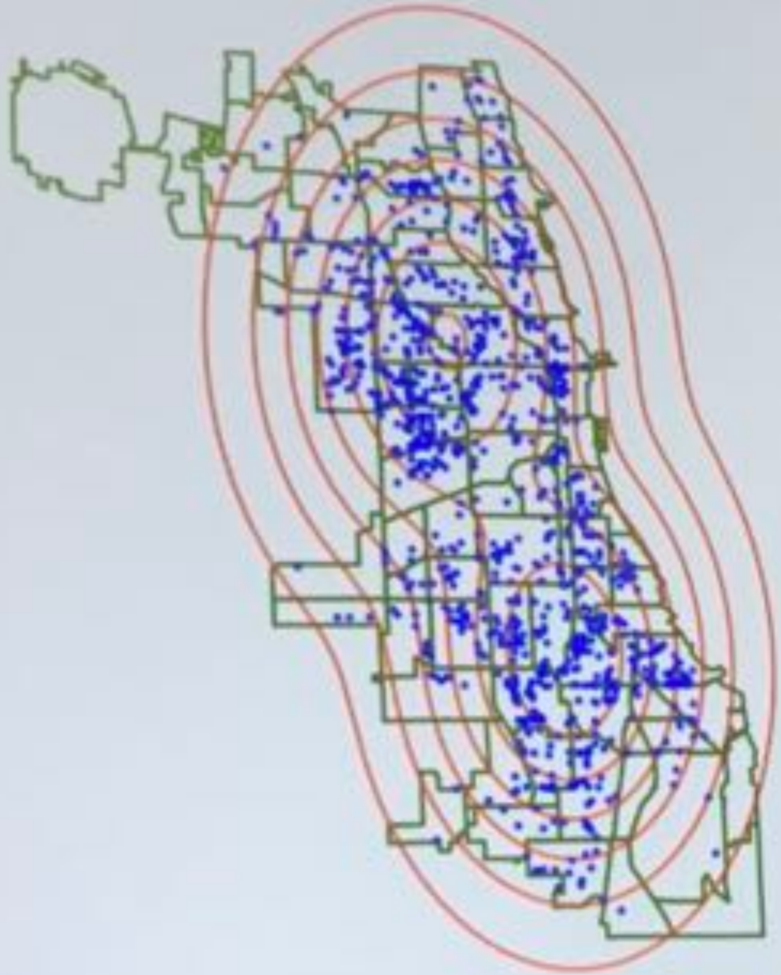
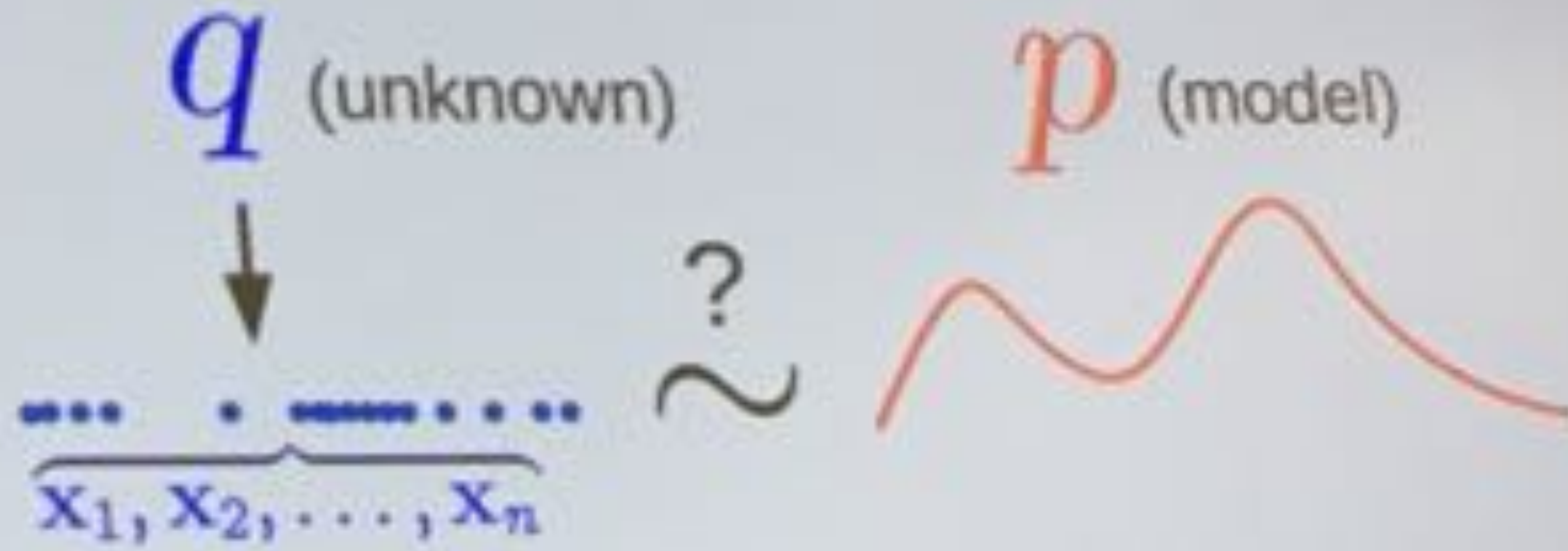
Model Criticism



Goals:

1. Test if a (complicated) model fits the data.
2. If it does not, show a location where it fails.

Problem Setting: Goodness-of-Fit Test



Test goal: Are data from the model p?

