Código MGPT: teoría de perturbaciones en gravedad modificada

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http://bitbucket.org/rodriguezmeza

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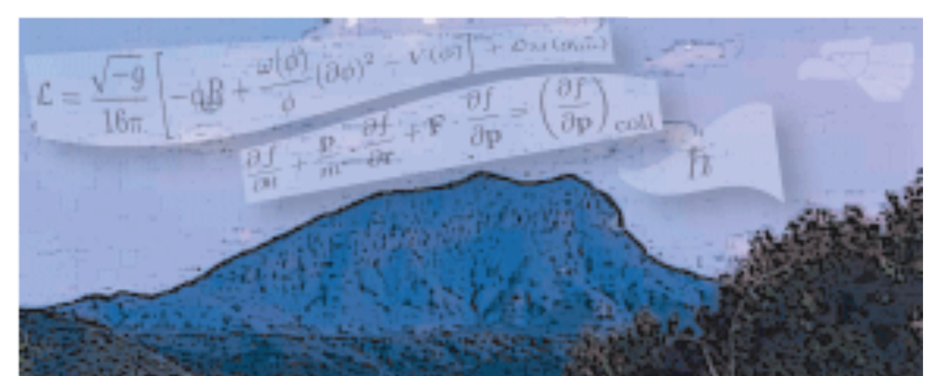
IV Taller de Métodos Numéricos y Estadísticos en Cosmología

Instituto de Ciencias Físicas, UNAM

30 de julio al 1 de agosto de 2018

Cuernavaca, Morelos



Content:

- Introduction
- NagBody kit
- Basic theory behind MGPT
- Programming: general structure, data structure and routines
- MGPT in action
- Example
- Conclusions

Useful links:

- NagBody kit: http://bitbucket.org/rodriguezmeza

  (git clone http://bitbucket.com/rodriguezmeza/nagbody_pkg-1.0.0.git)

- MGPT code: http://github.com/rodriguezmeza

  (git clone http://github.com/rodriguezmeza/mgpt-1.0.0.git)

- Slide presentations 2nd Mexican Cosmological Perturbation school (MEXPT2018): https://www.dropbox.com/sh/tcjip44dl1isnuu/AAAKF3bxidZDokh1Dvwjc-CRa?dl=0

- Youtube 2nd Mexican Cosmological Perturbation school (MEXPT2018):

  https://youtu.be/hkvRW6Mgj-k (JC-Hidalgo)

  https://youtu.be/s35rfx4DnrU (Josue De Santiago)

  https://youtu.be/vL9YP_9Q9p0 (Josue De Santiago)

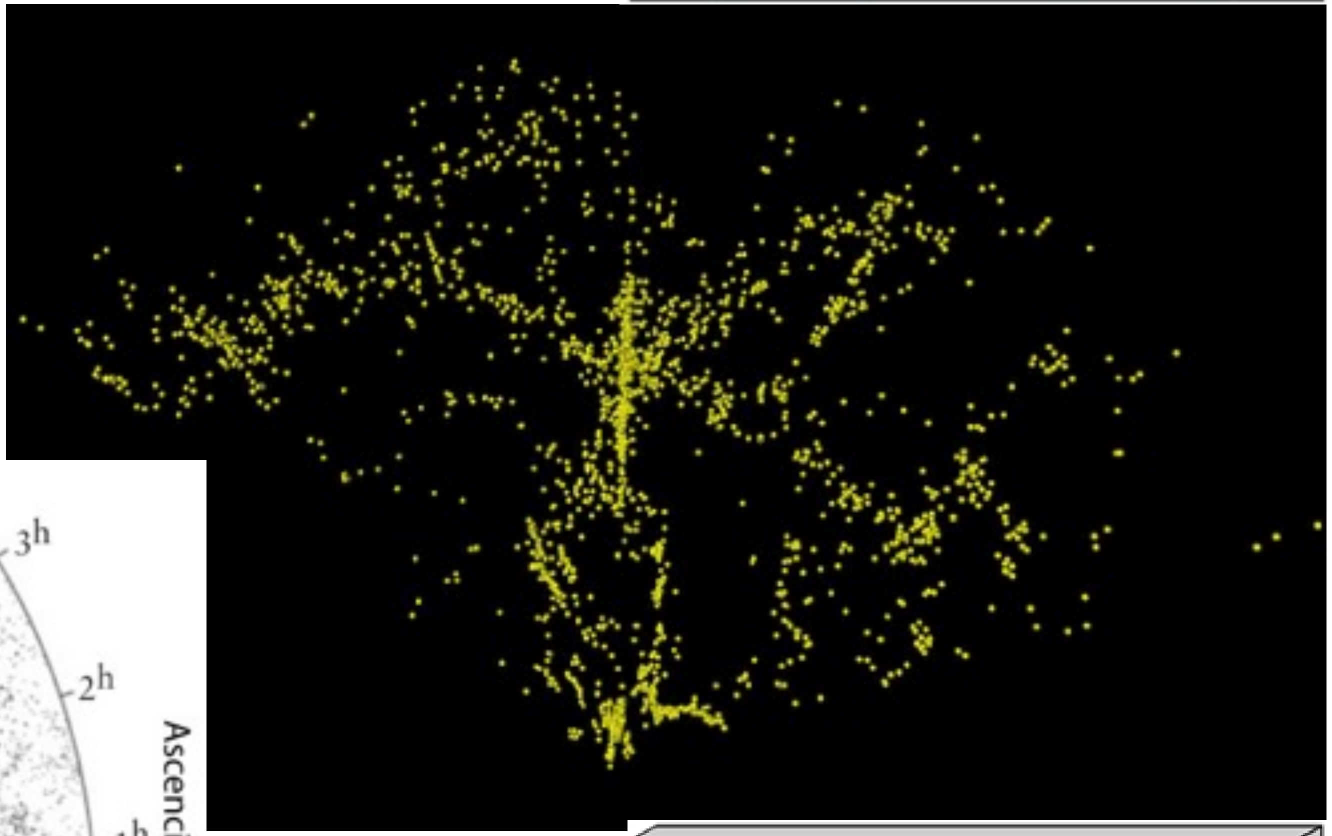
  https://youtu.be/sLASlHHp9NQ (A. Aviles)

  https://youtu.be/f_Xb13vmOuM (Mar)

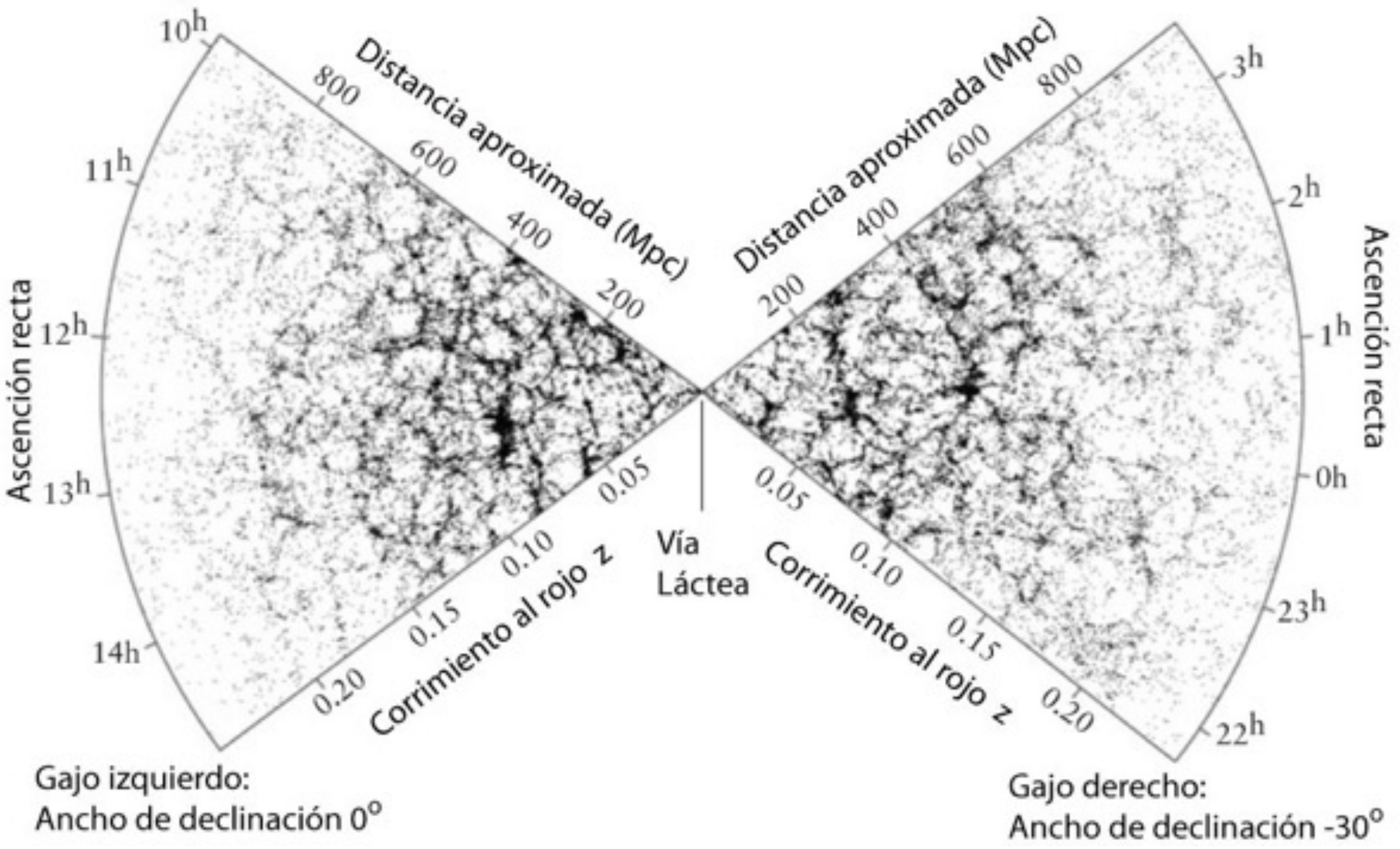
- Aviles, Cervantes, Lagrangian perturbation theory : arXiv:1705.10719

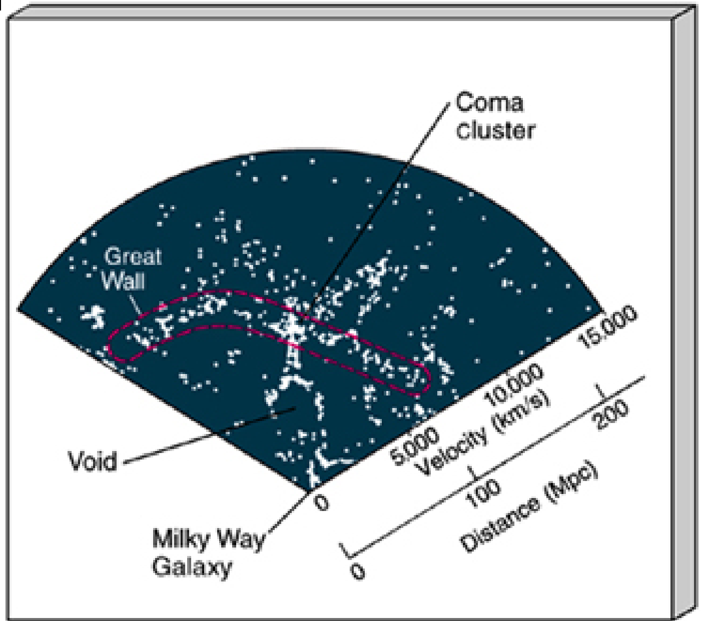
Observations: Large scale structure

- CfA
- 2dF (Redshift Survey Team/Anglo-Australian Observatory)
- 2MASS (Two Micron All Sky Survey)
- Durham/AAT redshift survey (included 200 galaxies at almost z=1/2!)
- SSDS (Sloan Sky Digital Survey)

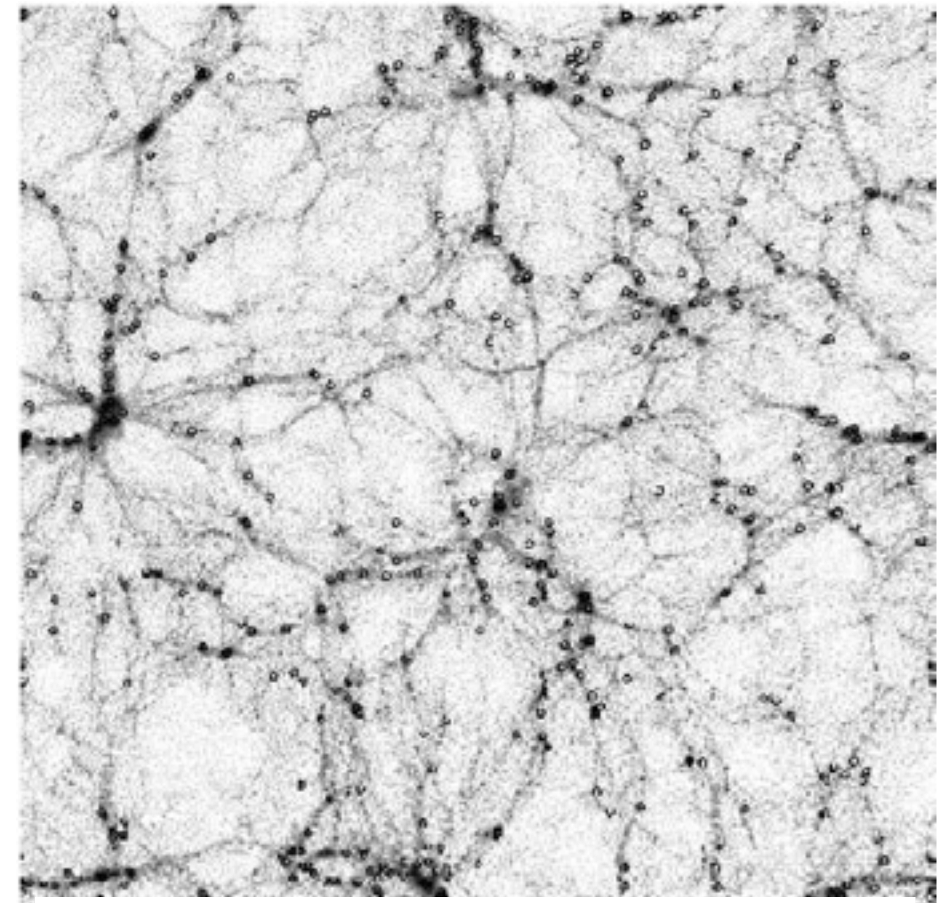


- 62,559 galaxias



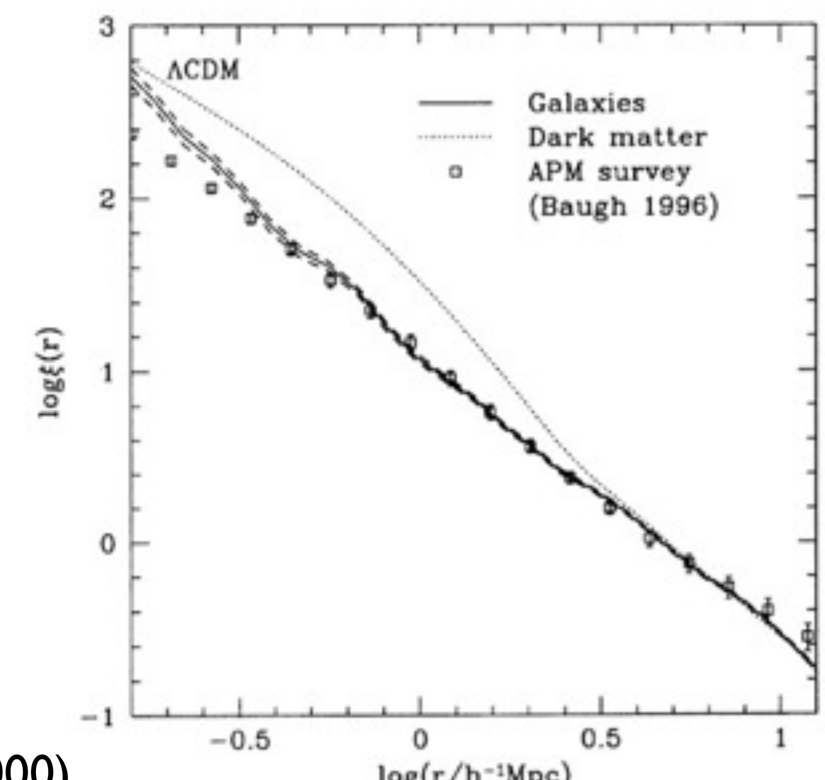


Simulations: characterising structure, voids, and clusters



- Top: The large scale structure obtained in a N-body simulation. Shown is only dark matter. Slab: 141x141x8 Mpc/h^3. The location of galaxies are shown by open circles. Galaxies were added by a semi-analytic model which assumes that dark matter haloes above a given mass threshold have at least one "central" galaxy located at the center of the halo. Higher mass haloes contain additional satellite galaxies.

