① $\log_2\left(\frac{8\sqrt{2}}{16}\right) + \log_2(32) - 2\log_2(4)$

simplify the fraction

- $\log_2\left(\frac{8\sqrt{2}}{16}\right)$

using identity of log → $\boxed{\log_b(b^x) = x}$ (base, exponent, argument)

$\cancel{\log_2 8\sqrt{2} - \log_2 16}$

$\frac{8\sqrt{2}}{16} = \frac{8\cdot 2^{\frac{1}{2}}}{16} = \frac{1}{2}\cdot 2^{\frac{1}{2}} = 2^{-1}\cdot 2^{\frac{1}{2}}$

combine exponents:

$-1 + \frac{1}{2} = \frac{-2+1}{2} = -\frac{1}{2} = 2^{-\frac{1}{2}}$

1st term becomes:

$\log_2\left(2^{-\frac{1}{2}}\right) = -\frac{1}{2}$

$\log_2(\underset{2^5}{32}) = 5$

- $2\log_2(\underset{2^2}{4}) = 2\log_2(2^2) = 2\cdot 2 = 4$

$-\frac{1}{2} + 5 - 4 = \frac{1}{2}$

Well known logarithmic formulas

② $\log_3(x-1) + \log_3(x+1) = 2$

multiply

$\log_3((x-1)\cdot(x+1)) = 2$

$\log_3(x^2-1) = 2$

convert from logarithmic to exponential form

$x^2 - 1 = 3^2 = 9$

- $x^2 - 1 = 9$

$x^2 = 10$

$x = \pm\sqrt{10}$,

біо

$x - 1 > 0$ + $x + 1 > 0$ +

$x - 1 > 0$ — $-\sqrt{10}$ $\quad$ $x + 1 > 0$ — $-\sqrt{10}$

Answer: $+\sqrt{10}$

(3) $f(t) = \text{init inv} \cdot (1 + \%)^{t}$ $\quad$ quarterly

A

$20{,}000 = 10{,}000 \cdot \left(1 + \frac{0.06}{4}\right)^{4t}$ $\quad$ $| : 10{,}000$

$2 = \left(1 + \frac{0.06}{4}\right)^{4t}$

$2 = (\cancel{1.006})$

$2 = 1.015^{4t}$ $\quad$ | take ln

$\ln(2) = \ln(1.015)^{4t}$

$\ln(2) = 4t \ln(1.015)$

$t = \frac{\ln(2)}{4 \ln(1.015)} \approx 11.64$ years

---

(4) Radioactive substance decays

$N(t) = N_0 e^{-kt}$

N_0 - initial amount

k - decay constant

t - time

find k if $t = 5$ years of half life

$N(t_{\frac{1}{2}}) = \frac{N_0}{2}$

$\frac{N_0}{2} = N_0 e^{-kt_{\frac{1}{2}}}$

$\frac{1}{2} = e^{-k \cdot 5}$

$\ln\left(\frac{1}{2}\right) = -5k$

$-0.6931 = -5k$

$k = 0.1386$ per year

(5)

100 grams decays to 70 grams in 3 h

What's the t, it will decay to 20 grams?

① $N(t) = N_0 e^{-kt}$

$N(3) = 100 e^{-k \cdot 3}$ $\quad | : 100$

70

$0.7 = e^{-k \cdot 3}$

take nat log

$\ln(0.7) = -3k$

$-0.3567 = -3k$

for 20 g how many h?

$k = 0.1189$

② $20 = 100 e^{-0.1189 \cdot t}$ $\quad | : 100$

$0.2 = e^{-0.1189t}$ $\quad |$ take ln

$\ln(0.2) = -0.1189t$

$-1.6094 = -0.1189t$

$t = 13.54$ hours

⑥ Find unit vector $\bar{u}$ $\quad A(1,2,3)$

$B(4,6,9)$

① Find vector $\overline{AB}$

$(4-1; 6-2, 9-3) = (3, 4, 6)$

② Find magnitude of $\vec{AB}$

$|\vec{AB}| = \sqrt{3^2 + 4^2 + 6^2} = \sqrt{9+16+36} = \sqrt{61} \approx 7.81$

③ Unit vector $\hat{u}$: $\frac{\vec{AB}}{|AB|} = \left(\frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}, \frac{9}{\sqrt{61}}\right)$