1. Nonparametric.
2. Linear-time. Runtime is $\mathcal{O}(n)$. Fast.
3. Interpretable. Model criticism by finding ★.

Model Criticism by Maximum Mean Discrepancy [Gretton et al., 2012]

- Find a location $\mathbf{v}$ at which q and p differ most [Jitkrittum et al., 2016].

$$\text{witness}(\mathbf{v}) = \mathbb{E}_{\mathbf{x}\sim q}[\quad k_{\mathbf{v}}(\mathbf{x}) \quad] - \mathbb{E}_{\mathbf{y}\sim p}[\quad k_{\mathbf{v}}(\mathbf{y}) \quad]$$

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$$\text{score}(\mathbf{v}) = \frac{|\text{witness}(\mathbf{v})|}{\text{standard deviation}(\mathbf{v})}.$$

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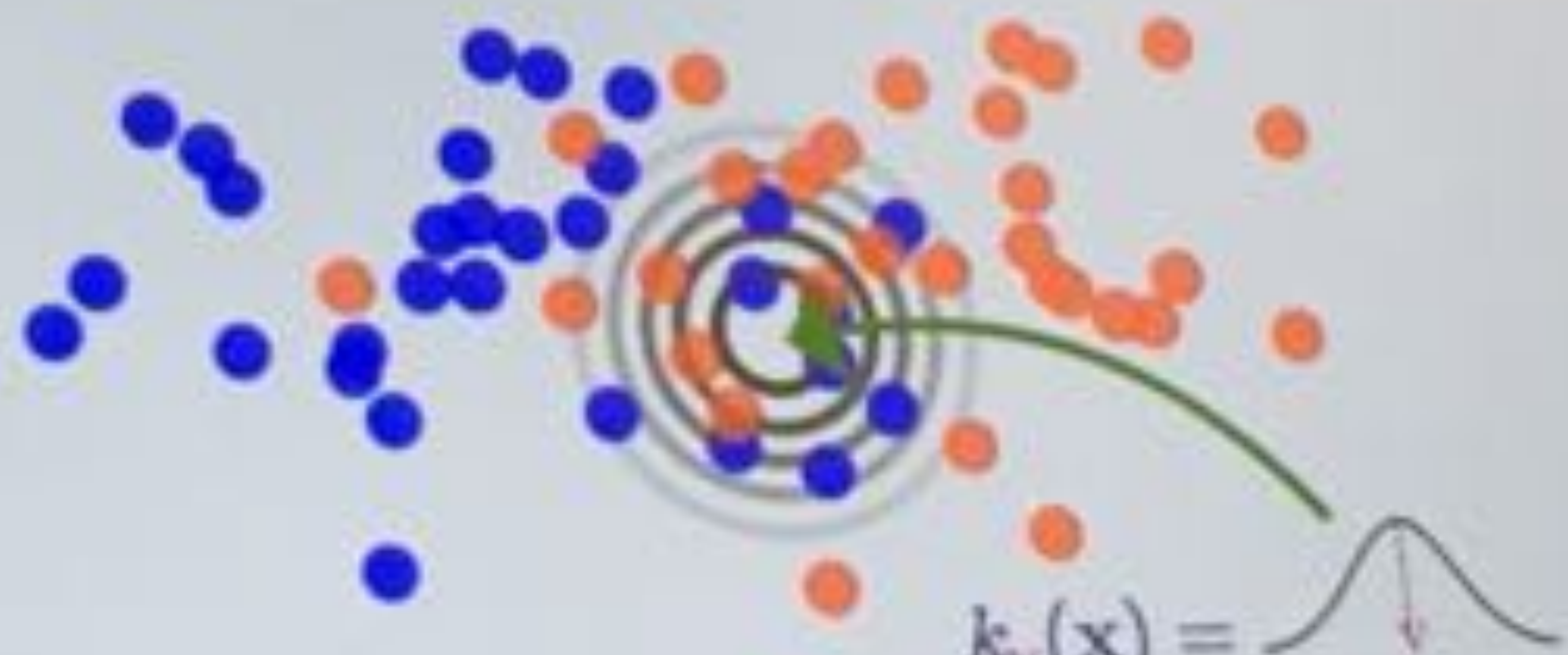
score: 0.008



$k_v(x) =$

$$\mathrm{witness}(v) = \mathbb{E}_{x\sim q}[\;\;] - \mathbb{E}_{y\sim p}[\;\;]$$

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Model Criticism by Maximum Mean Discrepancy [Gretton et al., 2012]

- Find a location v at which q and p differ most [Jitkrittum et al., 2016].

score: 25

$$\mathrm{witness}(v) = \mathbf{E}_{x\sim q}[\quad v \quad] - \mathbf{E}_{y\sim p}[\quad v \quad]$$

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No sample from p.
Difficult to generate.

