Exponential atomic functions

$\mathrm{eup}_a(x)$ function

Functional–differential equation (fde):

$$\frac{y'(x)}{\ln(a)} - y(x) = \frac{y(2x+1) - ay(2x-1)}{a-1}.$$

Support:

$$\operatorname{supp} \mathrm{eup}_a(x) = [-1; 1].$$

Definition:

$$\mathrm{eup}_a(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\mathrm{shc}(\ln(a)/2 - i\,u \cdot 2^{-k})}{\mathrm{shc}(\ln(a)/2)}\, dt, \quad \mathrm{shc}(x) := \frac{\sinh(x)}{x}.$$

$\mathrm{hup}_a(x)$ function

Link with $\mathrm{eup}_a(x)$ function:

$$\mathrm{hup}_a(x) \equiv \mathrm{eup}_a(x) * \mathrm{eup}_a(x).$$

Functional–differential equation:

$$\frac{y'(x)}{\ln^2(a)} - y(x) = \frac{a(y(2x+2) + y(2x-2)) - (a^2+1)y(2x)}{(a-1)^2}.$$

Support:

$$\operatorname{supp} \mathrm{hup}_a(x) = [-2; 2].$$

Definition:

$$\mathrm{hup}_a(x) = \frac{1}{\pi} \int\limits_{0}^{\infty} \cos(tx) \prod_{k=1}^{\infty} \frac{a^2 + 1 - 2a\cos(t2^{-k})}{(a-1)^2(1 + (u/(ln(a)2^k)^2)}\, dt.$$

$\mathrm{tup}_a(x)$ function

Link with $\mathrm{hup}_a(x)$ function:

$$\mathrm{tup}_a(x) \equiv \mathrm{hup}_a(x) * \mathrm{hup}_a(x).$$

Functional–differential equation (same to $\mathrm{eup}_a(x)$ fde with substitution $a^x \to e^{i2\pi ax}$):

$$\frac{y'(x)}{i2\pi a} - y(x) = \frac{y(2x+1) - i2\pi a y(2x-1)}{i2\pi a - 1}.$$

Support:

$$\operatorname{supp} \mathrm{tup}_a(x) = [-1; 1].$$

Definition:

$$\mathrm{tup}_a(x) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\mathrm{sinc}(\pi a - t \cdot 2^{-k})}{\mathrm{sinc}(\pi a)}\, dt.$$

$\mathrm{scup}_a(x)$ function

Link with $\mathrm{tup}_a(x)$ function:

$$\mathrm{scup}_a(x) \equiv \mathrm{tup}_a(x) * \mathrm{tup}_a(x).$$

Functional–differential equation:

$$\frac{y''(x)}{4\pi^2 a^2} + y(x) = \frac{y(2x+2) - \cos(2\pi a) y(2x) + y(2x-2))}{1 - \cos(2\pi a)}.$$

Support:

$$\operatorname{supp} \mathrm{scup}_a(x) = [-2; 2].$$

Definition:

$$\mathrm{scup}_a(x) = \frac{1}{\pi} \int_0^\infty \cos(tx) \prod_{k=1}^{\infty} \frac{\cos(t \cdot 2^{1-k}) - \cos(2\pi a)}{(1 - \cos(2\pi a))(1 - (t/\pi a 2^k)^2)}\, dt.$$

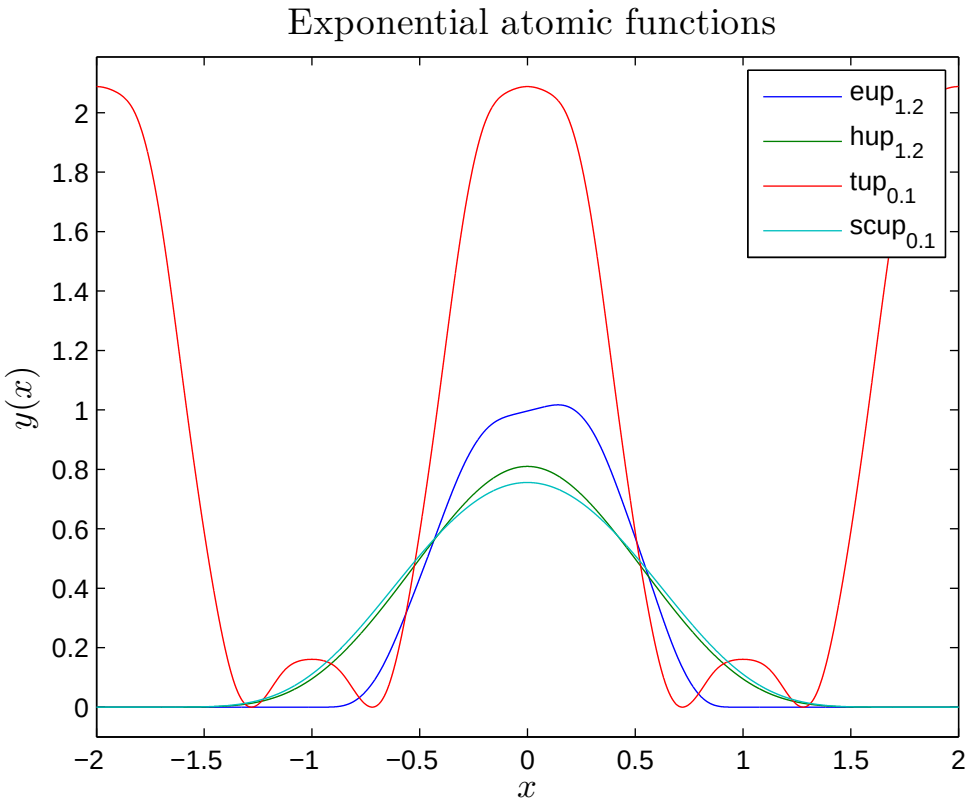

Figure 1: Exponential atomic functions plot

Remark 1

Atomic $\mathrm{tup}_a(x)$ is very strange.