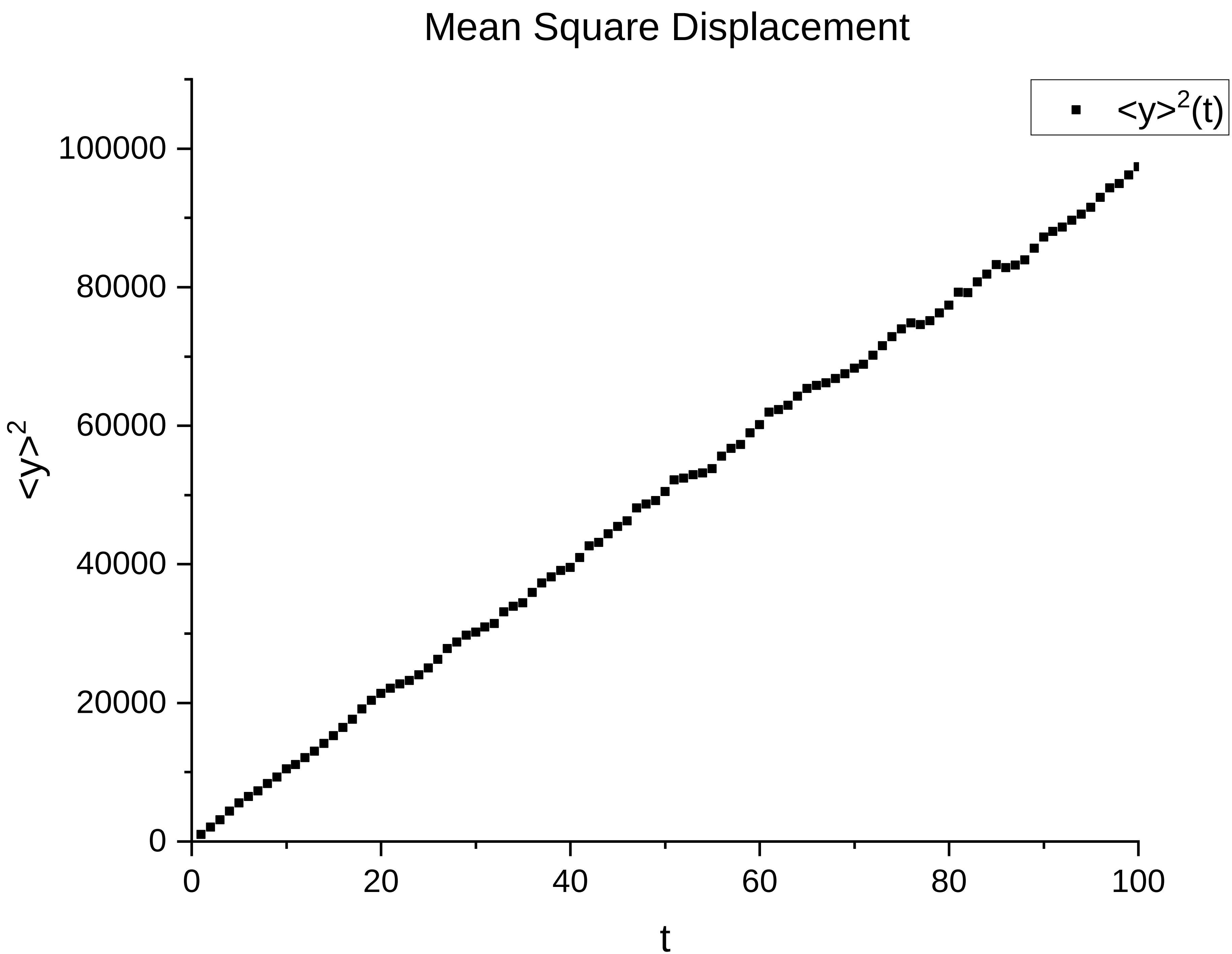
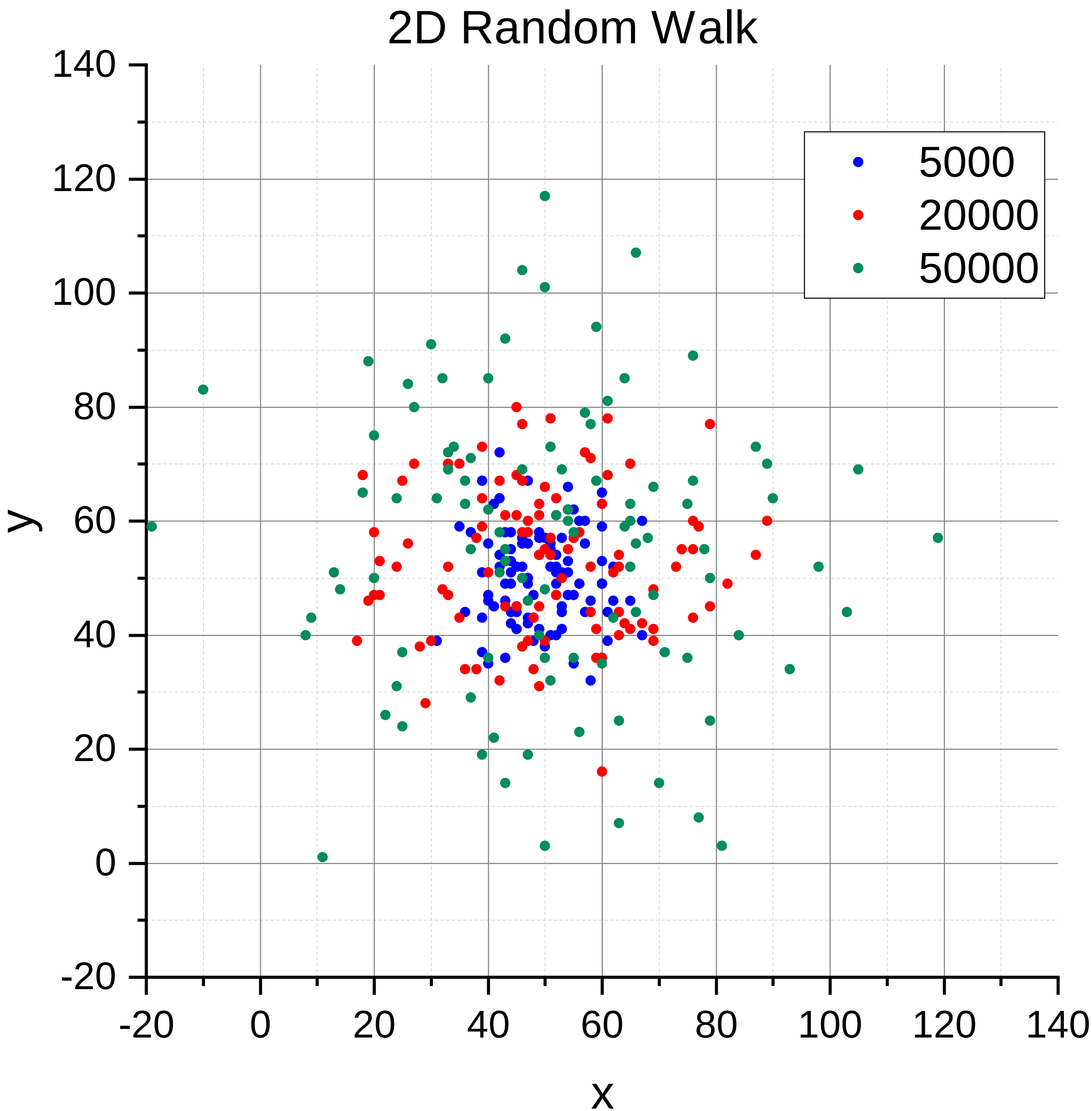