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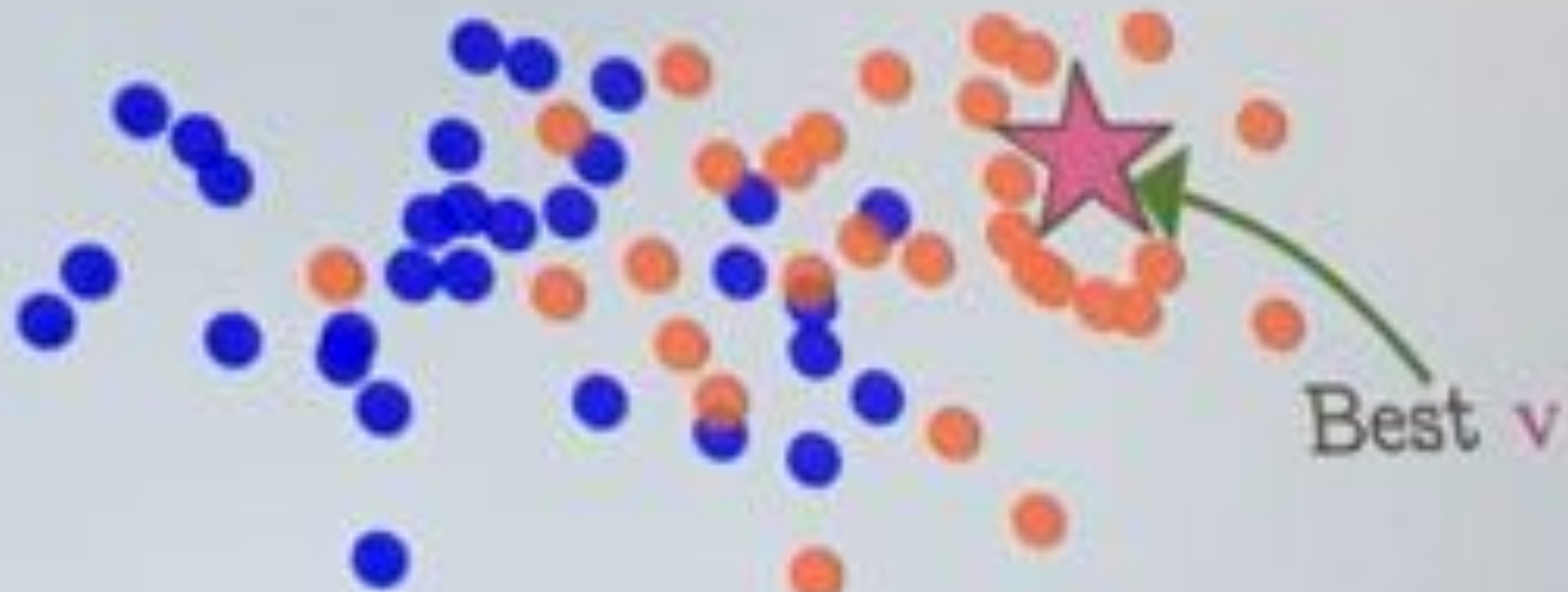

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The Stein Witness Function [Liu et al., 2016, Chwialkowski et al., 2016]

Problem: No sample from p. Cannot estimate $\mathbb{E}_{\mathbf{y}\sim p}[k_{\mathbf{v}}(\mathbf{y})]$.

