

Termination analysis of first order programs

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January 21, 2012

Overview

The syntax of Δ

```
<program> ::= <clause>+ <expression>
<expression> ::= <element> ( '.' <expression> ) ?
<element> ::= '0' | '(' <element> ')' | <name> | <application>
<application> ::= <name> <expression>*
<clause> ::= <name> <pattern>* ':' <expression> ';'
<pattern> ::= <pattern-element> ( '.' <pattern> ) ?
<pattern-element> ::= '0' | '_' | '(' <pattern> ')' | <name>
<name> ::= ['a'-'z'] ( ['-','a'-'z']* ['a'-'z'] ) ?
```

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Symbols

Description	Instance	Finite list	Space
Expression	x	X	\mathbb{X}
Element (of an expression)	e	E	\mathbb{E}
Function	f	F	\mathbb{F}
Clause	c	C	\mathbb{C}
Pattern	p	P	\mathbb{P}
Value (think “binary”)	b	B	\mathbb{B}
Name (think “variable”)	v	V	\mathbb{V}
Program (p was taken)	r	R	\mathbb{R}

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The syntax of Δ

$\langle \text{program} \rangle ::= \langle \text{clause} \rangle^+ \langle \text{expression} \rangle$	
$\langle \text{expression} \rangle ::= \langle \text{element} \rangle (\text{'.'} \langle \text{expression} \rangle) ?$	x
$\langle \text{element} \rangle ::= \text{'0'} \mid \text{'('} \langle \text{element} \rangle \text{'})' \mid \langle \text{name} \rangle \mid \langle \text{application} \rangle$	e
$\langle \text{application} \rangle ::= \langle \text{name} \rangle \langle \text{expression} \rangle^*$	$\langle v, X \rangle$
$\langle \text{clause} \rangle ::= \langle \text{name} \rangle \langle \text{pattern} \rangle^* \text{' := ' } \langle \text{expression} \rangle \text{' ; '}$	$c = \langle v, P, x \rangle$
$\langle \text{pattern} \rangle ::= \langle \text{pattern-element} \rangle (\text{'.'} \langle \text{pattern} \rangle) ?$	p
$\langle \text{pattern-element} \rangle ::= \text{'0'} \mid \text{'_'} \mid \text{'('} \langle \text{pattern} \rangle \text{'})' \mid \langle \text{name} \rangle$	p
$\langle \text{name} \rangle ::= [\text{'a'} - \text{'z'}] ([\text{'-' } \text{'a'} - \text{'z'}]^* [\text{'a'} - \text{'z'}]) ?$	v

(14)

Functions in Δ

$$f = \langle v, C \rangle \quad \text{s.t.} \quad \forall \langle v_1, P_1, _ \rangle, \langle v_2, P_2, _ \rangle \in C \ (v_1 = v_2 = v) \wedge (|P_1| = |P_2|)$$

Pattern matching is ensured **exhaustive** at compile time.

$$\forall b \in \mathbb{B} \ \exists c \in C \ c \succ b$$

Programs in Δ

$$r = \langle F, x \rangle$$

Demo

Disjoint shapes

$$s_1 \cap s_2 = \emptyset \quad \text{iff} \quad B_1 \cap B_2 = \emptyset$$

where

$$s_1, s_2 \in \mathbf{S} \wedge B_1 = \{b \mid b \in \mathbf{B} \wedge b \succ s_1\} \wedge B_2 = \{b \mid b \in \mathbf{B} \wedge b \succ s_2\}$$