

SBI

Simulations : Efficient methods for spatio-temporal models
estimation

Alexandre Loret

Contents



1 Model 1	3
1.a 2D Gaussian field	4
1.b Matérn covariance	5
1.c Examples with advection	6
1.d Examples with missing values	7
2 Model 2	8
2.a Example images	9

1 Model 1

2D Gaussian field



Initially, this model was a **temporary solution**

Simulates **spatio-temporal** data (1D space + 1D time for faster computation)

Examples : Solar radiation, air pollution

Data : Observations $y(x_i)$ at some points (x_1, \dots, x_n) .

Model : Observations are realizations of a random variables

$$Y(x_i) = \mu(x_i) + Z(x_i) + \tau \varepsilon_i, \tau > 0$$

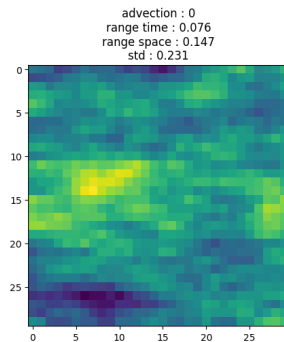
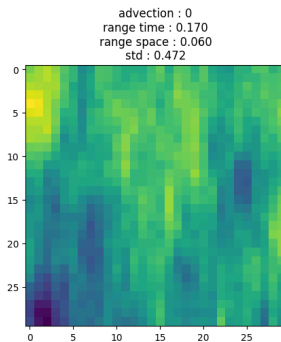
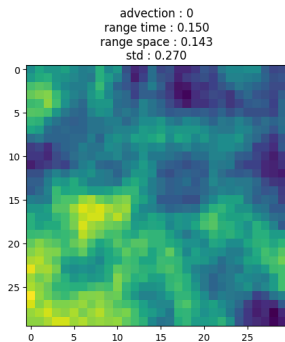
- μ : Fixed effect (regression covariates)
 - \emptyset in my simulation
- $Z \sim \mathcal{N}(0, \Sigma)$: Gaussian Random field
 - Matérn covariance in my simulation
- $\varepsilon_1, \dots, \varepsilon_n \sim \mathcal{N}(0, 1)$ iid noise

Matérn covariance

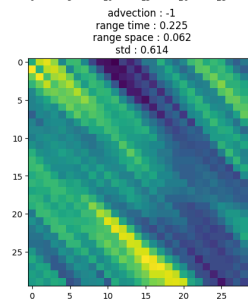
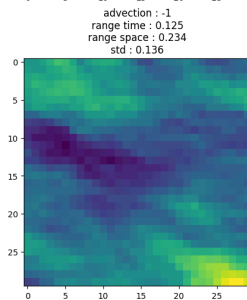
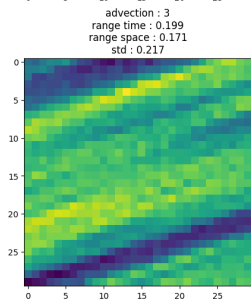
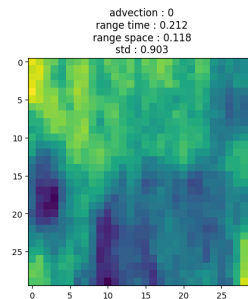
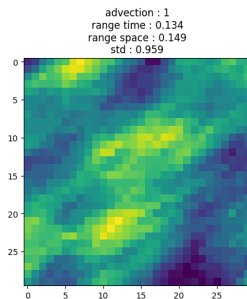
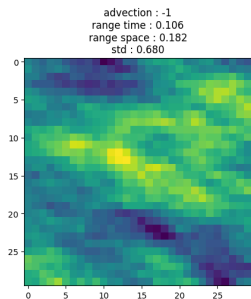


$$C_\nu(d) = \sigma^2 \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu} \frac{d}{\rho} \right)^\nu K_\nu \left(\sqrt{2\nu} \frac{d}{\rho} \right)$$

