Let the function describing the density of particles be $\rho(x, y, z, t)$. The density per unit volume per unit time is then proportional to the probability to find a particle given by equation $P(x, y, z, t)$. Thus, ρ and P obey the same equation. We find this equation by focusing on an individual walker. We also assume that the walker is confined to a cubic lattice making steps along the edges of the lattice.

The probability to find the walker at the site (i, j, k) at time n is $P(i, j, k, n)$. The walker has six neighboring nodes that it can arrive from to the site (i, j, k) each of which have a 1/6 chance to make a step into the (i, j, k) node.

Therefore, the following equation is derived:

$$P(i,j,k,n) = \frac{1}{6}[P(i+1,j,k,n-1) \ + P(i-1,j,k,n-1) \ + \ P(i,j+1,k,n-1) \\ + P(i,j-1.k.n-1) + \ P(i,j,k+1,n-1) \ + \ P(i,j,k-1,n-1)]$$

Rearranging the equation, we get:

$$P(i,j,k,n) - P(i,j,k,n-1) \\ = \frac{1}{6}\{[P(i+1,j,k,n-1) - 2P(i,j,k,n-1) + P(i-1,j,k,n-1)] \\ + [\ P(i,j+1,k,n-1) - 2P(i,j,k,n-1) + P(i,j-1.k.n-1)] \\ + [\ P(i,j,k+1,n-1) - 2P(i,j,k,n-1) + P(i,j,k-1,n-1)]\}$$

We then notice that the equation is in the form of a second order symmetric derivative, and, upon taking the limit, we arrive to the diffusion equation.

$$\frac{\partial P(x,y,z,t)}{\partial t} = D\nabla^2 P(x,y,z,t)$$