

FPTP Teller

Datalogisk institut, Copenhagen University (DIKU)
Supervised Project

Oleksandr Shturmov
oleks@diku.dk

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Chapter 1

FPTP

A First-Past-The-Post (FPTP) election is among the simplest types of elections. It is easy to understand, conduct and monitor on a small scale.

In its simplest incarnation, it is the “raising of hands” election. The winner is the candidate with the highest number of raised hands. To ensure anonymity, the voter may instead be asked to denote his choice on a piece of paper, and cast it in a ballot box. Once all the votes have been cast, the votes are tallied, and the candidate most frequently denoted, wins the election.

The name “first-past-the-post” stems from horse racing, where the first horse past a particular post wins the race. Unlike in horse racing however, there is often a high chance of a tie in an FPTP election.

On a national scale, an FPTP ballot is typically a list of candidates written out on a sheet of paper, and the voter is asked to mark their desired candidate, and cast the ballot into a ballot box. The ballots in all the ballot boxes are then tallied, and the candidate with the most marks wins the election.

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Chapter 2

Z

Votes are cast and tallied anonymously using a token system. A voter may chose among a set of available of options to express her intent. We assume a set of voter and option tokens as basic types. Issuing tokens is beyond the scope of this specification.

[VOTER, OPTION]

In an election, individuals are registered as voters, and candidates are registered as options. The process of voter and candidate registration is beyond the scope of this specification. We assume that everyone eligible to vote is registered as a voter, and everyone eligible to be elected is registered as a candidate. We assume a nonempty finite set of registered voters and candidates:

	voters : \mathbb{F}_1 VOTER
	candidates : \mathbb{F}_1 OPTION

Not all voter and option tokens are necessarily dealt. Token generation can happen offline, prior to registration, or online, during registration. It is beyond the scope of this specification to ensure that enough tokens are generated to accommodate all the registrations.

In an FPTP election, a voter always chooses one among her available options. The options are first and foremost a list of candidates. This list may be insufficient to express voter intent, and a “none of the above” option is typically added.

Aside from casting a valid vote, the voter may also:

1. not cast a vote;
2. cast a blank vote; or
3. cast an invalid vote.

Depending on the legislature, the above options may have different effects on the tally. For instance, not casting a vote may be legislatively equivalent to voting for “none of the above”. We leave it to further postprocessing to handle such discrepancies. We regard all of the above as separate options, and present a tally for each option.

$\text{nota, absent, blank, invalid} : \text{OPTION}$ $\text{options} : \mathbb{F}_1 \text{OPTION}$	
$\text{nota, novote, blank, invalid} \notin \text{candidates}$ $\text{options} = \{\text{nota, novote, blank, invalid}\} \cup \text{candidates}$	

Only registered voters can cast votes. A vote is a particular option chosen by a particular voter. We choose to encode the ballots in the system as a nonpartial function with adequately restricted types:

| $\text{ballots} : \text{voters} \rightarrow \text{options}$

Note, a voter always has a chosen option, even before an election occurs. In particular, before any votes are cast, all voters are absent. TODO: ballot registration.

After the voters, candidates, and ballots are in, we turn to the tallying itself.

The teller computes a tally for every option. Some of those options are candidates. The winners are those candidates that have attained a tally greater than or equal to every other candidate.

$\text{tally} : \text{options} \rightarrow \mathbb{N}$ $\text{winners} : \mathbb{F}_1 \text{candidates}$	
$\forall o : \text{options} \bullet \text{tally}(o) = \#(\text{ballots} \triangleleft \{o\})$ $\forall w : \text{winners} \bullet \forall c : \text{candidates} \bullet \text{tally}(w) \geq \text{tally}(c)$	

TODO: winners is a bad name. They're more like relative front runners.
Corollaries:

$\#(\text{winners}) \leq \#(\text{candidates})$ $\#(\text{ballots}) = \#(\text{voters})$	
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To show:

| $\#(\text{candidates}) = 1 \Rightarrow \#(\text{winners}) = 1$

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The canonical Z reference manual. Seconded only by the Z ISO standard itself.