$$\inf_{\substack{V,\rho,L,L_t,\\L_{0,i},S_k,L_{\mathcal{E},k}}} \sum_{k=1}^N \operatorname{vol}(\mathcal{E}(t_k))$$

$$\sum_{k=1}^N \operatorname{vol}(\{\bar{x} \mid \bar{x}^T S_k \bar{x} \leq 1\}) \qquad (1)$$

$$\sup_{\substack{such that}} \qquad (3)$$

$$p(t) - \dot{V}(t,\bar{x}) - L(t,\bar{x}) \left[V(t,\bar{x}) - \rho(t)\right] - L_t(t,\bar{x}) \left[t(T-t)\right] \qquad \text{is SOS} \qquad (4)$$

$$\rho(0) - V(0,\bar{x}) - \sum_i^N L_{0,i}(\bar{x}) g_{0,i}(\bar{x}) \text{is SOS} \qquad (5)$$

$$1 - \bar{x}^T S_k \bar{x} - L_{\mathcal{E},k}(\bar{x}) \left[\rho(t_k) - V(t_k,\bar{x})\right] \text{is SOS} \qquad (6)$$

$$\forall k \in \{1,\dots,N\} S_k \succeq 0 \qquad (7)$$

$$\forall k \in \{1,\dots,N\} L_t(t,\bar{x}), L_{0,i}(\bar{x}) \text{are SOS} \qquad (8)$$

$$\forall i \in \{1,\dots,N\}, \forall k \in \{1,\dots,N\} \qquad (9)$$

$$(10)$$

$$\min_{\substack{V, \rho, L, L_t, \\ L_{0,i}, S_k, L_{\mathcal{E},k}}} \sum_{k=1}^N \operatorname{vol}(\mathcal{E}(t_k)) = \sum_{k=1}^N \operatorname{vol}(\{\bar{x} \mid \bar{x}^T S_k \bar{x} \leq 1\})$$
s.t.
$$\dot{\rho}(t) - \dot{V}(t, \bar{x}) - L(t, \bar{x}) \left[V(t, \bar{x}) - \rho(t) \right] - L_t(t, \bar{x}) \left[t \left(T - t \right) \right] \text{ is SOS,}$$

$$\rho(0) - V(0, \bar{x}) - \sum_{i}^N L_{0,i}(\bar{x}) g_{0,i}(\bar{x}) \qquad \text{is SOS,}$$

$$1 - \bar{x}^T S_k \bar{x} - L_{\mathcal{E},k}(\bar{x}) \left[\rho(t_k) - V(t_k, \bar{x}) \right] \qquad \text{is SOS,}$$

$$S_k \succeq 0 \qquad \forall k \in \{1, \dots, N\},$$

$$L_t(t, \bar{x}), L_{0,i}(\bar{x}) \qquad \text{are SOS,}$$

$$\forall i \in \{1, \dots, N_0\} \forall k \in \{1, \dots, N\}$$

$$(11)$$