Mean Square Displacement
<y>^{2}(t)
100000
80000
60000
40000
20000
0
<y>^{2}
0
20
40
60
80
100
t


Random Walkers
$y_i(t)$
y_i
t
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100


2D Random Walk
5000
20000
50000
140
120
100
80
60
40
20
0
-20
-20
0
20
40
60
80
100
120
140
x
y

Let the function describing the density of particles be $\rho(x, y, z, t)$. The density per unit volume per unit time is then proportional to the probability to find a particle given by equation $P(x, y, z, t)$. Thus, ρ and P obey the same equation. We find this equation by focusing on an individual walker. We also assume that the walker is confined to a cubic lattice making steps along the edges of the lattice.

The probability to find the walker at the site (i, j, k) at time n is $P(i, j, k, n)$. The walker has six neighboring nodes that it can arrive from to the site (i, j, k) each of which have a 1/6 chance to make a step into the (i, j, k) node.

Therefore, the following equation is derived:

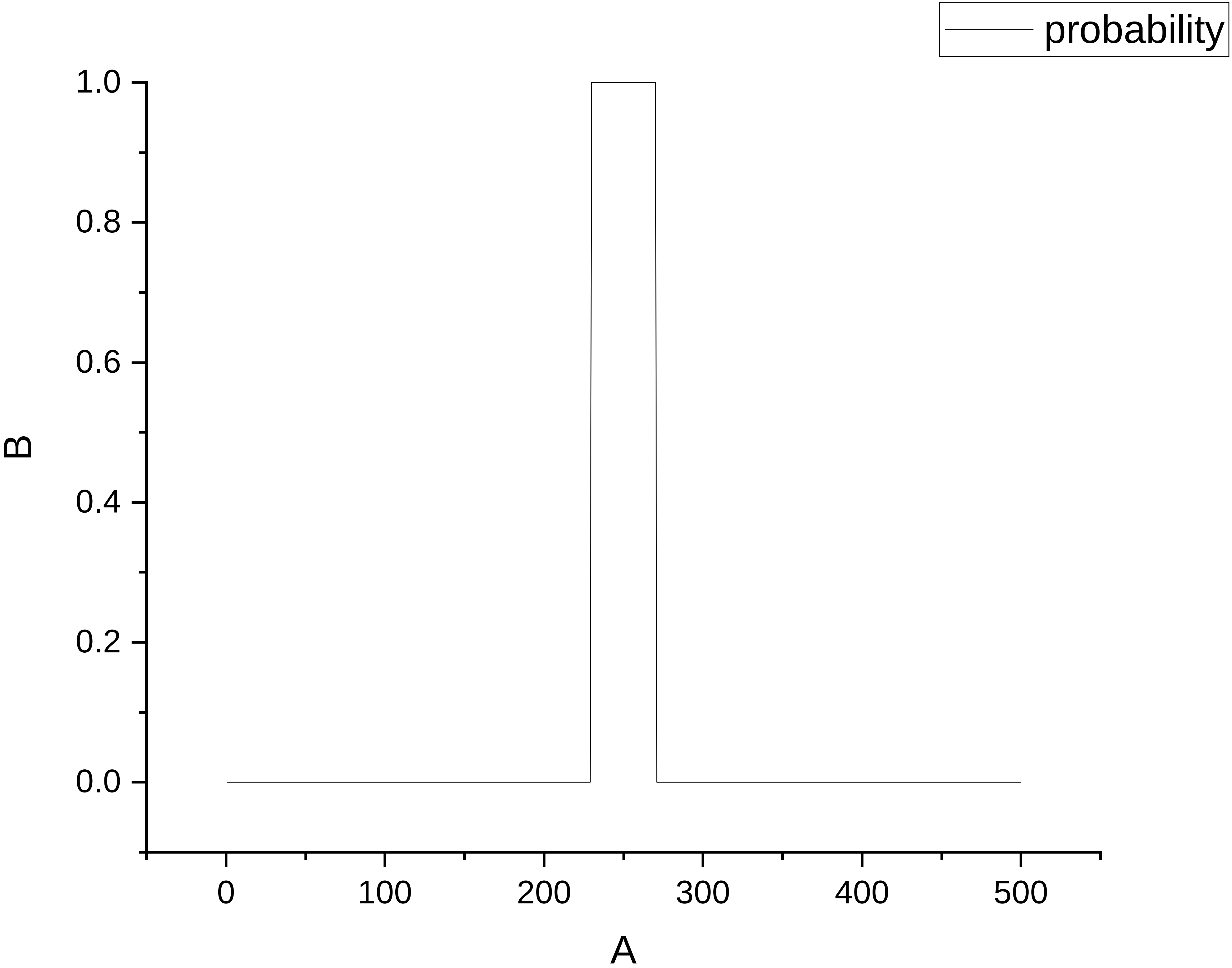
$$P(i,j,k,n) = \frac{1}{6}[P(i+1,j,k,n-1) \ + P(i-1,j,k,n-1) \ + \ P(i,j+1,k,n-1) \\ + P(i,j-1.k.n-1) + \ P(i,j,k+1,n-1) \ + \ P(i,j,k-1,n-1)]$$

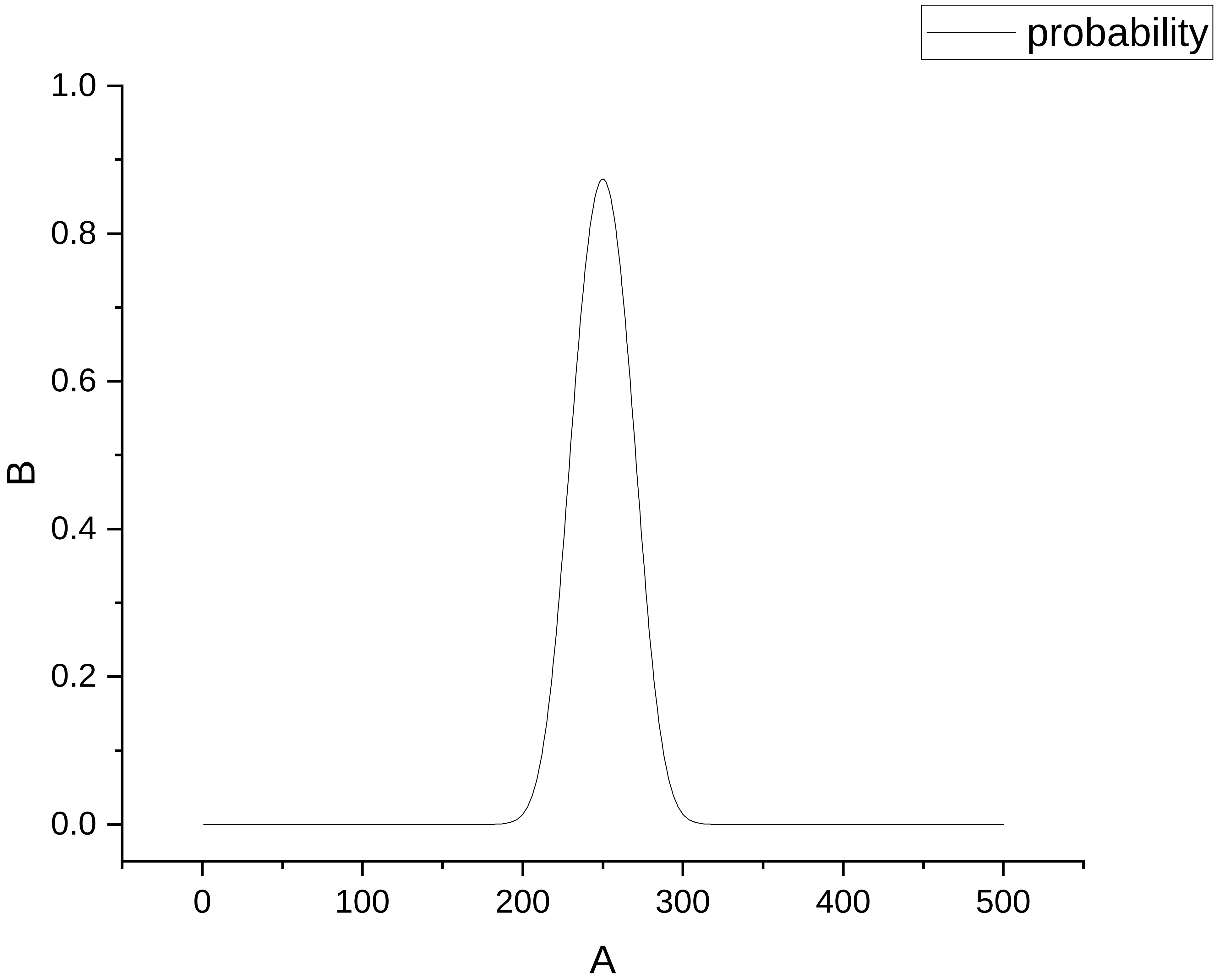
Rearranging the equation, we get:

