1.7
1.8

(25) $1, 2, \dots 2n$

n is odd int

Pick any 2 of nums, j and k, write $|j-k|$
and erase j and k

1. Sum

$$1+2+\dots+\underbrace{n+(n+1)}_{2n+1}+\dots+(2n-1)+(2n)$$

$2n-1$

$2n+1$

n pairs, each $(2n+1) \rightarrow \underbrace{n}_{odd}\underbrace{(2n+1)}_{odd} \rightarrow$ odd

(1) if $j, k \Rightarrow$ even

$-(j+k)+|j-k| \rightarrow$ parity didnt change

(2) $-(j+k)+|j-k| \rightarrow$ parity didnt change

odd + odd = even ↘ odd − odd = even

if j and k are odd

> parity doesnt change at the end of each operation

> we reduce number of numbers by 1 every time, until we have only a single num, it has the same parity, that original sum, which was odd, so it should also be odd.

n = is sequence of digits

(27) $d_1 d_2 d_3 \dots d_{m-1} d_m$

$= 19345 \dots$

Let $k = d_1d_2d_3 \dots d_{m-1}$ → Then $n = 10k + \underset{\text{last digit}}{d_m}$

d_m → any digit from 0 to 9

$n^4 = (10k+0)^4 = 10000k^4 = 10000k^4 + 0$

$n^4 = (10k+\boxed{1})^4 = 10000k^4 + ??\cdot k^3 + ??\cdot k^2 + ??\cdot k + 1$

$n^4 = (10k+\boxed{2})^4 = 10000k^4 + ??\cdot k^3 + ??k^2 + ??\cdot k + 16$

$n^4 = (10k+\underline{3})^4 = 10000k^4 + ??k^3 + ??\cdot k^2 + ??\cdot k + 81$

$n^4 = (10k+\underline{4})^4 = 10000k^4 + ??k^3 + ??k^2 + ??\cdot k + 256$

$n^4 = (10k+\underline{5})^4 = 10000k^4 + ??k^3 + ??k^2 + ??\cdot k + 625$

$n^4 = (10k+\underline{6})^4 = 10000k^4 + ??k^3 + ??k^2 + ??\cdot k + 1296$

$n^4 = (10k+\underline{7})^4 = 10000k^4 + ??k^3 + ??k^2 + ??\cdot k + 2401$

$n^4 = (10k+\underline{8})^4 = 10000k^4 + ??k^3 + ??k^2 + ??\cdot k + 4096$

$n^4 = (10k+\underline{9})^4 = 10000k^4 + ??k^3 + ??k^2 + 6561$

expansion

$(t+w)^4 = t^4 + 4t^3w^1 + 6t^2w^2 + 4t^1w^3 + \underline{w^4}$

↓ affects the last digit-n^4

$t = 10k$

$w = 0$ to 9

$t^4 = 10000k^4$

$\left.\begin{array}{l} 4t^3w^1 = \times 1000 \\ 6t^2w^2 = \times 100 \\ 4t^1w^3 = \times 10 \end{array}\right\}$ don't affect last digit

(35) $\left[\frac{p}{q} ; \frac{R}{S}\right]$ $\frac{R}{S} - \frac{p}{q} = d$

$\underset{\text{rational}}{\frac{p}{q}} + \underset{\text{irrational}}{\overset{\text{rational}}{x} \cdot \sqrt{2}} < \frac{R}{S} \Rightarrow \frac{p}{q} + x \cdot \sqrt{2} \Rightarrow$ irrational

rat + irr = irr

$x \cdot \sqrt{2}$ → always exists

$x \cdot \sqrt{2} < \frac{1}{5} - \boxed{d}$ the gap size

$x \cdot \sqrt{2} < d \quad , \quad d > 0$

$0 < x < \frac{d}{\sqrt{2}} \quad , \quad d > 0$