Propositional Equivalences

the implication is ~~False~~ T unless p is T, q is F

(1,3) (15) $(\neg q \land (p \to q)) \to \neg q$ and (true if both parts are true)

p	q	$\neg q$	$p \to q$	$\neg q \land (p \to q)$	$(\neg q \land (p \to q)) \to \neg q$
T	T	F	T	F	$\boxed{T}$
T	F	T	F	F	$\boxed{T}$
F	T	F	T	F	$\boxed{T}$
F	F	T	T	T	$\boxed{T}$

tautology

implication w/ false premise is always true

?

(42) (1) truth table

p	q	C (compound prop)
T	T	F
T	F	T
F	T	T
F	F	F

(2) conjunctions

$p = T \quad q = F \quad p \land \neg q$

$p = F \quad q = T \quad \neg p \land q$

(3) disjunctions

$C = (p \land \neg q) \lor (\neg p \land q)$

? *

(45)

DNF
CNF

(1) we can express logical AND ($\land$) operat using De Morgan's laws

$p \land q \equiv \neg(\neg p \lor \neg q)$

→ $p \land q$ can be expressed as "Not (Not p OR Not q)

(51)

logical AND can be expressed using only NOT & OR express logical implication →

(2) ~~[illegible]~~ using NOT and OR

$p \to q \equiv \neg p \lor q$

→ implication can be expressed directly using NOT and OR without needing additional operators

{NOT, OR} collection is functionally complete

* (51) Compound proposition = to $p \to q$

NOR (denoted by $\downarrow$)

(1) $\neg p$ is $p \downarrow q$ is $\neg(p \lor q)$, $p \downarrow q = T$ only when both p and q = F

(2) express Not using NOR

$\neg p$ is $p \downarrow p$, bc $p \downarrow p = \neg(p \lor p) = \neg p$

(3) express OR using NOR

$p \lor q$ is $(p \downarrow p) \downarrow (q \downarrow q)$, bc $p \downarrow p = \neg p$

$q \downarrow q = \neg q$

$(p \downarrow p) \downarrow (q \downarrow q) = \neg(\neg p \lor \neg q) = p \lor q$

(4) express implication, using NOR

$p \to p$ is $\neg p \lor q$, $p \to q = (p \downarrow p) \downarrow q$

⇒ Compound proposition is logically equivalent to $p \to q$ using only NOR operator:

$(p \downarrow p) \downarrow q$