

modular arithmetic
congruences

hash tables
hashing
has been used in DS

Chinese Theorem

Ch 4.2 ~~7~~, ~~12~~, 27, 37, 43

10 11 12 13 14 15
A B C D E F

(7) ¹⁴
(1) (80E)₁₆
_{8 4 2 1}
1000 0000 1110

(2) (ABBA)₁₆
1010 1011 1011 1010

(3) (135AB)₁₆
0001 0011 0101 1010 1011

(4) (DEFACED)₁₆
1101 1110 1111 1010 1100 1110 1101

(19) ↑ Each octal integer corresponds to
a block of 3 binary digits:

Ex: (12345)₁₀

$$12345 = 8 \cdot 1543 + 1$$

$$1543 = 8 \cdot 192 + 7$$

$$192 = 8 \cdot 24 + 0$$

$$24 = 8 \cdot 3 + 0$$

$$3 = 8 \cdot 0 + 3$$

(3007)₈

$(3007)_8$

~~0000 0000 0000 0000~~

$\begin{array}{r} 011\ 000\ 000\ 111 \\ \hline 421 \end{array}$

Each hexadecimal corresponds to a block of 4 binary digits

$0110\ 0000\ 0111$

$\begin{array}{r} 8421 \\ (607)_{16} \\ 2003 \end{array}$

(27) $3 \bmod 99$ (Fast Exponentiation) alg. 5
initially set $x=1$ and $\text{power} = 3 \bmod 99 = 3$

repeatedly square the base

(1) $2003_{10} = 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^5 + 2^1 + 2^0$

(2) calculate powers of 3 modulo 99, $\Rightarrow 3^{2^k} \bmod 99$

$$3^1 \bmod 99 = 3$$

$$3^2 \bmod 99 = 3^2 = 9$$

$$3^4 \bmod 99 = 81$$

$$3^8 \bmod 99 = 81^2 = 6561 \bmod 99 = 27$$

$$3^{16} \bmod 99 = 27^2 = 729 \bmod 99 = 36$$

$$3^{32} \bmod 99 = 36^2 = 1296 \bmod 99 = 6$$

$$3^{64} \bmod 99 = 6^2 = 36$$

$$3^{128} \bmod 99 = 36^2 = 6$$

$$3^{256} \bmod 99 = 6^2 = 36$$

$$3^{512} \bmod 99 = 36^2 = 6$$

$$3^{1024} \bmod 99 = 6^2 = 36$$

? Addition of integers

- (37)
- (1) add the numbers directly
 - (2) Check for carry out
 - (3) Check the result
 - (4) Determine overflow
 - (5) Interpret the result

$$A \rightarrow 5 \rightarrow \begin{matrix} 8 & 4 & 2 & 1 \\ 0 & 1 & 0 & 1 \end{matrix}$$

$$B \rightarrow 3 \rightarrow 0011$$

$$\begin{array}{r} + 0101 \\ 0011 \\ \hline \end{array}$$

$$1000 \rightarrow -7 \text{ (in a 4 bit ones complement system)}$$

$$3^{11} \Rightarrow 11 = (1011)_2 \Rightarrow 3^{11} = 3^8 3^2 3^1$$

$$3^2 = 9$$

$$3^4 = 9^2 = 81$$

$$3^8 = (81)^2 = 6561$$

$$3^{11} = 3^8 \cdot 3^2 \cdot 3^1 = 6561 \cdot 9 \cdot 3 = 177,147$$

(43) for 2's complement expansions