Digital Arithmetic

Computer Systems 2016

Oleks

DIKU

September 14, 2016

0

1

Truth Tables

NOT

A	$\overline{A}$
0	1
1	0

Truth Tables

NOT

A	$\overline{A}$
0	1
1	0

AND

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

An **algebra** is a collection of functions adhering to **laws**.

The functions **NOT**, **AND**, and **OR** form a **boolean algebra**:

$$\begin{aligned}\textbf{NOT} &: \mathbb{B}^1 \to \mathbb{B}\\ \textbf{AND} &: \mathbb{B}^2 \to \mathbb{B}\\ \textbf{OR} &: \mathbb{B}^2 \to \mathbb{B}\end{aligned}$$

Boolean Algebra Laws - AND form

Name	AND Form
Identity Law	$1 \cdot A = A$
Null Law	$0 \cdot A = 0$
Idempotent Law	$A \cdot A = A$
Inverse Law	$A \cdot \overline{A} = 0$
Commutative Law	$A \cdot B = B \cdot A$
Associative Law	$(A \cdot B) \cdot C = A \cdot (B \cdot C)$
Distributive Law	$A + B \cdot C = (A + B) \cdot (A \cdot C)$
Absorption Law	$A \cdot (A + B) = A$
De Morgan's Law	$\overline{A \cdot B} = \overline{A} + \overline{B}$

Boolean Algebra Laws - OR form

Name	OR Form
Identity Law	$0 + A = A$
Null Law	$1 + A = 1$
Idempotent Law	$A + A = A$
Inverse Law	$A + \overline{A} = 1$
Commutative Law	$A + B = B + A$
Associative Law	$(A + B) + C = A + (B + C)$
Distributive Law	$A \cdot (B + C) = A \cdot B + A \cdot C$
Absorption Law	$A + A \cdot B = A$
De Morgan’s Law	$\overline{A + B} = \overline{A} \cdot \overline{B}$

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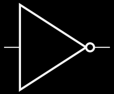
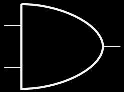
Gates

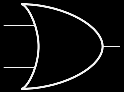
NOT



AND



OR



More Truth Tables

XOR

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

NAND

A	B	$A \uparrow B$
0	0	1
0	1	1
1	0	1
1	1	0

NOR

A	B	$A \downarrow B$
0	0	1
0	1	0
1	0	0
1	1	0

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Do we add these as basic functions to our algebra?

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Do we add these as basic functions to our algebra?

Do we need fancy new hardware?

Admissibility

A function is **admissible** if

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it can be derived from other functions

$\equiv$

it does not contradict the laws of the algebra

XOR is admissible

— Proof by reading the truth table.

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

$$\begin{aligned} A \oplus B &\equiv (\,\overline{A} \cdot B\,) + (\,A \cdot \overline{B}\,) && \text{(read 1s)} \\ &\equiv \overline{(\,\overline{A} \cdot \overline{B}\,) + (\,A \cdot B\,)} && \text{(read 0s)} \end{aligned}$$

"*A* or *B*, but not both *A* and *B*"

NAND is admissible

— Proof by reading the truth table.

A	B	$A \uparrow B$
0	0	1
0	1	1
1	0	1
1	1	0

$$A \uparrow B \equiv (\overline{A} \cdot \overline{B}) + (A \cdot \overline{B}) + (\overline{A} \cdot B) \qquad \text{(read 1s)}$$

$$\equiv \overline{(A \cdot B)} \qquad \text{(read 0s)}$$

"not both A and B"

NOR is admissible

— Proof by reading the truth table.

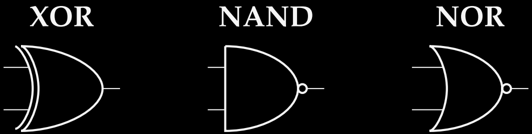
A	B	$A \downarrow B$
0	0	1
0	1	0
1	0	0
1	1	0

$$A \downarrow B \equiv \left(\overline{A} \cdot \overline{B} \right) \qquad \text{(read 1s)}$$

$$\equiv \overline{\left(A \cdot \overline{B} \right) + \left(\overline{A} \cdot B \right) + \left(A \cdot B \right)} \qquad \text{(read 0s)}$$

"neither A nor B"

More Gates



We like fancy new hardware!