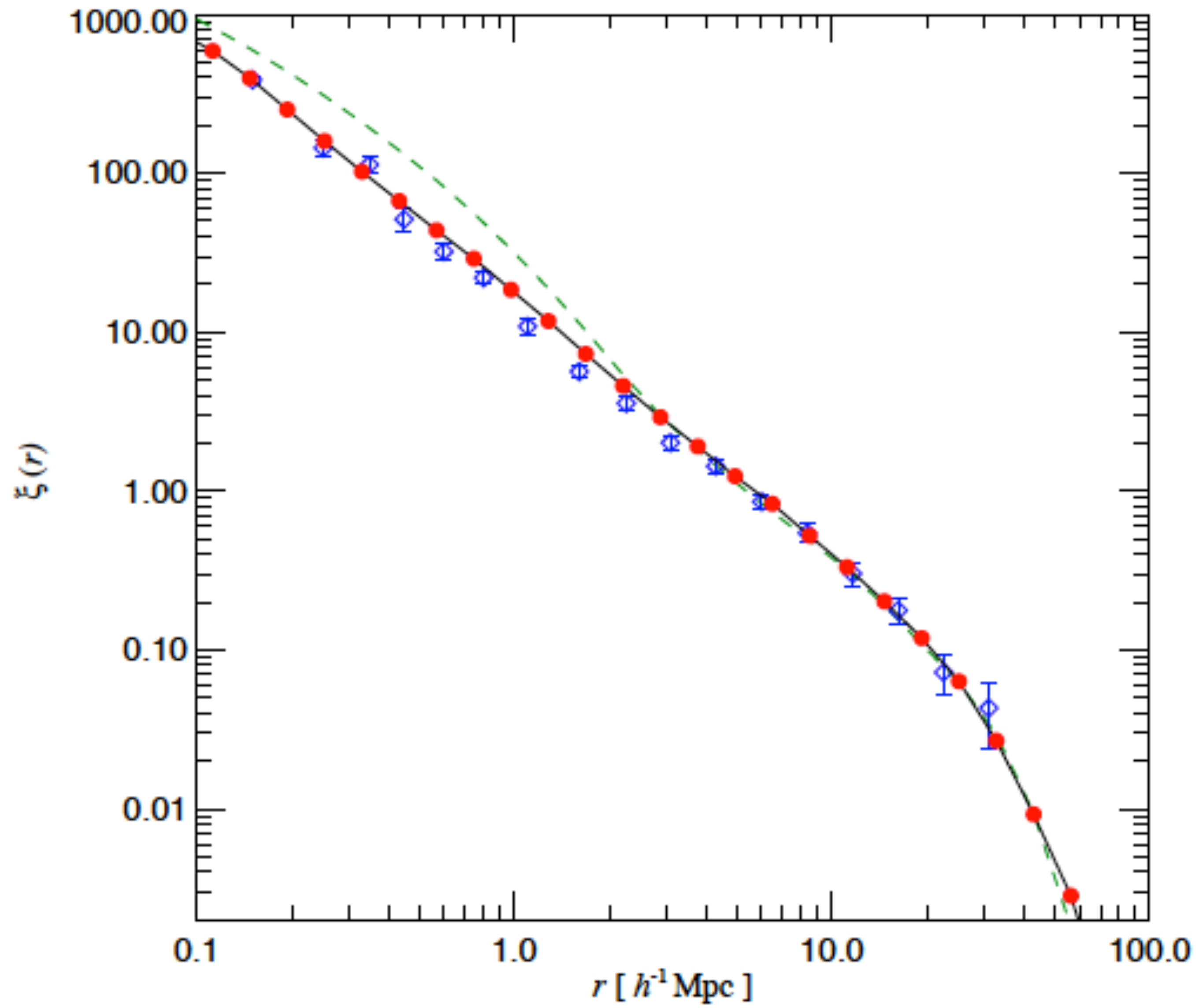
- Bottom: Two point correlation function for dark matter (dotted line); galaxies (solid lines, with dashed lines showing the Poisson error bars in the simulation. Open squares are APM survey.



- Benson et al (2000)
- Baugh (1996)

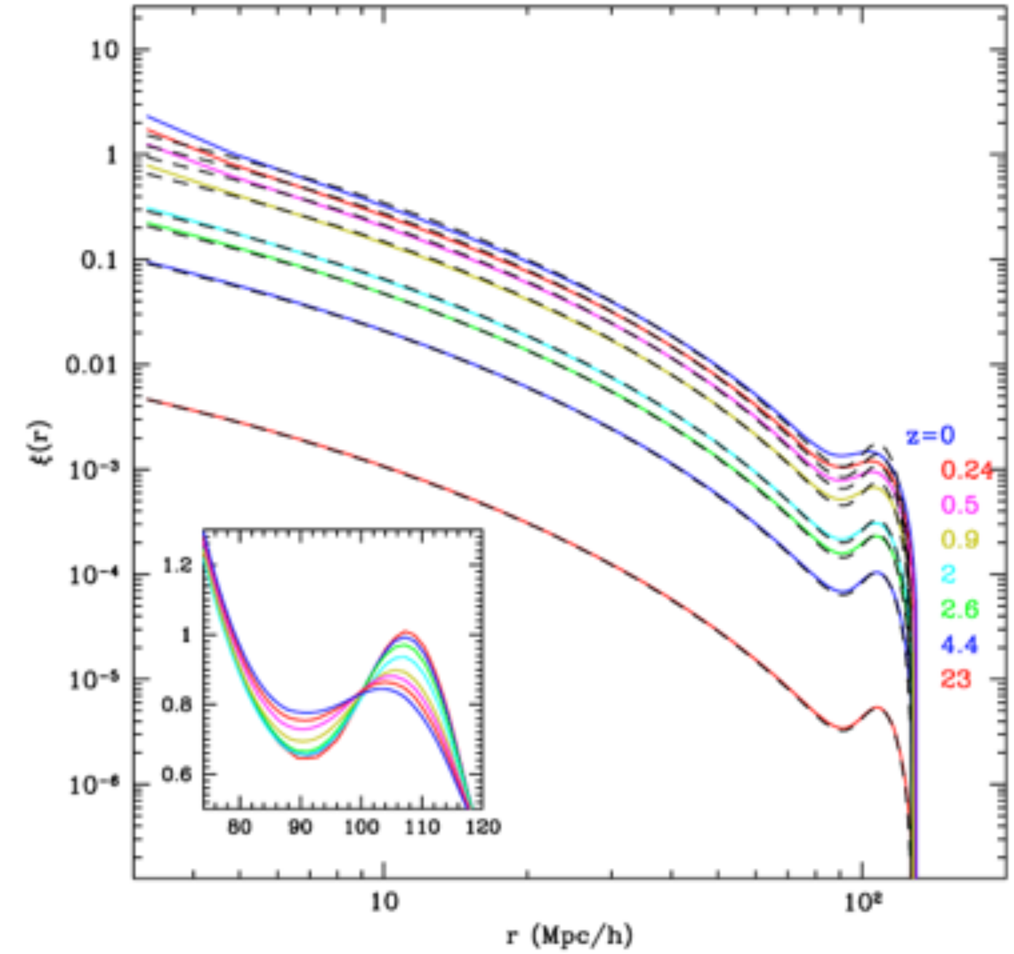
Millenium simulation I

- Galaxy 2-point correlation function at the present epoch. Red symbols (vanishingly small Poisson error-bars) show measurements for model galaxies brighter than M_K=-23. Data for the large spectroscopic redshift survey 2dFGRS are shown as blue diamonds. The SDSS and APM surveys give similar results. Both, for the observational data and for the simulated galaxies, the correlation function is very close to a power-law for r < 20 Mpc/h. By contrast the correlation function for the dark matter (dashed line) deviates strongly from a power-law

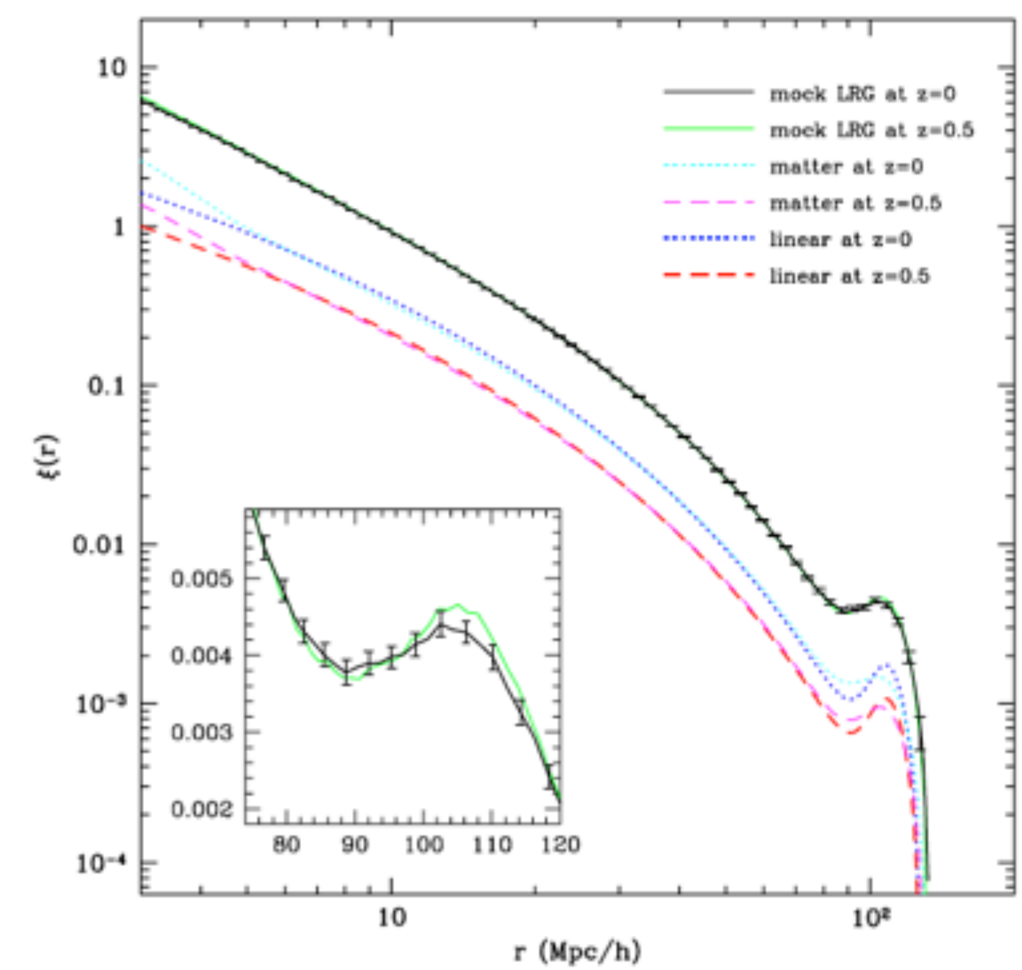


Horizon simulation

- Evolution of the correlation function of the matter density field at the epochs from z=0 to 23. Dashed curves are the linearly evolved correlation functions, and the coloured ones are the matter correlation functions measured from the horizon simulation. The inset box magnifies the matter correlation functions near the baryon oscillation bump with amplitudes to match at r=48 Mpc/h after scaling the peak of the baryonic bump of matter correlation at z=23 to unity.



- Top curves: the real space correlation functions of the mock LRGs in the whole cube at z=0 and 0.5. 3sigma error bars are attached to the correlation function at z=0. The matter density correlation functions and the linear theory correlation functions at z=0 and 0.5 (bottom curves), are also shown.