(Stein) witness($\mathbf{v}$) $= \mathbb{E}_{\mathbf{x}\sim q}[T_p$

$] - \mathbb{E}_{\mathbf{y}\sim p}[T_p$

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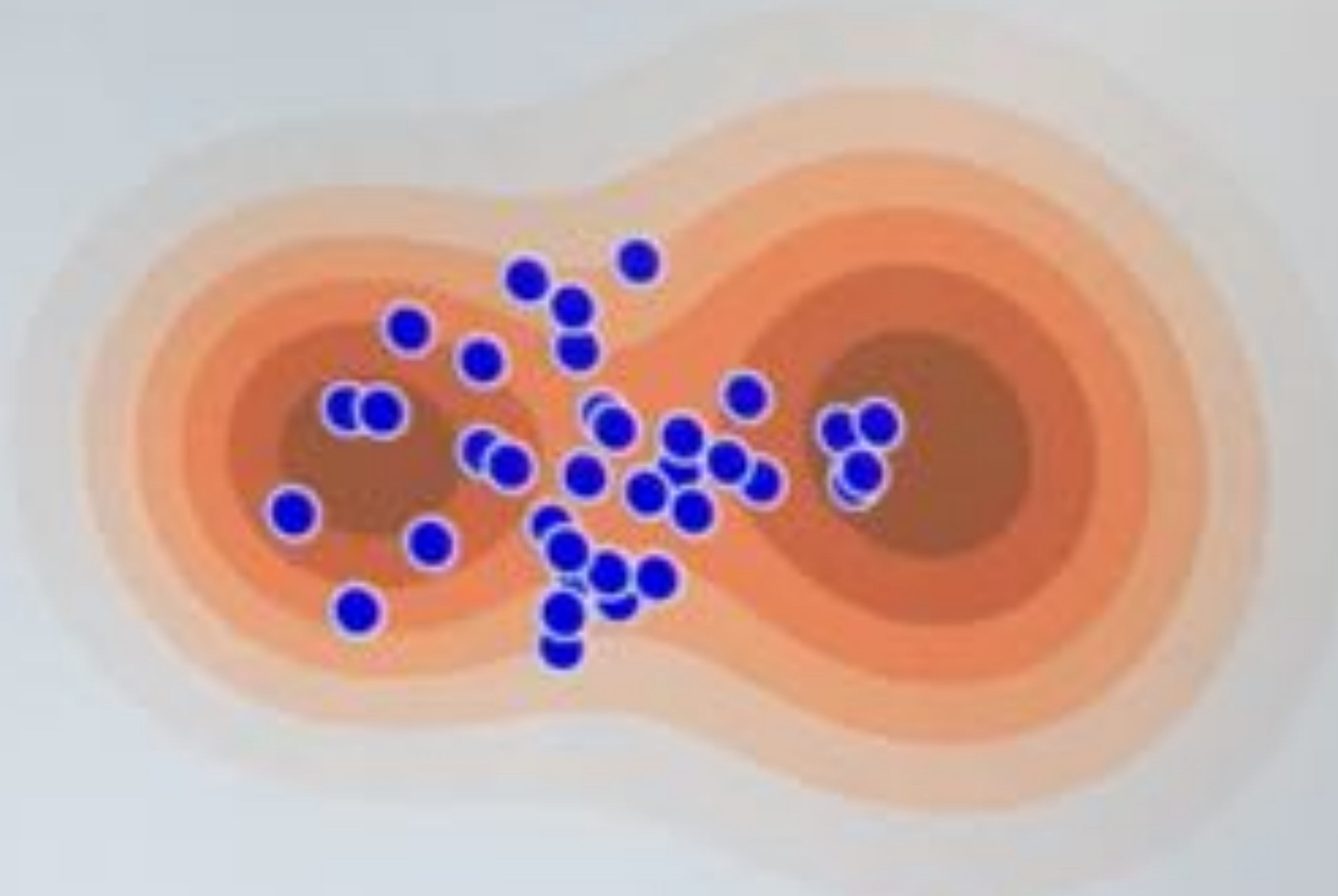
Idea: Define T_p such that $\mathbb{E}_{\mathbf{y}\sim\mathbf{p}}(T_p k_{\mathbf{v}})(\mathbf{y}) = 0$, for any $\mathbf{v}$.

