(c) ~~xx~~ x^2 is odd

(35) 2 and 3 - rational nums $\frac{2}{1}$ $\frac{3}{1}$

Rational nums are nums that can be expressed as fractions, where numerator, denominator are both integers

- Irrational nums are nums that can't be expressed as fractions.

$a = 2$ $b = 3$ $a < b$

num in between $(a+b)/2 = (3+2)/2$ - won't be a whole num, so it can't be expressed as fraction → therefore irrational number

(25) *Parity

If we start w/ even and subtract the odd, we get an odd num. If we start w/ odd and subtract another odd, we get an even.

If n is an odd num (1, 3, 5) we can take odd num of steps in our game. If n is even num (2, 4, 6) we're allowed to take even num of steps.

No matter we are allowed to take odd or even we end up w/ odd num

In all we get n sums of 2n+1, so total sum is n(2+1). If n is odd, it is product of two odd nums and is odd.

(27) Conjecture - the final decimal digit of 4th power of any integer, can only be 0, 1, 5, 6

Proof:

$(10k+0)^4 = 10000k^4 = 100000k^4 + 0$

$(10k+1)^4 = 10000k^4 + ?? \cdot k^3 + ? \cdot k^2 + ? \cdot k + 1$

$(10k+2)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 16$

$(10k+3)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 81$

$(10k+4)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 256$

$(10k+5)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 625$

$(10k+6)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 1296$

$(10k+7)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 2401$

$(10k+8)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 4096$

$(10k+9)^4 = 10000k^4 + ?? \cdot k^3 + ? k^2 + ? \cdot k + 6561$

the corresponding term has no effect on ones digit of the answer. The ones digits are: 0, 1, 6, 1, 6, 5, 6, 1, 6, 1, so it's always 0, 1, 5, 6

37 a) $\Sigma x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

~~$x_2 y_1 + x_2 y_2 = x_2 y_1 = x_j y_j = (x_2 - x_2)(y_2 - y_j)$~~ ?

swap $x_i y_j + x_j y_i$ to the sum and subtracted $x_i y_i + x_j y_j$. The net effect is to have added $x_i y_j + x_j y_i - x_i y_i - x_j y_j = (x_j - x_i)(y_i - y_j) \Rightarrow$ nonnegative by order of assumption

8) $y_i < y_j$ for some $i < j$. When swap y_i and y_j we increase the sum by $x_i y_j + x_j y_i - x_i y_j - x_j y_j = (x_j - x_i)(y_i - y_j)$, which is nonpositive by our ordering assumptions.