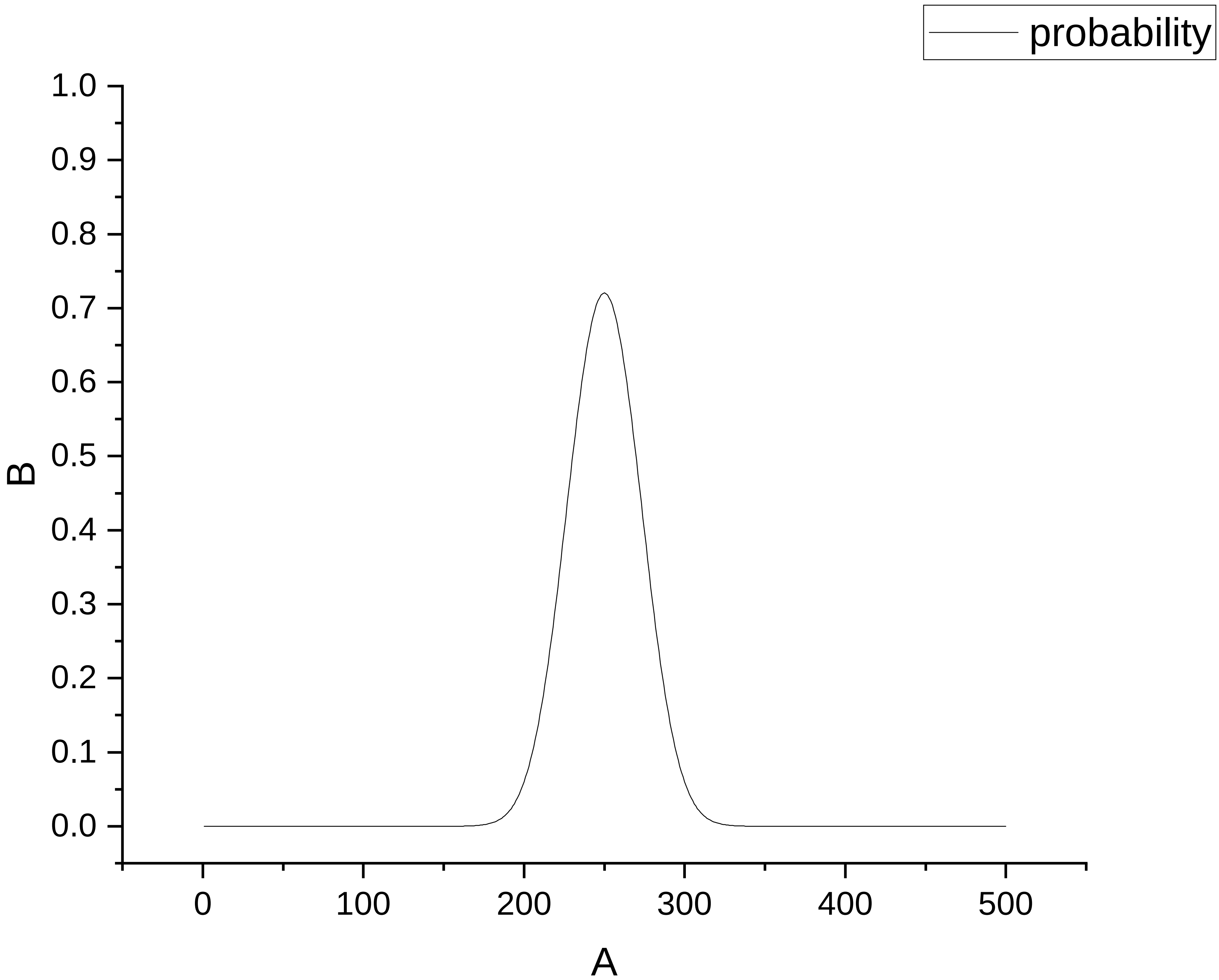
$$P(i,j,k,n) - P(i,j,k,n-1) \\ = \frac{1}{6}\{[P(i+1,j,k,n-1) - 2P(i,j,k,n-1) + P(i-1,j,k,n-1)] \\ + [\ P(i,j+1,k,n-1) - 2P(i,j,k,n-1) + P(i,j-1.k.n-1)] \\ + [\ P(i,j,k+1,n-1) - 2P(i,j,k,n-1) + P(i,j,k-1,n-1)]\}$$