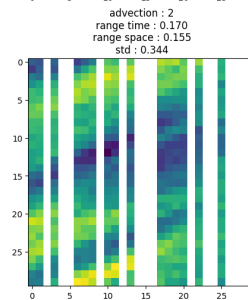
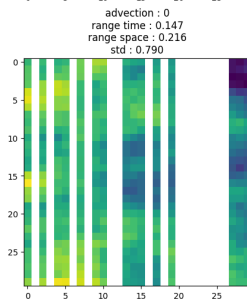
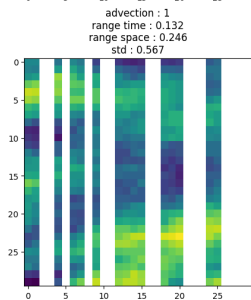
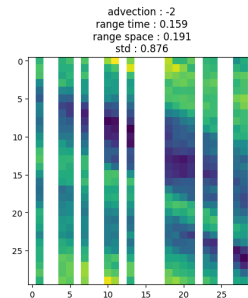
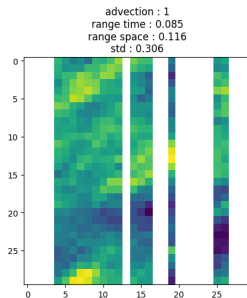
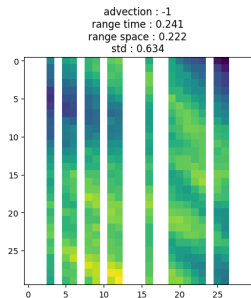
- ν is fixed
- The goal is to find the range ρ and the sill σ^2 (+ advection !)



Examples with advection



Examples with missing values

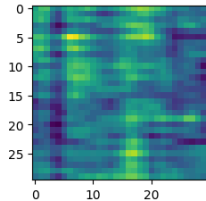


2 Model 2

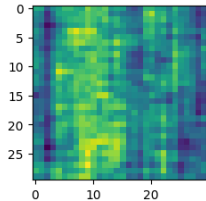
Example images



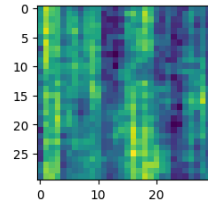
[0.15087784 1.76634852 4.34370943 0.44364371]



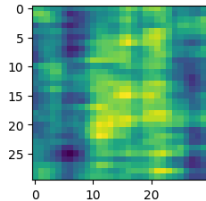
[0.22747285 4.78600877 1.53792955 0.26942486]



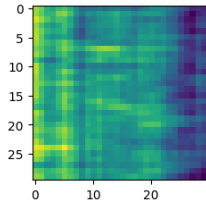
[0.14926448 3.6080421 1.14492359 0.57644445]



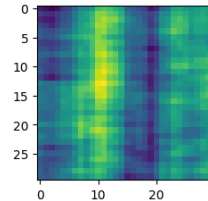
[0.15573314 3.51683721 4.95655694 0.13235375]



[0.09754914 1.97879003 4.49601297 0.86420287]



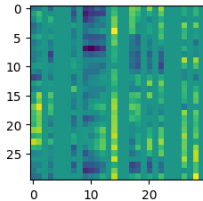
[0.08380741 2.14345196 3.95650453 0.46573915]



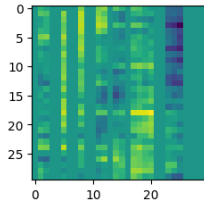
Example images (ii)



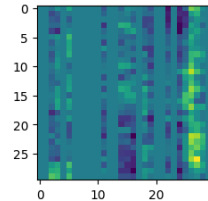
[0.29969249 4.54886881 2.59605922 0.49300348]



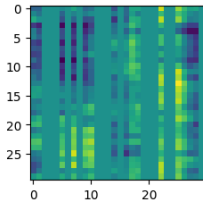
[0.17364503 2.47252642 3.47607238 0.06128045]



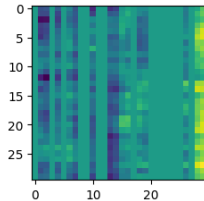
[0.25999614 1.47457849 1.66194617 0.11461999]



[0.26292579 2.71855617 3.61449134 0.91670586]



[0.22350107 4.90371507 3.57842928 0.11300251]



[0.14649946 4.56123837 2.25298373 0.03184421]

