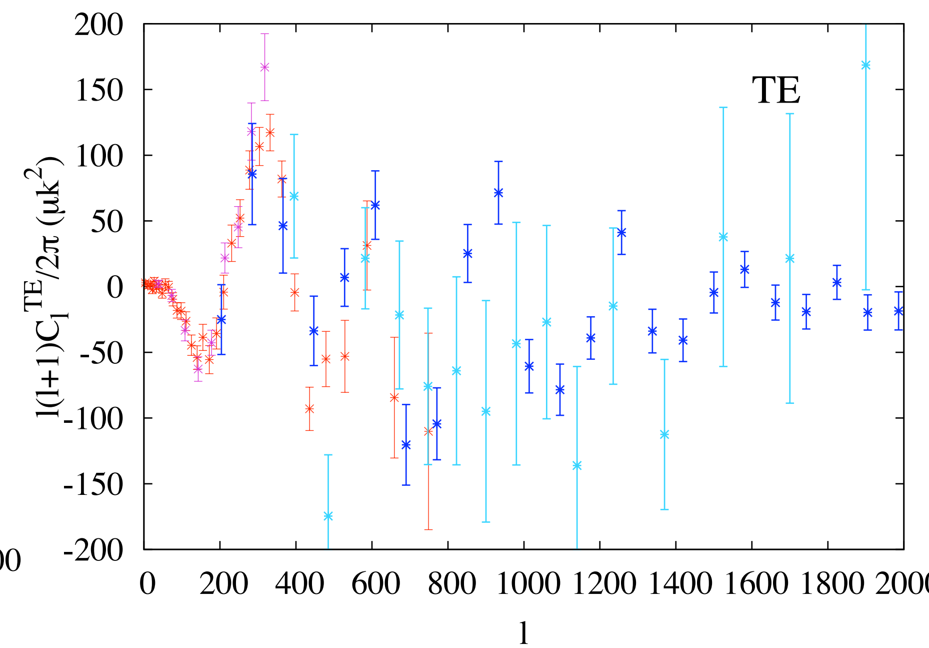
more observations

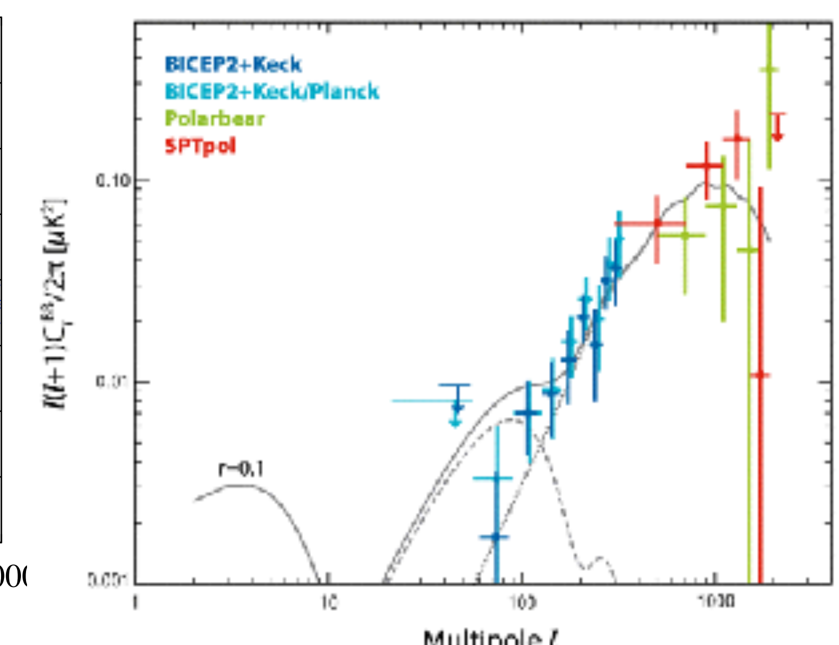
TT


Planck
WMAP9
ACT
SPT
$\mathcal{D}_\ell [\mu K^2]$
ℓ

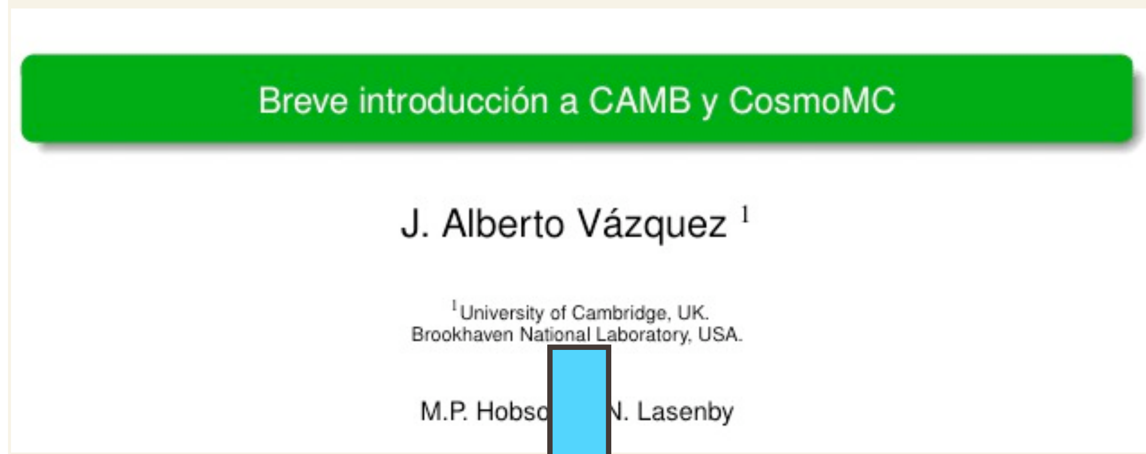

SPT 150 GHz
ACT 148 GHz
tSZ x CIB (abs.)
kSZ
CIB Clustering
tSZ
CIB Poisson
Radio Poisson
Lensed CMB
Total
Multipole Moment ℓ


EE
$l(l+1)C_l^{EE}/2\pi$ (μk^2)
l


TE
$l(l+1)C_l^{TE}/2\pi$ (μk^2)
l


BICEP2+Keck
BICEP2+Keck/Planck
Polarbear
SPTpol
r=0.1
Multipole ℓ