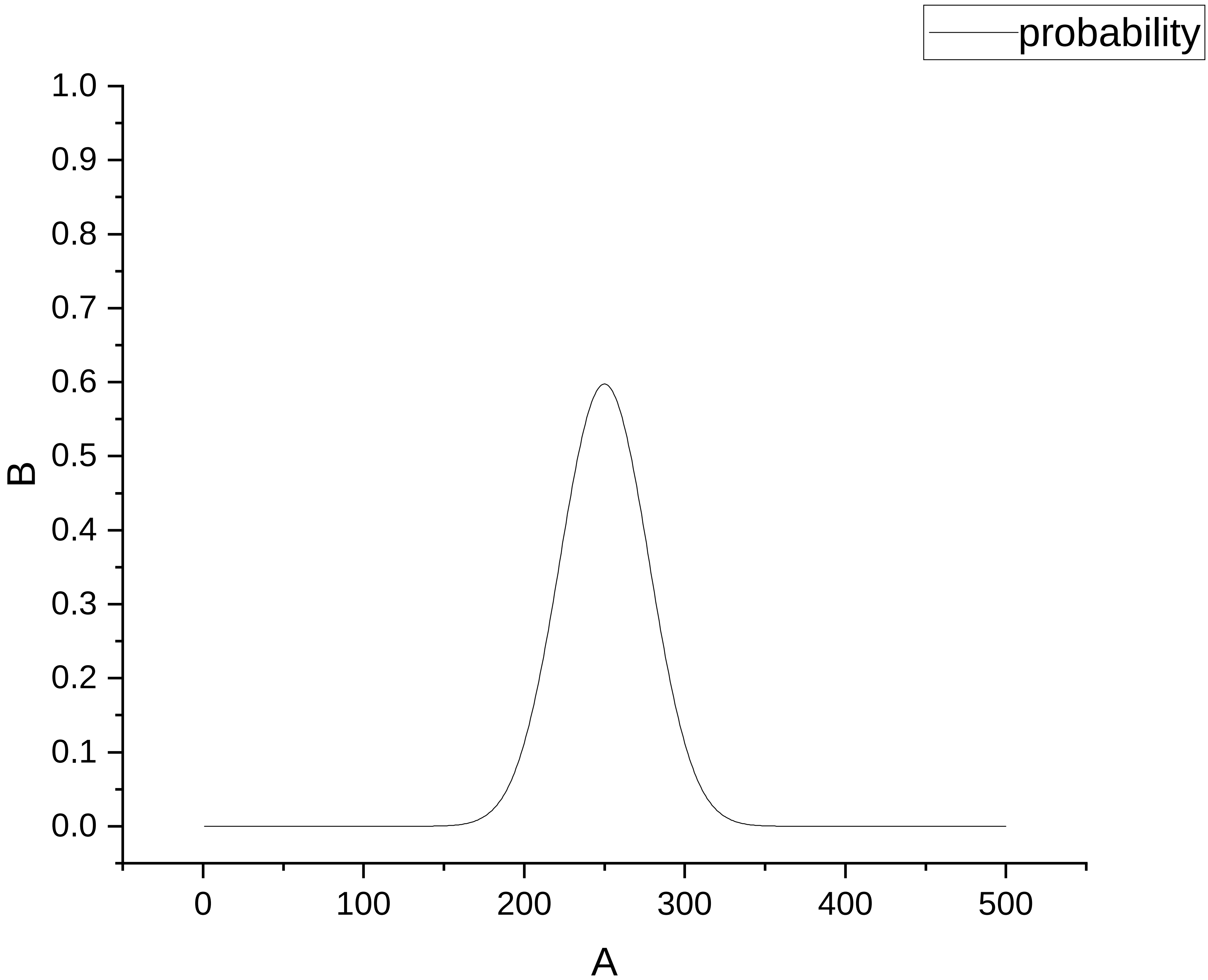
We then notice that the equation is in the form of a second order symmetric derivative, and, upon taking the limit, we arrive to the diffusion equation.

