

This specification describes a tally system for first-past-the-post (FPTP) elections.

In an FPTP election, citizens are registered as voters and candidates. Every candidate is a voter, but not every voter is a candidate.

This an anonymous teller, which means that every voter is issued a token, and a voter votes for a token among the set of candidate tokens.

It is outside the scope of this specification to ensure the anonymity of tokens. It is also outside the scope of this specification to ensure that all voters are handed out a token, and no duplicate ballots occur.

$[TOKEN]$

$ \begin{array}{l} voters : \mathbb{P} \, TOKEN \\ candidates : \mathbb{P} \, TOKEN \end{array} $	$ \begin{array}{l} candidates \subseteq voters \end{array} $
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$ \begin{array}{l} ballot : voters \rightarrow candidates \end{array} $
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We introduce a generalised reduction over sequences. The function takes as parameters an associative reduction operator, an initial accumulator value, and a sequence of elements. The accumulator need not have the same type as the elements of the sequence. For instance, we can use a reduction to count the number of occurrences of a particular value in a sequence.

$ \begin{array}{l} \overline{\overline{[S, T]}} \\ reduce : (S \times T \rightarrow S) \times S \times \text{seq } T \rightarrow S \\ \forall f : S \times T \rightarrow S \bullet \forall s : S \bullet \forall ts : \text{seq } T \bullet \\ \quad reduce(f, s, ts) = \\ \quad \quad \text{if } \# ts = 0 \\ \quad \quad \quad \text{then } s \\ \quad \quad \text{else } reduce(f, f(s, head \, ts), tail \, ts) \end{array} $
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This definition hints at a linear reduction, but an efficient implementation could perform a tree-like reduction on vector hardware.

$ \begin{array}{l} \overline{\overline{[T]}} \\ take1 : \mathbb{P}_1 \, T \rightarrow T \times \mathbb{P} \, T \\ \forall ts : \mathbb{P}_1 \, T \bullet \exists t : ts \bullet take1(ts) = (t, ts \setminus \{t\}) \end{array} $
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$[S, T]$	$reduce' : (S \times T \rightarrow S) \times S \times \mathbb{P} T \rightarrow S$
	$\forall f : S \times T \rightarrow S \bullet \forall b : S \bullet \forall as, s : \mathbb{P} T \bullet \forall a : T \bullet$ $(a, s) = take1(as) \Rightarrow$ $reduce'(f, b, as) = \mathbf{if} \# as = 0 \mathbf{then} b \mathbf{else} reduce'(f, f(b, a), s)$

	$sum : seq \mathbb{N} \rightarrow \mathbb{N}$
	$\forall plus : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N} \bullet \forall x, y : \mathbb{N} \bullet \forall as : seq \mathbb{N} \bullet$ $plus(x, y) = x + y \Rightarrow sum(as) = reduce(plus, 0, as)$

<i>Teller</i>	$tally : candidates \rightarrow \mathbb{N}$
	$\forall c : candidates \bullet \forall v : voters \bullet \forall i : \mathbb{N} \bullet$ $\forall votes : candidates \rightarrow seq \mathbb{N} \bullet$ $votes(c)(i) = \mathbf{if} ballot(v) = c \mathbf{then} 1 \mathbf{else} 0 \Rightarrow$ $tally(c) = sum(votes(c))$