EE TE BB



Breve introducción a CAMB y CosmoMC

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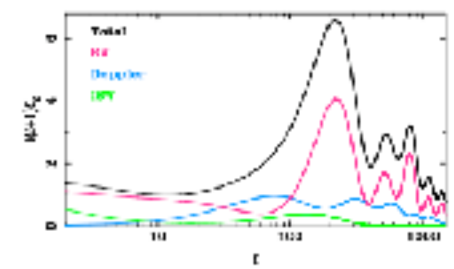


Camb: CMB spectrum

CosmoMC: parameter estimation $P(\mathbf{\Theta}|\mathbf{D}, M)$

CosmoNET (speed up of a factor of ~32)

Multinest: model selection $\mathcal{B}_{i,j} = \ln \frac{\mathcal{Z}_i}{\mathcal{Z}_j}.$

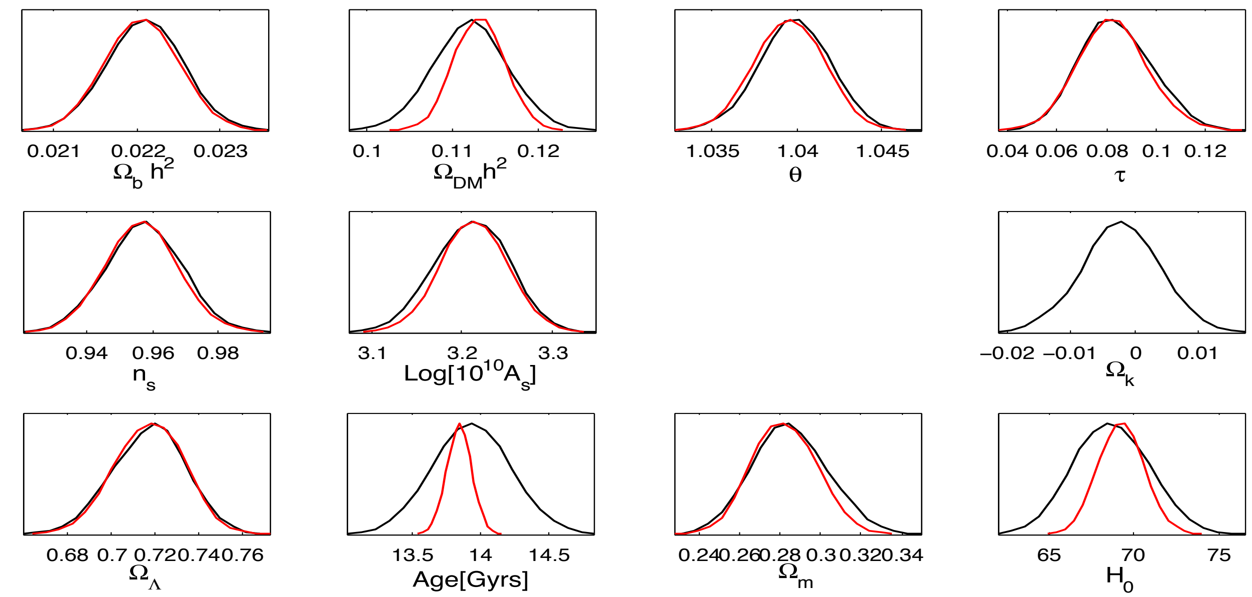
dataset consistency $R = \frac{\Pr(D|H)}{\prod_{i=1}^{n} \Pr(D_i|H)},$

