This document specifies a tally system for first-past-the-post (FPTP) elections in Z.

An FPTP election is the simplest possible type of election, and is frequently employed on a small scale. Pitfalls: minorities might be underrepresented in national elections with single-member legislative districts, certain groups might be discriminated into not participating, if parties choose the candidate to run in each district.

Votes are cast and tallied anonymously using a token system. Issuing tokens is beyond the scope of this specification. A voter may chose among a set of available of options to express her intent. We assume a set of voter and option tokens as basic types:

$$[VOTER, OPTION]$$

In an election, individuals are registered as voters and candidates. The process of voter and candidate registration is beyond the scope of this specification. We assume that everyone eligible to vote is registered as a voter. We assume a nonempty finite set of registered voters and candidates:

$$\begin{array}{|l} voters : \mathbb{F}_1\ VOTER \\ candidates : \mathbb{F}_1\ OPTION \end{array}$$

Not all voter and option tokens are necessarily dealt. Token generation can happen offline, prior to registration, or online, during registration. It is beyond the scope of this specification to ensure that enough tokens are generated to accommodate all the registrations.

In an FPTP election, a voter chooses one among the available options on a ballot. An FPTP ballot is first and foremost a list of candidates. This list may be insufficient to express voter intent. For this purpose, a "none of the above" option is typically added to the ballot.

Aside from casting a valid vote, the voter may also:

1. not cast a vote;
2. cast a blank vote; or
3. cast an invalid vote.

We take these options into consideration and say that a voter always chooses one among her possible options, in particular, a voter may chose the option to not cast a vote. A voter cannot not choose one of the options.

Depending on the legislature, some, or all of the above may be equivalent to voting for "none of the above", or casting an invalid vote. Likewise, it depends on the legislature what effect such votes (or lack thereof) have on the tally. To provide for these options, we introduce the following global variables:

$$hasNota : \{0, 1\}$$

hasNota indicates whether the ballot includes the special "none of the above" option.

$$nota, noVote, blank, invalid : OPTION$$
$$otherOptions : \mathbb{F}_1\ OPTION$$
$$\hline$$
$$otherOptions = \{nota, noVote, blank, invalid\}$$

$$noVoteIs : \{nota, noVote, blank, invalid\}$$
$$\hline$$
$$noVoteIs = nota \Rightarrow hasNota = 1$$

$$blankIs : \{nota, noVote, blank, invalid\}$$
$$\hline$$
$$blankIs = nota \Rightarrow hasNota = 1$$

$$invalidIs : \{nota, noVote, blank, invalid\}$$
$$\hline$$
$$invalidIs = 1 \Rightarrow hasNota = 1$$

- Has nota, no vote is nota, blank is nota, invalid is nota.
- Has nota, no

We provide a flag to specify whether not casting a vote should have an effect on the outcome of the election. If *voterMajority* is set to 0, then

$$voterMajority : \{0, 1\}$$

We summon these observations into the possible states of a ballot:

$$nota, blank, invalid : OPTION$$
$$options : \mathbb{F}_1\ OPTION$$
$$\hline$$
$$nota, blank, invalid \notin candidates$$
$$options = \{nota, blank, invalid\} \cup candidates$$

We should take care not to allow arbitrary tokens to register ballots, but only those that have been dealt to a voter. We assume a set of ballots available as a global variable:

$ballots : voters \rightarrow states$

The fact that ballots is a nonpartial function can be mitigated for by initializing it to map to *blank* for every voter.

We represent ballots as a function as multiple voters may vote for the same candidate, but every voter must vote for exactly one candidate.

This is a relative majority FPTP teller. For every candidate we provide a tally, and specify the winner of an election:

Teller

$$
\begin{array}{l}
tally : candidates \rightarrow \mathbb{N} \\
frontRunners : \mathbb{F}_1\ candidates \\
winner : maybeCandidates \\
\hline
\forall\, c : candidates \bullet tally(c) = \#(ballots \rhd \{c\}) \\
\forall\, f : frontRunners \bullet \forall\, c : candidates \bullet tally(f) \geq tally(c) \\
winner = \textbf{if}\ \#\, fronRunners = 1 \textbf{then}
\end{array}
$$