Scalar-tensor theory N-body simulations: Newtonian limit

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi G} R \longrightarrow \Phi_N = -G\frac{m}{r}$$

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \phi R \longrightarrow \Phi_N = -G\frac{m}{r}(1 + \alpha e^{-r/\lambda})$$

$$\mathcal{L} = \frac{\sqrt{-g}}{16\pi}\left[\phi R + \frac{\omega(\phi)}{\phi}(\partial\phi)^2 - V(\phi)\right] + \mathcal{L}_M(g_{\mu\nu})$$

Vlasov-Poisson-Helmoltz equations for *f*(**x**,**p**,*t*)

- Vlasov: $$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2} \cdot \frac{\partial f}{\partial \mathbf{x}} - m\nabla\Phi_N(\mathbf{x}) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0$$

- with: $$\Phi_N = \Psi - \frac{1}{2}\frac{G_N}{1+\alpha}\bar{\phi}$$

- Poisson: $$\nabla^2\Psi(\mathbf{x}) = 4\pi G_N \, a^2[\rho(\mathbf{x}) - \rho_b]$$
- Helmholtz: $$\nabla^2\bar{\phi}(\mathbf{x}) - \frac{1}{\lambda^2}\bar{\phi}(\mathbf{x}) = -8\pi\alpha \, a^2[\rho(\mathbf{x}) - \rho_b]$$

Structure formation: correlation function and power spectrum

- To study the structure formation in the universo, we compute the over-density:

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x})}{\rho_0} - 1$$

- Correlation function is:

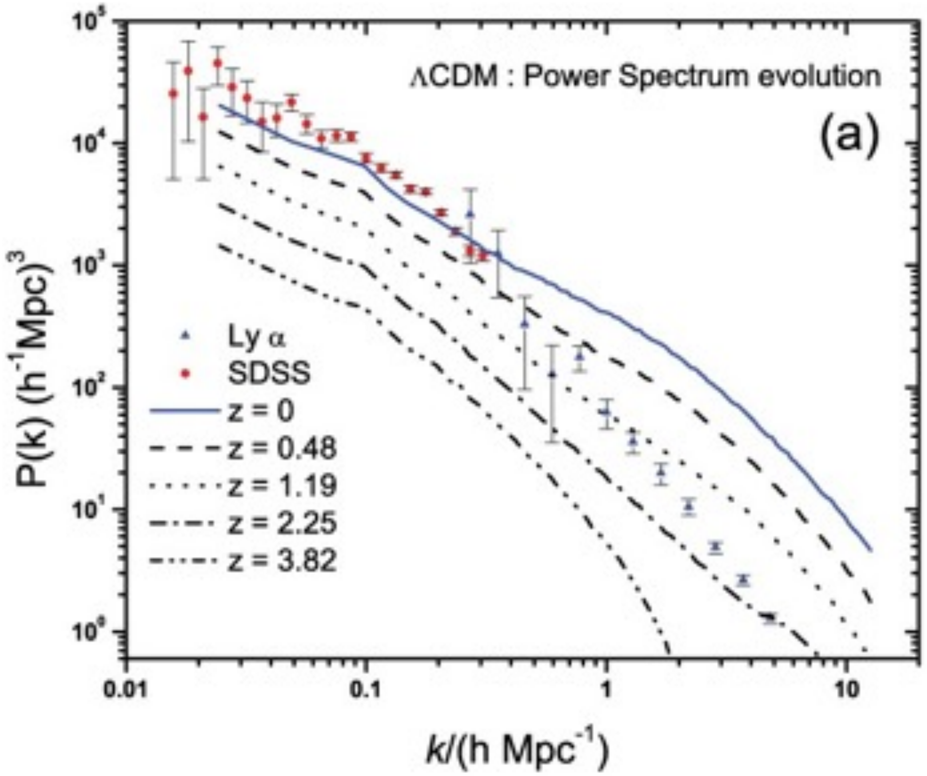
$$\xi(\mathbf{x}) \equiv \langle \delta(\mathbf{x}')\delta(\mathbf{x}' + \mathbf{x}) \rangle$$

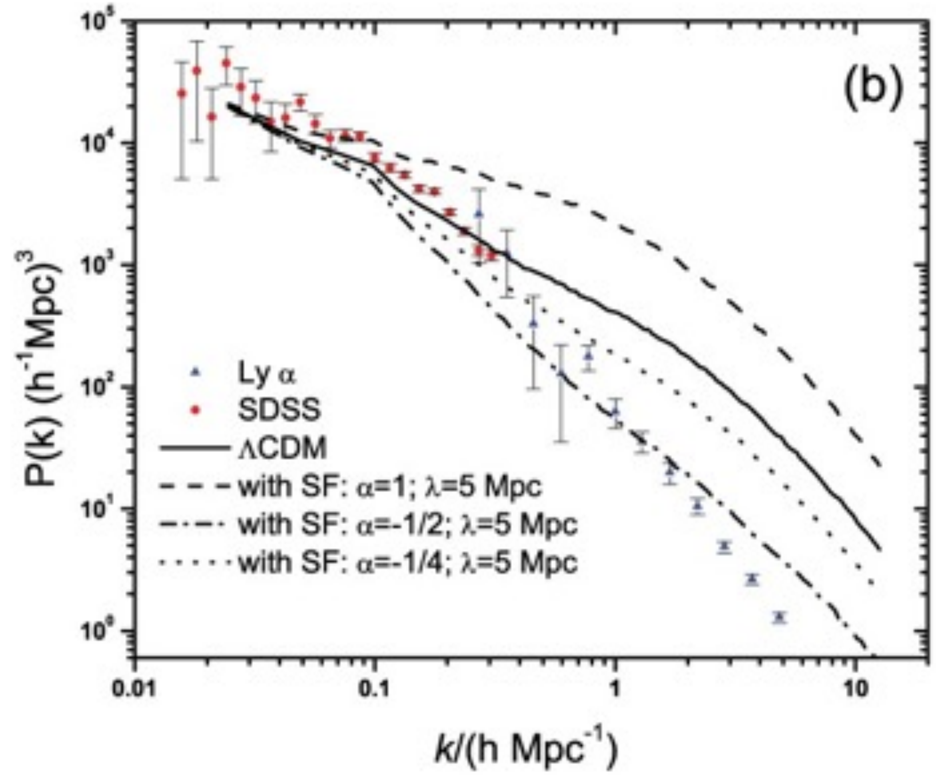
- Power spectrum is the Fourier transform of the correlation function:

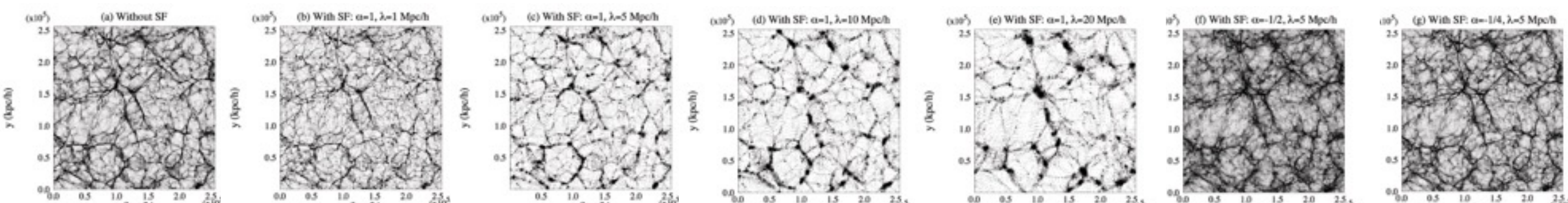
$$\xi(x) = \frac{1}{V} \sum_{\mathbf{k}} P(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}$$

The power spectrum of structure we see in the universe

with the fifth force







PS using powmes…

M.A. Rodriguez-Meza, Adv. Astron. 2012, 509682 (2012). arXiv: 1112.5201

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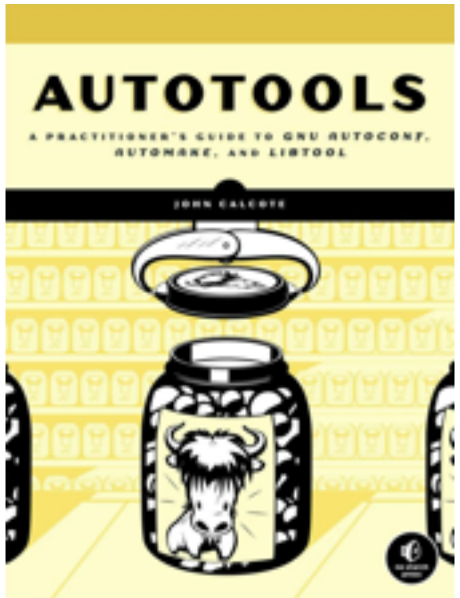


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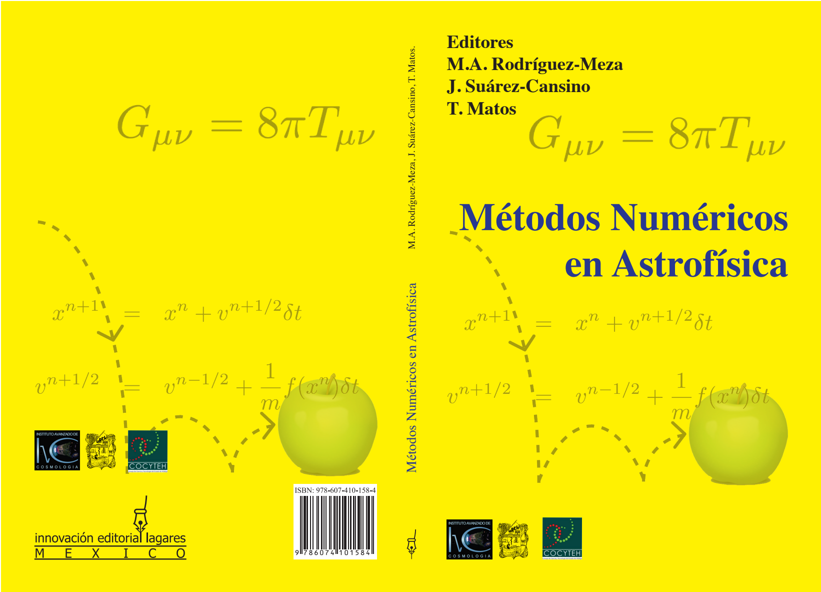
- 2D plots: **nplot2d**
- Templates coding start: **template06**
- And others for molecular dynamics…



gbsph (fifth force in galactic dynamics, Ruslan thesis, 2006)

gdgt_stt (LSSF with fifth force, the most simple screening, Alma first work, 2006…)

nmcmc (simple MCMC in C), CUTE, ROCKS,…

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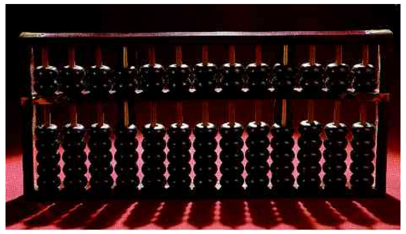

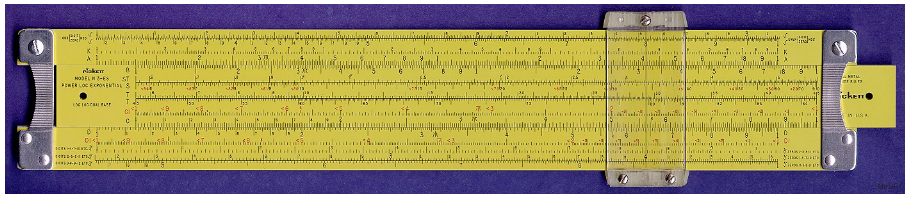

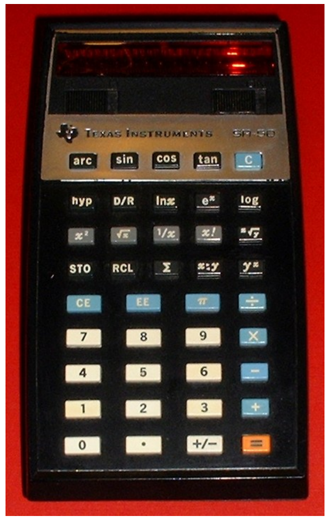

- Era del hielo:
- Edad de piedra
- Edad de bronce
- El oscurantismo: Bill Gates y Microsoft
- La manzana, pero no de Newton
- El renacimiento: Linux y GNU

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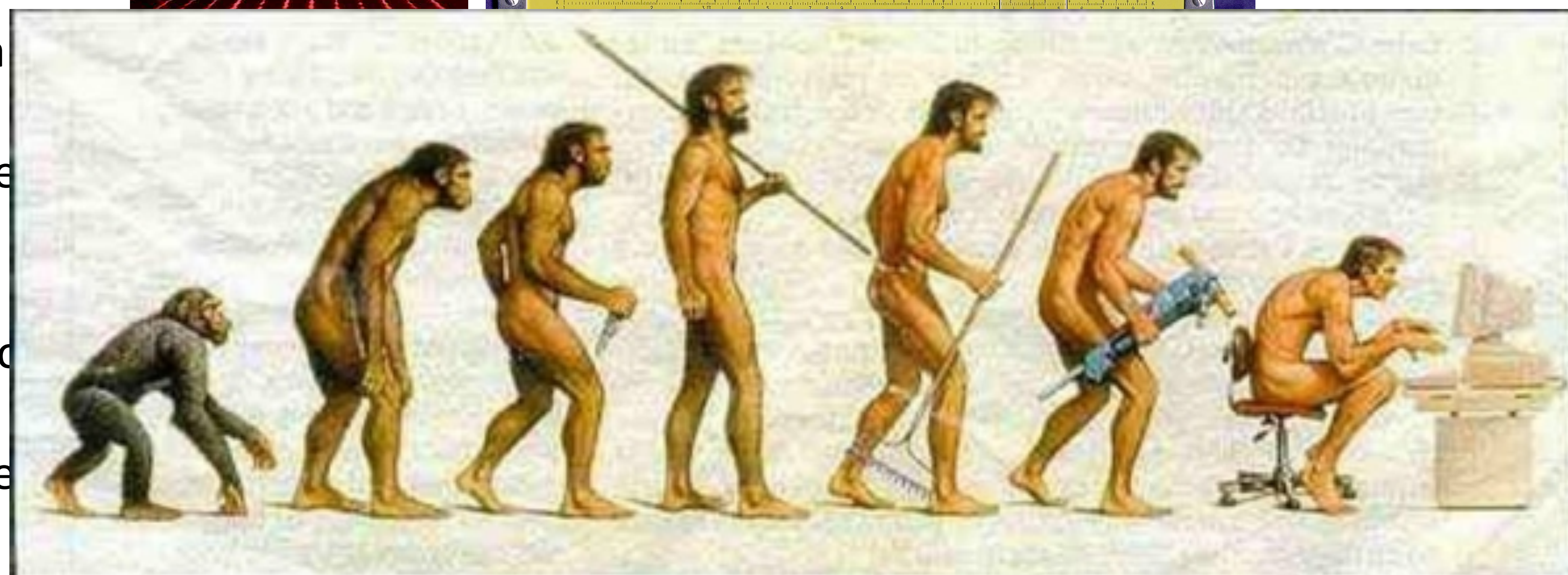

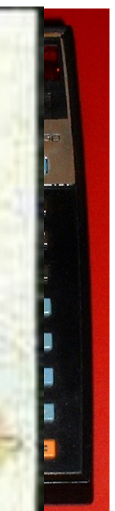

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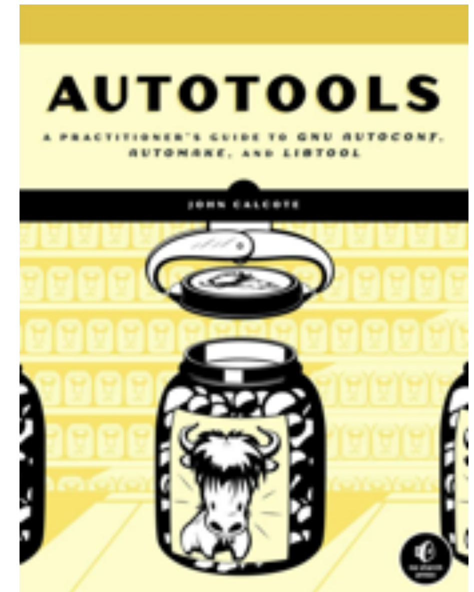