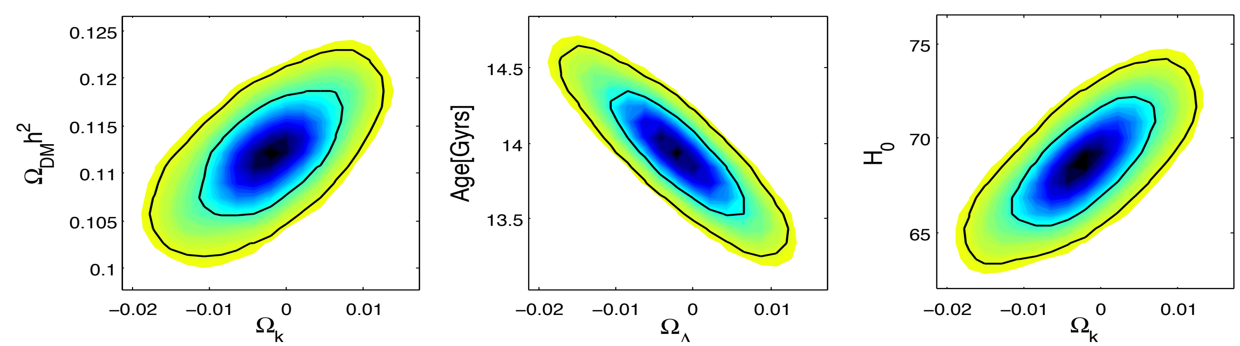
BAMBI: model averaging $P(\bar{\theta}|D) = \frac{\sum_k P(\bar{\theta}|D, \mathcal{M}_k) P(\mathcal{M}_k|D)}{\sum_k P(\mathcal{M}_k|D)}.$

MCMC Example

Description		Flat ΛCDM	Non-flat ΛCDM
	$\Omega_{\mathrm{b},0}h^2$	0.02206 ± 0.00042	0.0221 ± 0.00043
	$\Omega_{\mathrm{dm},0}h^2$	0.1130 ± 0.0028	0.112 ± 0.0041
Base	θ	1.039 ± 0.0019	1.039 ± 0.0020
parameters	τ	0.082 ± 0.013	0.083 ± 0.014
	n_{s}	0.956 ± 0.010	0.957 ± 0.011
	$\log[10^{10}A_{\mathrm{s}}]$	3.21 ± 0.035	3.21 ± 0.039
	$\Omega_{k,0}$	-	-0.0022 ± 0.0058
	$\Omega_{\mathrm{m},0}$	0.282 ± 0.015	0.285 ± 0.018
Derived	$\Omega_{\Lambda,0}$	0.717 ± 0.015	0.717 ± 0.016
parameters	H_0	69.2 ± 1.27	68.7 ± 2.13
	Age(Gyrs)	13.84 ± 0.086	13.93 ± 0.27
	$-2\ln\mathcal{L}_{\max}$	8240.46	8240.80
Bayes factor	$\mathcal{B}_{\Lambda,\Lambda+\Omega_k}$	$+1.6 \pm 0.4$	-
Dataset consistency	$\mathcal{B}_R$	$+5.06 \pm 0.4$	$+5.07 \pm 0.4$

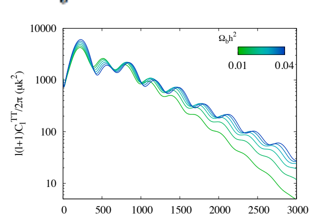
$\mathcal{B}_{k,\Lambda} = -1.90 \pm 0.35$





Theory

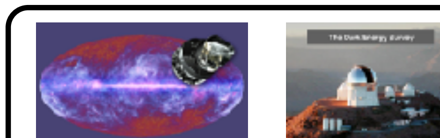
$$\int d^4x\sqrt{-g}\,R$$

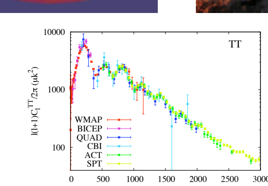


Analysis

Compare

Observations





JAVazquez

The end