D - Only one of two Editorial by en_translator

Let L be the least common multiplier of N and M.
Then, there are $\lfloor \frac{X}{L} \rfloor$ positive integers no greater than X that are divisible by $\lfloor \frac{X}{L} \rfloor$, so there are $\lfloor \frac{X}{N} \rfloor + \lfloor \frac{X}{M} \rfloor - 2 \times \lfloor \frac{X}{L} \rfloor$ integers 1 and X (inclusive) that are divisible by exactly one of N and M.

Additionally, the count monotonically increases with respect to X, so "the answer is at most X" if and only if "there are at least K integers between 1 and X that are divisible by exactly one of N and M," which in turn is equivalent to $\lfloor \frac{X}{N} \rfloor + \lfloor \frac{X}{M} \rfloor - 2 \times \lfloor \frac{X}{L} \rfloor \geq K$.

Thus, the problem can be solved with binary search using this property. Under the constraints of this problem, the answer is always at most 2×10^{18}, so one can set the lower and upper bounds to be 0 and 2×10^{18}, respectively, when starting binary search.

▶ Proof that the answer does not exceed 2×10^{18} or less

We assume $N < M$ without loss of generality. Let g be the greatest common divisor of N and M, $N = ng$, and $M = mg$ (n and m are integers with $1 \leq n < m$).
Then it holds that

$$\lfloor \frac{X}{N} \rfloor + \lfloor \frac{X}{M} \rfloor - 2 \times \lfloor \frac{X}{L} \rfloor > \frac{X}{N} + \frac{X}{M} - \frac{2X}{L} - 2 = \frac{(m+n-2)X}{gnm} - 2.$$

Let $X = 2 \times 10^{18}$; then the constraints guarantee that $\frac{m+n-2}{n} \geq 1$ and $\frac{X}{gm} \geq 2 \times 10^{10}$, so

$$\frac{X}{N} + \frac{X}{M} - \frac{2X}{L} - 2 = \frac{(m+n-2)X}{gnm} - 2 \geq 2 \times 10^{10} - 2,$$

meaning that at least $2 \times 10^{10} - 2$ integers no greater than X, i.e. at least K such integers, satisfy the conditions.
In fact, the upper bound under the constraints this time is $10^{18} - 5 \times 10^7$, when $N = 5 \times 10^7$, $M = 10^8$, and $K = 10^{10}$.
More specifically, one can set $L = 0$ and $R = 2 \times 10^{18}$, and repeat the following procedure while $R - L \geq 2$.

1. Let $X = \lfloor \frac{L+R}{2} \rfloor$.
2. If $\lfloor \frac{X}{N} \rfloor + \lfloor \frac{X}{M} \rfloor - 2 \times \lfloor \frac{X}{L} \rfloor \geq K$, then set $R = X$; otherwise, set $L = X$.

The answer is the resulting R.

For a fixed X, one can determine if $\lfloor \frac{X}{N} \rfloor + \lfloor \frac{X}{M} \rfloor - 2 \times \lfloor \frac{X}{L} \rfloor \geq K$ in $O(1)$ time, and the iteration loops at most 60 times. Hence, the problem has been solved.

Sample code in C++:


```
#include <bits/stdc++.h>
using namespace std;
long long gcd(long long x,long long y){
        if(x>y)swap(x,y);
        if(y%x==0)return x;
        return gcd(y%x,x);
}
int main() {
        long long n,m,x,k;
        cin>>n>>m>>k;
        x=(n*m)/gcd(n,m);
        long long l=0,r=(long long)2e+18,mid,y;
        while((l+1)<r){
                mid=(l+r)/2;
                y=(mid/n)+(mid/m)-2*(mid/x);
                if(y<k)l=mid;
                else r=mid;
        }
        cout<<r<<endl;
        return 0;
}
```