Week 9

Problem 1.9

3 shirts × 4 pairs of pants = 12 outfits

Problem 1.16

25 balls numbered from 1 to 25 are in bin
4 balls drawn one at a time
winning combinations consist of 4 numbers in the order selected

a) $25 \times 24 \times 23 \times 22 = 303,600$

b) $25 \times 25 \times 25 \times 25 = 390,625$

Problem 1.11

7 characters none = 'O'
1st → digit from 0 to 9
2nd → letter
remaining 5 either digit or letter but not letter 'O'

$$\downarrow\ \ \downarrow\ \ \underbrace{\_\ \_\ \_\ \_\ \_}_{25+10}$$

0 to 9 $26 - 1$ ('O') $= 25$

$10 \times 25 \times 35 \times 35 \times 35 \times 35 \times 35 = 13,130,468,750$

Problem 2.2

5 letters, every word no more than 3 letters

$5 \times 1 = 5$ (for 1 letter words)

$5 \times 5 = 25$ (for 2 letter words)

for 1st letter   for 2nd letter

$5 \times 5 \times 5 = 125$ for 3 letter words

$5 + 25 + 125 = 155$ words possible on Mumble

Problem 2.7

4 sons 3 daughters

7 chairs

at least 2 boys are next to each other

BB ~~BB~~ G BB

BBBGGGB

BBBBGGG

BGGGBBB

GGGBBBB

GBBGGBB

complementary counting $4! \times 3! = 144$

factorial denoted as $n!$, the product of all positive integers from 1 up to n

$4! = 4 \times 3 \times 2 \times 1 = 24$

$3! = 3 \times 2 \times 1 = 6$

total num to arrange all 7 (no restrictions)

$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5{,}040$

1. Count where no 2 boys sit next to each other BGBGBGB
2. ~~Count where~~ w/ restrictions

   $3! \times 4! = 144$
3. Complimentary counting

   $\underset{7!}{5{,}040} - \underset{3! \times 4!}{144} = 4{,}896$ ways to sit 7 chil so that at least 2 boys sit next to each other

Seat 1: A $\underline{\ }\ \underline{\ }$ B $\underline{\ }\ \underline{\ }$ C $\underline{\ }\ \underline{\ }$ 3 people
2 3 2 3 2 3

Seat 1, 2: AB$\underline{\ }$ AC$\underline{\ }$ BA$\underline{\ }$ BC$\underline{\ }$ CA$\underline{\ }$ CB$\underline{\ }$
3 3

Seats 1, 2, 3

ABC	ACB	BAC	BCA	CAB	CBA
1	2	3	4	5	6

$\frac{3}{1} \times \frac{2}{2} \times \frac{1}{3}$ ~~###~~ people / seat $\Rightarrow 3! = 6$

$\frac{5}{1} \times \frac{4}{2} \times \frac{3}{3} \times \frac{2}{4} \times \frac{1}{5}$ people / seats $\Rightarrow 5! = 120$ 5 people

If you start w/ that num and multiply to numb 1 less than that all the way to 1

factorial operation

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Problem 2.9 1. Use combinatorial approach

$x_1, x_2, x_3 \ldots x_7$

1 2 3 4 5 $\Rightarrow 5^7$ choices digits

adjacent $\rightarrow$ elements that are next to each other

$x_1 \neq x_2$ x_1 any int from 1 to 5

$x_2 \neq x_3$ from x_2 to x_7 must be diff

$x_3 \neq x_4$ from preceding x_{i-1}

$5 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$x_5 \neq x_6$ $5 \times 4^6 = 20,480$

$x_6 \neq x_7$

Q.14

20 members

3 officers : President, Vice, Treasurer

Ali hates Brenda

If Ali refuses to serve as an officer w/ Brenda

President VIP Treasurer
1. $20 \times 19 \times 18 =$ – no restrictions, total number of ways
6,840

2. Ali ← constructive counting

President Treasurer

VIP

18 left

Brenda

3 ways to choose 2 positions out of 3 for Ali and Brenda

$3 \times 2 \times 18 =$ offices
108

for each choice of 2 positions, there are 2 ways to assign Ali and Brenda to positions

3 total position

2 each pos taken by Brenda / Ali

3 ways to choose 2 pos out of 3 pos available

Ali Brenda
OR
Brenda Ali

3. $6{,}840 - 108 = 6{,}732$ Ali and Brenda are not both officers

3.4. distinct arrangements of PaPa ? $= 4!$ → if all 4 letters were unique
1 2 3 4 $4 \times 3 \times 2 \times 1$

2 P's $= 2! = 2 \times 1$

2 A's $= 2! = 2 \times 1$

number of distinct permutations

$$\frac{n!}{k_1! \times k_2! \times k_3! \ldots}$$

$$\frac{4!}{2! \times 2!} = 6.$$

3.8 *

pairs of vertices $\frac{n(n-1)}{2}$
However, n of these pairs correspond to edges of polygon rather than diagonals so we have to subtr these from our count, thus the num of diagonals in a convex polygon w/ n sides is $\frac{n(n-1)}{2} - n$

4.3

48 balls → 1 to 48

6 are chosen

when we care about order

$$\frac{48 \quad 47 \quad 46 \quad 45 \quad 44 \quad 43}{6!} = 12,271,512$$

we don't care about order so have to correct overcounting

5.4

12 players

Bob
Yogi

how many starting lineups of 5 players can coach Yellsalot make

$\overset{1}{\text{Bob}}$ $\overset{10}{\text{no Yogi}}$, rest of players $= \binom{10}{4}$

$\overset{1}{\text{Yogi}}$ $\overset{10}{\text{no Bob}}$, rest of people $= \binom{10}{4}$

$\frac{10}{5}$ rest of players no Yoy, no Bob

$\frac{10}{4} + \frac{10}{4} + \frac{10}{5} = \cancel{6}72$

OR

complimentary counting

$\frac{12}{5} - \frac{10}{3} = 672$

OR

no restrictions

$\frac{12!}{5! \times (12-5)!} = \frac{12!}{5! \times 7!} = 792$

no restrictions

Combination formula

$\frac{n!}{r!(n-r)}$

$\frac{10}{3} = \frac{10!}{3!(10-3)} = 120$

$792 - 120 = 672$

14-6 Coefficient of the term of $(x+2y^2)^6$ w/ y^8 in it?

Binomial theorem

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot a^{n-k} \cdot b^k$

where $\binom{n}{k} \longrightarrow$ binomial coeffic

$\frac{n!}{k!(n-k)!}$

$a = x$
$b = 2y^2$
$n = 6$

$T_k = \binom{6}{k} \cdot x^{6-k} \cdot (2y^2)^k$

we need exponent of y to be 8

since $(2y^2)^k = 2^k \cdot y^{2k}$

$$\binom{6}{4} x^2 (2y^2)^4 = 240 x^2 y^8 = 240.$$