## 1.5 Nested Quantifiers

**19** (a) $\forall x \forall y((x<0) \land (y<0) \rightarrow (x+y<0))$

(b) don't get it

(c) $\forall x \forall y (x^2+y^2 \geq (x+y)^2)$

(d) $\forall x \forall y (|xy| = |x||y|)$

**21** $\forall x \exists a \exists b \exists c \exists d ((x \geq 0) \rightarrow x = a^2+b^2+c^2+d^2)$

domain $\Rightarrow$ all integers

**31** (a) $\neg \forall x \exists y \forall z\, T(x,y,z) \equiv \exists x \neg \exists y \forall z\, T(x,y,z)$

$\equiv \exists x \forall y \neg \forall z\, T(x,y,z)$

$\equiv \exists x \forall y \exists z \neg T(x,y,z)$

(b) $\neg(\forall x \exists y\, P(x,y) \lor \forall x \exists y\, Q(x,y)) \equiv$

$\equiv \neg \forall x \exists y\, P(x,y) \land \neg \forall x \exists y\, Q(x,y)$

$\equiv \exists x \forall y \neg P(x,y) \land \exists x \forall y \neg Q(x,y)$

(c) $\neg \forall x \exists y (P(x,y) \land \exists z R(x,y,z)) \equiv$

$\equiv \exists x \neg \exists y\, P(x,y) \land \exists z R(x,y,z)$

$\equiv \exists x \forall y \neg P(x,y) \land \exists z R(x,y,z)$

$\equiv \exists x \forall y (\neg P(x,y) \lor \neg \exists z R(x,y,z))$

$\equiv \exists x \forall y (\neg P(x,y) \lor \forall z \neg R(x,y,z))$

(d) $\neg \forall x \exists y (P(x,y) \rightarrow Q(x,y)) \equiv \exists x \neg \exists y (P(x,y) \rightarrow Q(x,y))$

$\equiv \exists x \forall y \neg (P(x,y) \rightarrow Q(x,y))$

$\equiv \exists x \forall y (P(x,y) \land \neg Q(x,y))$

**49** $\forall x P(x) \land \exists x Q(x)$ for all x P(x) and exists x such that Q(x)

$= \forall x \exists y (P(x) \land Q(y))$

if left is T, then right naturally should be T, they are equivalent statements

If 2nd propos is T, if $P(x)$ is T for all $x$ then 1st prop is T, if not, then $P(x)$ fails for some $x$, but for this $x$ there must be $y$ such that $P(x) \vee Q(y)$ is T, so $Q(y)$ must be T, thus 1st proposition is T