⑦ Express vector $\vec{b} = 7\hat{i} - 2\hat{j} + 4\hat{k}$ in matrix form, find magnitude?

matrix form $\begin{bmatrix} 7 \\ -2 \\ 4 \end{bmatrix}$

magnitude $|\vec{b}| = \sqrt{7^2 + (-2)^2 + 4^2} = \sqrt{69} \approx 8.31$

(8) Adding scalling vectors

$\vec{a} = (2, -1, 3)$
$\vec{b} = (-1, 4, 2)$
compute $3\vec{a} - 2\vec{b}$

(1) $3\vec{a} = (6, -3, 9)$

$2\vec{b} = (-2, 8, 4)$

$(6, -3, 9) - (-2, 8, 4) = ((6-(-2)), (-3-8), (9-4))$

$= (8, -11, 5)$

(9) Dot Product:

find the angle btw products $\vec{p} \cdot \vec{q}$ (dot product)

$\vec{p} = (1, 2, 3)$

$\vec{q} = (4, -5, 6)$

(1) $(1 \cdot 4 + 2(-5) + 3 \cdot 6) = (4 + (-10) + 18) = 12$

(2) magnitude $= |\vec{pq}|$

$|\vec{p}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{q}| = \sqrt{4^2 + (-5)^2 + 6^2} = \sqrt{77}$

(3) Compute the angle θ

$$\cos\theta = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| \cdot |\vec{q}|} = \frac{12}{\sqrt{14} \cdot \sqrt{77}} = \frac{12}{\sqrt{1078}} = 0.3647$$

$\theta = \arccos(0.3647) \approx 68.58°$?

The angle btw $\vec{p}$ and $\vec{q}$ is $\approx 68.58°$

(10) Are orthogonal?

$\vec{u} = (2, -1, 4)$ $\quad \vec{v} = (-8, 4, -16)$

$\vec{u} \cdot \vec{v} = (2(-8) + (-1)4 + 4(-16)) = -84$

$\vec{u} \cdot \vec{v} \neq 0 \rightarrow$ vectors are not orthogonal

Matrices

(11) Sub $A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 4 & 5 \\ -2 & 1 \end{bmatrix}$

Compute $2A - 3B$

① $2A$

$A = \begin{bmatrix} 4 & -2 \\ 0 & 6 \end{bmatrix}$

$+3B$

$B = \begin{bmatrix} 12 & 15 \\ -6 & 3 \end{bmatrix}$

② $2A - 3B$

$$\begin{bmatrix} 4-12 & -2-15 \\ 0-(-6) & 6-3 \end{bmatrix}$$

$$\begin{bmatrix} -8 & \cancel{13}\ -17 \\ +6 & +3 \end{bmatrix}$$

(12) Multiply

$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ $E = CD$

$$E = \begin{bmatrix} (1\cdot 5)+(2\cdot 7) & (1\cdot 6)+(2\cdot 8) \\ \cancel{(3\cdot 6)+(4\cdot 8)} & \\ (3\cdot 5)+(4\cdot 7) & (3\cdot 6)+(4\cdot 8) \end{bmatrix}$$

$$\begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$\begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

(13) Row operations

$$\begin{cases} x+y+z=6 \\ 2x+4y+3z=14 \\ -3x+2y-2z=-10 \end{cases}$$

① Augmented matrix

α th

$$X\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 14 \\ -3 & 2 & -2 & -10 \end{array}\right]$$

(1) Use R_1 to eliminate x from R_2 and R_3

$R_2 = R_2 - 2R_1$

$R_2 = 2 - 2(1) = \underline{0}$

$-1 - 2(1) = \underline{-3}$

$3 - 2(1) = \underline{1}$

$14 - 2(6) = \underline{2}$

(2) $R_3 = R_3 + 3R_1$

$R_3 = -3 + 3(1) = 0$

$2 + 3(1) = 5$

$-2 + 3(1) = 1$

$-10 + 3(6) = 8$

(3)
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & 2 \\ 0 & 5 & 1 & 8 \end{array}\right]$$