Proposal: Model Criticism with the Stein Witness



$$\text{score}(v) = \frac{|\text{witness}(v)|}{\text{standard deviation}(v)}.$$

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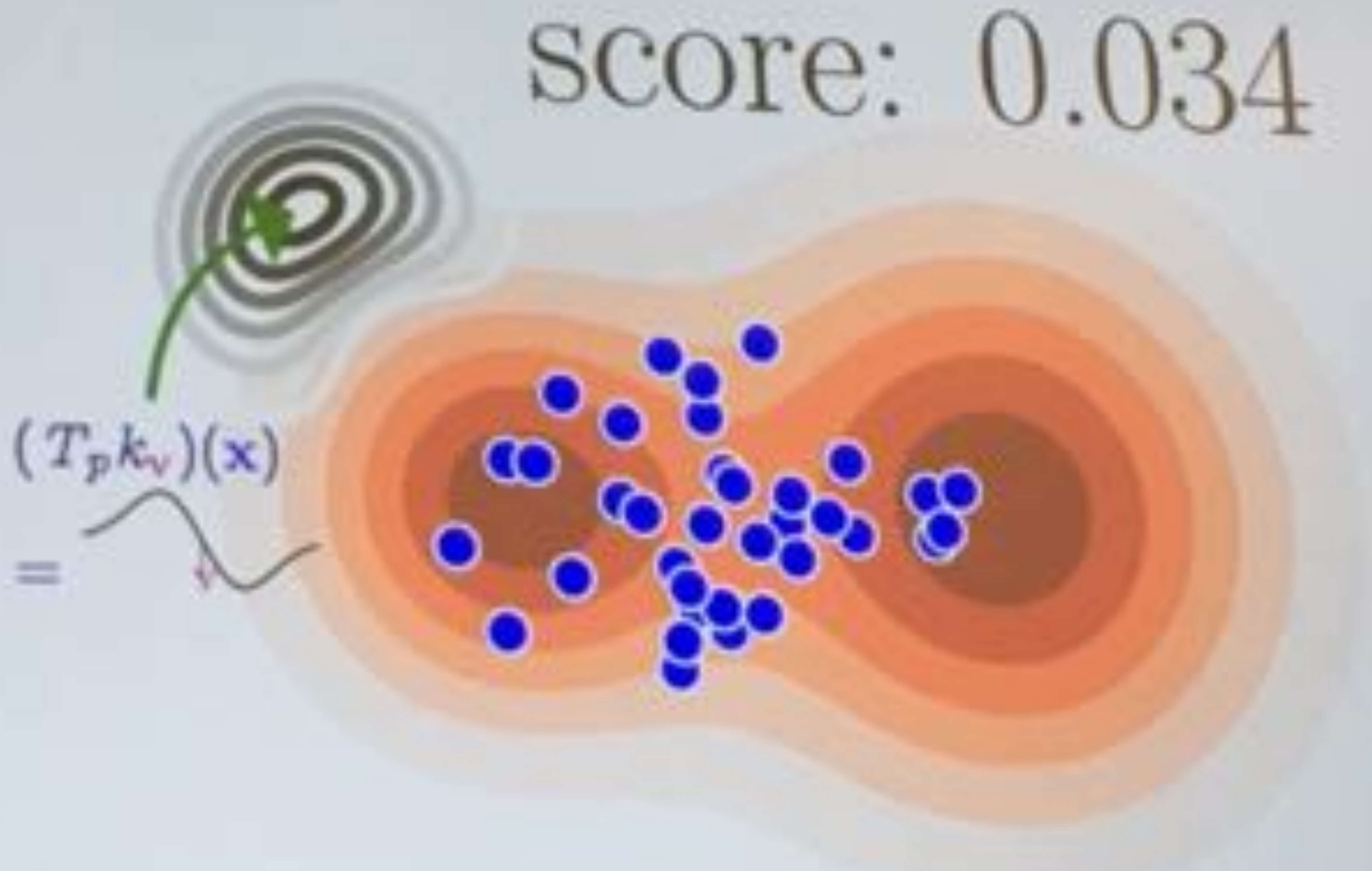
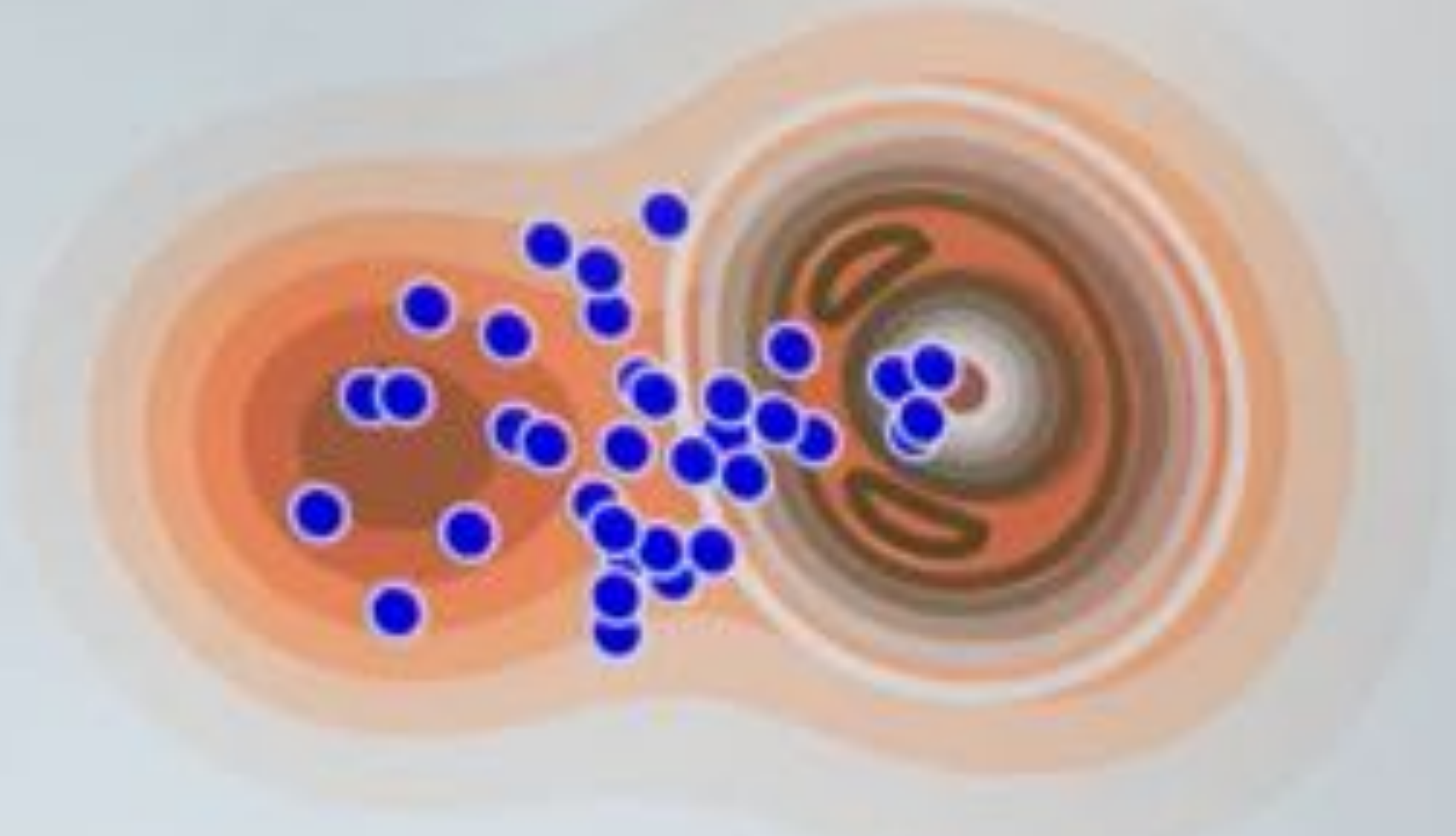


