(23) ~~a) 1 1 2~~ ~~b) 2~~ c) power set $|P| = 2^n$

a) $P(\{a, b, \{a, b\}\}) = 2^3 = 8$ elements

b) $P(\{\emptyset, a, \{a\}, \{\{a\}\}\}) = 2^4 = 16$ elements

c) $P(P(\emptyset)) = 2^0 = 1$ element, but this set is $2^1 = 2$ elements

$\{\emptyset\ \{\emptyset\}\}$

(37) How many diff elements A^n has when A has m elements and n is positive number

A^n - all possible sequences (ordered tuples) of length n where each position in sequence can be filled w/ any element from set A

If $A = \{1, 2\}$ and $n = 2 \Rightarrow$ all possible sequence
$(1,1), (1,2), (2,1), (2,2)$

m - num of elements in set A, if $A = \{a, b, c\}$ $m = 3$

How many choices there for each position in the sequence?

For each position in sequence of length n choose any one of m elements from A

(43) truth set of each predicates where domain is set of integers

a) $P(x): x^2 < 3$ $\{x \in \mathbb{Z} \mid x^2 < 3\} = \{-1, 0, 1\}$

b) $Q(x): x^2 > x$ all nonneg integers, except negat

$1^2 \not> 1$ $0^2 \not> 0$ $= \mathbb{Z} - \{0, 1\} =$

$= \{\ldots, -2, -1, 2, 3, 4, \ldots\}$

c) $R(x): 2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$ not in the domain

$\{x \in \mathbb{Z} \mid 2x + 1 = 0\} = \emptyset$ empty set

(47) Describe procedure for listing all subsets of finite set

S set, n elements, subsets $\to 2^n$

$S = \{x, y, z\} \to n = 3$

000 001 010 011 100 101 110 111

$\{\}$ $\{z\}$ $\{y\}$ $\{yz\}$ $\{x\}$ $\{xz\}$ $\{xy\}$ $\{xyz\}$

Algorithm:

1. start w/ S of n elements
2. compute 2^n (number of subsets)
3. For each num k from 0 to $2^n - 1$
   convert k to binary padded to n digits
   use binary digits to decode which elem of S are in subset, output each subset

(17) if A, B and C are sets, then $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$

a) each side is a subset of the other one

$x \in \overline{A \cap B \cap C}$, then $x \notin A \cap B \cap C$, so

x is not $\notin A$ or $x \notin B$ or $x \notin C$ $=$

$x \in \overline{A}$ or $x \in \overline{B}$ or $x \in \overline{C}$ $=$

$x \in \overline{A} \cup \overline{B} \cup \overline{C}$

$- x \in \overline{A} \cup \overline{B} \cup \overline{C}$, then $x \in \overline{A}$ or $x \in \overline{B}$

$x \in \overline{C}$ $=$ $x \notin A$ or $x \notin B$ or $x \notin C$

so x cannot be in intersection of A B C

Since, $x \in \overline{A \cap B \cap C}$, we conclude

$x \in \overline{A \cap B \cap C}$

b) build membership table

(37) If $m = 3$ and $n = 2$
position length

For each position in sequence of length n you can choose any one of the m elements from A

1 pos — 3 element
2 pos — 3 elements

Total num of sequences $= m^n : A^n$

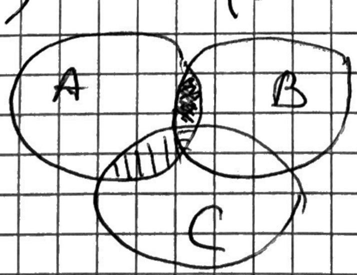
(17) 8)

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A} \cup \bar{B} \cup \bar{C}$
1	1	1	1	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	0	1	0	1	0	1	0	1
1	0	0	0	1	0	1	1	1
0	1	1	0	1	1	0	0	1
0	1	0	0	1	1	0	1	1
0	0	1	0	1	1	1	0	1
0	0	0	0	1	1	1	1	1

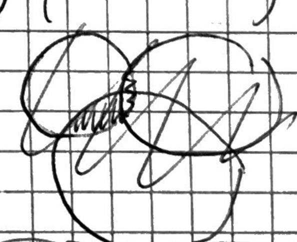
? * Venn Diagram ?

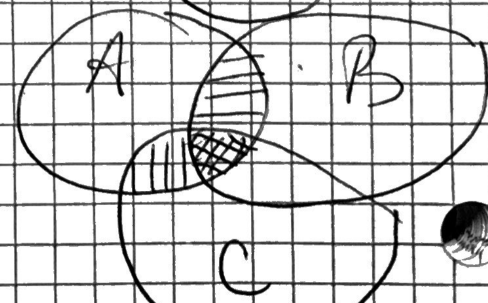
(27) a) $A \cap (B - C)$

$\cancel{A}\cap A$



b) $(A \cap B) \cup (A \cap C)$





c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$

(29) a) $A \cup B = A \Rightarrow B \subseteq A$
set B is a subset of set A
every elem of B is an elem of A

(b) $A - B = A \Rightarrow$ ~~A⊂B~~ $A \cap B = \emptyset$
none of elements A are in B
if there were A - B wouldn't be A

c) $A \cap B = A \Rightarrow A \subseteq B$
all elem of A are in B

d) $A \cap B = B \cap A \Rightarrow$ equality holds

e) $A - B = B - A$
every element of A-B must be in ~~B~~ A
and every element of B-A must not be in ~~B~~ A. No item can be in A and not A at the same time, there are no elements in both A-B and B-A. For sets to be equal they have to be $\emptyset \Rightarrow A = B$
?

(51) Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ for every positive int i

a) $A_i = \{-i, -i+1, \dots, -1, 0, 1, \dots, i-1, i\}$
as i increase, set gets larger
$A_1 \subset A_2 \subset A_3 \dots$ all sets are subsets of set int and all int included in
$\bigcup_{i=1}^{\infty} A_i = \mathbb{Z}$ because A_i is a subs

of each of others
$\bigcap_{i=1}^{\infty} A_i = A_1 = \{-1, 0, 1\}$

b) $A_i = \{-i, i\}$
all sets are subset of int and every nonzero int is in one of sets, so $\bigcup_{i=1}^{\infty} A_i = \mathbb{Z} - \{0\}$, no element is common to all sets

c) $A_i = [-i, i]$, that is, the set of real numbers x w/ $-i \leq x \leq i$
similar to (a) but we work w/ real nums: $\bigcup_{i=1}^{\infty} A_i = \mathbb{R}$, and $\bigcap_{i=1}^{\infty} A_i = A_1 = [-1, 1]$

d) $A_i = [i, \infty)$ that is, the set of real numbers x w/ $x \geq i$
sets get smaller as i increase:
$\ldots \subset A_3 \subset A_2 \subset A_1$
A_1 includes all the others
? $\bigcup_{i=1}^{\infty} A_i = A_1 = [1, \infty)$

*41) Suppose A, B, C are sets that $A \oplus C = B \oplus C$. Must it be the case that $A = B$?
Yes, to show $A = B$, we need $x \in A$ implies $x \in B$ and conversely.
$A \oplus C = B \oplus C$ and let $x \in A$ be given. 2 cases to consider
1) $x \in C$, ~~If~~ then we can't conclude

$x \notin A \oplus C$. Therefore $x \notin B \oplus C$
Now if x not in B, then $x \in B$ as desired

2. $x \notin C$, then $x \in A \oplus C$, therefore $x \in B \oplus C$ as well.
if x is not in B, then x is not $B \oplus C$ (since $x \notin C$ by assumption)
we conclude that $x \in B$, and proof is complete