(17) Does everyone want coffee?
1st professor wants coffee Yes } coffee
2nd professor wants coffee Yes
~~No~~ 3rd Not all No

(7)

conjunction

- Jan is rich and happy Negation?
→ Jan is not rich
Jan is not happy OR

disjunction

- Carlos will bicycle or run tomorrow.
→ Carlos will Not bicycle and
Carlos will Not run tomorrow.
(Carlos neither run nor bicycle.)

disjunction

+ Mei walks or takes the bus to class
→ Mei doesn't walk and doesn't take the bus to class.

conjunction

+ Ibrahim is smart and hard working
→ Ibrahim is Not smart OR
is Not hard working.

29 ⓐ $p \to \neg p$ $2^1 = 2$ + $2^n \leftarrow var$ ↑ row

ⓑ $(p \vee \neg r) \wedge (q \vee \neg s)$ − $2^4 = 16$

ⓒ $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$ + $2^6 = 64$

ⓓ $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$ − $2^4 = 16$

⑨ tautology, prove by truth table

a) $(p \wedge q) \to p$ b) $p \to (p \vee q)$

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

a [Tautology]

$p \vee q$	$p \to (p \vee q)$
T	T
T	T
T	T
F	T

b [Tautology]

c) $\neg p \to (p \to q)$

$\neg p$	$p \to q$	$\neg p \to (p \to q)$
F	T	T
F	F	T
T	T	T
T	T	T

c [Tautology]

d) $(p \wedge q) \rightarrow (p \rightarrow q)$ e) $\neg(p \rightarrow q) \rightarrow p$

f) $\neg(p \rightarrow q) \rightarrow \neg q$

$(p \wedge q) \rightarrow (p \rightarrow q)$

T T

T

F F

T

F T

T

F T

T

d

p	q	$(p \rightarrow q)$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

e

$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
F	T
T	T
F	T
T	T

f

[tautology]

p - hypothesis q - conclusion

(11) ? a) the implication is true if conclusion is true or both of parts false or both true, in this example we see 1st 2 options (false, when p = true, q = false)

Conditional statement

b) if p = true, then q = true by def of "V"

c) in all cases hypothesis is true and conclusion is true, the only time hypothesis is false is the last case, by conclusion is true, ⇒ T T

d) if p = true, then q = true by def of ~~B~~ "∧"

e) hypothesis is alway ~~true~~ false ~~of~~ the conclusion is true

f) If the hypothesis is true, then $p \to q$ must be false. This can happen only if q is false, which is precisely what we wanted to show!

(23) ? Show that below logically equivalent?

$(p \to R) \land (q \to R)$

AND

$(p \lor q) \to R$

both must be true

1. one of $(p \to R) \land (q \to R)$ must be false for statement to be false
2. for implication to be false, the conclusion must be false R = false

So, R = false, when either p or q is ~~~~ true

$(p \underset{T}{\vee} q) \to R$

$F \qquad F$

So, if R = false, then

$\underset{T}{(p} \to \underset{F}{R)} \wedge \underset{T}{(q} \to \underset{F}{R)}$

$F \wedge F \equiv F$

p	q	R	$p \vee q$	$(p \vee q) \to R$	$(p \to R)$	$(q \to R)$
T	T					
T	F					
F	T					
F	F					

10 a) $P(x)$ "x - can speak Russian"

$Q(x)$ "x - knows computer language C++"

$P(x)$, $Q(x)$, quantifiers, logical connectives?

$D = \forall$ all students at my school

1. There's a student at my school, who speak Russian and knows C++

$\exists x \to (P(x) \wedge Q(x))$ T

2. $\exists x (P(x) \wedge \neg Q(x))$ T

3. $\forall x (P(x) \vee Q(x))$ T

4. $\neg \exists x (P(x) \overset{\vee}{\cancel{\wedge}} (Q(x)))$ -

(15) Determine truth value if D = all variables consists of all integers

a) T c) T

b) F d) F

(17)* D of $P(x)$ = ints: 0, 1, 2, 3, 4

a) $\exists x\, P(x)$ => either $P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$.

Existential quantifiers are like disjunctions
Universal quantifiers are like conjunctions

b) $\forall x\, P(x)$ => all $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$.

