Week 2

9 $\sum_{k=3}^{12} 4\left(\frac{1}{2}\right)^k$ as a sum starting from $k=0$

1. We're summing $4\left(\frac{1}{2}\right)^k$ starting from $\underline{3}$ up to 12
2. We need to start sum from $\underline{0}$
3. now j (helper, new index) $= k-3$
4. Now range shifts -3 ~~k~~ $j=0$, $k=9$
5. Rewrite in terms of j ~~=~~ k ~~= 3~~ $j+3$

$$\boxed{\begin{array}{l} j=k-3 \\ k=j+3 \end{array}}$$

$$\sum_{k=3}^{12} = 4\left(\frac{1}{2}\right)^k = \sum_{j=0}^{9} 4\left(\frac{1}{2}\right)^{j+3}$$

$$4\left(\frac{1}{2}\right)^{j+3} = 4\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^j \qquad a^m \cdot a^n = a^{m+n} \text{ add exponents}$$

$$= 4 \cdot \frac{1}{8} \cdot \left(\frac{1}{2}\right)^j$$

simplify

$$= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^j = \left(\frac{1}{2}\right)^1 \cdot \left(\frac{1}{2}\right)^j = \left(\frac{1}{2}\right)^{1+j} = \frac{1}{2^{j+1}}$$

$$= \frac{1}{2^{j+1}}$$

10. Find 10^{th} term in geom sequence - each term is found by * the prev term by constant ratio R. Formula:

$a_n = a_1 \cdot R^{n-1}$

a_1 = 1st term

R = common ratio

1. $a_2 = -6 \qquad a_5 = 48$

$a_2 = a_1 \cdot R^{2-1} = a_1 \cdot R^1 = -6$

$a_5 = a_1 \cdot R^{5-1} = a_1 \cdot R^4 = 48$

2. find R?

$$\frac{a_1 \cdot R^4}{a_1 \cdot R^1} = \frac{48}{-6}$$

$$R^3 = -8$$

$$\underline{R = -2}$$

3. find a_1?

$a_1 \cdot -2 = -6$

$a_1 = 3$

3. Find 10^{th} term?

$a_{10} = a_1 \cdot R^{10-1}$

$a_{10} = 3 \cdot (-2)^9 = 3 \cdot (-512) = \underline{-1536}$ – 10^{th} term of the sequence

(11) Find common ratio R?

$a_4 = 54$ and $a_7 = 1458$

~~solve for a_1~~

1. $a_n = a_1 \cdot R^{n-1}$ 3.

$a_4 = a_1 \cdot R^{4-1} = 54$

$a_1 \cdot R^3 = 54$

$a_7 = a_1 \cdot R^{7-1} = 1458$

$a_7 = a_1 \cdot R^6 = 1458$

2. divide 2nd by 1st

$\frac{\cancel{a_1} \cdot R^{\cancel{6}}}{\cancel{a_1} \cdot R^{\cancel{3}}} = \frac{1458}{54}$

$R^3 = 27$

$\underline{\underline{R = 3}}$

(12) Calcul sum of 1st 15 terms of geom sequence where $\underline{a_1 = 8}$ and $\underline{R = \frac{3}{4}}$

Formula: $S_n = a_1 \cdot \frac{1 - R^n}{1 - R}$

$S_{15} = 8 \cdot \frac{1 - \left(\frac{3}{4}\right)^{15}}{1 - \frac{3}{4}} = 8 \cdot 4 \cdot \left(1 - \left(\frac{3}{4}\right)^{15}\right) =$

$= 32 \cdot (1 - 0.0134) \approx \underline{\underline{31.57}}$

$\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

(13) $P(x) = x^5 - 4x^3 + x^2 - 7$ classify by degree, numb of terms

degree

num of terms → 4 (quartic)

(14) $(2x^4 - 3x^3 + x - 5) + (x^3 - 2x^2 + 4x + 7) =$ Simplify?

$= 2x^4 - 3x^3 + x - 5 + x^3 - 2x^2 + 4x + 7 =$

$= 2x^4 - 2x^3 - 2x^2 + 5x + 2$

(15) Multiply polinomials

$\cancel{(x^2)(x^2 + x + 1)}$ $(x^2 - x + 2)(x^2 + x + 1) =$

$= (x^2) \cdot (x^2 + x + 1) + (-x) \cdot (x^2 + x + 1) + (2) \cdot (x^2 + x + 1) =$

$= (x^4 + x^3 + x^2) + (-x^3 - x^2 - x) + (2x^2 + 2x + 2) =$

$= x^4 + \cancel{x^3} + \cancel{x^2} - \cancel{x^3} - \cancel{x^2} - x + 2x^2 + 2x + 2 =$

$= \underline{x^4 + 2x^2 + x + 2}$

(16) GCD, LCM of monomials?