$$\text{score}(v) = \frac{|\text{witness}(v)|}{\text{standard deviation}(v)}.$$

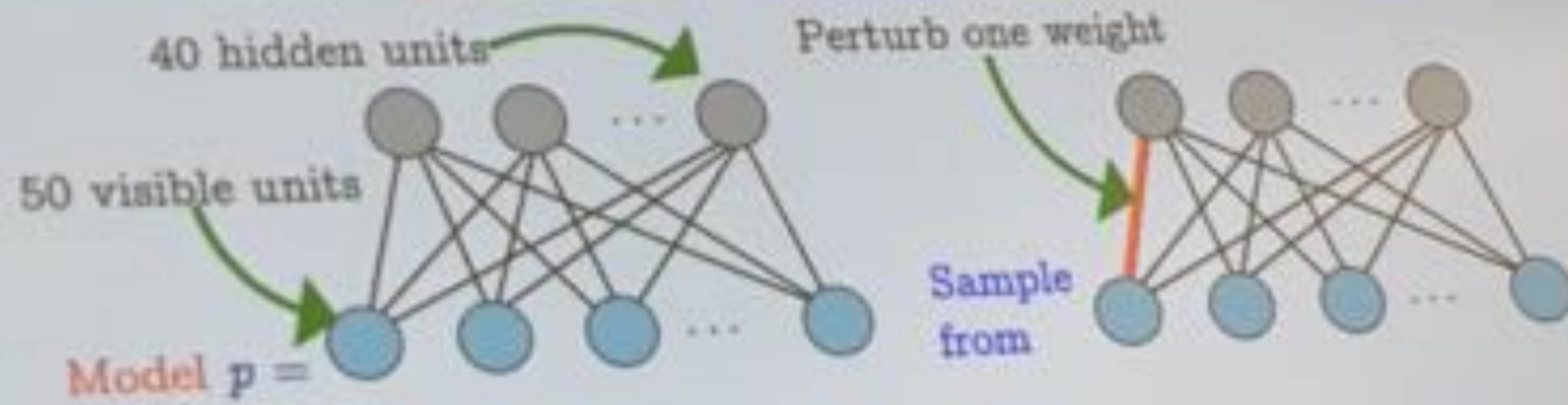
Proposal: Model Criticism with the Stein Witness

score: 0.44



$$\text{score}(\mathbf{v}) = \frac{|\text{witness}(\mathbf{v})|}{\text{standard deviation}(\mathbf{v})}.$$

Experiment: Restricted Boltzmann Machine (RBM)