* c) $\exists x \neg P(x)$ => one of / either
$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

* d) $\forall x \neg P(x)$ => all
$\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

* e) $\neg \exists x\, P(x)$ => will equal "d", since we have to flip the sign and negate each propos
$\neg(P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$
=

? * f) $\neg \forall x\, P(x)$ => will equal "b", since we have to
$\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

why we don't flip sign? only when we change from $\forall x$ to $\exists x$?

(23) Translate in 2 options: D = students in my class

D = all people

a) Someone in my class can speak Hindi

1. $\exists x\, P(x) \to P(x)$, where x can speak Hindi

2. $\exists x\, (C(x) \wedge P(x))$ —

proposition
function "x is in your class"

*everyone in your class is friendly

b) 1. $\forall x\, P(x) \to P(x)$, where x is friendly

2. $\forall x\, (C(x) \to P(x))$

if in my class then is friendly

+

c) There's a person in your class who wasn't born in California

1. $\exists x\, (\neg P(x))$, where $P(x)$ "x was born in CA"

2. $\exists x\, (C(x) \wedge \neg P(x))$ —

d) A student in your class has been in a

1. $\exists x\, (P(x))$

2. $\exists x\, (C(x) \wedge P(x))$ +

e) No student in your class has taken a course in logical programming

1. $\forall x\, (\neg P(x))$

2. $\cancel{\forall}(C(x) \to \neg P(x))$

(13) * Let $M(x,y) \to$ "x has sent y an e-mail message"

$T(x,y) \to$ "x has telephoned y", where

D = all students in my class

Use quantifiers

1. Ch has never sent email to Koko

$\neg M(x,y)$
Ch Koko

2. An has never sent email or telephoned to S

$\neg(M(Ar,S) \lor T(Ar,S)) =$

$\neg M(Ar,S) \land \neg T(Ar,S)$

3. Jose has never received message from Deborah

$\neg M(\text{Deborah}, \text{Jose})$

~~Everyone~~ student in my class has sent em to Ken

4. $\forall x\, M(x, \text{ken})$

every student

5. $\forall x \neg \cancel{M}(x, \text{Nina})$ T — No one in class has telephoned Nina

OR

$\neg \exists x\, T(x, \text{Nina})$

6. $\forall x (M(x, \text{Avi}) \lor \forall x\, T(x, \text{Avi}))$

Everyone either sent em or teleph to Avi

g) $\exists x\, M(x, \forall y)$ — There's student who sent everyone email

$\exists x \forall y (y \neq x \rightarrow M(x,y))$

h) Someone who either sent email OR telephoned everyone

— $\exists x\, M(x, \forall y) \vee T(x, \forall y)$

+ $\exists x \forall y (y \neq x \rightarrow (M(x,y) \vee T(x,y)))$

↓

need to make sure they are different

i) $\exists x \exists y (y \neq x \wedge M(x,y) \wedge M(y,x))$

assert existence of 2 peopel who sent email both ways

There are 2 diff students who who have sent each other emails

*

j) $\exists x\, M(x,x) \rightarrow$ same student

There's a student who has sent himself an email

*

k) $\exists x \forall y (y \neq x \rightarrow \neg M(x,y) \wedge \neg T(x,y))$

There's a student who hasn't received email and never been called

*

l) $\forall x \forall y (y \neq x \rightarrow M(x,y) \vee T(x,y))$

— every student has received email from anyone or telephoned

*

m) $\exists x \exists y (y \neq x \rightarrow M(x,y) \leftrightarrow T(x,y))$

There're at least 2 students such that 1 has sent the other email and 2nd telephoned the 1st

n) There are 2 diff students who btw them sent email | telephoned to everyone

$\exists x \exists y \, (x \neq y \wedge \forall z((z \neq x \wedge z \neq y) \to$

all different

$(M(x,z) \vee M(y,z)$

$\vee T(x,z) \vee T(y,z)))))$

l) $\forall x \exists y \, (x \neq y \wedge (M(y,x) \vee T(y,x)))$

y = another student

m) $\exists x \exists y \, (x \neq y \wedge M(x,y) \wedge T(\cancel{y}, x))$

n) tricky since "everyone else" means 3rd var