GCD – greatest common diviser

LCM – least common multiple

$24x^3y^2z^5$

$36x^5y^3z^2$

coefficients: 24, 36

x terms: x^3, x^5

y terms: y^2, y^3

z terms: z^5, z^2

1. GCD of coefficients

factors of $24 = 2^3 \cdot 3$

factors of $36 = 2^2 \cdot 3^2$

GCD – product of lowest powers of common factors

$GCD(24, 36) = 3 \cdot 2^2 = \underline{12}$

2. GCD of x terms: x^3, x^5

GCD – x is raised to the lowest power $= x^3$

3. GCD of y terms: y^2

4. GCD of z terms: z^2

$12x^3y^2z^2$ – GCD

5. Find LCM - product of highest power of all prime factors

$LCM(24, 36) = 2^3 \cdot 3^2 = \underline{72}$

$LCM(x^3, x^5) = \underline{x^5}$

$LCM(y^2, y^3) = \underline{y^3}$

$LCM(z^5, z^2) = \underline{z^5}$

$\underline{72x^5y^3z^5}$ - LCM

17. Factoring Quadratics ?

$x^4 - 13x^2 + 36$ $\qquad i = x^2$

$i^2 - 13i + 36 = 0$

$i \quad -9$

$i \quad -4$

$(i-9)\cdot(i-4) = 0$

$(x^2-9)(x^2-4) = \underline{(x-3)(x+3)\cdot(x-2)(x+2)}$

18 Special Binomial products

Expand $(2x+3y)^5$ using Binomial Theorem!

Formula: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

$a = 2x$

$b = 3y$ $\quad \binom{5}{k}(2x)^{5-k}\cdot(3y)^k$ $\quad$ k goes from 0 to 5

$n = 5$

$\binom{5}{0}(2x)^5\cdot(3y)^0 = 1 \cdot 32x^5 \cdot 1 = 32x^5$

$\binom{5}{1}(2x)^4\cdot(3y)^1 = 5 \cdot 16x^4 \cdot 3y = 240x^4y$

$\binom{5}{2}(2x)^3\cdot(3y)^2 = 10 \cdot 8x^3 \cdot 9y^2 = 720x^3y^2$

$\binom{5}{3}(2x)^2(3y)^3 = 10 \cdot 4x^2 \cdot 27y^3 = 1{,}080x^2y^3$

$\binom{5}{4}\cdot(2x)^1(3y)^4 = 5 \cdot 2x \cdot 81y^4 = 810xy^4$

$\binom{5}{4}\cdot(2x)^0\cdot(3y)^5 = 1 \cdot 1 \cdot 243y^5 = 243y^5$

$(2x+3y)^5 = 32x^5 + 240x^4y + 720x^3y^2 + 1{,}080x^2y^3 + 810xy^4$
$+ 243y^5$

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(19) $6x^3 + 11x^2 - 31x + 15$ by $3x-2$

dividend $\dfrac{6x^3 + 11x^2 - 31x + 15}{3x-2}$ divisor $=$ $2x^2$

$3x-2 \;\; | \; 6x^3 + 11x^2 - 31x + 15$

1. $\dfrac{6x^3}{3x} = \underline{2x^2}$ $*(3x-2) = 6x^3 - 4x^2$

$3x-2 \;\; | \; 6x^3 + 11x^2 - 31x + 15$

$$
\begin{array}{r|l}
 & \quad 2x^2 + 5x - 7 \\
3x-2 & \;\; 6x^3 + 11x^2 - 31x + 15 \\
 & -(6x^3 - 4x^2) \\
\hline
 & \quad + 15x^2 - 31x + 15 \\
 & -(15x^2 - 10x) \\
\hline
 & \quad -21x + 15 \\
 & -(-21x + 14) \\
\hline
 & \quad 1
\end{array}
$$

: $\dfrac{15x^2}{3x} = 5x$

: $\dfrac{-21x}{3x} = -7$

$2x^2 + 5x - 7 + \dfrac{1}{3x-2}$

List all possible rational zeroes of $f(x) = 2x^4 - 5x^3 + x^2 - 4$

(2a) possible zeroes are quotients of
factors of last term -4
factors of leading coefficient 2

(1) constant term $= -4$
factors : $\pm 1, \pm 2, \pm 4$

(2) leading coeficient 2
factors : $\pm 1, \pm 2$

$\frac{p}{q} = \pm \frac{1}{1}, \pm \frac{1}{2}, \pm \frac{2}{1}, \pm \frac{1}{2}, \cancel{\frac{2}{2}}, \pm \frac{4}{1}, \pm \frac{4}{2}$

$\frac{p}{q} = \frac{\text{factors of last}}{\text{factors of first}} = \pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$