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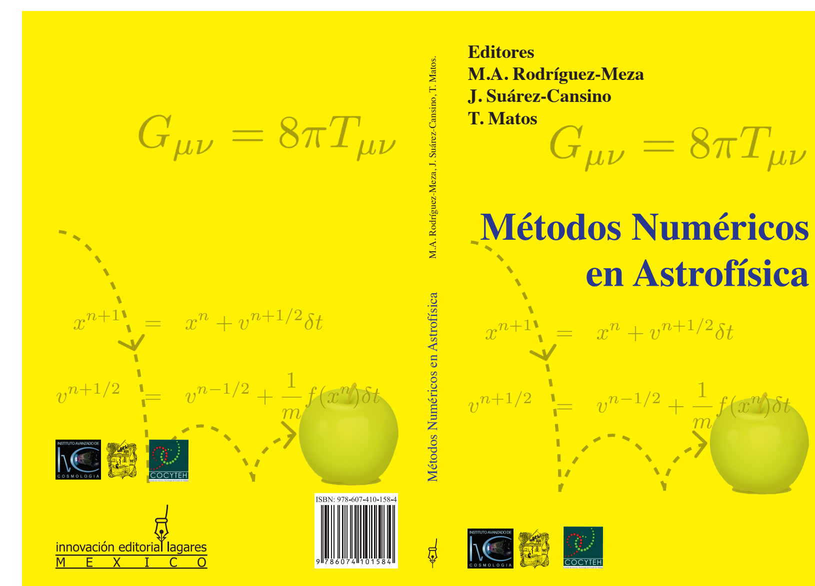

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¡Fue tal el éxito que se fue a Viena para estudiar opera!…

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Na

Python, no…

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  co
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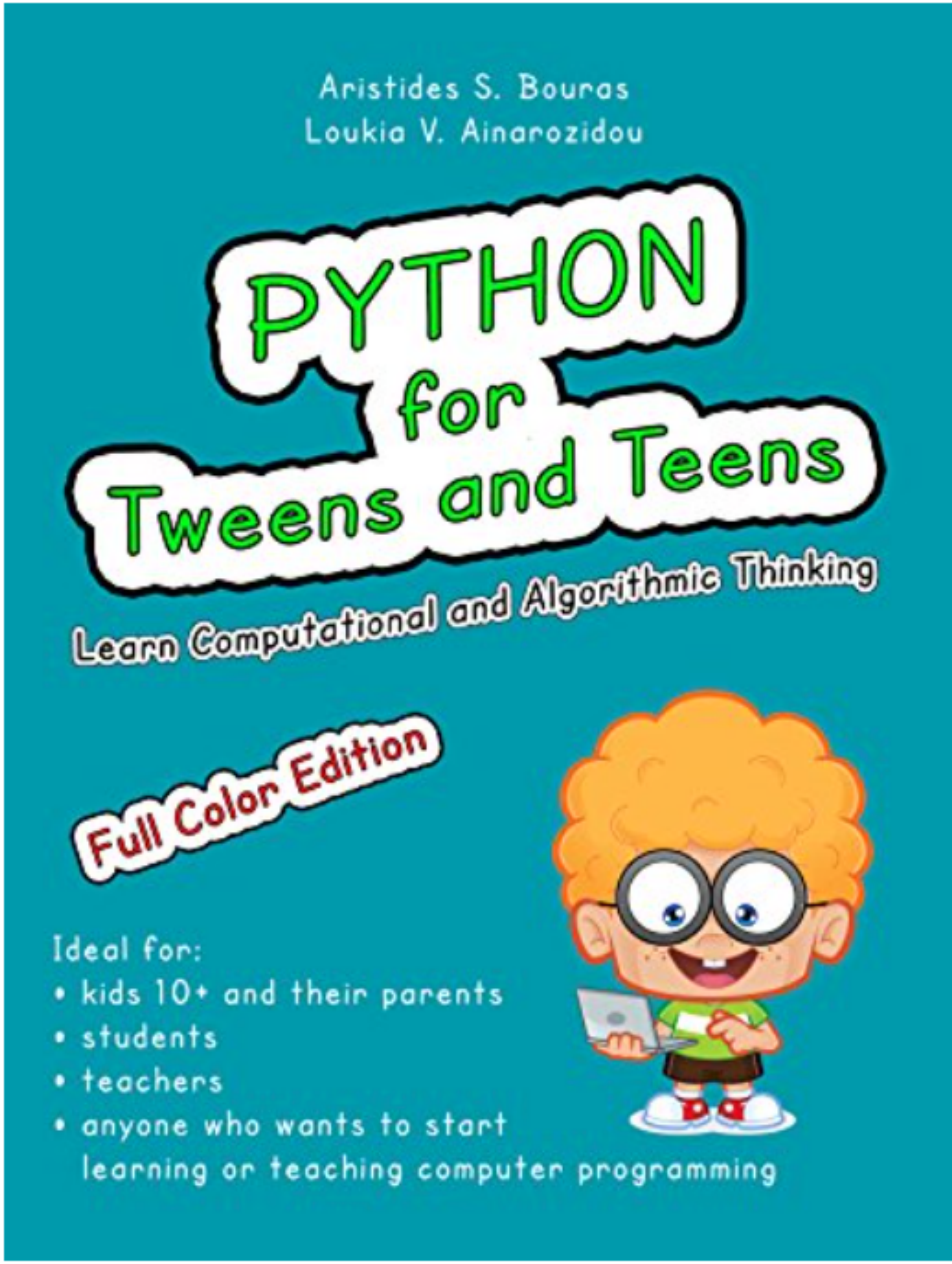

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Clasificación de computologos

- Gurus: Knuth, Linus Torvald, ...
- Genios (breakthroughs): El inventor de FFT
- Los que hacen códigos: treecode, P-Mesh, SPH, ...
- Los que modifican los códigos ya hechos
- Los que usan códigos ...
- Los que usan paquetes como Gaussian, MatLab, Maple, Mathematica, Serius, COMSOL, ...

- El cómputo científico en México ¿Usuarios o desarroladores?

… y además, ¡Somos bien díscolos!… no presentamos ni lo que no es nuestro.

(… Umberto Eco, "El número cero";
Julio Córtazar, "Los premios")

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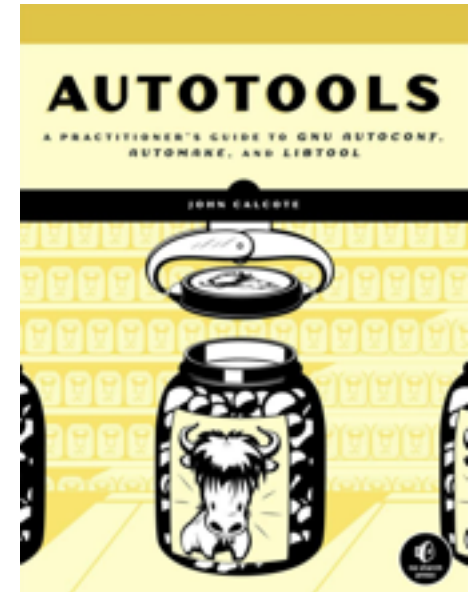


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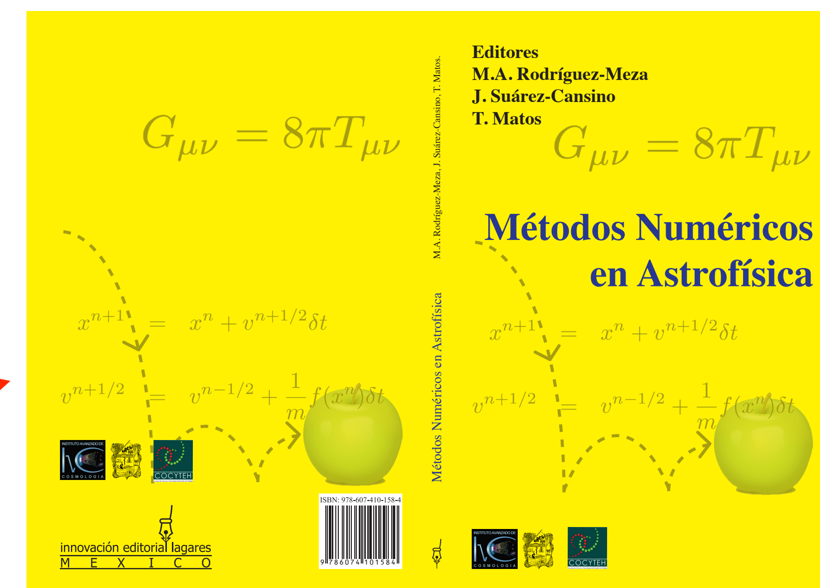

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MGPT: programming start template06

(MainLoop)

- main.c

  (MainLoop)

- cmdline_defs.h
- data_structure.h
- globaldefs.h
- protodefs.h
- startrun.c

- template.c
- template_io.c

- models.c
- models.h

- Data structures
- Routines and functions

MGPT: equations to solve

- Slide presentations 2nd Mexican Cosmological Perturbation school (MEXPT2018): https://www.dropbox.com/sh/tcjip44dl1isnuu/AAAKF3bxidZDokh1Dvwjc-CRa?dl=0
- Youtube 2nd Mexican Cosmological Perturbation school (MEXPT2018):

  https://youtu.be/hkvRW6Mgj-k (JC-Hidalgo)

  https://youtu.be/s35rfx4DnrU (Josue De Santiago)

  https://youtu.be/vL9YP_9Q9p0 (Josue De Santiago)

  https://youtu.be/sLASIHHp9NQ (A. Aviles)

  https://youtu.be/f_Xb13vmOuM (Mar)

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}\left[\varphi R - \frac{\omega_{\rm BD}(\varphi)}{\varphi}\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi)\right] + \mathcal{L}_m(\Psi, g_{\mu\nu}),$$

$$\mathcal{L}_R = \sqrt{-g}(R + f(R)).$$

Linear perturbations:

$$\frac{1}{a^2}\nabla^2\psi = 4\pi G\bar{\rho}\delta - \frac{1}{2a^2}\nabla^2\varphi,$$

$$(3 + 2\omega_{\rm BD})\frac{1}{a^2}\nabla^2\varphi = -8\pi G\bar{\rho}\delta + \mathrm{NL},$$

When going to the Fourier space, the Klein-Gordon equation can be written as

$$(3 + 2\omega_{\rm BD})\frac{k^2}{a^2}\varphi = 8\pi G\rho\delta - \mathcal{I}(\varphi). \quad (7)$$

Here the self-interaction term $\mathcal{I}$ may be expanded following [60] as $\mathcal{I}(\varphi) = M_1(k)\varphi + \delta\mathcal{I}(\varphi)$ with

$$\delta\mathcal{I}(\varphi) = \frac{1}{2}\int\frac{d^3k_1 d^3k_2}{(2\pi)^3}\delta_D(\mathbf{k} - \mathbf{k}_{12})M_2(\mathbf{k}_1, \mathbf{k}_2)\varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2) + \frac{1}{6}\int\frac{d^3k_1 d^3k_2 d^3k_3}{(2\pi)^6}\delta_D(\mathbf{k} - \mathbf{k}_{123}) \times M_3(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)\varphi(\mathbf{k}_1)\varphi(\mathbf{k}_2)\varphi(\mathbf{k}_3) + \cdots, \quad (8)$$

Aviles-Cervantes, PRD (2017) 96, 123526.

MGPT: equations to solve

Now we consider in more detail $f(R)$ gravity, that was first used to explain the current cosmic acceleration in [96,97]. Variations with respect to the metric of the action constructed with the Lagrangian density of Eq. (3), lead to the field equations

$$G_{\mu\nu} + f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R - \left(\frac{f}{2} - \Box f_R\right) g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (9)$$

where $f_R \equiv \frac{df(R)}{dR}$. By taking the trace to this equation, we obtain

$$3\Box f_R = R(1 - f_R) + 2f - 8\pi G\rho, \quad (10)$$

where we use a dust-like fluid with $T^\mu_\mu = -\rho$. In cosmological pertubative treatments, one splits $f_R = \bar{f}_R + \delta f_R$ and $R = \bar{R} + \delta R$, where the bar indicates background quantities and $\bar{R} \equiv R(\bar{f}_R)$. Necessary conditions to get a background cosmology consistent with the ΛCDM model are $|\bar{f}/\bar{R}| \ll 1$ and $|\bar{f}_R| \ll 1$ [71].

In the quasistatic limit, the trace equation can be written as

$$\frac{3}{a^2}\nabla^2 \delta f_R = -8\pi G \bar{\rho}\delta + \delta R, \quad (11)$$

MGPT: equations to solve

Since $\delta R = R(f_R) - R(\bar{f}_R)$, we may expand the potential as

$$\delta R = \sum_i \frac{1}{n!} M_n (\delta f_R)^n, \qquad M_n \equiv \left. \frac{d^n R(f_R)}{d f_R^n} \right|_{f_R = \bar{f}_R}, \quad (12)$$

from which the mass associated to the perturbed field δf_R can be read as

$$m = \sqrt{\frac{M_1}{3}}. \quad (13)$$

We consider the case $n = 1$. Using Eqs. (12) and (15), we find the functions M_1, M_2 and M_3:

$$M_1(a) = \frac{3}{2} \frac{H_0^2}{|f_{R0}|} \frac{(\Omega_{m0} a^{-3} + 4\Omega_\Lambda)^3}{(\Omega_{m0} + 4\Omega_\Lambda)^2}, \quad (17)$$

$$M_2(a) = \frac{9}{4} \frac{H_0^2}{|f_{R0}|^2} \frac{(\Omega_{m0} a^{-3} + 4\Omega_\Lambda)^5}{(\Omega_{m0} + 4\Omega_\Lambda)^4}, \quad (18)$$

$$M_3(a) = \frac{45}{8} \frac{H_0^2}{|f_{R0}|^3} \frac{(\Omega_{m0} a^{-3} + 4\Omega_\Lambda)^7}{(\Omega_{m0} + 4\Omega_\Lambda)^6}. \quad (19)$$

These functions depend only on the background evolution since they are the coefficients of the expansion of a scalar field potential about its background value.