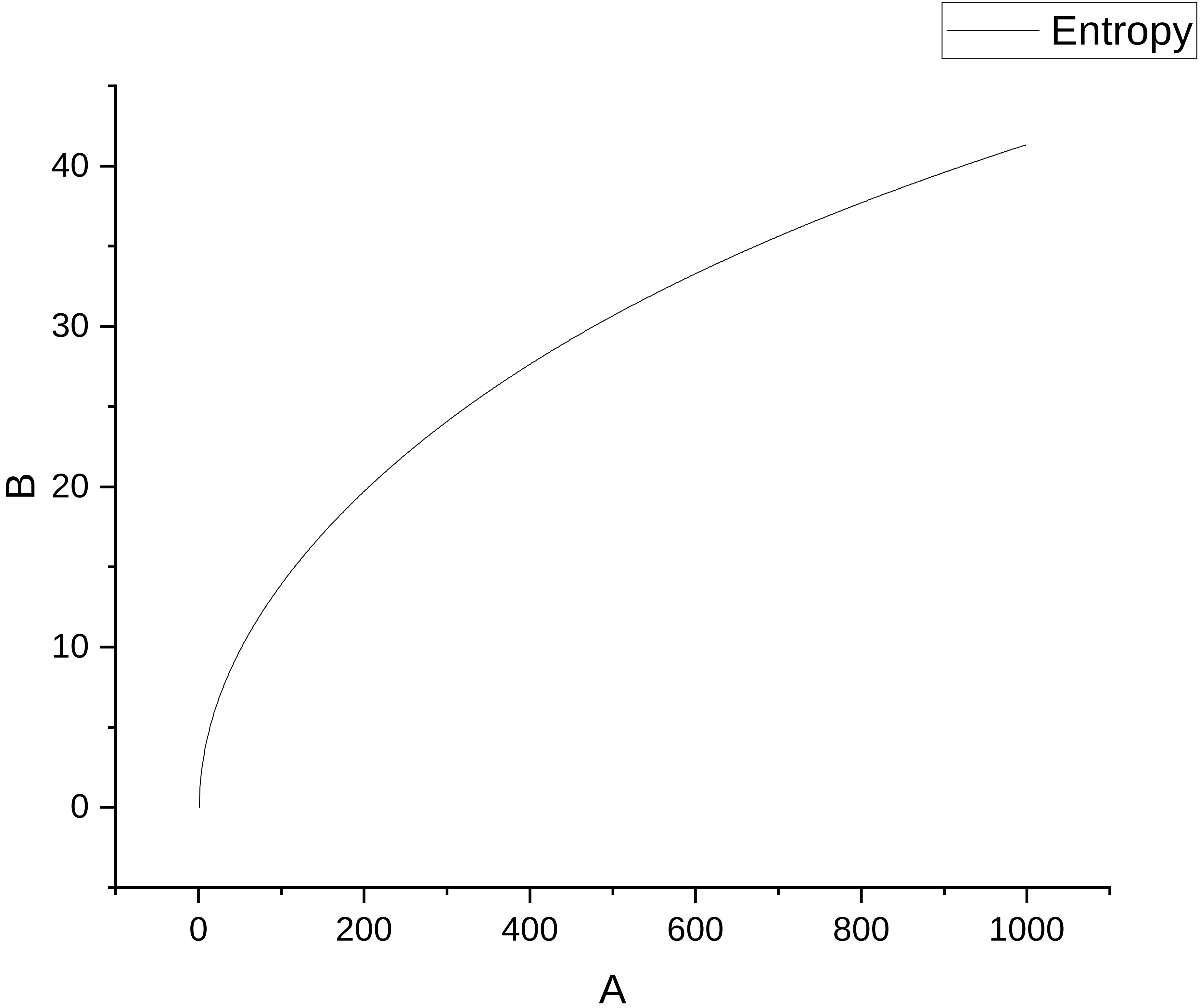
$$\frac{\partial P(x,y,z,t)}{\partial t} = D\nabla^2 P(x,y,z,t)$$