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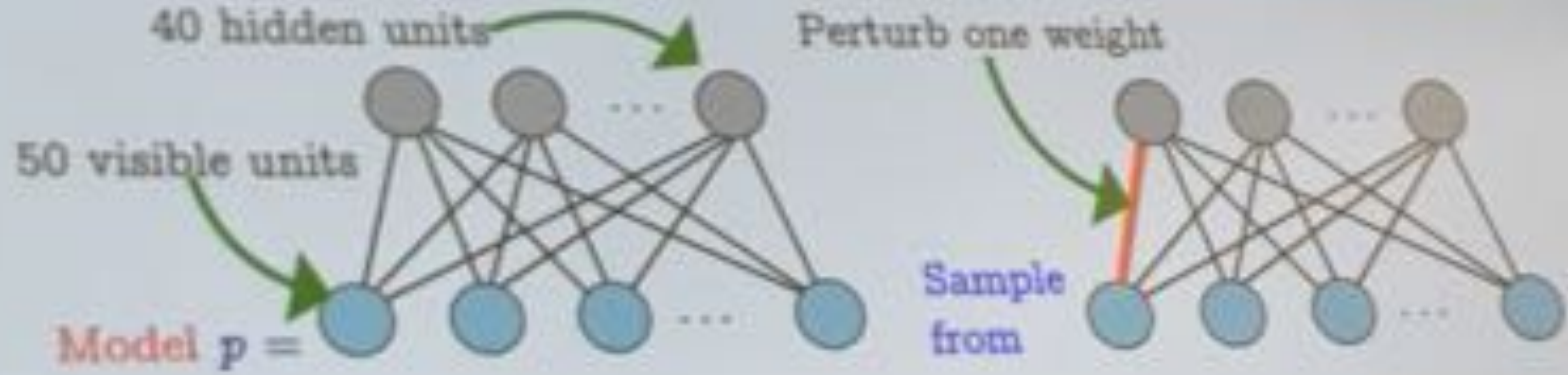
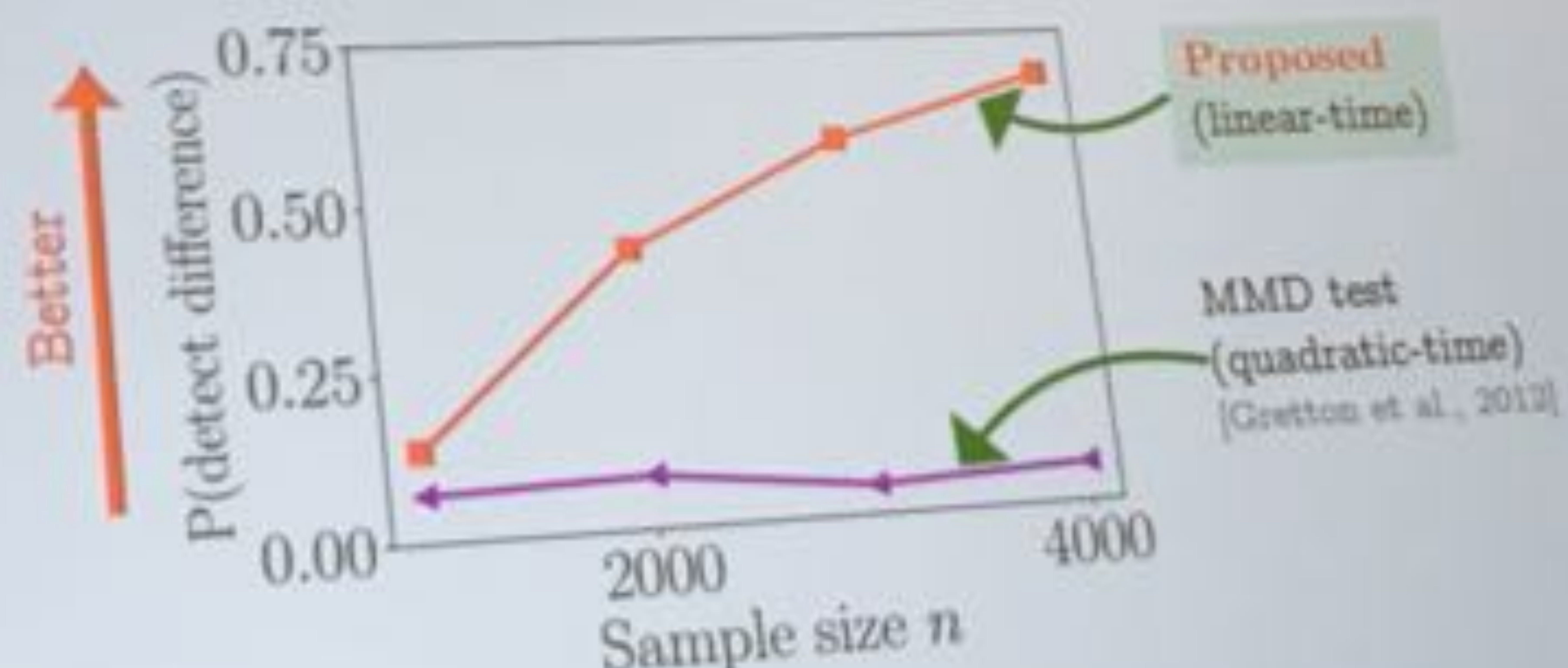
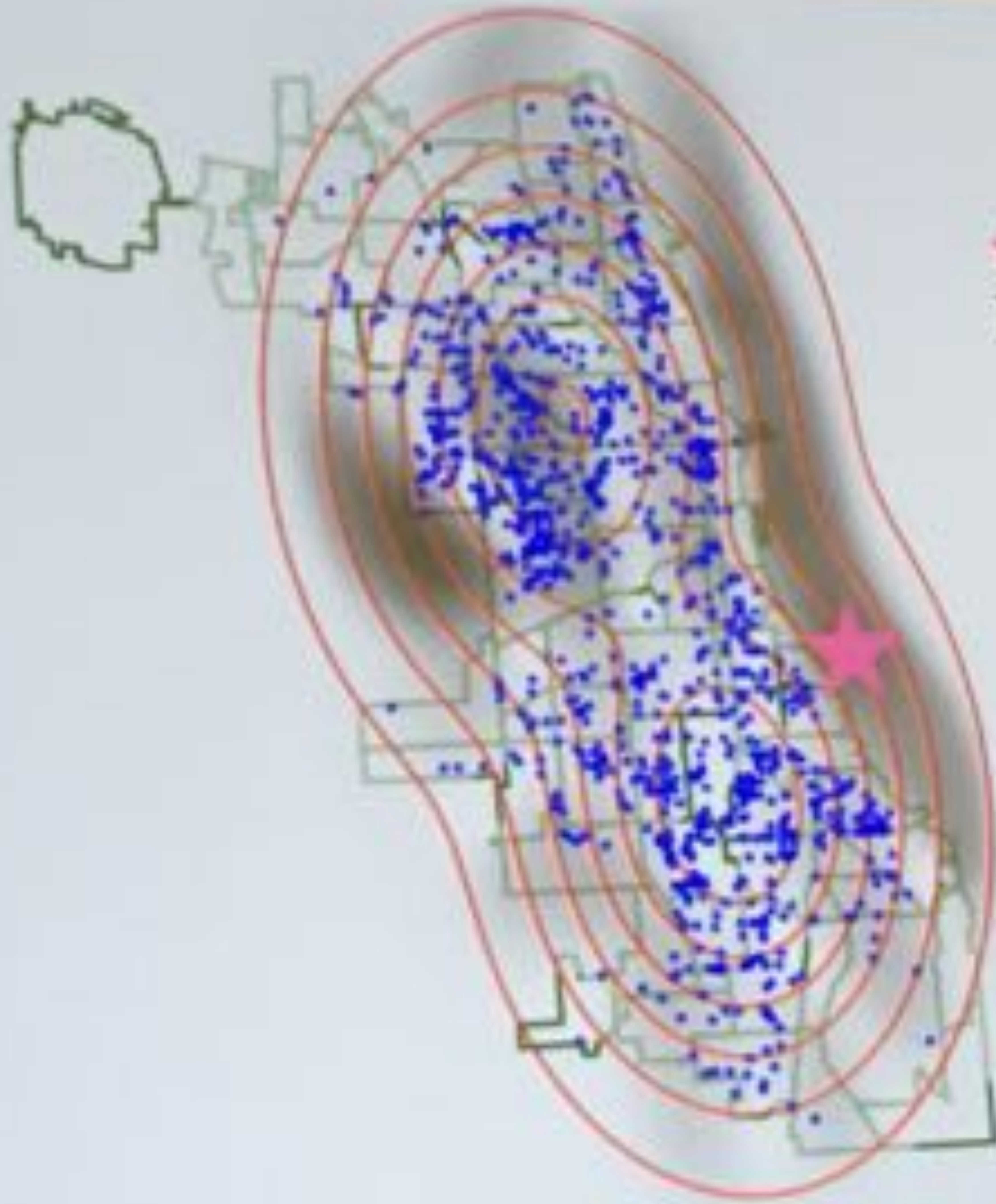