In this work, we focus our attention to the Hu-Sawicky model [71], defined by

$$f(R) = -M^2 \frac{c_1 (R/M^2)^n}{c_2 (R/M^2)^n + 1}, \quad (14)$$

where the energy scale is chosen to be $M^2 = H_0^2 \Omega_{m0}$. In this parametrized model, at high curvature ($R \gg M^2$) the function $f(R)$ approaches a constant, recovering GR with cosmological constant, while at low curvature it goes to zero, recovering GR; the manner these two behaviors are interpolated is dictated by the free parameters. Given that $f_{RR} > 0$ for $R > M^2$ the solutions are stable and the mapping to a scalar tensor gravity is allowed [98]. The latter also implies that this gravitational model introduces a single one extra degree of freedom. In order to mimic the background evolution of the ΛCDM model, it is also necessary that $c_1/c_2 = 6\Omega_\Lambda/\Omega_{m0}$ [71], thus leaving two parameters to fix the model. Noting that the background value is about $\bar{R} \gtrsim 40 M^2 \gg M^2$, Eq. (14) simplifies and leads to

$$\boxed{f_R \simeq f_{R0} \left(\frac{\bar{R}}{R} \right)^{n+1},} \quad (15)$$

with

$$\bar{R} = 6(\dot{H} + 2H^2) = 3H_0^2 (\Omega_{m0} a^{-3} + 4\Omega_\Lambda). \quad (16)$$

MGPT: equations to solve

$$\mathbf{x}(\mathbf{q},t) = \mathbf{q} + \mathbf{\Psi}(\mathbf{q},t)$$

We consider the cases for which the MG theory can be written as a Brans-Dicke like model at linear level. For longitudinal modes we have to solve the system

$$\nabla_{\mathbf{x}} \cdot (\ddot{\Psi}(\mathbf{q},t) + 2H\dot{\Psi}) = -\frac{1}{a^2}\nabla^2_{\mathbf{x}}\psi(\mathbf{x},t) \quad (23)$$

$$\frac{1}{a^2}\nabla^2_{\mathbf{x}}\psi = 4\pi G\bar{\rho}\delta(\mathbf{x},t) - \frac{1}{2a^2}\nabla^2_{\mathbf{x}}\varphi. \quad (24)$$

In x-Fourier space, and in the quasistatic limit, the Klein Gordon equation is

$$(3+2w_{\mathrm{BD}})\frac{k_x^2}{a^2}\varphi(\mathbf{k}_x,t) = 8\pi G\bar{\rho}\delta(\mathbf{k}_x,t) - M_1(\mathbf{k}_x,t)\varphi(\mathbf{k}_x,t) - \delta I(\varphi). \quad (25)$$

we define the operator

$$\hat{\mathcal{T}} = \frac{d^2}{dt^2} + 2H\frac{d}{dt}; \quad (26)$$

thus, Eqs. (23) and (24) can be written as

$$\nabla_{\mathbf{x}} \cdot \hat{\mathcal{T}}\Psi = -4\pi G\bar{\rho}\delta(\mathbf{x},t) + \frac{1}{2a^2}\nabla^2\varphi + \frac{1}{2a^2}(\nabla^2_{\mathbf{x}}\varphi - \nabla^2\varphi), \quad (27)$$

where $\nabla_i = {}_{,i}$ is partial derivative with respect to q^i. With this splitting of the Laplacian we recognize the scalar field as a function of Lagrangian coordinates, the term $\nabla^2_{\mathbf{x}}\varphi - \nabla^2\varphi$ has a geometrical nature that arises when transforming the Klein-Gordon equation from Eulerian to Lagrangian coordinates. We shall call it the *frame-lagging term*, and we show below that is non-negligible and it is necessary for recovering ΛCDM at sufficiently large scales, where the fifth force mediated by the scalar field is essentially zero.

MGPT: equations to solve

In perturbation theory, the relevant fields are formally expanded as

$$\boxed{\Psi = \lambda\Psi^{(1)} + \lambda^2\Psi^{(2)} + \lambda^3\Psi^{(3)} + \mathcal{O}(\lambda^4), \qquad (30)}$$

and analogously for δ and φ. From now on the control parameter λ is absorbed in the definitions of the perturbed fields.

The Fourier transform of $\varphi(\mathbf{q})$ leads to

$$-\frac{k^2}{2a^2}\varphi(\mathbf{k}) = -(A(k)-A_0)\tilde{\delta}(\mathbf{k}) + \frac{k^2/a^2}{6\Pi(k)}\delta I(\mathbf{k}) - \frac{(3+2\omega_{\mathrm{BD}})k^2/a^2}{3\Pi(k)}\frac{1}{2a^2}[(\nabla_{\mathbf{x}}^2\varphi - \nabla^2\varphi)](\mathbf{k}), \qquad (32)$$

where $[(\cdots)](\mathbf{k})$ means Fourier transform of $(\cdots)(\mathbf{q})$ and we defined

$$A(k) = 4\pi G\rho\left(1+\frac{k^2/a^2}{3\Pi(k)}\right), \qquad (33)$$

$$\Pi(k) = \frac{1}{3a^2}((3+2\omega_{\mathrm{BD}})k^2 + M_1 a^2), \qquad (34)$$

$$A_0 = A(k=0,t) = 4\pi G\bar{\rho}. \qquad (35)$$

$A(k)$ is the gravitational strength in the MG cases, while A_0 is for GR.

Now, using Eqs. (22), (27), and (32), we obtain the equation of motion

$$\boxed{[(J^{-1})_{ij}\hat{\mathcal{T}}\Psi_{i,j}](\mathbf{k}) = -A(k)\tilde{\delta}(\mathbf{k}) + \frac{k^2/a^2}{3\Pi(k)}\delta I(\mathbf{k}) + \frac{M_1}{3\Pi(k)}\frac{1}{2a^2}[(\nabla_{\mathbf{x}}^2\varphi - \nabla^2\varphi)](\mathbf{k}), \qquad (37)}$$

and up to third order the frame lagging term is

$$\boxed{[(\nabla_{\mathbf{x}}^2\varphi - \nabla^2\varphi)](\mathbf{k}) = [-2\Psi_{i,j}\varphi_{,ij} - \Psi_{i,ij}\varphi_{,j} + 3\Psi_{i,j}\Psi_{j,k}\varphi_{,ki} + 2\Psi_{i,j}\Psi_{j,ik}\varphi_{,k} + \Psi_{l,li}\Psi_{i,j}\varphi_{,j}](\mathbf{k}), \qquad (38)}$$

which is obtained from Eqs. (29) and (31). We see below that this term has a key role to understand the correct contributions to LPT. We write it as a Fourier expansion as follows[3]

$$\frac{1}{2a^2}[(\nabla_{\mathbf{x}}^2\varphi - \nabla^2\varphi)](\mathbf{k}) = -\frac{1}{2}\int_{\mathbf{k}_{12}=\mathbf{k}} \mathcal{K}_{\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\delta^{(1)}(\mathbf{k}_1,t)\delta^{(1)}(\mathbf{k}_2,t) - \frac{1}{6}\int_{\mathbf{k}_{123}=\mathbf{k}} \mathcal{K}_{\mathrm{FL}}^{(3)}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)\delta^{(1)}(\mathbf{k}_1,t)\delta^{(1)}(\mathbf{k}_2,t)\delta^{(1)}(\mathbf{k}_3,t). \qquad (40)$$

$$\mathcal{K}_{\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = 2\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}(A(k_1)+A(k_2)-2A_0) + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}(A(k_1)-A_0) + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_2^2}(A(k_2)-A_0).$$

MGPT: equations to solve

$$
\begin{aligned}
\mathcal{K}^{(3)}_{\rm FL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) &= 3\left(2\frac{(\mathbf{k}_1\cdot\mathbf{k}_{23})^2}{k_1^2k_{23}^2}+\frac{\mathbf{k}_1\cdot\mathbf{k}_{23}}{k_1^2}\right)\frac{D^{(2)}(\mathbf{k}_2,\mathbf{k}_3)}{D_+(k_2)D_+(k_3)}(A(k_1)-A_0)\\
&+3\left(2\frac{(\mathbf{k}_1\cdot\mathbf{k}_{23})^2}{k_1^2k_{23}^2}+\frac{\mathbf{k}_1\cdot\mathbf{k}_{23}}{k_{23}^2}\right)\frac{D^{(2)}_\varphi(\mathbf{k}_2,\mathbf{k}_3)}{D_+(k_2)D_+(k_3)}(A(k_{23})-A_0)\\
&+6\left(3\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)}{k_2^2k_3^2k_1^2}+2\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^2(\mathbf{k}_3\cdot\mathbf{k}_1)}{k_2^2k_3^2k_1^2}+\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)}{k_3^2k_1^2}\right)(A(k_1)-A_0).
\end{aligned}
$$

$$
D^{(2)}_\varphi(\mathbf{k}_1,\mathbf{k}_2)=D^{(2)}(\mathbf{k}_1,\mathbf{k}_2)+\left(1+\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2}\right)D_+(k_1)D_+(k_2)-\frac{2A_0}{3}\frac{M_2(\mathbf{k}_1,\mathbf{k}_2)+J_{\rm FL}(\mathbf{k}_1,\mathbf{k}_2)(3+2\omega_{\rm BD})}{3\Pi(k_1)\Pi(k_2)}D_+(k_1)D_+(k_2).
$$

MGPT: equations to solve

Lagrangian displacement equation in Fourier space for third-order perturbation theory:

$$
\begin{aligned}
(\hat{\mathcal{T}} - A(k))[\Psi_{i,i}](\mathbf{k}) &= [\Psi_{i,j}\hat{\mathcal{T}}\Psi_{j,i}](\mathbf{k}) - \frac{A(k)}{2}[\Psi_{i,j}\Psi_{j,i}](\mathbf{k}) - \frac{A(k)}{2}[(\Psi_{l,l})^2](\mathbf{k}) \\
&- [\Psi_{i,k}\Psi_{k,j}\hat{\mathcal{T}}\Psi_{j,i}](\mathbf{k}) + \frac{A(k)}{6}[(\Psi_{l,l})^3](\mathbf{k}) + \frac{A(k)}{2}[\Psi_{l,l}\Psi_{i,j}\Psi_{j,i}](\mathbf{k}) \\
&+ \frac{A(k)}{3}[\Psi_{i,k}\Psi_{k,j}\Psi_{j,i}](\mathbf{k}) + \frac{k^2/a^2}{6\Pi(\mathbf{k})}\delta I(\mathbf{k}) + \frac{M_1}{6\Pi(k)}\frac{1}{a^2}[(\nabla_{\mathbf{x}}^2\varphi - \nabla^2\varphi)](\mathbf{k}). \qquad (47)
\end{aligned}
$$

At linear order the right-hand side of Eq. (47) is set to zero, leading to the Zel'dovich solution

$$\Psi^i(\mathbf{k}, t) = i\frac{k^i}{k^2}D_+(k,t)\delta^{(1)}(\mathbf{k}, t = t_0), \qquad (48)$$

where we choose t_0 to be the present time and $D_+(k)$ is the fastest growing solution to

$$(\hat{\mathcal{T}} - A(k))D_+(k) = 0, \qquad (49)$$

MGPT: equations to solve

A straightforward computation of Eq. (47), see Appendix B 1, gives the Lagrangian displacement to second order:

$$\mathbf{k}_i\Psi^{i(2)}(\mathbf{k}) = \frac{i}{2}\int_{\mathbf{k}_{12}=\mathbf{k}} \frac{3}{7}\Big(\bar{D}_a^{(2)}(\mathbf{k}_1,\mathbf{k}_2) - \bar{D}_b^{(2)}(\mathbf{k}_1,\mathbf{k}_2) \times \frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2k_2^2} - \bar{D}_{\delta I}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + \bar{D}_{\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\Big) \times D_+(k_1)D_+(k_2)\delta_1\delta_2. \tag{54}$$

Hereafter δ_1 and δ_2 denote the linear density contrasts with wave numbers $\mathbf{k}_1$ and $\mathbf{k}_2$ evaluated at present time.

Momentum conservation implies $\mathbf{k} = \mathbf{k}_{12} \equiv \mathbf{k}_1 + \mathbf{k}_2$, as it is explicit in the Dirac delta function. The growth functions are given by

$$D_a^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = (\hat{\mathcal{T}} - A(k))^{-1}(A(k)D_+(k_1)D_+(k_2)) \tag{55}$$

$$D_b^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = (\hat{\mathcal{T}} - A(k))^{-1}((A(k_1)+A(k_2) - A(k))D_+(k_1)D_+(k_2)), \tag{56}$$

$$D_{\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = (\hat{\mathcal{T}} - A(k))^{-1}\left(\frac{M_1(k)}{3\Pi(k)}\mathcal{K}_{\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2)D_+(k_1)D_+(k_2)\right), \tag{57}$$

$$D_{\delta I}^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = (\hat{\mathcal{T}} - A(k))^{-1}\left(\left(\frac{2A_0}{3}\right)^2\frac{k^2}{a^2} \times \frac{M_2(\mathbf{k}_1,\mathbf{k}_2)D_+(k_1)D_+(k_2)}{6\Pi(k)\Pi(k_1)\Pi(k_2)}\right), \tag{58}$$

and the normalized growth functions are defined as

$$\bar{D}_{a,b,\delta I,\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,t) = \frac{7}{3}\frac{D_{a,b,\delta I,\mathrm{FL}}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,t)}{D_+(k_1)D_+(k_2)}. \tag{59}$$

[3]Throughout this paper, we adopt the shorthand notations

$$\int_{\mathbf{k}_{12\cdots n}=\mathbf{k}} = \int\prod_{i=1}^{n}\frac{d^3k_i}{(2\pi)^3}(2\pi)^3\delta_D(\mathbf{k}-\mathbf{k}_{12\cdots n}), \tag{39}$$

and $\mathbf{k}_{12\ldots n} = \mathbf{k}_1 + \mathbf{k}_2 + \cdots \mathbf{k}_n$.