(4) Use R_2 to eliminate y from R_3

$R_2 \cdot \frac{5}{-3} + R_3$

$\rightarrow R_3 = R_3 + \left(\frac{5}{-3} R_2\right)$

$R_3[2] = 5 + \left(\frac{5}{-3} \cdot (-3)\right) = 10$

$1 + \left(\frac{5}{-3} \cdot (1)\right) = -\frac{2}{3}$

$\rightarrow R_3[4] = 8 + -\frac{10}{3} = \frac{24}{3} - \frac{10}{3}$

$R_3 = [0, 0, -\frac{2}{3} \mid \frac{14}{3}] = \frac{14}{3}$

(5) Solve for z

$-\frac{2}{3}z = \frac{14}{3}$

$z = \frac{14}{3} \cdot \left(-\frac{2}{3}\right)$

$z = \frac{14}{3} \cdot \left(-\frac{3}{2}\right)$

$\underline{z = -7}$

(6) Sub z in R_2

$-3y + 1(-7) = 2$

$-3y - 7 = 2$

$-3y = 9$

$\underline{y = -3}$

(7) Sub y and z in R_1

$x + (-3) + (-7) = 6$

$x - 10 = 6$

$\underline{x = 16}$

(14) Reduced Row Echelon form

$$B = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

① eliminate -1 ~~R_3~~ in R_1 by using R_3

$R_1 = R_1 + R_3 \times 1$

$-1 + 1(1) = -1 + 1 = 0$

$0 + (-1) = -1$

$R_1[4] = R_1[4] + 1 \cdot R_1[4]$

$0 + 1(-1) = -1$

$R_1 = [1, 2, 0, -1]$

② Eliminate 3 in $R_{23}[3]$

$R_2 = R_2 - 3 \cdot R_3$

$= 3 - 3(1) = 0$

$= 5 - 3(-1) = 8$

$R_2 = [0, 1, 0, 8]$

③ Eliminate 2 in $R_1[2]$

$R_1 = R_1 - 2R_2$

$= 2 - 2(1) = 0$

$= -1 - 2(8) = -17$

$R_1 = [1, 0, 0, -17]$

New matrix is in Reduced Row-Echelon form

$$\begin{bmatrix} 1 & 0 & 0 & -17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

(15) Matrix Inverse

RREF Relationship

$A = \begin{matrix} 2 & 1 \\ 5 & 3 \end{matrix}$, find A^{-1} using Row operations

$[A \mid I]$

$$\left[\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{array}\right] \Rightarrow \text{A into I}$$

① $a_{11} = 1$

$R_1 = R_1$ ~~7~~ $/ 2$

$R_1 = \left[1, \frac{1}{2} \mid \frac{1}{2}, 0\right]$

② eliminate a_{21}

$R_2 = R_2 - 5R_1$

$5 - 5(1) = 0$

$3 - 5\left(\frac{1}{2}\right) = \frac{1}{2}$

$0 - 5\left(\frac{1}{2}\right) = 0 - \frac{5}{2} = -\frac{5}{2}$

$1 - 5(0) = 1$

R_2 becomes $[0, \frac{1}{2} \mid -\frac{5}{2}, 1]$?

(3) $a_{22} = 1$

$R_2 = R_2 \times 2$

$R_2 = [0, 1 \mid -5, 2]$

elemenate a_{12}

$R_1 = R_1 - (\frac{1}{2} R_2)$

$1 - \frac{1}{2}(0) = 1$

$\frac{1}{2} - \frac{1}{2}(1) = 0$

$\frac{1}{2} - \frac{1}{2}(-5) = 3$

$0 - \frac{1}{2}(2) = 0 - 1 = -1 \implies$

$R_1 = [1, 0 \mid 3, -1]$

Augmented matrix

$$\left[\begin{array}{cc|cc} 1 & 0 & 3 & -1 \\ 0 & 1 & -5 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$