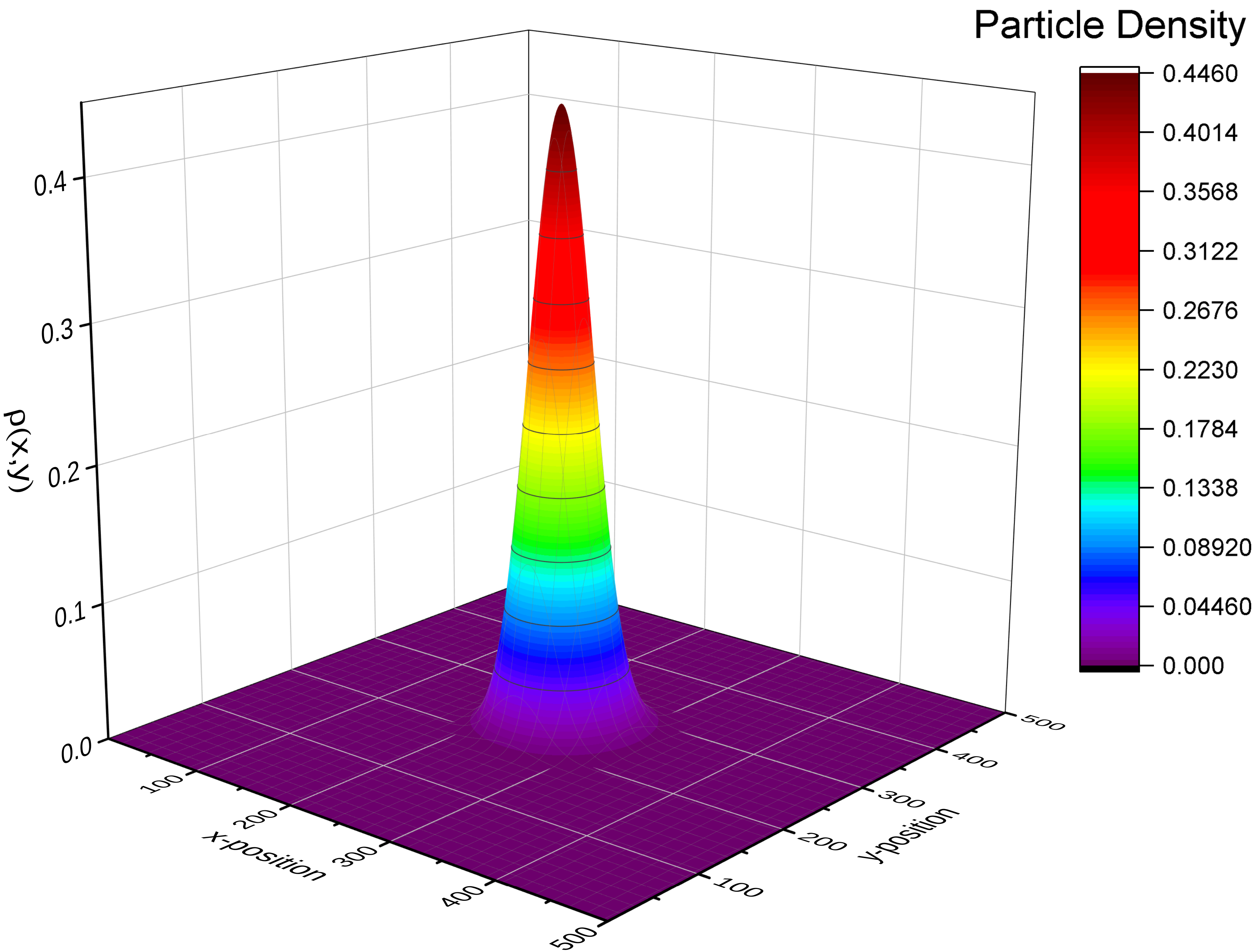

probability
1.0
0.8
0.6
0.4
0.2
0.0
B
0
100
200
300
400
500
A


probability
1.0
0.8
0.6
0.4
0.2
0.0
B
0
100
200
300
400
500
A

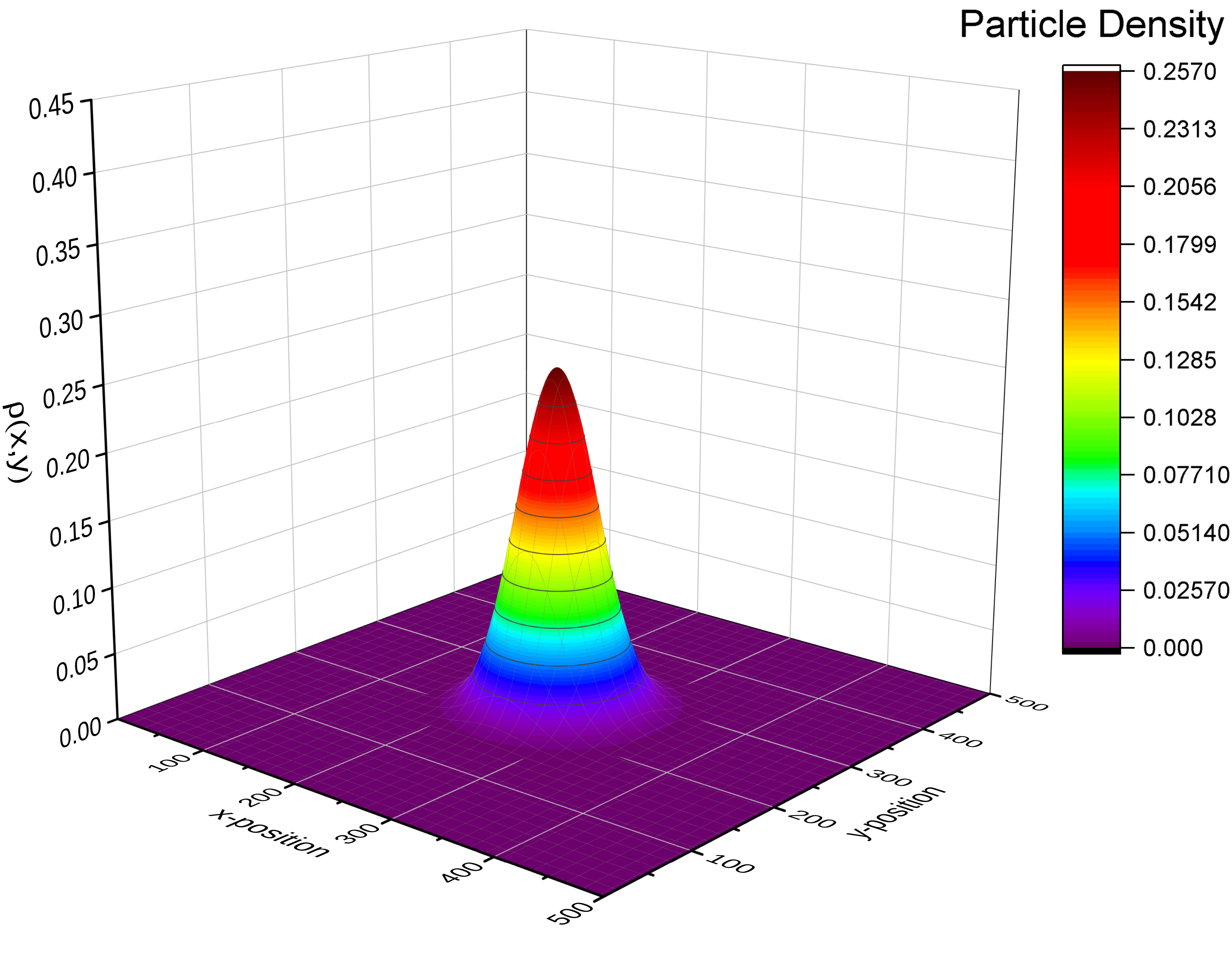

probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
B
0
100
200
300
400
500
A