MGPT: equations to solve

A straightforward but lengthy computation, see Appendix B 2, leads to

$$k_i\Psi^{(3)i}(\mathbf{k}) = \frac{i}{6}\int_{\mathbf{k}_{123}=\mathbf{k}} D_+(k_1)D_+(k_2)D_+(k_3)\delta_1\delta_2\delta_3\left\{\frac{5}{7}\left(\bar{D}^{(3)}_A - \bar{D}^{(3)}_B\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^2}{k_2^2k_3^2} + \bar{D}^{(3)}_{CTa}\right)\left(1-\frac{(\mathbf{k}_1\cdot\mathbf{k}_{23})^2}{k_1^2k_{23}^2}\right)\right.$$
$$\left. -\frac{1}{3}\left(\bar{D}^{(3)}_C - 3\bar{D}^{(3)}_D\frac{(\mathbf{k}_2\cdot\mathbf{k}_3)^2}{k_2^2k_3^2} + 2\bar{D}^{(3)}_E\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_2\cdot\mathbf{k}_3)(\mathbf{k}_3\cdot\mathbf{k}_1)}{k_1^2k_2^2k_3^2} + \bar{D}^{(3)}_{CTb}\right) - \bar{D}^{(3)}_{\delta I} + \bar{D}^{(3)}_{\rm FL}\right\}, \tag{70}$$

with normalized growth functions

$$\bar{D}^{(3)}_{A,B,CTa}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{7}{5}\frac{D^{(3)}_{A,B,CTa}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{D_+(k_1)D_+(k_2)D_+(k_3)}, \tag{71}$$

$$\bar{D}^{(3)}_{C,D,E,\delta I,{\rm FL},CTb}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{D^{(3)}_{C,D,E,\delta I,{\rm FL},CTb}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)}{D_+(k_1)D_+(k_2)D_+(k_3)}, \tag{72}$$

and growth functions

$$D^{(3)}_A = (\hat{\mathcal{T}} - A(k))^{-1}(3D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-A(k))D^{(2)}_a(\mathbf{k}_2,\mathbf{k}_3)), \tag{73}$$

$$D^{(3)}_B = (\hat{\mathcal{T}} - A(k))^{-1}(3D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-A(k))D^{(2)}_b(\mathbf{k}_2,\mathbf{k}_3)), \tag{74}$$

$$D^{(3)}_{CTa} = (\hat{\mathcal{T}} - A(k))^{-1}(3D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-A(k))(D^{(2)}_{\rm FL}(\mathbf{k}_2,\mathbf{k}_3) - D^{(2)}_{\delta I}(\mathbf{k}_2,\mathbf{k}_3))), \tag{75}$$

$$D^{(3)}_C = (\hat{\mathcal{T}} - A(k))^{-1}(9D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-2A(k))D^{(2)}_a(\mathbf{k}_2,\mathbf{k}_3) - 3A(k)D_+(k_1)D_+(k_2)D_+(k_3)), \tag{76}$$

$$D^{(3)}_D = (\hat{\mathcal{T}} - A(k))^{-1}(3D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-2A(k))D^{(2)}_b(\mathbf{k}_2,\mathbf{k}_3) + 3A(k)D_+(k_1)D_+(k_2)D_+(k_3)), \tag{77}$$

$$D^{(3)}_E = (\hat{\mathcal{T}} - A(k))^{-1}(3(3A(k_1)-A(k))D_+(k_1)D_+(k_2)D_+(k_3)), \tag{78}$$

$$D^{(3)}_{CTb} = (\hat{\mathcal{T}} - A(k))^{-1}(9D_+(k_1)(A(k_1)+\hat{\mathcal{T}}-2A(k))(D^{(2)}_{\rm FL}(\mathbf{k}_2,\mathbf{k}_3) - D^{(2)}_{\delta I}(\mathbf{k}_2,\mathbf{k}_3))), \tag{79}$$

$$D^{(3)}_{\delta I} = (\hat{\mathcal{T}} - A(k))^{-1}\left(\frac{k^2/a^2}{6\Pi(k)}\mathcal{K}^{(3)}_{\delta I}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)D_+(k_1)D_+(k_2)D_+(k_3)\right), \tag{80}$$

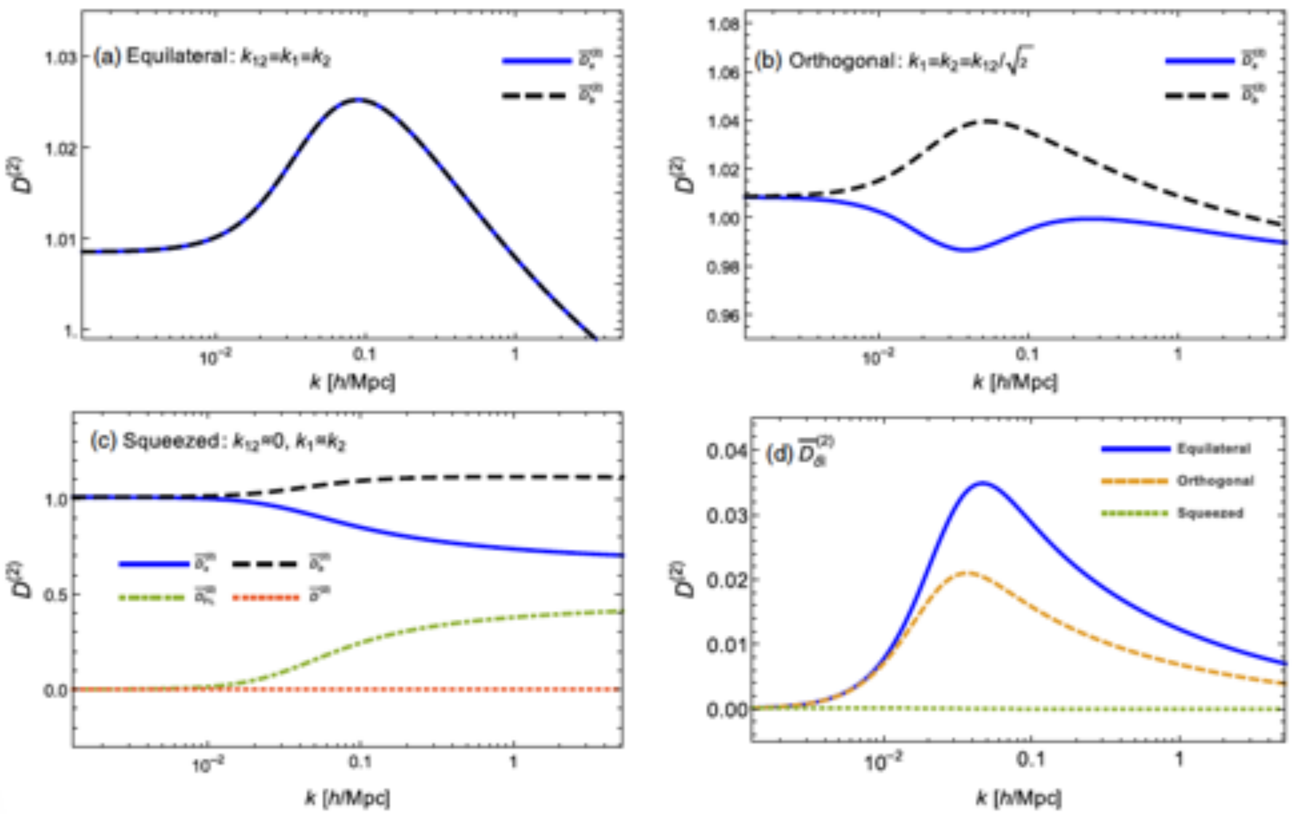
$$D^{(3)}_{\rm FL} = (\hat{\mathcal{T}} - A(k))^{-1}\left(\frac{M_1}{3\Pi(k)}\mathcal{K}^{(3)}_{\rm FL}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)D_+(k_1)D_+(k_2)D_+(k_3)\right). \tag{81}$$

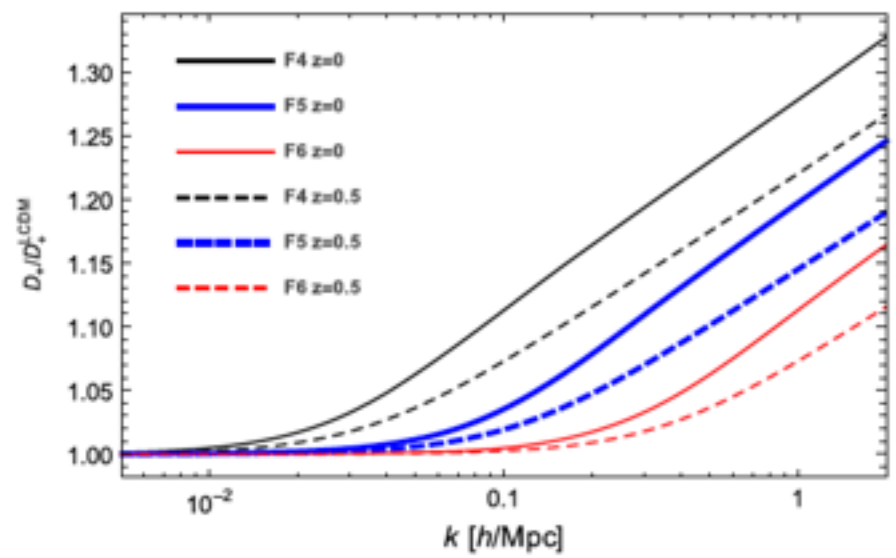
MGPT: equations to solve

In one-loop statistics, the third-order growth function should be symmetrized by summing over all permutations—contrary to cyclic permutations which are sufficient in ΛCDM. By doing this, we obtain

$$
\begin{aligned}
&D^{(3)symm}(\mathbf{k},-\mathbf{p},\mathbf{p}) \\
&= (\hat{\mathcal{T}}-A(k))^{-1}\Bigg\{D_+(p)(A(p)+\hat{\mathcal{T}}-A(k))D^{(2)}(\mathbf{p},\mathbf{k})\left(1-\frac{(\mathbf{p}\cdot(\mathbf{k}+\mathbf{p}))^2}{p^2|\mathbf{p}+\mathbf{k}|^2}\right) \\
&\quad - D_+(p)(A(p)+A(|\mathbf{k}+\mathbf{p}|)-2A(k))D^{(2)}(\mathbf{p},\mathbf{k}) + (2A(k)-A(p)-A(|\mathbf{k}+\mathbf{p}|))D_+(k)D_+^2(p)\frac{(\mathbf{k}\cdot\mathbf{p})^2}{k^2p^2} \\
&\quad - (A(|\mathbf{k}+\mathbf{p}|)-A(k))D_+(k)D_+^2(p) - \left(\frac{M_1(\mathbf{k}+\mathbf{p})}{3\Pi(|\mathbf{k}+\mathbf{p}|)}\mathcal{K}^{(2)}_{\mathrm{FL}}(\mathbf{p},\mathbf{k}) - \left(\frac{2A_0}{3}\right)^2\frac{M_2(\mathbf{p},\mathbf{k})|\mathbf{k}+\mathbf{p}|^2/a^2}{6\Pi(|\mathbf{k}+\mathbf{p}|)\Pi(k)\Pi(p)}\right)D_+(k)D_+^2(p) \\
&\quad + \frac{M_1(k)}{3\Pi(k)}\Bigg[\left(\frac{(\mathbf{p}\cdot(\mathbf{k}+\mathbf{p}))^2}{p^2|\mathbf{p}+\mathbf{k}|^2}-\frac{\mathbf{p}\cdot(\mathbf{k}+\mathbf{p})}{p^2}\right)(A(p)-A_0)D^{(2)}(\mathbf{p},\mathbf{k})D_+(p) + \left(\frac{(\mathbf{p}\cdot(\mathbf{k}+\mathbf{p}))^2}{p^2|\mathbf{p}+\mathbf{k}|^2}-\frac{\mathbf{p}\cdot(\mathbf{k}+\mathbf{p})}{|\mathbf{k}+\mathbf{p}|^2}\right) \\
&\quad \times (A(|\mathbf{k}+\mathbf{p}|)-A_0)D^{(2)}_{\varphi}(\mathbf{p},\mathbf{k})D_+(p) + 3\frac{(\mathbf{k}\cdot\mathbf{p})^2}{k^2p^2}(A(k)+A(p)-2A_0)D_+(k)D_+^2(p)\Bigg] \\
&\quad - \frac{1}{2}\frac{k^2/a^2}{6\Pi(k)}\mathcal{K}^{(3)symm}_{\delta I}(\mathbf{k},-\mathbf{p},\mathbf{p})D_+(k)D_+^2(p)\Bigg\} + (\mathbf{p}\to-\mathbf{p}),
\end{aligned}
\tag{84}
$$

MGPT: equations to solve





MGPT: equations to solve

- Lagrangian displacements and 3-point functions

Lagrangian displacement power spectra and bispectra are 2- and 3-rank tensors, of the form $\langle \Psi_i(k_1)\Psi_i(k_2)\rangle$ and $\langle \Psi_i(k_1)\Psi_j(k_2)\Psi_k(k_3)\rangle$, that we contract with related momenta to construct scalars. In this section, we are interested in those combinations that are necessary for matter statistics at one loop. These special combinations are defined in Appendix A, Eqs. (A8)–(A12). Straightforward calculations using the Lagrangian displacements up to third order yield

$$Q_1(k) = \frac{k^3}{4\pi^2}\int_0^\infty dr r^2 P_L(kr)\int_{-1}^1 dx P_L(k\sqrt{1+r^2-2rx})\left(\bar{D}_a^{(2)} - \bar{D}_b^{(2)}\left(\frac{x^2+r^2-2rx}{1+r^2-2rx}\right) - \bar{D}_{\delta I}^{(2)} + \bar{D}_{\mathrm{FL}}^{(2)}\right)^2, \quad (85)$$

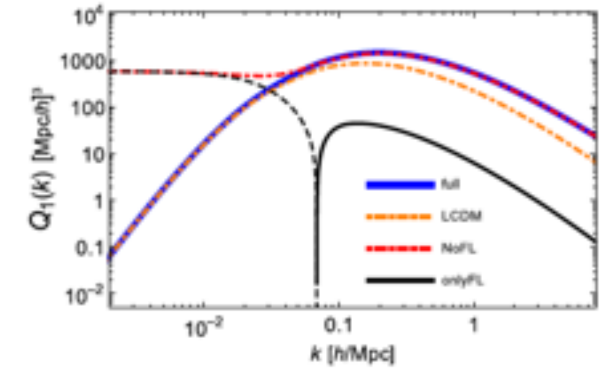
$$Q_2(k) = \frac{k^3}{4\pi^2}\int_0^\infty dr P_L(kr)\int_{-1}^1 dx P_L(k\sqrt{1+r^2-2rx})\frac{rx(1-rx)}{1+r^2-2rx}\left(\bar{D}_a^{(2)} - \bar{D}_b^{(2)}\frac{(x-r)^2}{1+r^2-2rx} + \bar{D}_{\mathrm{FL}}^{(2)} - \bar{D}_{\delta I}^{(2)}\right), \quad (86)$$

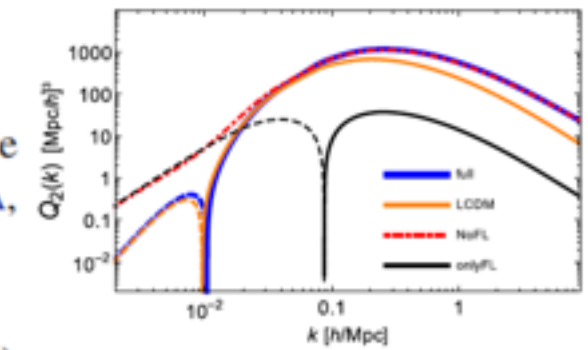
$$Q_3(k) = \frac{k^3}{4\pi^2}\int_0^\infty dr P_L(kr)\int_{-1}^1 dx \frac{x^2(1-rx)^2}{(1+r^2-2rx)^2} P_L(k\sqrt{1+r^2-2rx}), \quad (87)$$

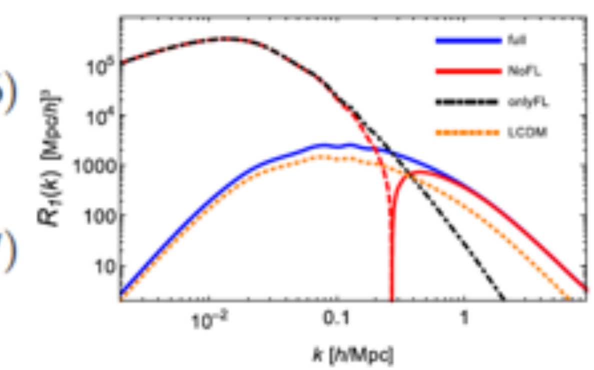
$$R_1(k) = \frac{k^3}{4\pi^2} P_L(k)\int_0^\infty dr P_L(kr)\int_{-1}^1 dx \frac{D^{(3)symm}(\mathbf{k},-\mathbf{p},\mathbf{p})}{D_+(k)D_+^2(p)}, \quad (88)$$

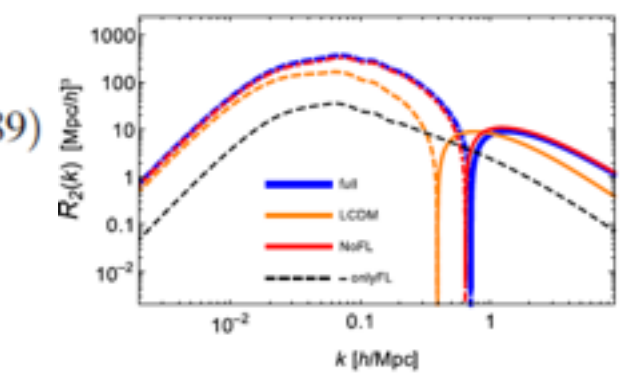
$$R_2(k) = \frac{k^3}{4\pi^2} P_L(k)\int_0^\infty dr P_L(kr)\int_{-1}^1 dx \frac{rx(1-rx)}{1+r^2-2rx}(\bar{D}_a^{(2)} - \bar{D}_b^{(2)}x^2 + \bar{D}_{\mathrm{FL}}^{(2)} - \bar{D}_{\delta I}^{(2)}), \quad (89)$$

with $r = p/k$ and $x = \hat{\mathbf{k}}\cdot\hat{\mathbf{p}}$.





