F - S = 1 Editorial by en_translator

An important fact is that the area of a triangle whose vertices are $(0,0), (X,Y)$ and (A,B) is $\frac{|AY-BX|}{2}$. Thus we can consider that, given integers X and Y, this problem asks to find an integer pair (A,B) such that

$$|AY-BX|=2.$$

We explain how to solve this problem. Here, we let $g=\gcd(X,Y)$. (Note: $\gcd(a,b)$ is defined as the greatest common divisor of $|a|$ and $|b|$.)
If g is 3 or greater, then the answer does not exist because $AY-BX$ is always a multiple of g.
If $g=1,2$, then one can always obtain a solution using an algorithm called extended Euclidean algorithm. The extended Euclidean algorithm is an algorithm that, given an integer pair (a,b) as input, finds an integer pair (x,y) such that $ax+by=\pm\gcd(a,b)$ in $\mathrm{O}(\log\min(|a|,|b|))$ time. (Here, the integer pair x,y is guaranteed to be integers satisfying $\max(|x|,|y|)\leq\max(|a|,|b|)$.)

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```cpp
// Sample implementation of extended Euclidean algorithm
pair<long long, long long> extgcd(long long a, long long b) {
  if (b == 0) return make_pair(1, 0);
  long long x, y;
  tie(y, x) = extgcd(b, a % b);
  y -= a / b * x;
  return make_pair(x, y);
}
```

By feeding $(Y,-X)$ as the input of extended Euclidean algorithm, one can obtain a pair (c,d) such that

$$cY-dX=\pm g$$

and $|c|,|d|\leq 10^{17}$ as the return value. What we originally wanted is a pair (A,B) such that $AY-BX=\pm 2$, which can be obtained by multiplying c and d by $\frac{2}{g}$. This (A,B) safely satisfies $|A|,|B|\leq 10^{18}$.

Hence, the problem has been solved. The computational complexity is about $\mathrm{O}(\log\min(X,Y))$, which is fast enough.