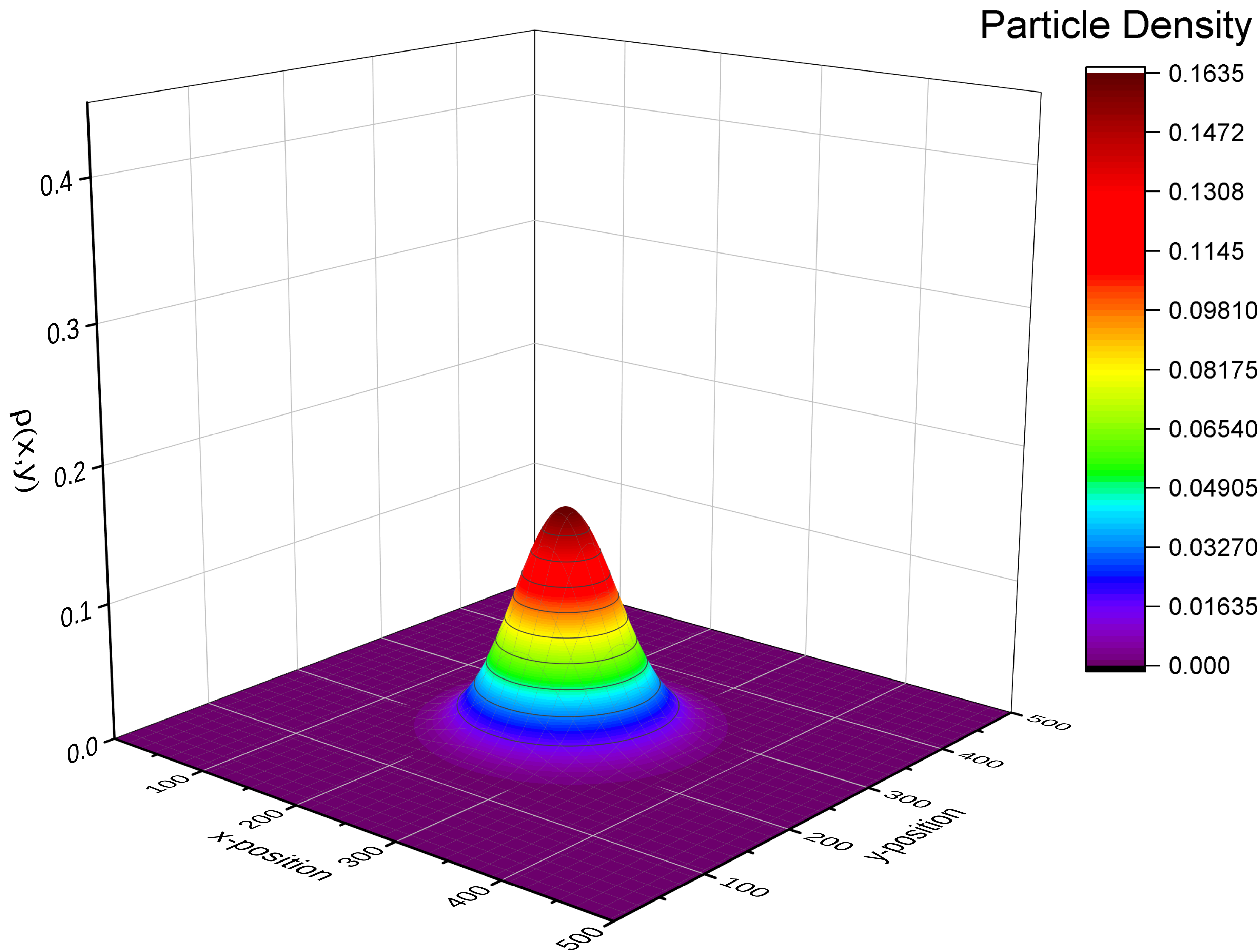

probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
B
0
100
200
300
400
500
A


Entropy
B
40
30
20
10
0
0
200
400
600
800
1000
A



Particle Density
0.4460
0.4014
0.3568
0.3122
0.2676
0.2230
0.1784
0.1338
0.08920
0.04460
0.000
ρ(x,y)
0.0
0.1
0.2
0.3
0.4
x-position
100
200
300
400
500
y-position
100
200
300
400
500


Particle Density
0.2570
0.2313
0.2056
0.1799
0.1542
0.1285
0.1028
0.07710
0.05140
0.02570
0.000
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
ρ(x,y)
100
200
300
400
500
x-position
y-position


Particle Density
0.1635
0.1472
0.1308
0.1145
0.09810
0.08175
0.06540
0.04905
0.03270
0.01635
0.000
ρ(x,y)
0.4
0.3
0.2
0.1
0.0
100
200
300
400
500
x-position
y-position