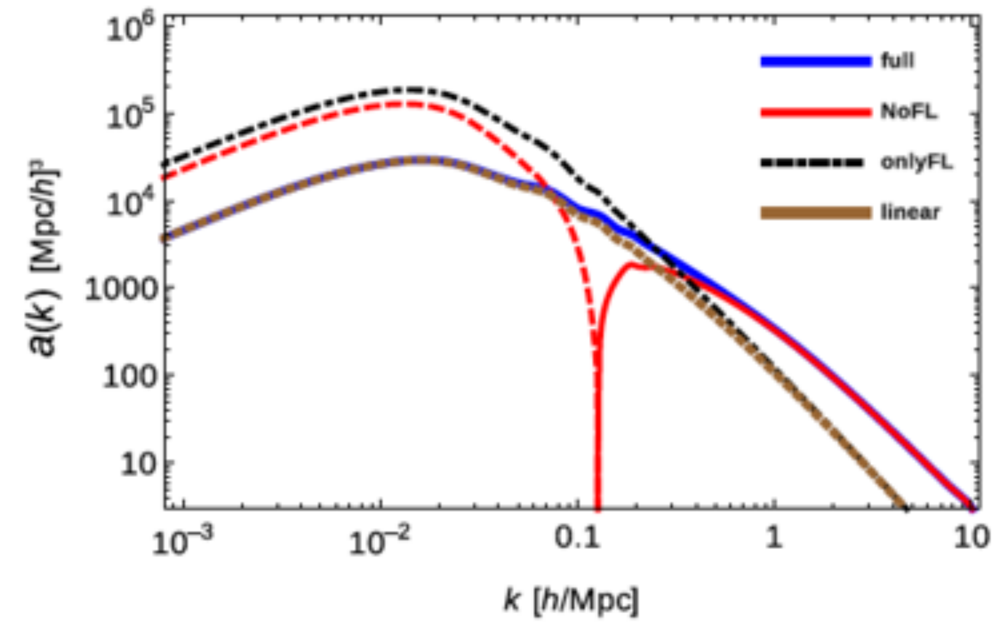


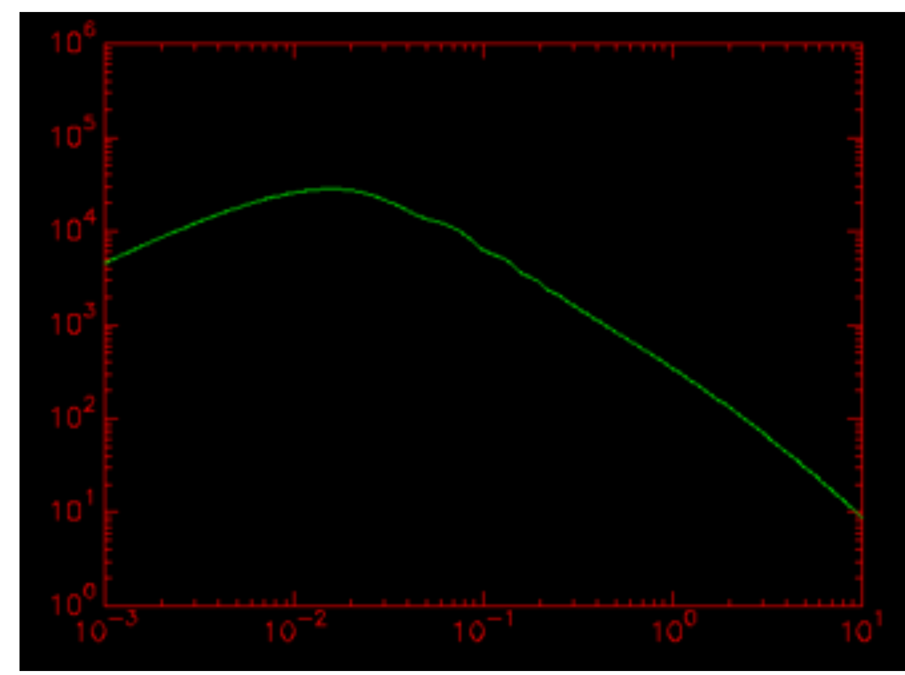


MGPT: equations to solve

We finalize this section by showing the power spectrum $a(k) \equiv P_{\nabla\cdot\Psi\nabla\cdot\Psi}$ of the divergence of Lagrangian displacement fields, $\nabla_i \Psi^i$, at one loop. This is given by $a(k) = k_1^i k_2^j \langle \Psi_i^{(1)}(k_1)\Psi_j^{(1)}(k_2)\rangle_c{}' + k_1^i k_2^j \langle \Psi_i^{(2)}(k_1)\Psi_j^{(2)}(k_2)\rangle_c{}' + 2k_1^i k_2^j \langle \Psi_i^{(1)}(k_1)\Psi_j^{(3)}(k_2)\rangle_c{}'$, where the notation $\langle(\cdots)\rangle'$ means that we omit a Dirac delta function. Using Eqs. (A8) and (A11), this is

$$\boxed{a(k) = P_L(k) + \frac{9}{98} Q_1(k) + \frac{10}{21} Q_2(k), \qquad (105)}$$





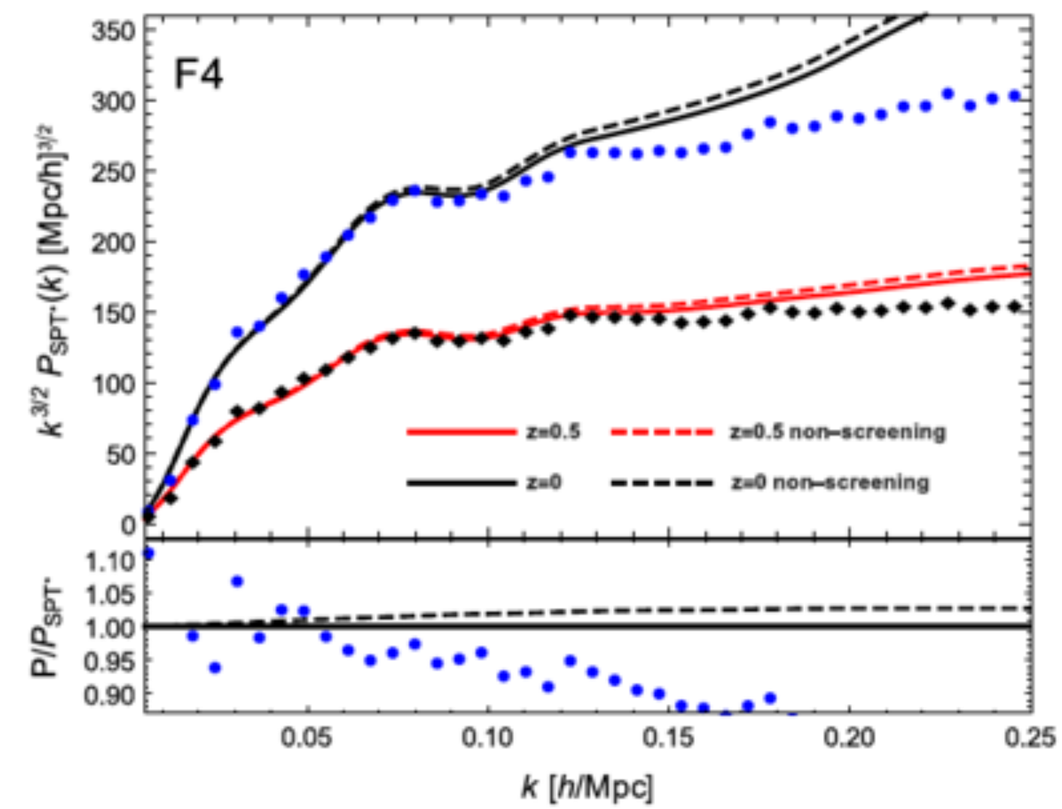
MGPT: equations to solve

The SPT* power spectrum is usually written as $P_{\mathrm{SPT}*}(k) = P_L(k) + P_{22}(k) + P_{13}(k)$, in terms of Q and R functions these are

$$P_{\mathrm{SPT}*} = P_L(k) + \frac{10}{21}R_1(k) + \frac{6}{7}R_2(k) + \frac{9}{98}Q_1(k) + \frac{3}{7}Q_2(k) + \frac{1}{2}Q_3(k) - \sigma_L^2 k^2 P_L(k) \quad \text{(A29)}$$

where

$$\sigma_L^2 = \frac{1}{2}X_L(q \to \infty) = \frac{1}{6\pi^2}\int_0^\infty dk P_L(k) \quad \text{(A30)}$$



MGPT: general structure

(MainLoop)

- **main.c**

  (MainLoop)

- cmdline_defs.h
- data_structure.h
- globaldefs.h
- protodefs.h
- startrun.c

- mglpt.c
- mglpt_io.c
- mglpt_fns.c
- mglpt_diffeqs.c
- mglpt_quads.c

- models.c
- models.h

- mglpt_postprocess.c
- mglpt_bias.c

- tests_mglpt.c
- tests_mglpt_diffeqs.c
- tests_mglpt_quads.c
- tests_models.c
- tests.h

- Data structures
- Routines and functions

Mathematica notebook

Modified Gravity Lagrangian Perturbation Theory (MG-LPT)

General parameters

$f_{R0} = 10^{-5}$

$z_0 = 0$ (*Final z for evaluation zev*)

$h = 0.697$

$\Omega_{m0} = 0.281$

$S_c = 1$ (*Screening :: 1 ON;   0 OFF*)

$\eta_0 = \mathrm{Log}\left[\frac{1}{1+z_0}\right]$

inputFile = "BLi_matterpower_Extended.dat" (*ΛCDM PS*)

$\mathrm{fourpi2} = 4.0\,\pi^2$

Set of auxiliary functions

$$\frac{1}{1+z} = \mathrm{Exp}[\eta]$$

ΛCDM definitions

$$H[\eta] = \sqrt{\Omega_{m0}\,\mathrm{Exp}[-3\,\eta] + (1-\Omega_{m0})}$$

$$\Omega_m[\eta] = \frac{1}{1+((1-\Omega_{m0})/\Omega_{m0})\,\mathrm{Exp}[3\,\eta]}$$

Auxiliary functions

$$f_1[\eta] = \frac{3}{2\left(1+\frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)} = \frac{3}{2}\,\Omega_m[\eta]$$

$$f_2[\eta] = 2 - f_1[\eta]$$

$$A_0[\eta] = \frac{3}{2}\,\frac{\Omega_m[\eta]\,H[\eta]^2}{H_0^{\,2}} \quad \text{(*verificar el cuadrado de H0*)}$$

and $H_0 \equiv 1/H[z=0] = 2997.92458$ (*invH0*) in Mpc / h.

Mathematica notebook

Model definitions

For the Hu-Sawicky model and ΛCDM parameters

$$m[\eta] = \frac{1}{H_0}\left(\frac{1}{2\,f_{R0}}\right)^{1/2}\frac{(\Omega_{m0}\,\mathrm{Exp}[-3\,\eta]+4\,(1-\Omega_{m0}))^{(2+n_{HS})/2}}{(\Omega_{m0}+4\,(1-\Omega_{m0}))^{(1+n_{HS})/2}}$$

Define

$$A\,(k) = \frac{3}{2}\,\Omega_m\,H^2\,\mu[\eta,\,k]$$

where

$$\mu[\eta,\,k] = 1+\frac{2\,\beta^2\,k^2}{k^2+\mathrm{Exp}[2\,\eta]\,m[\eta]^2} \quad (*\text{mu is from } A(k) = \tfrac{3}{2}\Omega_m H^2\mu\ *)$$

Also

$$M_1[\eta] = 3\,m[\eta]^2$$

$$M_2[\eta] = \frac{9/4}{H_0{}^2}\left(\frac{1}{f_{R0}}\right)^2\frac{(\Omega_{m0}\,\mathrm{Exp}[-3\,\eta]+4\,(1-\Omega_{m0}))^5}{(\Omega_{m0}+4\,(1-\Omega_{m0}))^4}$$

$$M_3[\eta] = \frac{45/8}{H_0{}^2}\left(\frac{1}{2\,f_{R0}}\right)^3\frac{(\Omega_{m0}\,\mathrm{Exp}[-3\,\eta]+4\,(1-\Omega_{m0}))^7}{(\Omega_{m0}+4\,(1-\Omega_{m0}))^6}$$

Try $\beta^2 = 1/6$, $n_{HS} = 1$ and $\omega_{BD} = 0$.

Mathematica notebook

Model definitions (continuation)

$$K_{FL}^{(2)}[\mathbf{k}_1, \mathbf{k}_2] = 2\,\frac{(\mathbf{k}_1\cdot\mathbf{k}_2)^2}{k_1^2\,k_2^2}\,(A(k_1) + A(k_2) - 2A_0) + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}\,(A(k_1) - A_0) + \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_2^2}\,(A(k_2) - A_0)$$

$$K_{FL}^{(2)}[\eta, k_f, k_1, k_2] = \frac{1}{2}\,\frac{\left(k_f^2 - k_1^2 - k_2^2\right)^2}{k_1^2\,k_2^2}\,(\mu[\eta, k_1] + \mu[\eta, k_2] - 2) + \frac{1}{2}\,\frac{k_f^2 - k_1^2 - k_2^2}{k_1^2}\,(\mu[\eta, k_1] - 1) + \frac{1}{2}\,\frac{k_f^2 - k_1^2 - k_2^2}{k_2^2}\,(\mu[\eta, k_2] - 1)$$

$$A[k] = A_0\left(1 + \frac{k^2/a^2}{3\,\Pi[k]}\right)$$

$$\Pi[\eta, k] = \frac{1}{3\,a^2}\left((3 + 2\,\omega_{BD})\,k^2 + M_1\,a^2\right) = \frac{k^2}{\mathrm{Exp}[2\,\eta]} + m[\eta]^2$$

where $m[\eta]$ is the mass of the scalar field.