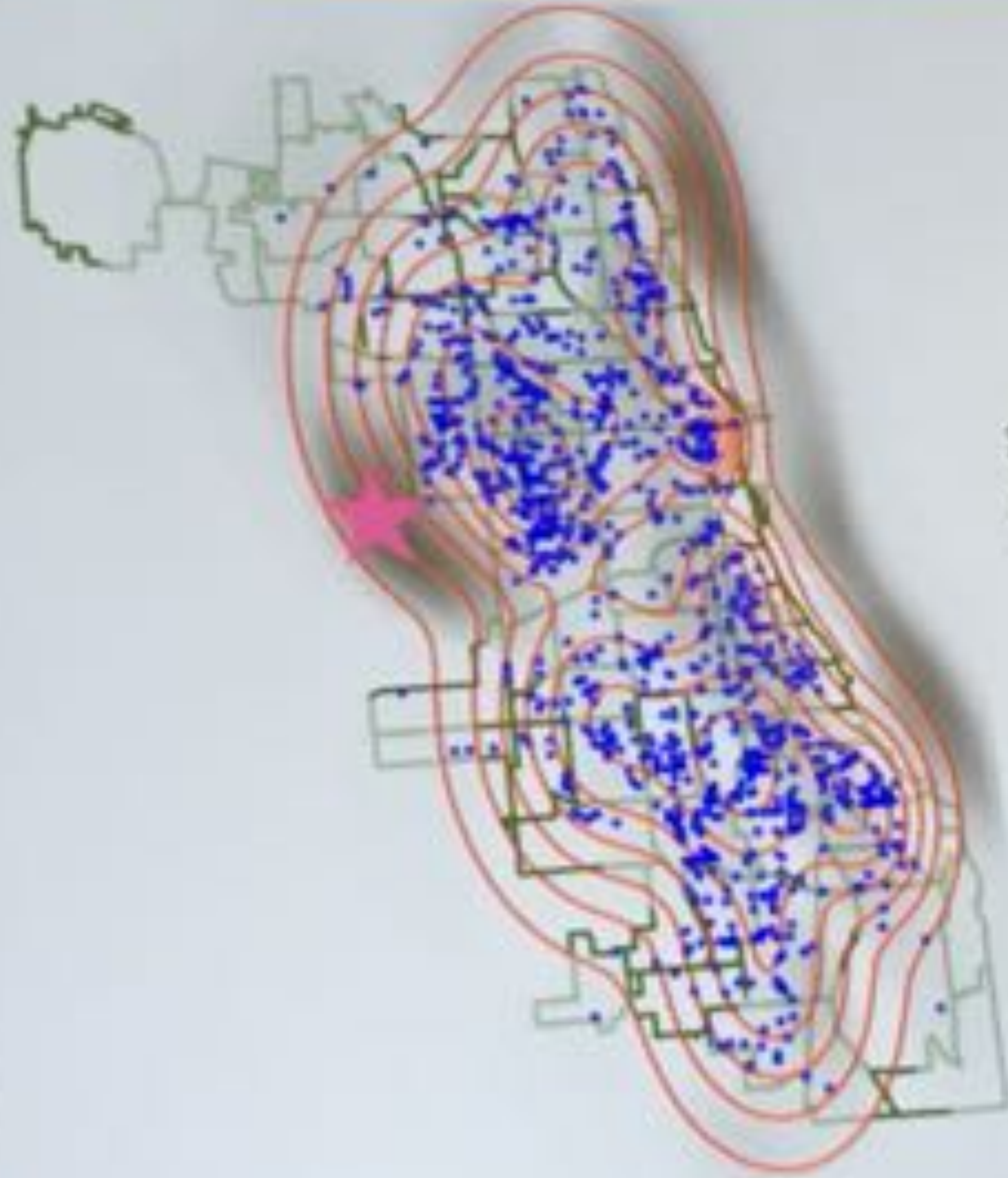
Interpretable Features: Chicago Crime



★ = optimized v.

No robbery in Lake Michigan.

Interpretable Features: Chicago Crime



Still, does not capture the left tail.

Conclusions

Proposed a new goodness-of-fit test.

1. Nonparametric. Normalizer not needed.
2. Linear-time
3. Interpretable

Poster #57 tonight

Python code: `https://github.com/wittawatj/kernel-gof`