The corresponding definitions for $D^{(3)\,\mathrm{symm}}$:

$$\mathrm{kpp}[x, k, p] = \sqrt{k^2 + p^2 + 2\,k\,p\,x}$$

$$K_{FL}^{(2)}[\eta, x, k, p] = 2\,x^2\,(\mu[\eta, k] + \mu[\eta, p] - 2) + \frac{p\,x}{k}\,(\mu[\eta, k] - 1) + \frac{k\,x}{p}\,(\mu[\eta, p] - 1)$$

$$J_{FL}^{(2)}[\eta, x, k, p] = \frac{9}{2\,A_0[\eta]}\,K_{FL}^{(2)}[\eta, x, k, p]\,\Pi[\eta, k]\,\Pi[\eta, p]$$

Mathematica notebook

Set of sources

Le us also define the sources

$$S_a[\eta, k] = A[k] = f_1[\eta]\,\mu[\eta, k]$$

$$S_b[\eta, k, k_1, k_2] = (A[k_1] + A[k_2] - A[k]) = (f_1[\eta]\,\mu[\eta, k_1] + f_1[\eta]\,\mu[\eta, k_2] - f_1[\eta]\,\mu[\eta, k])$$
$$= f_1[\eta]\,(\mu[\eta, k_1] + \mu[\eta, k_2] - \mu[\eta, k])$$

$$S_{FL}[\eta, k, k_1, k_2] = \frac{M_1[k]}{3\,\Pi[k]}\,K_{FL}{}^{(2)}[\mathbf{k}_1, \mathbf{k}_2]$$

$$= f_1[\eta]\,\frac{\text{mass}[\eta]^2}{\Pi[\eta, k]}\,K_{FL}{}^{(2)}[\eta]$$

$$S_{\delta I}[\eta, k, k_1, k_2] = \left(\frac{2\,A_0}{3}\right)^2 \frac{k^2}{a^2}\,\frac{M_2[\mathbf{k}_1, \mathbf{k}_2]}{6\,\Pi[k]\,\Pi[k_1]\,\Pi[k_2]}$$

$$= \frac{1}{6}\left(\frac{\Omega_m[\eta]\,H[\eta]}{\text{Exp}[\eta]\,(2997)}\right)^2 \frac{k^2\,M_2[\mathbf{k}_1, \mathbf{k}_2]}{\Pi[\eta, k]\,\Pi[\eta, k_1]\,\Pi[\eta, k_2]}$$

Mathematica notebook

Linear theory to get linear PS in MG

Equation to solve

$$D_+''[\eta] + f_2[\eta]\, D_+'[\eta] - f_1[\eta]\, \mu[\eta, k]\, D_+[\eta] = 0$$

with the initial conditions

$$D_+[\eta_0] = \mathrm{Exp}[\eta_0]$$

$$D_+'[\eta_0] = \mathrm{Exp}[\eta_0]$$

We obtain $D_+[\eta, k]$.

Now, given a ΛCDM power spectrum (PS) we obtain the linear PS in MG:

$$P_{LMG}[\eta, k] = \left(\frac{D_+[\eta, k]}{D_+[\eta, 0]}\right)^2 P_{\Lambda CDM}[\eta, k]$$

Mathematica notebook

Second order

The equations to solve are

$$D_a{}^{(2)}{}''[\eta] + \left(2 - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\right) D_a{}^{(2)}{}'[\eta] - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\,\mu[\eta, k_f]\, D_a{}^{(2)}[\eta] = A[k]\, D_+[\eta, k_1]\, D_+[\eta, k_2]$$

$$D_b{}^{(2)}{}''[\eta] + \left(2 - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\right) D_b{}^{(2)}{}'[\eta] - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\,\mu[\eta, k_f]\, D_b{}^{(2)}[\eta] = (A[k_1] + A[k_2] - A[k])\, D_+[\eta, k_1]\, D_+[\eta, k_2]$$

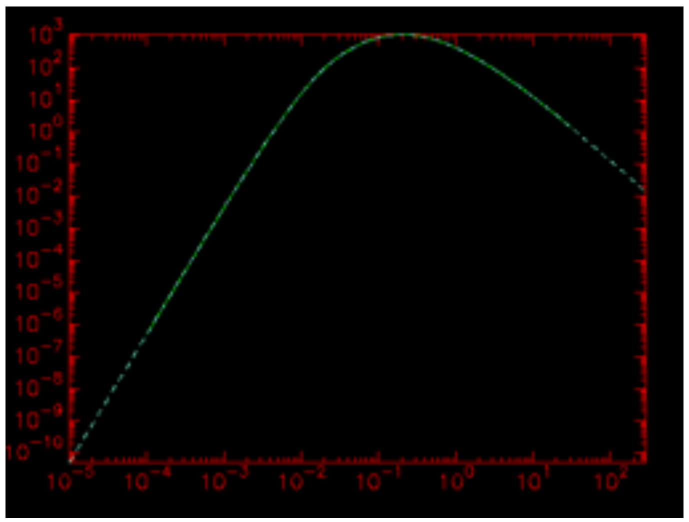
$$D_{FL}{}^{(2)}{}''[\eta] + \left(2 - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\right) D_{FL}{}^{(2)}{}'[\eta] - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\,\mu[\eta, k_f]\, D_{FL}{}^{(2)}[\eta] = \frac{M_1[k]}{3\,\Pi[k]}\, K_{FL}{}^{(2)}[\mathbf{k}_1, \mathbf{k}_2]\, D_+[\eta, k_1]\, D_+[\eta, k_2]$$

$$D_{\delta I}{}^{(2)}{}''[\eta] + \left(2 - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\right) D_{\delta I}{}^{(2)}{}'[\eta] - \frac{3}{2\left(1 + \frac{1-\Omega m}{\Omega m}\,\mathrm{Exp}[3\,\eta]\right)}\,\mu[\eta, k_f]\, D_{\delta I}{}^{(2)}[\eta] = \left(\frac{2\,A_0}{3}\right)^2 \frac{k^2}{a^2}\, \frac{M_2[\mathbf{k}_1, \mathbf{k}_2]\, D_+[\eta, k_1]\, D_+[\eta, k_2]}{6\,\Pi[k]\,\Pi[k_1]\,\Pi[k_2]}$$

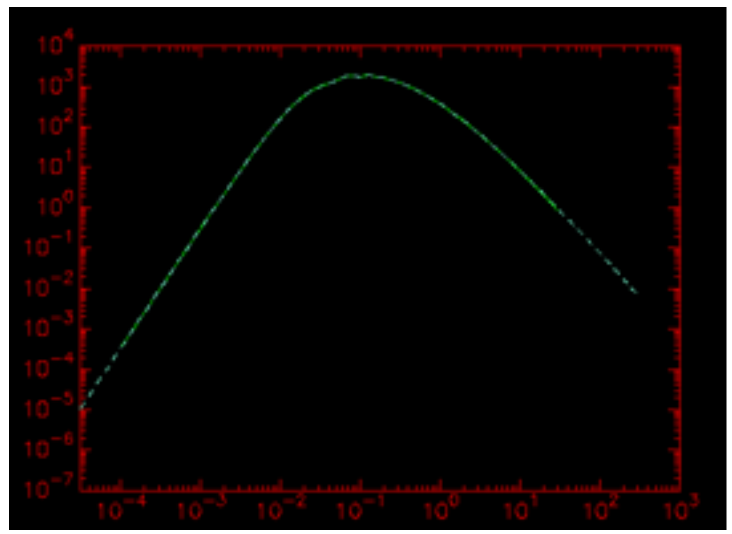
MGPT in action

- Go to…

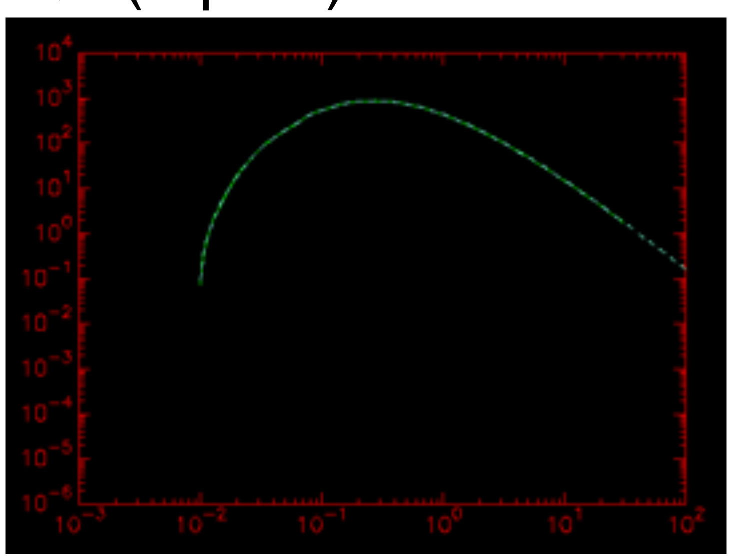
Q1 (Mpc/h)^3



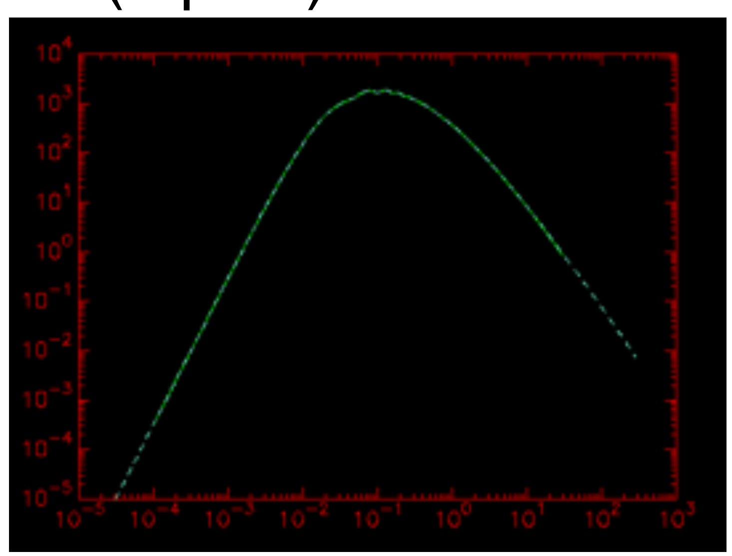
R1 (Mpc/h)^3



Q2 (Mpc/h)^3



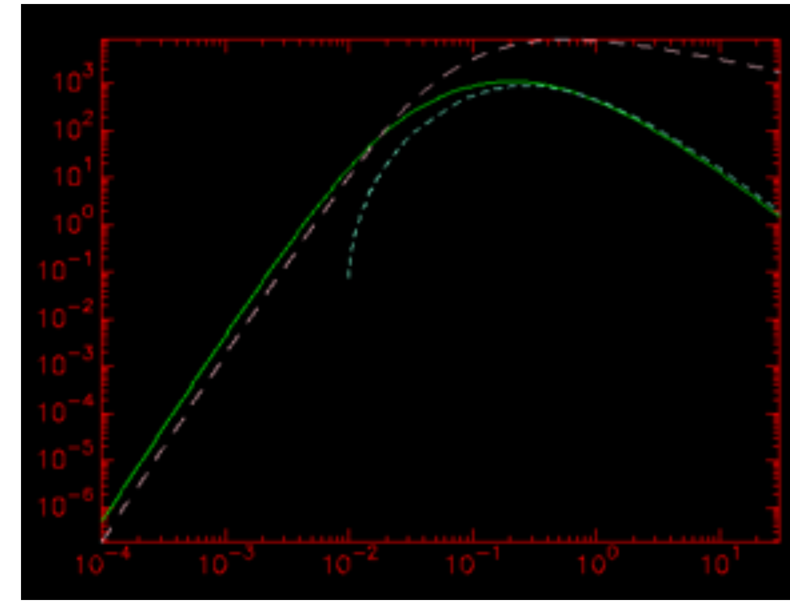
RI (Mpc/h)^3



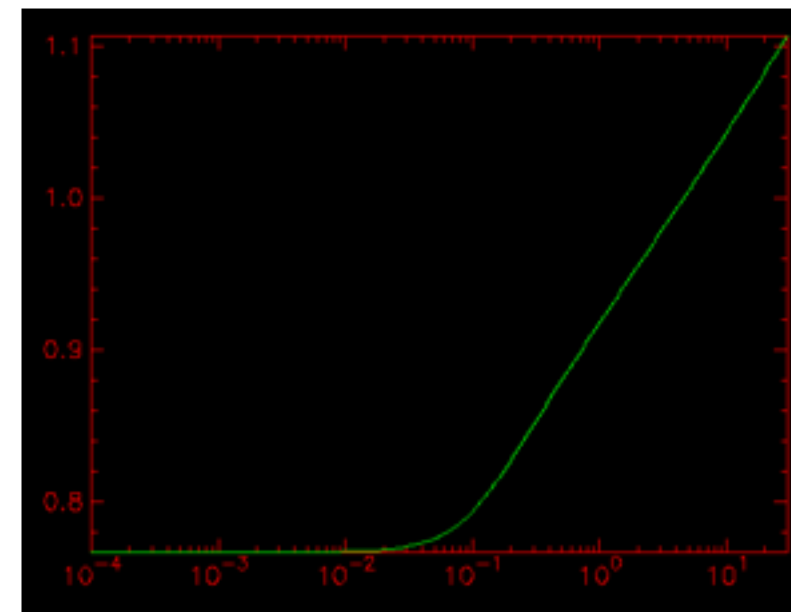
k (h/Mpc)

k (h/Mpc)

Q1, Q2, Q3 (Mpc/h)^3

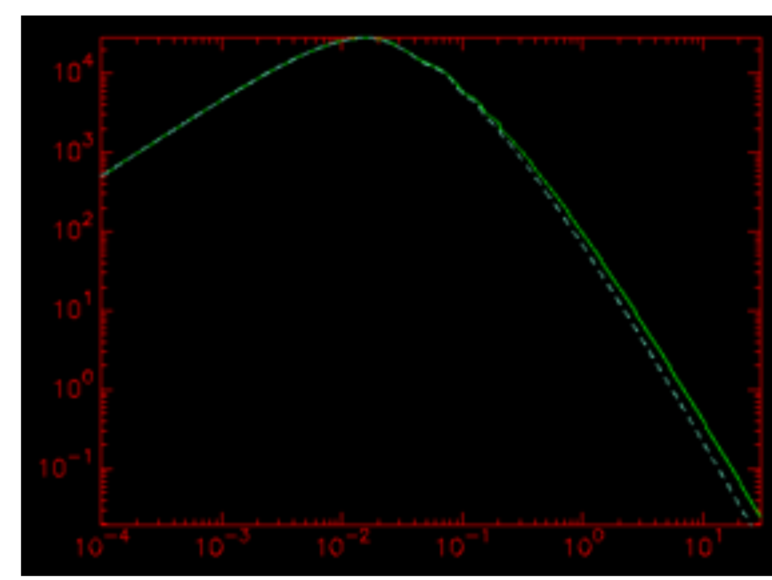


D+



PSMG

PSLCDM



k (h/Mpc)

Thanks…